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MANUAL OF PETROGRAPHIC METHODS

BY

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PREFACE

The desire of an increasing number of students for more complete information in regard to modern petrographic-microscopic methods than is to be found in any English work on the subject, has led the author to prepare this book. While the preliminary portions of many excellent elementary and intermediate text-books on optical mineralogy, and certain portions of most crystallographies and mineralogies, are devoted to microscopic methods, none makes any pretense at completeness, and even in the present work of over 600 pages those parts devoted to microchemical methods, examination of opaque minerals and mineral grains, etc., might be expanded, and there might be added chapters on photomicroscopy, projection apparatus for polarized light, etc., etc.

Owing to the fact that many students who take up the subject of petrography are weak in their preliminary training in physics and mathematics, the author has thought it best to treat the subjects of harmonic motion, light, and lenses somewhat more fully that he otherwise would have done. The mathematical demonstrations may have a somewhat formidable look to the non-mathematician, but this is due more to the fact that they are carried in detail through the various steps, and consequently are more easily followed, than that they are actually difficult. Likewise for the non-mathematician, the more cumbersome algebraic methods have been used in certain demonstrations rather than those of the calculus.

This book is not intended for beginners working without instructors, for to such the great variety of methods described will only bring confusion. The investigator and advanced student, however, should be familiar with all methods, old and new; for methods, once abandoned, may serve as preliminary stages to new lines of thought and further improvements. Unfamiliarity with what had been done in the past has frequently led to duplication of work, perhaps with a considerable expenditure of time that might have been used to better advantage.

The data for this book have been brought together from widely scattered sources, as may be seen from the footnotes. Much of the original material is in foreign publications, inaccessible to the majority of students. It has not been thought sufficient to take these references, even for the general bibliographies, at second hand, but the original works have been consulted. In every case where the reference seemed to be of sufficient importance to insert but the original work was not accessible, the footnote has been marked with an asterisk(*).

In this, the first attempt to give in English a comprehensive review of petrographic methods, it cannot be otherwise than that there should be many omissions—it is to be hoped not many errors. If any is found the author will be extremely glad to have his attention called to it, as he will also be for criticisms, or suggestions for additional material that should be included.

For more or less general information the author is indebted to the standard works of Rosenbusch and Wülfing, Duparc and Pearce, Groth, Iddings, Miers, Tutton, and Wright. For permission to use certain figures he wishes to express his thanks to Professors Becke, Duparc, and Miers, and Doctor Wright, and to the manufacturers of certain apparatus for the use of electrotypes. The half-tones of the interference figures are reproduced from the late Doctor Hauswaldt's magnificent *Interferencerscheinungen im polarisirten Lichte*, and are given with the kind consent of Frau Hauswaldt. The author has to express his appreciation to the attendants at the John Crerar Library for their uniform courtesy in obtaining for him the innumerable volumes consulted in this work, and especially for their efforts to find the proper volumes of the numerous publications to which wrong citations were given by other writers. To Professor G. W. Myers he is indebted for certain mathematical demonstrations. Most especially he desires to thank Professor A. C. Lunn, of the University of Chicago, who has placed him under great obligations by critically reading those parts of the manuscript dealing with lenses and light, and for giving him valuable suggestions. Finally his thanks, and the thanks of all petrographers to whom this book may prove useful, are due to the publishers for their willingness to issue a work of this kind, which must necessarily have a limited circulation.

ALBERT JOHANNSEN.

THE UNIVERSITY OF CHICAGO,
December 24, 1913.

CONTENTS

PAGE

PREFACE v
TABLE OF CONTENTS vii
LIST OF ABBREVIATIONS xxiii

CHAPTER I

MINERALOGICAL PRINCIPLES 1
Crystals 1
Crystallographic axes 1
The Weiss parameters 3
The Naumann system 3
The Miller indices 3
Zones 4

CHAPTER II

STEREOGRAPHIC PROJECTION 5
Introductory 5
Definitions 5
Locating points 6
Circles drawn upon a sphere appear as circles in stereographic projection . . 8
Spherical angles appear in their true values in stereographic projection . . 9
Graphical solutions of problems 10
Protractors and scales 14
Calculating the location of points in stereographic projection 19
Accuracy of stereographic projection 22
Problems solved by means of a stereographic net 22
Various accessories used in stereographic projection 25

CHAPTER III

A FEW PRINCIPLES OF OPTICS 29
The nature of light 29
Corpuscular or emission theory 29
The undulatory or wave theory of Huygens 30
The electromagnetic theory 30
The ether 31
Wave motion 31
The movements of oscillation 31
Simple harmonic motion 33
Isochronism and angular velocity 33
Harmonic curves 34
Combinations of simple harmonic motions 37
Combinations of harmonic curves 41

CHAPTER IV

PAGE

ISOTROPIC MEDIA 48
Definitions 48
Wave motion in isotropic media 48
Intensity of light 49
Color of light 49
Velocity and wave length of light 49
Wave front and wave surface 49
Reflection of waves 50
Passage of light into a medium of different density 52
Refraction of light upon passing into an isotropic medium of different density 52
Index of refraction 54
Passage of light into different isotropic media 54
Relation between indices of refraction and velocity of propagation of light . 56
Total reflection and the critical angle 56
Polarization, and light polarized by reflection 57
Angle of polarization 58
Variation in intensity—Malus' law 59
Polarization by refraction 59
Arago's law 60

CHAPTER V

ANISOTROPIC MEDIA 61
Single refraction and double refraction 61
Optically uniaxial crystals 62
Double refraction in calcite 62
Optic axis 70
Principal optic section 70
Positive and negative uniaxial crystals 70
Velocity of any intermediate ray 71
Velocity of any intermediate wave 72
Vibration directions 73
Ray surface and wave surface 75
Graphic development of ray and wave surfaces 76
Curve of ease of vibration (Fresnel's curve of elasticities) 79
Fresnel's ellipsoid 80
The optical indicatrix 80
Huygens' construction for double refraction in uniaxial crystals 82
Summary of the optical properties of uniaxial crystals 89

CHAPTER VI

ANISOTROPIC MEDIA (continued) 91
Optically biaxial crystals 91
Vibration axes 91
Fletcher's indicatrix 92
Ray surface 94
Wave surface 98
Optic biradials or secondary optic axes 99

PAGE
Optic binormals or primary optic axes . . . 100
Interior conical refraction . . . 100
Exterior conical refraction . . . 102
Optic axial angle, true and apparent . . . 102
Equations expressing the value of the true axial angle . . . 103
Relation between the true and apparent axial angles . . . 104
Plane of the optic axes . . . 105
Bisectrices . . . 105
Positive and negative biaxial crystals . . . 105
Polarization by double refraction . . . 106
Circular and elliptical polarization . . . 107
Rotary polarization . . . 108
Summary of optical principles . . . 110

CHAPTER VII

LENSES . . . 114
Definitions . . . 114
Axis, vertices and thickness of a lens . . . 114
Optical center . . . 114
Principal focal point . . . 115
Conjugate foci of convex lenses . . . 116
Refraction through simple lenses . . . 116
Focus of combined lenses . . . 118
Gauss' method . . . 119
Application of Gauss' cardinal points to the determination of the image formed by a lens . . . 121
Equations for the determination of the cardinal points of any lens system . . . 121
Lateral magnification . . . 121
Convergence of a lens . . . 122
Formation of images by lenses . . . 122
System of two faces . . . 123
Aberration . . . 129
Angular aperture . . . 131
Numerical aperture . . . 131
Table of numerical apertures for various angular apertures . . . 132
Apertometer . . . 132
Magnifying power . . . 133

CHAPTER VIII

THE MICROSCOPE . . . 136
Simple microscope . . . 136
Hand lens . . . 136
Compound microscope . . . 138
Formation of the image . . . 138
Optical and mechanical tube lengths . . . 138
Focal length . . . 140
Magnifying power . . . 140
Field of view . . . 140
The petrographic microscope . . . 141

PAGE
Description . . . 141
The mechanical parts of a petrographic microscope . . . 142
Foot . . . 142
Pillar or post . . . 142
Limb or arm . . . 142
Stages, simple and mechanical . . . 142
Verniers . . . 144
Body tube . . . 145
Objective holders . . . 146
Slot for accessories . . . 148
Centering device for objective . . . 148
Coarse and fine adjustment . . . 149
Sub-stage . . . 150
Diaphragms . . . 151

CHAPTER IX

THE MICROSCOPE (CONTINUED) . . . 154
The Optical Parts of a Petrographic Microscope . . . 154
Illuminating apparatus . . . 154
Polarizing prisms . . . 158
Introduction . . . 158
Nicol prism . . . 158
Sang prism . . . 164
Foucault prism . . . 165
Hartnack-Prazmowski prism . . . 165
Talbot prism . . . 167
Glan prism . . . 167
Thompson prisms . . . 167
Fuessner prisms . . . 168
Bertrand prisms . . . 169
Ahrens prism (1884) . . . 170
Madan prism . . . 171
Ahrens prism (1886) . . . 171
Grosse double-slit air prism . . . 172
Leiss prism . . . 172
Von Lommel prism . . . 173
Von Fedorow polarizer . . . 173
Halle prisms . . . 174
Glass polarizing prisms . . . 174
Summary of properties of polarizing prisms . . . 175
Polarizer and analyzer . . . 176
Determination of the vibration directions in the nicol prisms . . . 178
Bertrand lens . . . 178

CHAPTER X

THE MICROSCOPE (CONTINUED) . . . 180
The optical parts of a petrographic microscope (continued) . . . 180
The objective . . . 180
Introductory . . . 180

PAGE

Definition . . . 180
Depth of definition (depth of focus) or penetration. . . . 180
Flatness of field . . . 181
Illuminating power . . . 181
Resolving power . . . 181
Working distance . . . 182
Magnifying power . . . 182
Dry and immersion objectives . . . 183
Classification of objectives according to correction for aberration . . . 185
Effect of cover-glasses of different thicknesses upon objectives . . . 185
Comparative table ot dry achromatic objectives of different makers . . . 189
Aperture table . . . 190
Testing the objective . . . 191
Cost of objectives . . . 193
The ocular or eyepiece . . . 193
Huygens ocular . . . 193
Ramsden ocular . . . 194
Compensating oculars . . . 194
Comparative table of Huygens oculars of different makers . . . 195
Oculars for special purposes . . . 195
Demonstration oculars . . . 196
Focussing cross-hairs in the ocular . . . 197
Replacing cross-hairs . . . 197
Magnification of the compound microscope . . . 197

CHAPTER XI

VARIOUS MODERN MICROSCOPES . . . 199
Introduction . . . 199
Leitz stand AM . . . 199
Leitz Berkey model . . . 200
Leitz new stand . . . 200
Seibert microscope . . . 202
Fuess stand VIa . . . 202
Fuess stand IIIa . . . 203
Fuess microscope, Model Ib . . . 205
Fuess microscope, Model IIa . . . 207
Zeiss crystallographic and petrographic microscope III MD . . . 208
Zeiss small mineralogical stand VM . . . 209
Reichert mineralogical stand MI . . . 209
Reichert mineralogical microscope M VIII . . . 211
Bausch & Lomb LCH petrographic microscope . . . 211
Nachet microscope . . . 213
Swift improved Dick petrographic microscope . . . 215
Swift large petrographic microscope . . . 215
Beck London petrographic microscope . . . 217
Société Genevoise universal microscope . . . 218
Fuess microscope for the theodolite method . . . 218
Beck Rosenhain metallurgical microscope . . . 221

CHAPTER XII

PAGE

SELECTING, USING, AND TAKING CARE OF A MICROSCOPE 222
Selecting a microscope 222
Use and care of a microscope 223
Light 223
Table 225
Method of working 225
Position 225
Proper eye to use 225
Eye shade 226
Amount of light 227
Proper magnifying power to use 227
Objective clutch 227
Focussing 227
Changing the ocular 228
Hints on the care of a microscope 228
Care of stand 228
Care of nicols and lenses 228
Testing and adjusting the microscope and the accessories 229
Cross-hairs 229
Bertrand ocular 230
Bertrand lens, Centering 230
Nicol prisms 230
Accessories 231

CHAPTER XIII

OBSERVATIONS BY ORDINARY LIGHT 233
Ordinary light 233
Determination of crystal form 233
Cleavage and parting 235
Determination of refractive indices 237
Relief 237
The method of the Duc de Chaulnes 238
Brewster's method for determining the refractive index of a liquid 241
Becquerel and Cahour's method for determining the refractive index of a liquid 241
Bertin's method 242
Sorby's method 244

CHAPTER XIV

OBSERVATIONS BY ORDINARY LIGHT (CONTINUED) 249
Determination of the refractive indices of a mineral by the immersion or embedding method 249
Maschke 249
Sorby 250
Thoulet 250
Stephenson 251
Rohrbach 251
Brauns 251

PAGE

Bertrand . . . 251
Klein . . . 252
Schroeder van der Kolk . . . 252
Zirkel . . . 252
Retgers . . . 252
Ambronn (1893) . . . 253
Ambronn (1896) . . . 254
Marpmann . . . 254
Schroeder van der Kolk (1898) . . . 255
Schroeder van der Kolk (1900) . . . 256
Immersion fluids . . . 259
Determination of the refractive indices of fluids . . . 265
Introductory . . . 265
Smith's method . . . 265
Pauly's method . . . 266
Michel-Lévy's indicators . . . 268
De Souza-Brandao's indicators . . . 268
Clerici's method . . . 270

CHAPTER XV

OBSERVATIONS BY ORDINARY LIGHT (CONTINUED) . . . 271
Determination of the refractive indices of a mineral by the Becke method . . . 271
Becke's explanation . . . 271
Hotchkiss' explanation . . . 272
Grabham's explanation . . . 274
Inclined illumination . . . 275
Viola-deChaulnes-Becke method . . . 276
Viola-Becke method . . . 276
Practical applications of the Becke method . . . 277
Refractive index of Canada balsam . . . 283
Relation between refractive index and density . . . 285
The examination of opaque minerals . . . 285

CHAPTER XVI

MEASUREMENTS UNDER THE MICROSCOPE . . . 287
Measurement of enlargement . . . 287
Measurement of the field of view . . . 287
Measurement of lengths . . . 288
Measurement of areas . . . 290
Measurement of thicknesses . . . 293
Measurement of plane angles . . . 293
Measurement of optic axial angles . . . 295

CHAPTER XVII

DRAWING APPARATUS . . . 296

CHAPTER XVIII

ROTATION APPARATUS . . . 300

CHAPTER XIX

PAGE
THE COLOR OF MINERALS 309
Idiochromatic and allochromatic minerals 309
Determination of color 310
Determination of the color of opaque minerals 311

CHAPTER XX

MONOCHROMATIC LIGHT 313
The production of monochromatic light 313
Ray filters 314
Incandescent vapors of solids 316
Incandescent gases 317
Dispersed white light produced by a monochromator 317

CHAPTER XXI

EXAMINATION BY PLANE POLARIZED LIGHT 320
Absorption, dichroism, pleochroism 320
Absorption of light in crystals 320
Isotropic substances 320
Anisotropic substances 320
Uniaxial crystals 321
Biaxial crystals 322
Pleochroic halos 323
Pseudo-pleochroism, pseudo-dichroism, or pseudo-absorption 324
Interference phenomena, without the analyzer, produced by an overlying pleochroic mineral 324
Determination of pleochroism 325
Determination of the absorption coefficient 326

CHAPTER XXII

INTERFERENCE COLORS 328
Interference 328
Color of thin plates 328
Newton's color scale 330
Color scale according to Quincke 331
Color scale according to Kraft 332

CHAPTER XXIII

EXAMINATION BETWEEN CROSSED NICOLS 336
Isotropic substances 236
Anisotropic substances 336
Retardation in anisotropic media 337
Phasal difference 337
Interference of polarized light 337
Extinction angles 339

PAGE
Passage of monochromatic light through two nicol prisms and a mineral section . . . 341
The intensity of the emerging light . . . 343
Two superposed mineral plates . . . 346
Examination by white light. Interference colors . . . 348
Calculation of the value of the birefringence in any section . . . 351
Lines of equal birefringence . . . 355
Abnormal birefringence . . . 359

CHAPTER XXIV

EXAMINATION BETWEEN CROSSED NICOLS (CONTINUED) . . . 361
Determination of the vibration directions in mineral plates . . . 361
Optical character of the elongation . . . 361
Accessories used for the determination of the vibration directions of a mineral . 362
Kinds of accessories . . . 362
Simple plane parallel plates . . . 362
Quarter undulation mica plate . . . 362
Unit retardation plate . . . 365
Retardation wedges . . . 365
Simple quartz of gypsum wedge . . . 365
Von Fedorow mica comparator . . . 366
Wright combination wedge . . . 366
Johannsen quartz-mica wedge . . . 367

CHAPTER XXV

EXAMINATION BETWEEN CROSSED NICOLS (CONTINUED) . . . 369
Determination of the order of birefringence . . . 369
Compensating wedge for the determination of birefringence . . . 369
Michel-Lévy chart of birefringences . . . 370
Babinet compensator . . . 373
Von Chrustschoff twin compensator . . . 376
Michel-Lévy comparateur . . . 377
Von Fedorow method for determining low interference colors . . . 378
Cesàro wedge . . . 379
Amann birefractometer . . . 379
Von Fedorow mica comparator . . . 379
Salomon's method for computing the value of $\omega - \epsilon$ in uniaxial minerals . . . 383
Wallerant's method for measuring slight double refraction . . . 383
Nikitin's method . . . 383
Joly's method . . . 383
Wright combination wedge . . . 383
Evans simple quartz wedge . . . 383
Evans double quartz wedge . . . 384
Seidentopf quartz wedge compensator . . . 384
Wright double combination wedge . . . 385
Nikitin quartz compensator . . . 385

CHAPTER XXVI

PAGE

EXAMINATION BETWEEN CROSSED NICOLS (CONTINUED) 386
Determination of very slight double refraction 386
Sensitive violet 386
Biot quartz plate 386
Savart plate 386
Soleil bi-quartz plate 387
Bravais twinned mica plate 387
Klein quartz plate 387
Bertrand ocular 388
Calderon ocular 388
Traube bi-mica plate 388
Brace half-shade elliptical polarizer and compensator 388
Sommerfeldt twinned gypsum plate 388
Königsberger ocular 388
Half-shade plates 388

CHAPTER XXVII

EXAMINATION BETWEEN CROSSED NICOLS (CONTINUED) 390
Practical methods for the determination of extinction angles 390
Relation of the optical ellipsoid to the crystallographic axes. Parallel and inclined extinction 390
Methods for measuring 392
Unit retardation plate 393
Bravais twinned mica plate 393
Kobell stauroscope 394
Klein quartz plate 394
Bertrand ocular 394
Calderon plate 395
Von Fedorow's method by means of the universal stage 395
Wiedemann double double-quartz wedge 395
Stöber quartz double plate 396
Traube bi-mica plate 396
Macé de Lépinay half-shade plate 396
Sommerfeldt twinned gypsum plate 397
Wright artificially twinned quartz plate 397
Wright bi-quartz wedge plate 398

CHAPTER XXVIII

EXAMINATION BETWEEN CROSSED NICOLS (CONTINUED) 399
Calculation of extinction angles in random thin sections 399
Zones 399
Calculation of extinction angles for any face of the 100–010 zone of a monoclinic crystal 399
Calculation of the extinction angle for any face of any zone of any crystal, 403
Graphical methods for the determination of extinction angles on any plane . . . 406
Extinction diagram and curves of equal extinction 410
Influence of dispersion upon extinction angles 412

CHAPTER XXIX

PAGE

OBSERVATIONS BY CONVERGENT POLARIZED LIGHT 413
Polariscope, conoscope 413
Interference figures 415
Isotropic crystals 415
Random sections 415
Anisotropic crystals 416
Uniaxial crystals 416
Section perpendicular to the optic axis 416
Section oblique to the optic axis 418
Sections parallel to the optic axis 419
Biaxial crystals 420
Sections cut at right angles to the acute bisectrix 420
Sections cut at right angles to the obtuse bisectrix 423
Sections inclined to the bisectrices 423
Sections at right angles to an optic axis 424
Sections parallel to the plane of the optic axes 425
Locating the point of emergence of an optic axis 425
Uniaxial crystals 425
Biaxial crystals 426

CHAPTER XXX

OBSERVATIONS BY CONVERGENT POLARIZED LIGHT (CONTINUED) 429
Isotaques, skiodromes, and isogyres 429
Isotaques or curves of equal velocity 429
Skiodromes 430
To construct the skiodromes for a random section 433
Deduction of the isogyres from the skiodromes 434
Skiodromes of uniaxial crystals 435
Sections cut at right angles to the optic axis 435
Sections inclined to the optic axis 436
Sections parallel to the optic axis 436
Skiodromes of biaxial crystals 437
Sections perpendicular to the principal vibration axes 437
Sections perpendicular to the acute bisectrix 437
Sections perpendicular to the obtuse bisectrix 437
Sections perpendicular to the optic normal 438
Sections perpendicular to an optical plane of symmetry 438
Sections perpendicular to the plane of the optic axes 438
Inclined sections 439
Random sections 439
Equations for the isogyres or neutral curves 440

CHAPTER XXXI

DISPERSION OF LIGHT IN CRYSTALS 442
Normal and anomalous dispersions 442
Dispersion in orthorhombic crystals 443
Dispersion of the optic axes 443
Crossed axial plane dispersion 444

PAGE
Dispersion in monoclinic crystals . . . 445
Dispersion of the bisectrices . . . 445
Inclined dispersion (of both bisectrices) . . . 445
Horizontal dispersion (of the acute bisectrix) . . . 446
Crossed dispersion (of the obtuse bisectrix) . . . 447
Dispersion in triclinic crystals . . . 448
Unsymmetrical dispersion . . . 448
Effect of temperature change on dispersion . . . 448

CHAPTER XXXII

THE PETROGRAPHIC MICROSCOPE AS A CONOSCOPE AND THE METHODS FOR OBSERVING INTERFERENCE FIGURES . . . 449
Observing interference figures with the microscope . . . 449
Lasaulx method . . . 449
Bertrand method (1878) . . . 449
Klein method . . . 450
Laspeyres method . . . 451
Bertrand method (1880) . . . 451
Schroeder van der Kolk method . . . 452
Czapski ocular . . . 453
Becke-Klein magnifier . . . 453
Lenk-Lasaulx method . . . 453
Sommerfeldt condenser . . . 454
Wright-Lasaulx method . . . 454
Johannsen auxiliary lens . . . 454
Orientation of image in relation to object . . . 456

CHAPTER XXXIII

DETERMINATION OF THE OPTICAL CHARACTER OF A CRYSTAL BY MEANS OF ITS INTERFERENCE FIGURE . . . 457
Positive and negative minerals . . . 457
Uniaxial crystals . . . 457
Quarter undulation mica plate . . . 457
Unit retardation plate . . . 459
Quartz or gypsum wedge . . . 460
Inclined sections . . . 461
Sections parallel to the optic axis . . . 462
Biaxial crystals . . . 462
Mica plate, gypsum plate, and quartz wedge . . . 462

CHAPTER XXXIV

MEASUREMENT OF THE OPTIC AXIAL ANGLE BY CONVERGENT POLARIZED LIGHT . . . 466
Introduction . . . 466
Mallard method for sections showing the points of emergence of both optic axes . . . 467
Becke's graphical solution of sin $E=n$ sin V . . . 468
Schwarzmann axial angle scale . . . 469
Schwarzmann ocular . . . 470

PAGE
De Souza-Brandao axial angle diagram 471
Michel-Lévy method for sections perpendicular to a bisectrix 472
Viola method. 474
Becke method for determining graphically the axial angle in sections which do not show the points of emergence of both optic axes 476
Determination of the point of emergence of an optic axis. 476
Becke's rotating drawing stage 478
Becke method for determining, by means of the curvature of the isogyres, the value of the axial angle in sections which show the point of emergence of but a single optic axis 480
Wright's modification of the Becke method for determining the axial angle by means of the curvature of the isogyres 483
Modifications of Becke's method 485
Wright 485
Stark 485
Tertsch 485

CHAPTER XXXV

MEASUREMENT OF THE OPTIC AXIAL ANGLE BY MEANS OF A ROTATION APPARATUS . . 487
The rotation apparatus 487
Locating one optic axis 489
Determination of the position of an optic axis by means of the optical curves. . 494
Locating the point of emergence of the second optic axis 495
Locating the symmetry planes and the axes of the optical ellipsoid within the crystal. 497
Determination of the position of the second optic axis when the first is determinable by optical curves 498
Approximate determination of the optic axes when the section lies nearly parallel to the plane of the optic axes 499
Simplified methods 500
Both optic axes appear in the field of the microscope at the most satisfactory angle 500
One optic axis makes an angle of less than 20° with the normal to the section . 500
One optic axis makes an angle of between 20° and 55° with the normal to the section, the other lies beyond 55° 501
Both optic axes are inclined more than 55° to the normal to the section . . 501

CHAPTER XXXVI

DETERMINATION OF OTHER PROPERTIES THAN 2V BY MEANS OF THE UNIVERSAL STAGE 503
Opaque minerals 503
Isotropic, uniaxial, or biaxial character 503
Positive or negative character 503
Maximum extinction angle 504
Mean refractive index of a mineral 504
Orientation of the crystal section with reference to the axes of the optical ellipsoid 504
Determination of the maximum birefringence of an unknown mineral from that of one which is known. 504
Graphical representation of the variation in the double refraction in different directions 506

CHAPTER XXXVII

PAGE
OPTICAL ANOMALIES 508
The cause of optical anomalies 508

CHAPTER XXXVIII

DETERMINATION OF SPECIFIC GRAVITY 515
Specific gravity 515
Hydrostatic balance 515
Jolly balance 516
Pycnometer for determining the specific gravity of powders 517
Smeeth's method for mineral powders 517
Specific gravity of porous substances 518
Specific gravity of substances soluble in water 518
Determination of specific gravity by heavy solutions 518
Sonstadt (or Thoulet) solution 519
Klein solution 521
Rohrbach solution 524
Methylene iodide (Braun) 525
Retgers' heavy fluids 526
Tabulation of the properties of heavy fluids 528
Schroeder van der Kolk 529
Muthmann 529
Clerici 529
Joly 530
Hubbard 530
Streng 530
Retgers 531
Behr 531
Determination of the specific gravity of the heavy solution 532
Goldschmidt's method 532
Sprengel tube 532
Sollas' modification of the Sprengel tube 533
Westphal balance 533
Salomon's apparatus 534
Sollas' hydrostatic float 534
Merwin's method by refractive indices 534
Molten substances as specific gravity fluids 535
Determination of the specific gravity of a mineral whose density is greater than that of the fluid 535
Thoulet 535

CHAPTER XXXIX

THE MECHANICAL SEPARATION OF ROCK CONSTITUENTS 537
Preliminary examination 537
Separation by means of the electromagnet 538
Separation by means of water 541
Separation by means of heavy fluids 542
Indicators 542
Table of specific gravities 544

PAGE
Heavy solutions . . . 545
Heavy melts . . . 545
Separating apparatus . . . 547
Thoulet . . . 547
Goldschmidt . . . 548
Harada . . . 549
Oebbeke . . . 550
Van Werveke . . . 550
Brögger . . . 551
Smeeth . . . 552
Diller . . . 553
Laspeyres . . . 553
Wülfing . . . 553
Luedecke . . . 554
Separation apparatus for heavy melts . . . 554
Causes likely to produce errors in separating minerals or in determining specific gravities by means of heavy fluids . . . 556
Separation of thin flakes and fine needles . . . 557
Separation by hand . . . 557
Separation by chemical means . . . 558

CHAPTER XL

MICROCHEMICAL REACTIONS . . . 559
General microchemical reactions . . . 559
Chemical reactions on rock slices . . . 559
Apparatus . . . 559
Preparing the slide . . . 560
Microchemical filtrations . . . 561
Gelatinizing and staining minerals . . . 562
Special reactions, chiefly on thin sections . . . 563
Hauynite, noselite, sodalite, melilite, and zeolites . . . 563
Nephelite, cancrinite, and hydronephelite . . . 564
Olivine family . . . 565
Apatite . . . 565
Carbonates . . . 565
Separating quartz from feldspar . . . 568

CHAPTER XLI

PREPARATION OF THIN SECTIONS OF ROCKS . . . 572
Early history . . . 572
Section cutting machines . . . 574
Diamond saws . . . 580
Sawing a rock slice . . . 583
Grinding a section . . . 585
Various grinding machines . . . 588
Orienting devices . . . 592
Mounting the section . . . 593
Special methods for preparing sections of unusual material . . . 599
Friable material . . . 599

PAGE
Vesicular rocks 600
Coal 601
Clays and soft powders 601
Sand and other loose grains 602
Hydrous minerals 602
Minerals soluble in water 602
The preparation of polished faces on rocks 602
Rims 602

CHAPTER XLII

PETROGRAPHIC COLLECTIONS 605
Field work 605
Working tools 605
Hand specimens 607
Wrappers and labels 608
Packing specimens for shipment 608
Office work 609
Accession catalogue 609
Permanent labels for hand specimens 609
Labels for thin sections 610
Marking thin sections 611
Cases for thin sections 612
Card catalogue 613

APPENDIX

Greek alphabet 619
Useful formulæ 619
Trigonometric 619
Cartesian geometric 621
Conversion tables for weights and measures 623
Useful recipes 624
Table of natural sines and cosines 626
Table of natural tangents and cotangents 628
INDEX 631

LIST OF ABBREVIATIONS[1]

Abh. Akad. Wiss. Berlin = Abhandlungen der königlich preussischen Akademie der Wissenschaften, Berlin. I (1770)+.

Abh. geol. Specialkarte Elsass-Loth. = Abhandlungen zur geologischen Specialkarte von Elsass-Lothringen. Strassburg i. E.

Amer. Geol. = The American Geologist. Minneapolis, Minn. I (1888)–XXXVI (1905). Merged in Economic Geology in 1906.

Amer. Jour. Microsc. = The American Journal of Microscopy and Popular Science. New York. I (1875)–XII (1881).

Amer. Jour. Sci. = The American Journal of Science. New Haven, Conn. I (1818)+. 50 volumes to a series.

Amer. Mon. Microsc. Jour. = The American Monthly Microscopical Journal. Washington, D. C. I (1880)–XXIII (1902). Preceded by Amer. Quart. Microsc. Jour.

Amer. Nat. = The American Naturalist. New York. I (1867)+.

Amer. Quart. Microsc. Jour. = The American Quarterly Microscopical Journal. New York. 1878–1879. Continued as Amer. Mon. Microsc. Jour.

Ann. Chim. et Phys. = Annales de chimie et de physique. Paris. I (1788)+.

Ann. d. Phys. = Annalen der Physik. Leipzig. I (1799)+. 1st series, 76 vols. 1799–1824, edited by L. W. Gilbert. 2nd series, 160 vols., 1824–1876, edited by J. C. Poggendorff. 3d series, 69 vols., 1877–1899, edited by G. Wiedemann (Vols. 48–69 with E. Wiedemann). 4th series, continued from I (1900)+.

Ann. d. k. k. naturhist. Hofmuseum = Annalen des k. k. naturhistorischen Hofmuseums. Wien. I (1886)+.

Ann. d. Mines = Annales des mines. Paris. I (1816)+. 10th series begun in 1902. Continuation of Jour. d. Mines.

Anz. Akad. Wiss. Krakau = Anzeiger der Akademie der Wissenschaften in Krakau. I (1901)+. (Akademija umiejetnosci). Same as Bull. Acad. Sci. Cracovie.

Arch. d. naturwiss. Landesdurchf. Böhmen = Archiv der naturwissenschaftlichen Landesdurchforschung von Böhmen. Prague.

Arch. d. sciences, physiques et naturelle, see Bibliotheque universelle. Genève.

Arch. f. Mikrosk. Anatomie = Archiv für mikroskopische Anatomie und Entwicklungsgeschichte. Bonn. I (1865)+.

Arch. Néer. = Archives néerlandaises des sciences exactes et naturelles. Haarlem. I (1866)+.

Astron. and Astrophys. = Astronomy and Astrophysics. Northfield, Minn. I (1882)–XII (1894). Continued as The Astrophysical Journal. Chicago. I (1895)+.

Ber. deutsch. bot. Gesell. = Berichte der deutschen botanischen Gesellschaft. Berlin. I (1882)+.

Ber. deutsch. chem. Gesell. = Berichte der deutschen chemischen Gesellschaft. Berlin. I (1868)+.

Ber. Gesell. Wiss. Leipzig = Berichte über die Verhandlungen der königlich sächsichen Gesellschaft der Wissenschaften zu Leipzig. I (1846)+.

[1] In most cases in the following list, the date of the first volume issued is given for bibliographic information. If a second date appears it indicates that the publication has been discontinued. A + sign indicates that the series continues to date.

Ber. oberhess. Gesell. = Bericht der oberhessischen Gesellschaft für Natur- und Heilkunde. Giessen. I (1847)+.

Bibliothèque universelle, Genève = Originally Bibliothèque britannique. Genève. 1796–1815. Continued as Bibliothèque universelle des sciences, belles-lettres et arts, Genève. 1816–1835. Continued further as Bibliothèque universelle de Genève. 1836–1845. Now Bibliothèque universelle. Archives des sciences, physiques et naturelles, Partie scientifique. Genève. 1846+.

Biol. Centralbl. = Biologisches Centralblatt. Leipzig.

Bot. Centralbl. = Botanisches Centralblatt. Cassel.

Bull. Acad. Sci. Cracovie = Bulletin international de l'Académie des sciences de Cracovie. I (1901). Same as Anz. Akad. Wiss. Krakau.

Bull. Acad. Roy. Belgique = Bulletins de l'Académie royale des sciences, des lettres et des beaux-arts de Belgique. Classe de Sciences. Bruxelles. I (1832)+.

Bull. Soc. Belge de Micr. = Bulletin de la société Belge de microscopie. Bruxelles.

Bull. Soc. Chem. Paris = Bulletin de la société chimique de France. Paris. I (1858)+.

Bull. Soc. Min. France = Bulletin de la société français de minéralogie. Paris. I (1878)+. Previous to 1886 Société minéralogique de France.

Carl's Repertorium = Repertorium der Physik. Edited by Philip Carl. München. I (1866)–XXVII (1891).

Centralbl. f. Min., etc. = Centralblatt für Mineralogie, Geologie und Paläontologie. Stuttgart. I (1900)+.

Chem. News = The Chemical News and Journal of Physical Science. London. Originally The Chemical Gazette. I (1843)–(1859). Chemical News. 1860+.

Comptes Rendus = Comptes rendus hebdomadaires des séances de l'Académie des Sciences. Paris. I (1835)+.

Chem. Zeitschr. = Chemische Zeitschrift. I (1901)+.

Denkschr. Akad. Wiss. Wien = Denkschriften der mathematisch-natur wissenschaftliche Classe der kaiserliche Akademie der Wissenschaften zu Wien. I (1848)+.

Deutsche Mechan. Zeitung = Deutsche Mechaniker Zeitung. Berlin. I (1898)+.

Econ. Geol. = Economic Geology, n. p. I (1905)+.

Edinburgh New Phil. Jour. = The Edinburgh New Philosophical Journal. Edinburgh (1826–1864). Originally Edinburgh Philosophical Journal (1819–1825). The Edinburgh New Philosophical Journal (1826–1854). New Series (1855–1864). Merged in The Quarterly Journal of Science, 1864.

English Mechanic = English Mechanic and World of Science. London. I (1865)+.

Földtani Közlöny = Földtani Közlöny (Geological Communications). Budapest.

Fortschritte der Min., Kryst., und Petrog. = Fortschritte der Mineralogie, Kristallographie und Petrographie. Jena. I (1911)+.

Gilbert's Ann. = Gilbert's Annalen der Physik. See Ann. d. Phys.

Geol. Fören. i Stockholm Förh. = Geologiska föreningens i Stockholm förhandlingar. Stockholm. I (1872)+.

Geol. Mag. = The Geological Magazine. London. Originally The Geologist. I (1858–1863). . The Geological Magazine, I (1864)+.

Grunert's Arch. = Archiv der Mathematik und Physik. Leipzig und Berlin. I (1841)+. Founded by J. Grunert.

Jahresh. d. Ver. f. vaterl. Naturk. Württemberg = Jahreshefte des Vereins für vaterländische Naturkunde in Württemberg.

Jour. and Proc. Roy. Soc. New So. Wales = Journal and Proceedings of the Royal Society of New South Wales. Sydney.

Jour. Appl. Microsc. = Journal of Applied Microscopy and Laboratory Methods. Rochester, N. Y. I (1898)–VI (1903).

Jour. Chem. Soc. London = The Journal of the Chemical Society. London. I (1849)+.

Jour. d. Mines = Journal des mines (1795–1815). Continued as Ann. d. Mines, 1816+.

Jour. d. Phys. = Journal de physique théoretique et appliquée. Paris. I (1872)+.

Jour. Geol. = The Journal of Geology. Chicago. I (1893)+.

Jour. Microsc. = The Journal of Microscopy and Natural Science. See Jour. Postal Microsc. Soc.

Jour. N. Y. Microsc. Soc. = The Journal of the New York Microscopical Society. I (1885)–XIV (1898).

Jour. Postal Microsc. Soc. = The Journal of the Postal Microscopical Society. London. I (1882)–II (1883). Succeeded by The Journal of Microscopy and Natural Science. III (1883)–XVI (1897).

Jour. Roy. Microsc. Soc. = The Journal of the Royal Microscopical Society. London. Preceded by Transactions of the Mineralogical Society (1844–1868), The Monthly Microscopical Journal (1869–1877). The Journal of the Royal Microscopical Society, 1878+.

Jour. Roy. Soc. Arts = The Journal of the Royal Society of Arts. London. I (1852)+.

Jour. Washington Acad. Sci. = The Journal of the Washington Academy of Science. Washington, D. C. I (1911)+.

Knowledge = Knowledge. London. I (1881)+.

Mém. Acad. France = Mémoires de l'Académie des sciences de l'Institut de France. I (1796)+. Various slight variations in the title.

Mém. Acad. Sci. Belgique = Mémoires couronnés et mémoires des savants étrangers publiés par l'Académie royale des sciences, des lettres et des beaux-arts de Belgique. I (1817)–LXII (1904). Bruxelles.

Mémoires couronnés et autres mémoires publiés par l'Academie royale des sciences, des lettres et des beaux-arts de Belgique. Collection in 8vo. I (1840)–LXVI (1904). Bruxelles.

Beginning with 1906 all the Memoirs of the Academy are published in two series. A, sciences, B, Lettres, sciences morales et politiques. Each series includes two collections, one in 4to and one in 8vo.

Mem. Accad. Sci. Napoli = Memorie dell'accademia delle scienze fisiche e matematiche. Napoli.

Mem. and Proc. Chem. Soc., London = Memoirs and Proceedings of the Chemical Society of London. I (1841)–III (1848). Continued as Jour. Chem. Soc. London.

Microsc. Bull. = The Microscopical Bulletin and Science News. Philadelphia. I (1883)–XVIII (1901).

Microsc. News = The Microscopical News. See Northern Microsc.

Microscope = The Microscope. Washington, D. C. and v. p. I (1881)–V (1897).

Mineralog. Mag. = The Mineralogical Magazine and Journal of the Mineralogical Society of Great Britain and Ireland. I (1876)+.

Mon. Microsc. Jour. = The Monthly Microscopical Journal. London. Continued as Jour. Roy. Microsc. Soc., *quod vide.*

Morphol. Jahrb. = Morphologisches Jahrbuch. Leipzig.

Nature = Nature, a Weekly Illustrated Journal of Science. London. I (1869)+.

Nachr. Gesell. Wiss. Göttingen = Nachrichten der kgl. Gesellschaft der Wissenschaften zu Göttingen.

National Druggist = National Druggist. St. Louis.

Neues Jahrb. = Originally Leonhard's Taschenbuch für die Gesammte Mineralogie, Frankfurt a.M. (1807–1824), Leonhard's Zeitschrift für Mineralogie (1825–1829), Leonhard und Bronn's Jahrbuch für Mineralogie, Geognosie, Geologie, und Petrefaktenkunde, Heidelberg (1830–1832), Neues Jahrbuch für Mineralogie, Geognosie, Geologie, und Petrefaktenkunde, Heidelberg (1833–1862), Neues Jahrbuch für Mineralogie, Geologie, und Palaeontologie. Stuttgart. (1879)+.

Neues Jahrb., B.B. = Neues Jahrbuch, etc., Beilage Band. I (1883)+.

Nicholson's Journal = A Journal of Natural Philosophy, Chemistry, and the Arts. London. I (1797)–V (1801), N. S. I (1802)–XXXVI (1813).

Northern Microsc. = The Northern Microscopist. London. (1881.) Followed by The Microscopical News and Northern Microscopist. London. 1882–1883.

Notizbl. Ver. Erdk. Darmstadt = Notizblatt des Vereins für Erdkunde zu Darmstadt und des mittelrheinischen geologischen Vereins. Darmstadt. I (1858)+.

Phil. Mag. = The Philosophical Magazine. London. 1798–1832. United in 1832 with the Edinburgh Journal of Science under the title London and Edinburgh Philosophical Magazine and Journal of Science, I (1832)–(1850), followed by London, Edinburgh and Dublin Philosophical Magazine and Journal of Science, I (1851)+.

Phil. Trans. Roy. Soc. London = The Philosophical Transactions of the Royal Society of London. I (1665)+.

Physical Review = The Physical Review. New York. I (1894)+.

Pogg. Ann. = See Ann. der Phys.

Proc. Amer. Acad. = Proceedings of the American Academy of Arts and Sciences. Boston. I (1846)+.

Proc. Amer. Microsc. Soc. = Proceedings of the American Microscopical Society. v.p. Originally Transactions of the American Microscopical Society (1878). Proceedings of the National Microscopical Congress, Vols. I to II; Proceedings of the American Society of Microscopists, Vols. II to XIV; Proceedings of the American Microscopical Society, XV to XVII.

Proc. Cambridge Phil. Soc. = Proceedings of the Cambridge Philosophical Society. Cambridge (England). I (1843)+.

Proc. Liverpool Geol. Asso. = Proceedings of the Liverpool Geological Association. Liverpool. I (1860)+.

Proc. Rochester Acad. Sci. = Proceedings of the Rochester Academy of Science. Rochester, N.Y. I (1889)+.

Proc. Geol. Soc. London = Proceedings of the Geological Society of London. 1826–1845. Continued in Quart. Jour. Geol. Soc., London. 1845+.

Proc. Roy. Soc. Edinburgh = Proceedings of the Royal Society of Edinburgh. I (1845)+.

Proc. Roy. Soc. Dublin = Scientific Proceedings of the Royal Dublin Society. I (1856)+.

Proc. Roy. Soc. London = Proceedings of the Royal Society of London. I (1800)+.

Proc. Roy. Soc. Victoria = Proceedings of the Royal Society of Victoria. Melbourne. I (1897)+.

Prometheus = Prometheus. Illustrirte Wochenschrift über die Fortschritte in Gewerbe, Industrie und Wissenschaft. I (1889)+.

Quart. Jour. Geol. Soc. London = The Quarterly Journal of the Geological Society of London. I (1845)+.

Quart. Jour. Microsc. Sci. = The Quarterly Journal of Microscopical Science. London. I (1853)+.

Rend. Accad. Napoli. = Rendiconto dell'Accademia delle Scienze Fisiche e Mathematiche. Napolio.

Rend. Accad. Lincei, Roma = Rendiconti della Reale Accademia dei Lincei, Roma. I (1840 ?)+. Continuation of Atti and Transunti della, etc.

Rep. Brit. Asso. Adv. Sci. = Report of the British Association for the Advancement of Science. London. I (1831)+.

Rivista di Min. Crist. Ital. = Rivista di Mineralogia e Cristallografia Italiana. Padua. Vol. XVIII is 1897+.

Schlömilch's Zeitschr. = Zeitschrift für Mathematik und Physik. Founded in 1856 by O. Schlömilch. Leipzig. I (1856)+.

Science = Science. New York. I (1880)+.

Sci. Gossip = Science Gossip. London. I (1865)–(1902).

Sitzb. Akad. Wiss. Berlin = Sitzungsberichte der königlich preussischen Akademie der Wissenschaften. Berlin. I (1836)+.

Sitzb. Akad. Wiss. Heidelberg = Sitzungsberichte der Heidelberger Akademie der Wissenschaften.

Sitzb. Akad. Wiss. München. = Sitzungsberichte der königlich Bayerischen Akademie der Wissenschaften zu München. Vol. I (1860)+. Since 1870 the Math.-Phys. Cl. and the Phil.-Histor. Cl. publish separate Sitzungsberichte.

Sitzb. Akad. Wiss. Wien = Sitzungsberichte der mathematisch-naturwissenschaftlichen Klasse der kaiserlichen Akademie der Wissenschaften. Wien. I (1848)+.

Sitzb. Gesell. Wiss. Prag. = Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe der königlich böhmischen Gesellschaft der Wissenschaften. Prag.

Sitzb. niederrhein. Gesell. Bonn = Sitzungsberichte der niederrheinischen Gesellschaft für Natur- und Heilkunde zu Bonn. 1854–1906. Continued in Sitzungsberichte herausgegeben von Naturhistorischen Verein der preussischen Rheinlande und Westfalens.

Trans. Amer. Inst. Mining Eng. = Transactions of the American Institute of Mining Engineers. New York. I (1871)+.

Trans. Liverpool Geol. Asso. = Transactions of the Liverpool Geological Association.

Trans. Roy. Irish Acad. = Transactions of the Royal Irish Academy. Dublin. I (1787)+.

Trans. Roy. Soc. Edinburgh = Transactions of the Royal Society of Edinburgh. I (1783)+.

T. M. P. M. = Tschermak's Mineralogische und Petrographische Mitteilungen, Vienna. Originally Mineralogische Mittheilungen, 1871–1877, continued as above. I (1878)+.

U. S. G. S., Ann. Rept. = Annual Report of the United States Geological Survey. Washington, D. C. I (1880)+.

U. S. G. S., Bull. = Bulletin of the United States Geological Survey. Washington, D. C. No. I (1883)+.

U. S. G. S., Mono. = Monograph of the United States Geological Survey. Washington, D. C. I (1890)+.

U. S. G. S., P. P. = Professional Paper of the United States Geological Survey. Washington, D. C. No. I (1902)+.

Versl. en Meded. Akad. Weten. Amsterdam = Verslagen en Mededeelingen der Koninklijke Akademie van Wetenschappen te Amsterdam. Afdeeling natuurkunde. I (1855)–IX (1892).

Verh. k. k. Geol. Reichsanst. Wien. = Verhandlungen der k. k. geologischen Reichsanstalt. Wien. I (1867)+.

Verh. Russ. Min. Gesell, St. Petersburgh = Verhandlungen der russisch-kaiserlichen Mineralogischen Gesellschaft zu St. Petersburgh.

Verh. Phys. Med. Gesell. Würzburg = Verhandlungen der physikalisch-medicinischen Gesellschaft zu Würzburg. I (1850)+.

Verh. Naturf. Gesell. Basel = Verhandlungen der naturforschende Gesellschaft. Basel. Vol. VII (1885)+.

Verh. Naturhist. Ver. Preuss. Rheinl. Bonn. = Verhandlungen des naturhistorischen Vereins der preussischen Rheinlande und Westfalens. Bonn. I (1844)+.

Wiedem. Ann. Weidemann's Annalen. See Ann. der Phys.

Zeitschr. f. analyt. Chemie. = Zeitschrift für analytische Chemie. Wiesbaden. I (1862)+.

Zeitschr. f. angew. Mikrosk. = Zeitschrift für angewandte Mikroskopie, u. s. w. Berlin, Leipzig und Weimar. I (1895)+.

Zeitschr. d. deutsch. geol. Gesell. = Zeitschrift der deutschen geologischen Gesellschaft. Berlin. I (1849)+.

Zeitschr. f. Instrum. = Zeitschrift für Instrumentenkunde. Berlin. I (1881)+.

Zeitschr. f. Kryst. = Zeitschrift für Krystallographie und Mineralogie. Leipzig. I (1877)+.

Zeitschr. f. wiss. Mikrosk. = Zeitschrift für wissenschaftliche Mikroskopie und für mikroskopische Technik. Leipzig. I (1884)+.

Zeitschr. f. gesammten Naturwiss. = Zeitschrift für die gesammten Naturwissenschaften. Halle. I (1853)+.

Zeitschr. f. physik. Chemie = Zeitschrift für physikalische Chemie, Stöchiometrie und Verwandtschaftslehre. Leipzig. I (1887)+.

MANUAL OF PETROGRAPHIC METHODS

CHAPTER I

MINERALOGICAL PRINCIPLES

1. Crystals.—Minerals may occur either crystallized or amorphous. When crystallized, they possess certain properties which are alike in parallel directions; when amorphous, the properties show no regular or uniform variations. Substances which crystallize, when left free to grow as they will, tend to assume definite forms which are characteristic for that mineral.

Not only do crystals tend to build up regular forms, but there is a definite molecular arrangement throughout their mass, so that, as we shall see, we are enabled, by certain optical examinations, to determine their characteristics regardless of accidental, favorable conditions of growth.

2. Crystallographic Axes.—The faces which develop upon a crystal may be referred to certain imaginary axes, generally regularly arranged, always, however, having a definite position in a given mineral. In general these axes are three in number, and the various faces may be defined by their intercepts upon them. According to the kinds of axes, we may divide all crystals into six (or seven) groups. Without going into the question of symmetry, it is simplest to describe the different systems in the order of decreasing complexity.

I. In the **isometric**[1] system the faces are referred to three interchangeable axes at right angles to each other. In ideal crystals and in drawings, these axes are represented as of equal lengths; in nature they are usually not alike. It is customary to consider one axis (c) vertical, one extending from left to right (b), and one from front to back (a). The angles between these axes are expressed by α for that between c and b, by β for that between c and a, and by γ for that between a and b. In this system they are all 90° (Fig. 1).

II. In the **tetragonal**[2] system the faces are referred to three axes at right angles to each other, two of them being interchangeable, the other either longer or shorter. The two equal axes are the a and b, the unequal axis is the c. $a=b\lessgtr c$, $\alpha=\beta=\gamma=90°$;

III. In the **hexagonal**[3] system there are four axes. The three horizontal axes are interchangeable and inclined 60° to each other; the vertical one (c)

[1] *Tessular*, Mohs; *Isometric*, Hausmann; *Tesseral*, Naumann; *Regular*, Weiss, Rose; *Cubic*, Dufrénoy, Miller, des Cloizeaux; *Monometric*, Dana's original system.

[2] *Pyramidal*, Mohs; *Viergliedrige* oder *Zwei-und-einaxige*, Weiss; *Tetragonal*, Naumann; *Monodimetric*, Hausmann; *Quadratic*, von Kobell; *Dimetric*, Dana originally.

[3] *Rhombohedral*, Mohs; *Sechsgliedrige* oder *Drei-und-einaxige*, Weiss; *Hexagonal*, Naumann; *Monotrimetric*, Hausmann.

is at right angles to the plane of the other three, and is either longer or shorter. One of the short axes (a_2) is conventionally considered as extending from left to right. The intercepts are written in the order a_1, a_2, a_3, c (Fig. 2).

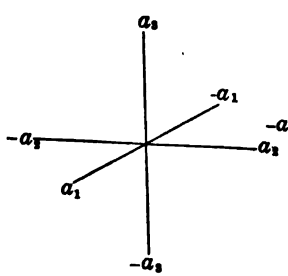

FIG. 1.

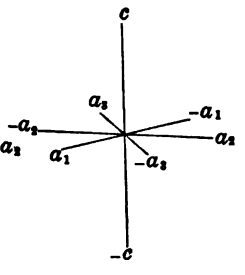

FIG. 2.

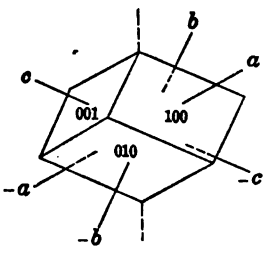

FIG. 3.

FIG. 1.—The crystallographic axes in the isometric system. $a=b=c$, $\alpha=\beta=\gamma=90°$.
FIG. 2.—The crystallographic axes in the hexagonal system. $a_1=a_2=a_3 \lesseqgtr c$.
FIG. 3.—The crystallographic axes in the trigonal system when referred to three axes. $a=b=c$.

IIIa. The **trigonal**[1] system is sometimes considered as independent of the preceding, and includes its hemimorphic forms. It is usually referred to four axes arranged as in the hexagonal system. Originally, however, it was

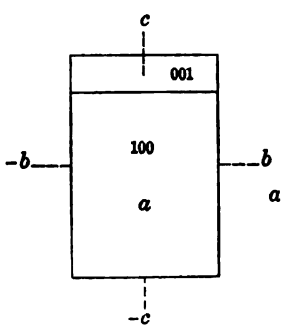

FIG. 4.

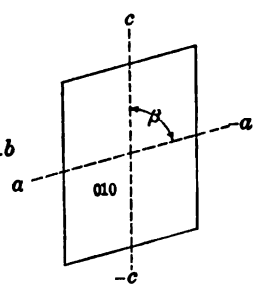

FIG. 5.

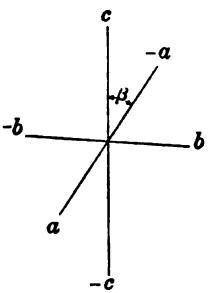

FIG. 6.

FIGS. 4 TO 6.—The crystallographic axes in the monoclinic system. $a \lesseqgtr b \lesseqgtr c$, $\alpha=\gamma=90°$, $\beta<90°$.

referred by Miller to three, and this method is still followed occasionally. The axes are interchangeable and oblique (Fig. 3).

IV. In the **orthorhombic**[2] system the faces are referred to three unequal axes at right angles to each other. $a \lesseqgtr b \lesseqgtr c$, $\alpha=\beta=\gamma=90°$.

V. In the **monoclinic**[3] system the faces are referred to three unequal

[1] *Trigonal*, Groth.

[2] *Prismatic* or *Orthotype*, Mohs; *Ein-und-einaxige*, Weiss; *Rhombic* or *Anisometric*, Naumann; *Trimetric* or *Orthorhombic*, Hausmann; *Trimetric*, Dana originally.

[3] *Hemi-prismatic* and *Hemi-orthotype*, Mohs; *Zwei-und-eingliedrige*, Weiss; *Monoclinohedral*, Naumann; *Clinorhombic*, von Kobell, Hausmann, des Cloizeaux; *Augitic*, Haidinger; *Oblique*, Miller; *Monosymmetric*, Groth.

axes, one of which (a) is inclined in the plane of the vertical axis (c); the other two (c, b) are at right angles to each other. The inclined axis (a) projects downward from back to front, and the acute angle between it and c is called β. $a \lesseqgtr b \lesseqgtr c$, $\alpha = \gamma = 90°$, $\beta < 90°$ (Figs. 4–6).

VI. In the **triclinic**[1] system there are three unequal axes, none of which is at right angles to any other. $a \lesseqgtr b \lesseqgtr c$.

3. The Weiss Parameters.—It was stated above that the various crystallographic forms are defined by their intercepts upon the axes. These **parameters,** as the intercepts are called, have been variously expressed by different writers, but at the present time only three systems are more or less used. The first of these is that of Weiss,[2] who denoted the semi-crystallographic axes by the letters a, b, and c, and indicated the position of any face by the ratio of its intercepts upon them. For example, $1a : 1b : 2c$ indicates that the face cuts the a and b axes at unity and the c at twice that distance, $1a : 2b : 1c$ indicates that it cuts the a and c axes at unity and the b at twice that distance, and $1a : \infty b : 2c$ indicate that it cuts the a axis at unity, the b at infinity—that is, it is parallel to b—, and the c at twice unity.

4. The Naumann System.—The Weiss system was simplified by Naumann[3] who omitted the designation of the axes, wrote the intercepts in inverse order—that is c, b, a—, made one of the axes, usually a, unity and omitted writing it, and inserted, after the number referring to the c axis, the letter O in the isometric system and the letter P in the others. By his method the three forms given above become, $2P$, $P2$, and $2P\infty$.

5. The Miller Indices.—The two preceding systems have been gradually superseded by the so-called Miller system. This is the one in common use at the present time and is the one used in this book. It was proposed by Whewell[4] in 1825 and soon after, independently, by Grassmann[5] and by Frankenheim.[6] It did not come into common use, however, until Professor

[1] *Tetarto-prismatic*, Mohs; *Ein-und-eingliedrige*, Weiss; *Triclinohedral*, Naumann; *Clinorhomboidal*, von Kobell; *Anorthic*, Haidinger, Miller; *Anorthic or doubly oblique*, des Cloizeaux; *Asymmetric*, Groth.

[2] C. S. Weiss: *Krystallographische Fundamentalbestimmung des Feldspathes.* Abh. Akad. Wiss. Berlin, Physik. Kl, 1816–17, 231–285, especially footnote p. 244.

Idem: *Ueber eine verbesserte Methode für die Bezeichnung der verschiedenen Flächen eines Krystallisationssystems.* Ibidem, 1816–17, 286–336.

[3] Carl Fr. Naumann: *Grundriss der Krystallographie.* Leipzig, 1826.

[4] W. Whewell: *A general method of calculating the angles made by any planes of crystals and the laws according to which they are formed.* Phil. Trans. Roy. Soc. London, Pt. I (1825), 87–130.

[5] J. G. Grassmann: *Zur physischen Krystallonomie und geometrischen Combinationslehre*, Stettin, 1829.*

[6] M. L. Frankenheim: *De crystallorum cohaesione.* Vratislaviae, 1829.*

Miller[1] of Cambridge adopted it in his writings, more especially in his Crystallography. In this system the intercepts are written in the order a, b, c and are expressed as reciprocals of the values given in the Weiss system. That is, the Weiss parameters may be obtained by considering the Miller indices as the denominators of fractions and then reducing them to whole numbers. The Weiss forms $a:b:2c$, $a:2b:c$, $a:\infty b:2c$, $a:\infty b:\infty c$, become (221), (212), (201), and (100). Intercepts on the negative ends of the axes are written with a minus sign above the figures thus, $1\bar{1}1$, $1\bar{2}2$, etc.

6. Zones.—When the intersections of certain faces are mutually parallel, and consequently parallel to the same line—called the **zone-axis**—drawn through the intersection of the crystallographic axes, the faces are said to lie in a zone. Examples of zones are 100, 101, 001, $\bar{1}01$, $\bar{1}00$, $\bar{1}0\bar{1}$, $00\bar{1}$, $10\bar{1}$; and 001, 010, $00\bar{1}$, $0\bar{1}0$.

[1] W. H. Miller: *On the forms of sulphuret of nickel and other substances.* Phil. Mag., VI (1835), 104–107.

Idem: *Ueber die Krystallform des Schwefelnickels und anderer Substanzen.* Pogg. Ann., XXXVI (1835), 475–479.

Idem: *A treatise on crystallography.* Cambridge, 1839.*

See also E. von Fedorow: *Die Millerschen sind die allein zulässigen Symbole. Zeitschr. f. Kryst.*, XXIV (1894–5), 132–136.

CHAPTER II

STEREOGRAPHIC PROJECTION[1]

7. Introductory.—If a crystal be placed in the center of a sphere, and any point upon it be connected by a straight line with the center and the surface, this point will be definitely located by the latter intersection. Since it is generally not practicable to use a sphere to show crystal properties, various methods of projection upon a flat surface have been devised. Among these the most common are orthographic, gnomonic,[2] and stereographic projections; and, of these, the latter is the one which has been found most convenient in crystallography.

The method of representing the surface of a globe in stereographic projection appears to have been invented by the astronomer Hipparchus about the middle of the second century before Christ. It was used by Ptolemy about three hundred years later in map making, and has been used, more or less, until the present time. In crystallography it was first used by Neumann,[3] whose book does not appear to have been appreciated, however, for only the first part was issued. The method was later quite extensively used by Miller[4] in his Crystallography.

8. Definitions.—In the following discussion it will be convenient to use certain terms with definite meanings. If we consider the line connecting the north and south poles of a sphere as vertical, the north pole will be uppermost in the projection, and we may say:

A *great circle* is one whose plane passes through the center of the sphere. It is the largest circle that can be described upon it.

A *small circle* is any circle less than a great circle.

[1] See general bibliography at end of chapter.

[2] For gnomonic projection see:

E. Mallard: *Traité de cristallographie.* Paris, 1879, I, 63–66.*

H. A. Miers: *The gnomonic projection.* Mineralog. Mag., VII (1887), 145–149.

V. Goldschmidt: *Projection und graphische Krystallberechnung.* Berlin, 1887.*

N. Story-Maskelyne: *Crystallography. A treatise on the morphology of crystals.* Oxford, 1895, 492–499.*

G. F. Herbert Smith: *On the advantages of the gnomonic projection and its use in the drawing of crystals.* Mineralog. Mag., XIII (1913), 309–321.

Idem: *Ueber die Vorzüge der gnomonischen Projection und über ihre Anwendung beim Krystallzeichnen. Zeitschr. f. Kryst.*, XXXIX (1903–4), 142–152.

Harold Hilton: *The gnomonic net.* Mineralog. Mag., XIV (1904), 18–20.

H. E. Boeke: *Die gnomonische Projektion in ihrer Anwendung auf kristallographische Aufgaben.* Berlin, 1913.

[3] F. Neumann: *Beiträge zur Krystallonomie.* Berlin und Posen, 1823.*

[4] W. H. Miller: *A treatise on crystallography.* Cambridge, 1839.*

Idem: *On the employment of the stereographic projection of the sphere in crystallography.* Phil. Mag., XIX (1860), 325–328.

Vertical great circles are those which pass through the north and south poles. Their projections are straight lines, and their centers lie in the equatorial plane. These lines may be called *meridians*.

Vertical small circles are circles whose centers lie on the equator and whose radii are less than 90°. They are projected as circles.

The *horizontal great circle* is the equator.

Horizontal small circles correspond to parallels of latitude and, consequently, may be called *parallels*.

Antipodal points are points on the sphere at opposite ends of lines passing through the center. Thus the north and south poles are antipodal points.

The *pole* of a face is the point where a line, drawn at right angles to the face and passing through the center of the sphere, pierces the latter. The term is also applied to the stereographic projection of this point.

9. Locating Points.—In making a stereographic projection, all lines and points of a crystal must first be imagined as projected upon a sphere

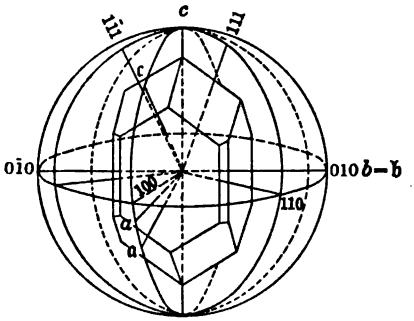

FIG. 7.—Perspective view of a sphere surrounding a crystal of diopside, showing the location of the poles, etc.

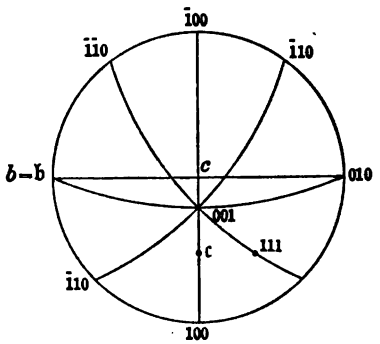

FIG. 8.—Stereographic projection of the same.

(Fig. 7), crystal faces being represented by the piercing points of lines extending at right angles to them and through the center of the sphere. If, now, the eye be placed at the south pole, all of these lines and points can be traced upon a transparent plane lying in the plane of the equator (Fig. 8).

Let the circle in Fig. 9 represent a north and south section through a sphere along a meridian. The eye being placed at the point *S*, we will observe the intersections of the meridian with the 10°, 20°, 30°, etc., parallels, as points upon the line *WE*, the intersections in the southern hemisphere being represented by points beyond the circle. Seen in stereographic projection, these points will appear at *a*, *b*, *c*, etc., as shown in Fig. 10 on the line *J'J*. On some other meridian the intersections of the same parallels will appear at the points *a'*, *b'*, *c'*, etc. The distances of these points from the center are measured by the tangents of half the angles made by the lines from the south pole through them, the radius being taken as unity, for, by geometry, the angle $BSN = 1/2\ BON$ (Fig. 11). Since *BON* is measured by

the arc NB, BSN is equal to one-half the arc NB, and its tangent is equal to $\frac{OT}{SO} = \frac{OT}{r}$, where r is the radius of the circle. From this relationship it is easy to locate, mathematically, the intersections of these points in the projection. This is of value in determining the points where the lines extended through points in the southern hemisphere cut the projection plane. The relative positions of these points are fixed; the actual distances will depend upon the scale used.

Since all the intersections between meridians and any parallel lie at the same distance from the center (Fig. 10), the parallel itself will appear, in the projection, as a circle through these points, consequently each circle in the figure represents a distance 10° farther from the north pole than the adjacent one. The points N and O (Fig. 9) will be projected at M (Fig. 10), consequently the line NOS (Fig. 9) will be projected upon the same point. Since a meridian is the intersection of a sphere and a plane passing through its north and south poles, and since this plane must contain the NOS line which is vertical, the plane itself must be vertical, and its intersection with the sphere must be projected as a straight line passing through the center. All great circles, therefore, which pass through the poles of the sphere, appear, in stereographic projection, as straight lines passing through the center.

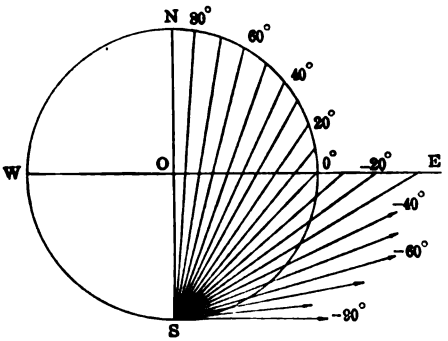

FIG. 9.

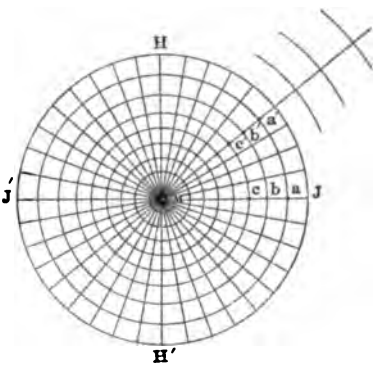

FIG. 10.

FIGS. 9 AND 10.—Method of locating points in stereographic projection. FIG. 9. Vertical section through a sphere; FIG. 10. —Stereographic projection showing positions of parallels and meridians.

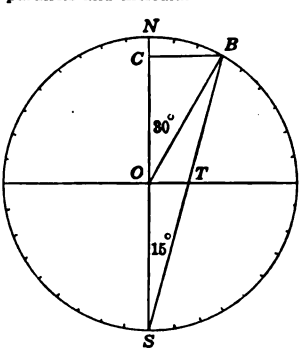

FIG. 11.—Tangent relations in stereographic projection.

10. Circles drawn upon a Sphere appear as Circles in Stereographic Projection.—One of the chief advantages of stereographic projection over other projections is the fact that all circles traced upon the sphere appear as true circles in the drawing, the limiting case of meridians appearing as straight lines being the case of circles with centers at infinity.

FIG. 12.—Orthographic projection of a circle drawn upon a sphere.

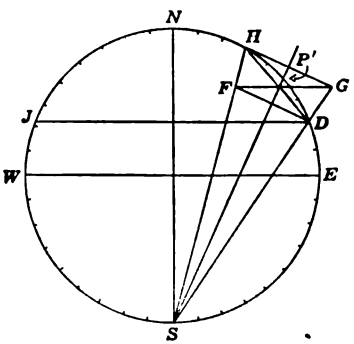

FIG. 13.—Vertical section through *N H P' D E S* of preceding figure.

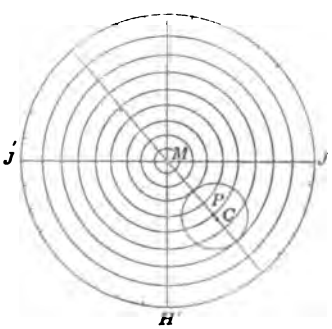

FIG. 14.—Stereographic projection of the small circle *H D*.

Let Fig. 12 represent a sphere in orthographic projection, Fig. 13 a section through the meridian *NP'E*, and Fig. 14 a stereographic projection through *WE*. Upon the sphere, about the point *P'*, a circle is described having a radius, for example, measured by 20° of its surface. The upper and lower points of this circle will appear, in Fig. 13, at *H* and *D*, and the center at *P'*. The triangle *HSD* (Fig. 13) is a section of the inclined cone *HSD* of Fig. 12. It has, by construction, a circular base at the surface of the sphere, and has its apex at *S*. If the line *SD* be extended to *G* so that *SG* = *SH*, then *HSG* is the section of a symmetrical cone having an elliptical base. If, now, this cone be rotated through 180° on its axis *SP'*, so that the major and minor axes are parallel to their former positions, the circle *HD* will be in the position *FG* (Fig. 13). All sections through the cone parallel to either of these sections, consequently, will have similar circular sections. In other words, there will be two series of circular sections in the cone, namely, sections parallel to *HD* and to *FG*. The latter sections are parallel, also, to the equator *WE*; for if a line *JD* be drawn parallel to *FG*, we have, by construction, the angle *JDS* = *FGS*. We also have *FGS* = *DHS*, since it is the same angle in a revolved position. The angle *JDS* lies on the circumference of a circle and, by geometry, we know its value to be one-half the arc *JS*. *DHS* also lies on the circumference, and its value is one-half the arc *DS*. The included angles being equal, the arcs *JS* and *DS* are equal, consequently the line *DJ* is

at right angles to the line NS, that is, it is horizontal, and any geometrical figure drawn upon the plane of which this line is the projection, will appear as a similar figure in the projection.

From this demonstration we may see that *the stereographic projection of any circle which may be described upon a sphere will be a true circle.*[1] The stereographic projection of the center of the small circle (P, Fig. 14) will not, however, be the center of the projected circle (c), but will lie somewhat within it.

The explanation here given will apply also to great circles, which are likewise projected as true circles.

11. Spherical Angles appear in Their True Values in Stereographic Projection.—Another advantage of stereographic projection is the fact that the

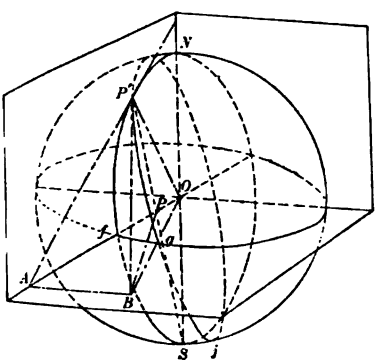

FIG. 15.—Perspective view of sphere and intersecting planes.

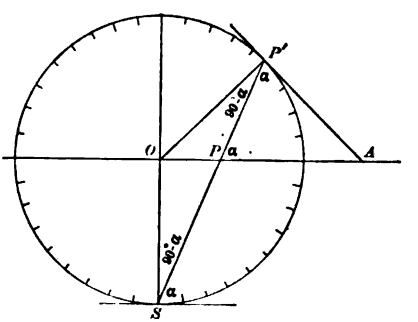

FIG. 16.—Geometric relations between angles.

angle at which two circles cross on the sphere appears in its true value in the drawing.

Let $P'fS$ and $P'gj$ (Fig. 15) be two great circles on the sphere. It is to be proved that the angle $fP'g$, which lies on the surface of the sphere, will appear in its true value in the projection. As the simplest case consider first one side of this angle to be formed by a great circle passing through the pole N; then $P'fg$ is a right angle. Since a spherical angle is measured by the angle between the tangents to the great circles which form the angle, AP' and BP', tangents to the two great circles $P'fS$ and $P'gj$, will measure the angle $fP'g$, whereby $fP'g = AP'B$. Now the stereographic projection of P' is P, and since the angle $AP'P = APP'$, as may be seen from Fig. 16, we have an isosceles

[1] For analytical demonstration see Thos. Craig: *A treatise on projections.* U. S. Coast and Geodetic Survey, Washington, 1882, 13–28, 187–191.

For graphical demonstration see E. Gelcich und F. Sauter: *Kartenkunde geschichtlich dargestellt.* Leipzig. 2te Aufl. von Paul Dinse, 1897, 42–44.

See also V. Goldschmidt: *Ueber stereographische Projection. Zeitschr. f. Kryst.*, XXX (1899), 260–271.

triangle in which $AP'=AP$. If we pass a plane through the two tangents, AP' and BP', its trace on the horizontal plane will be the line AB, which must necessarily lie at right angles to the line AP, for, by geometry, the trace of a tangent plane lies at right angles to the shortest line between it and the center of the sphere. We have, now, two triangles, $AP'B$ and APB, in which one side (AB) is common to both, one side equal in each ($AP'=AP$), and one angle a right angle ($P'AB=PAB=90°$). The triangles, consequently, are equal; $P'B=PB$, and the angle $AP'B=APB$. The angle in the projection, therefore, is the same as the angle on the sphere.

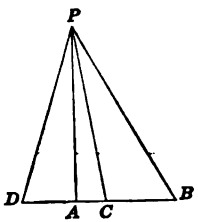

FIG. 17.

In a similar manner, another right triangle, as DPA or CPA, Fig. 17, may be proved to be projected in its true value. The algebraic sum of APB and DPA or CPA ($=DPB$ or CPB), being thus projected in its true value, any angle, however placed and of whatever value, will also so appear.

12. Graphical Solutions of Problems.—(1) *Given a pole, to find the corresponding great circle.* Let the required pole be 30° above the horizon and 130° to the left front. Let Fig. 18 be a vertical section through the sphere along

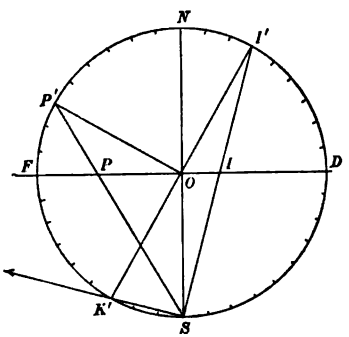

FIG. 18.—Vertical section through a sphere, showing locations of points.

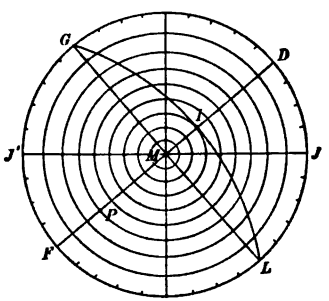

FIG. 19.—Stereographic projection of preceding.

the 130° meridian, then P', Fig. 18, 30° above F, will be its vertical projection, and P, Fig. 19, 130° to the front and on the parallel through P, its stereographic projection. $K'I'$, Fig. 18, is the trace of the plane which passes through the center of the sphere and lies at right angles to the line OP'. Its intersection with the surface of the sphere is the required great circle.

In the vertical section (Fig. 18), the point O represents the line GL of the stereographic projection—the piercing points (G and L) through the sphere, being two antipodal points on the great circle. The point I' and its projection I, 90° from P, represent a third point on the circle. It is now only necessary to pass a great circle through the three points G, I, L, Fig. 19. The

center of the circle will lie half way between I and the projection of K'. The latter point, however, falls too far to the left to make it possible to determine the center by taking half the distance, IK. If one uses circles of uniform size for the projection, it is possible to construct scales giving the positions of the centers for various great circles, a method used by Penfield[1] for projection circles 14 cm. in diameter. The center may be located without scales, however, since it must lie at equal distances from L, I and G. With these three points as centers, describe two sets of equal arcs, such as g', l', i', and g'', l'', i'' (or g''', l''', i'''), Fig. 20. If the arcs drawn with I

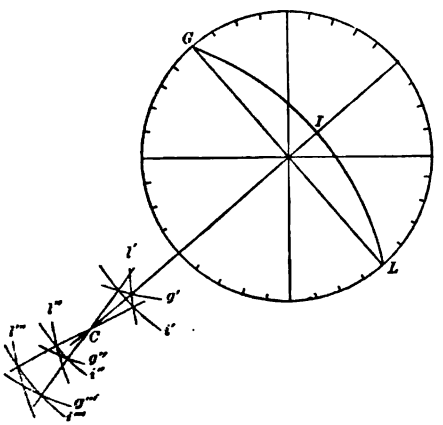

FIG. 20.—Method for locating the center of a circle when three points upon the arc are given.

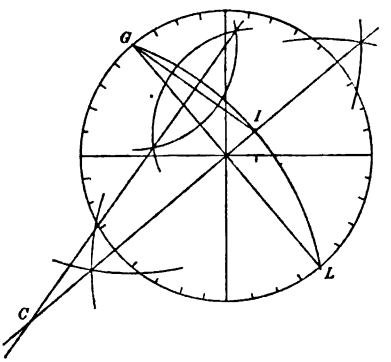

FIG. 21.—Another method for locating the center of a circle when three points upon the arc are given.

as a center fall on opposite sides of the intersection of the other two (i', l', and i'', l''), connect opposite angles; if they fall on the same side (i'', l'', and i''', l'''), connect angles on the same side. The desired center (C) is where these straight lines cross. Another method of finding the center is to erect perpendiculars to two chords (GL and GI, Fig. 21). The intersection is the desired point.

(2) *To pass a great circle through two points which fall within the equatorial circle.* Let O and D, Fig. 22, be any two points within the equatorial circle. The center of the projected circle passing through these points must lie at equal distances from each, consequently it must lie on a line at right angles to the line connecting them. Construct this line (GC) by drawing equal arcs from the two points, and connect the intersections. Other points on the great circle are the antipodal points to O and D, either one being sufficient to determine it. The position of the antipodal point, say of O, Fig. 22, can be determined by making use of an auxilliary great circle. Draw a vertical great circle, or meridian, through O ($AOMB$), and measure the elevation of

[1] See references, page 16, *infra*.

this point above the equator by making use of the projected parallels. In Fig. 22 this distance (*A* to *O*) is 30°. The desired antipodal point (*E*) is 30° below the equator on the same meridian, that is, 180° from *O*. Con-

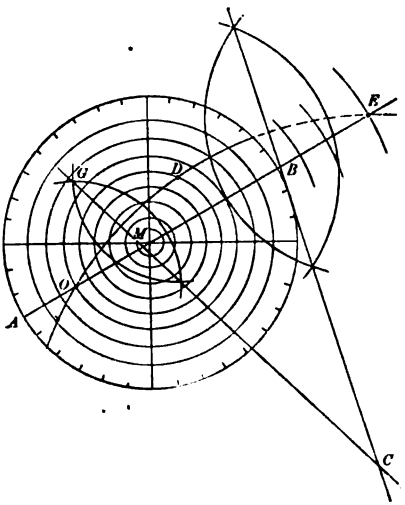

FIG. 22.—Construction for determining the center of a great circle passing through two given points.

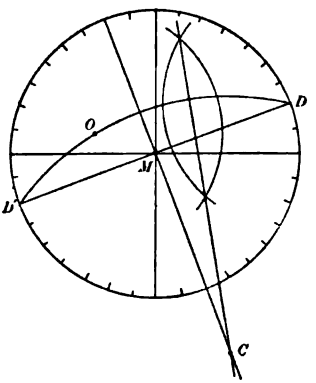

FIG. 23.—Construction for determining the center of a great circle through two points, one of which lies on the equator.

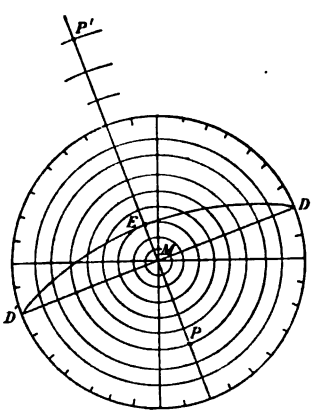

FIG. 24.—Construction for locating the poles of a given great circle.

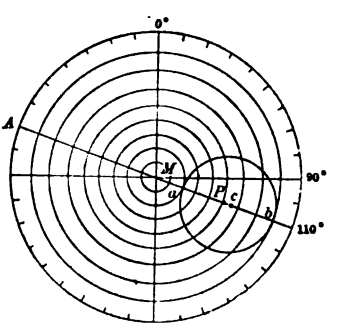

FIG. 25.—Projection of a small circle.

struct a perpendicular to *DE*; its intersection with the extension of *GM* is the desired center (*C*).

(3) *To pass a great circle through two points, one of which lies on the equator.* The desired circle must pass through *O* and *D*, Fig. 23. It must also pass

through the antipodal point of D. Since the latter lies upon the equator, its antipodal point (D') must also lie upon it. Construct the vertical great circle or meridian DMD' to locate D'. The desired center must lie on a line at right angles to this meridian, that is, on a line through M, intersecting the equator 90° from D and from D'. It must also lie on the medial line between O and D. The intersection of these two lines marks the location of the center C.

(4) *To find the poles of a given great circle.* By means of the projected parallels, measure 90° each way from the point where the great circle crosses the bisecting meridian. Thus the distances from E to the poles P and P' (Fig. 24) are each 90°.

(5) *To draw a small circle, its size and the location of its center on the sphere being given.* Let it be required to draw a small circle with a radius of 30°, and

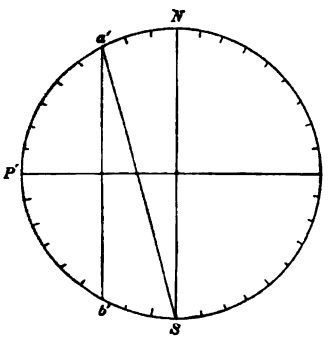

FIG. 26.—Vertical section of a vertical small circle.

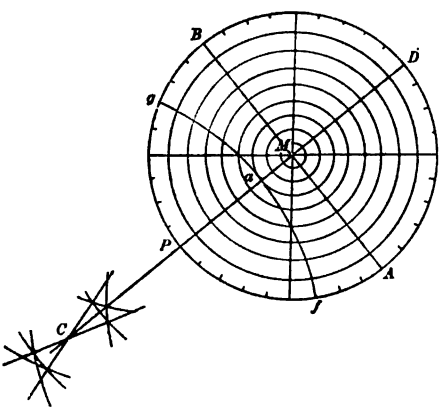

FIG. 27.—Construction for projecting vertical small circles.

with its center 40° above the equator and at the right on the 110° meridian. Draw first the 110° meridian (AMb, Fig. 25). Since the center of the desired circle is 40° above the equator and its radius is 30°, its lower point b will fall at the intersection of this meridian with the 10° parallel. The upper point a of the circle will fall $40°+30°=70°$ above the equator; the pole P, 40° above it. The actual center c of the circle in the projection will lie half way between the points a and b.

(6) *To draw a vertical small circle of given size. Let* $a'b'$, Fig. 26, be a vertical section through a vertical small circle of 60° radius. One point (a, Fig. 27) can be located 60° from P and on the desired meridian PD by means of the parallels. Two points, f and g, each 60° from P and on the equator, represent two other points on the vertical small circle. The problem now becomes that of constructing a circle through three points, which may be done as in Case 1. If the vertical small circle in the projection is an arc of long

radius, the center of the circle *fag* may lie off the paper. In such cases the line may be drawn best by means of the curved ruler described below (Art. 13).

(7) *To measure the angles of a spherical triangle.*—From trigonometry we know that a spherical triangle is one formed by the intercepts, on the surface of a sphere, of a triedral angle with its vertex at the center. As the angular distance between two points on a globe is measured in degrees on the arc of a great circle, so also are the angles in the projection of a spherical triangle measured by the arcs of great circles at a distance of 90° from the angle.

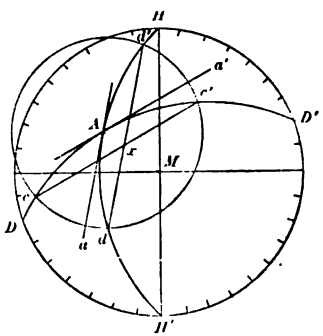

FIG. 28.—Measurement of the projection of a spherical triangle.

The diedral angles of the triedral angle are the angles of the spherical triangle, and these have their original values in the stereographic projections. Thus, to measure the angle $H'AD'$ (Fig. 28), draw tangents to each circle at A, and measure the angle between them by means of a transparent protractor. To draw an accurate tangent, lay off equal distances on each arc, as cc' and dd', connect these points, and draw, through A, lines parallel to the chords thus located. The angle dxc', between the chords, is, of course, the same as the angle aAa' between the tangents.

A much simpler method, involving the use of a stereographic net, is described below.

13. Protractors and Scales.—As mentioned above, the process of making the measurements required in stereographic projection can be much simplified by the use of suitable protractors and scales. So long ago as 1867 there was used, in the U. S. Hydrographic Office, a protractor divided into degrees by great and small circles, and known as Professor Chauvenet's Great Circle

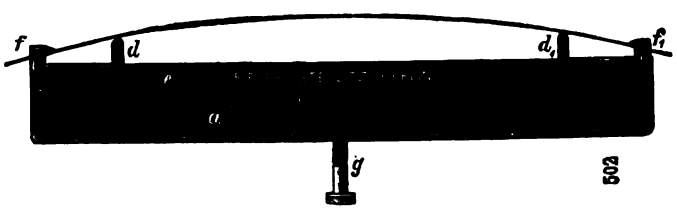

FIG. 29.—Curved ruler, after Wulff and von Fedorow. 2/7 natural size. (Fuess.)

Protractor. An illustration of it is given by Sigsbee.[1] Wulff,[2] in 1893, used the stereographic projection in showing the optical properties of crystals, and gave an illustration of a curved ruler to be used in drawing arcs of large

[1] Capt. C. D. Sigsbee: *Graphical methods for navigators.* U. S. Hydrographic Office, Washington, D. C., 1896.*

[2] Georg Wulff: *Ueber die Vertauschung der Ebene der stereographischen Projection und deren Anwendung.* Zeitschr. f. Kryst., XXI (1893), 249–254.

circles. Von Fedorow,[1] in a series of articles on determinative methods beginning the same year, made much use of this projection. He gave, in his first paper, a mathematical explanation of why the curve in the curved

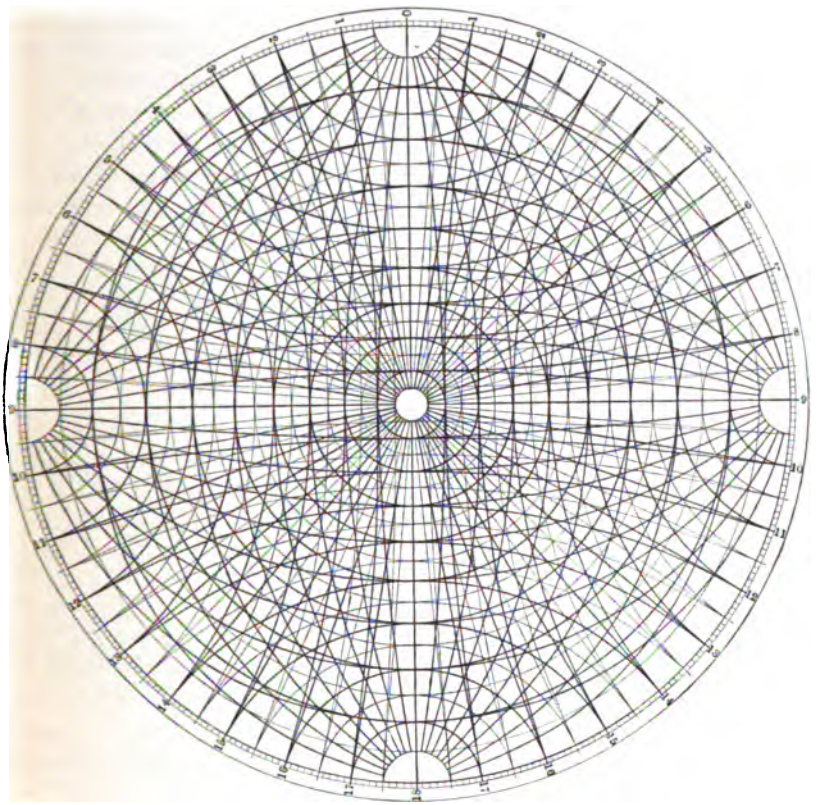

FIG. 30.—The von Fedorow net for stereographic projection. 1/2 size of original.

[1] E. von Fedorow: *Universal-(Theodolith-)Methode in der Mineralogie und Petrographie. I. Universalgeometrische Untersuchungen.* Zeitschr. f. Kryst., XXI (1893), 574–714.

Idem: *Universal-(Theodolith-)Methode in der Mineralogie und Petrographie. II. Krystalloptische Untersuchungen.* Ibidem, XXII (1894), 229–268.

Idem: *Universalmethode und Feldspathstudien. I. Methodische Verfahren.* Ibidem, XXVI (1896), 225–262.

Idem: *Universalmethode und Feldspathstudien. II. Feldspathbestimmungen.* Ibidem, XXVII (1897), 337–398.

Idem: *Universalmethode und Feldspathstudien. III. Die Feldspäthe des Bogoslowskschen Bergreviers.* Ibidem, XXIX (1898), 604–658.

Idem: *Universalgoniometer mit mehr als zwei Drehaxen und genaue graphische Rechnung.* Ibidem, XXXII (1899), 468–478.

Idem: *Zur Theorie der Krystallographischen Proejctionen.* Ibidem., XXXIII (1900), 589–598.

ruler is always the arc of a circle. This instrument has since been improved[1] and now possesses a scale from which one can read directly the curvature of the arc (*s*, Fig. 29).

In his paper in 1897, von Fedorow published a stereographic net which greatly simplifies both drawing and computation. It is printed in pale blue or gray ink on tracing paper, and shows divisions, 5° apart, of two sets of stereographically projected great circles and vertical small circles at right angles to each other, one series of horizontal small circles, and one of vertical great circles, also 5° apart. The original net is 20 cm. in diameter and is shown, half size, in Fig. 30. It is used by placing it over the drawing and pricking through to locate desired points, or by rotating it to read angles.

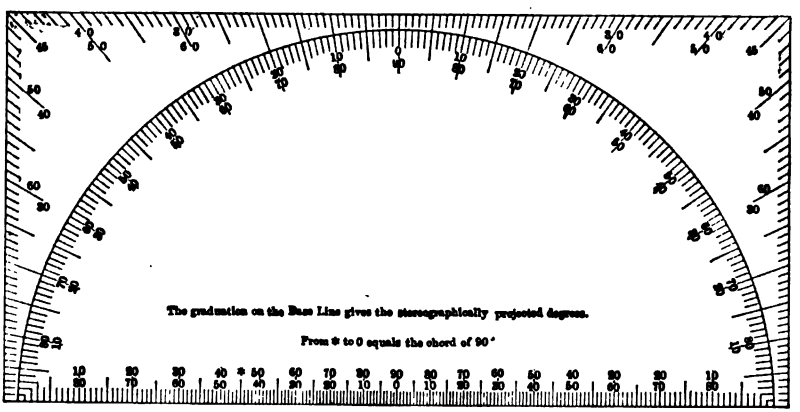

FIG. 31.—Protractor used by Penfield.

Further directions for its use will be given below. A similar net was used by Michel-Lévy[2] in 1894 and later.

In 1901 appeared the first of a series of articles by Penfield[3] on stereographic projection; a series which has done more than any other publication in English to bring the method before mineralogists and petrologists. In

[1] E. von Fedorow: *Ueber die Anwendung des Dreispitzzirkels für krystallographische Zwecke.* Zeitschr. f. Kryst., XXXVII (1902–3), 138–142.

[2] A. Michel-Lévy: *Étude sur la détermination des feldspaths dans les plaques minces.* Paris, 1894.

Idem: *Ibidem.* Deuxième fascicule, 1896.

Idem: *Ibidem.* Troisème fascicule, 1904.

[3] Samuel L. Penfield: *The stereographic projection and its possibilities from a graphical standpoint.* Amer. Jour. Sci., XI (1901), 1–24, 115–144.

Idem: *On the use of the stereographic projection for geographical maps and sailing charts.* Ibidem, XIII (1902), 245–276, 347–376.

Idem: *On the solution of problems in crystallography by means of graphical methods based upon spherical and plane trigonometry.* Ibidem, XIV (1902), 249–284.

his first paper, Penfield[1] gave rather an elaborate discussion of principles and methods, and described a series of celluloid and paper protractors. His instruments and scales include (1) a protractor (Fig. 31) whose circle, 14 cm. in diameter, is divided into degrees, and whose base shows the stereographically projected positions of these points, (2) a protractor for measuring the arcs of great circles, and consisting of a series of vertical small circles 1° apart, (3) a

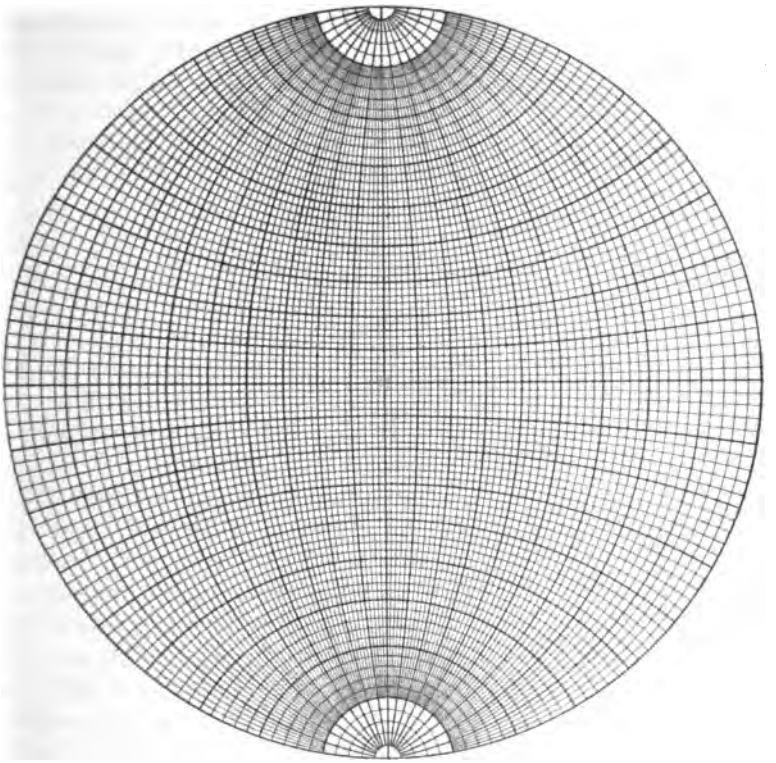

FIG. 32.—The Wulff stereographic net. 1/2 size of original.

protractor giving great circles and vertical small circles 5° apart, and (4) a protractor giving great circles 2° apart. Several of these protractors might well be combined into one, as is done is the Fedorow net.

Besides these protractors, Penfield made a series of very useful scales. One gives the radii of stereographically projected arcs of great circles, and is used to determine the centers of these circles in the projection without determining them graphically. Another gives the radii of stereographically projected arcs of vertical small circles, and a third gives the stereographic projections of the intersections of a vertical great circle with parallels, up to

[1] Samuel L. Penfield: *Op. cit.*, XI (1901), 138. Penfield's protractors and scales are for sale by E. L. Washburn & Co., New Haven, Conn.

and including 156° from the north. The latter is a continuation of the base line of Fig. 31. A curved ruler, with the curved strip made of wood, thus differing from the von Fedorow model mentioned above, is also described.

In 1902, Wulff,[1] in a very important paper on the optical properties of isomorphous crystals, published the lithograph net (Fig. 32) which is most commonly used at the present time. Like von Fedorow's, it is 20 cm. in diameter, but it is printed on heavy paper, and over it is laid the tracing paper upon which the drawing is to be made. It shows the stereographic projections of great circles and of vertical small circles, 2° apart.

Hutchinson,[2] in 1908, prepared a protractor and a net like the one just

FIG. 33.—The Hutchinson stereographic protractor.

described except that it has a diameter of only 5 in. (12.6 cm.). For the use of students, and where extreme accuracy is not required, the 5-in. circle is much more convenient than the larger one, since it can be constructed on a sheet of paper of moderate size and most of its circles can be drawn with an ordinary pair of compasses. The protractor (Fig. 33) is adapted to the 5-in. circle.[3] It is made of boxwood, is 2.5 in. in width, and about 12 in. in length. The intersection of the edge of the protractor with the zero line, which extends across it about 2 5/8 in. from one end, forms the center of the circle. The divisions toward the shorter end represent the stereographic projections of every second degree. The longer end is divided into degrees like an ordinary rectangular protractor and may be used for setting off angles. By multiplying these divisions by two it likewise gives the stereographic projections of points lying below the equator. For the sake of clearness, the finer divisions have been omitted from both ends in the figure.

Johannsen,[4] in 1911, constructed a drawing-board which greatly simpli-

[1] Georg Wulff: *Untersuchungen im Gebiete der optischen Eigenschaften isomorpher Krystalle.* Zeitschr. f. Kryst., XXXVI (1902), 1–28.

[2] A. Hutchinson: *On a protractor for use in constructing stereographic and gnomonic projections of the sphere.* Mineralog. Mag., XV (1908), 93–112.

[3] In a letter to the author, Doctor Hutchinson states that protractors may now be had of the following radii: 10 cm. for use with the Wulff net, 7 cm. for use with Penfield's circles, 5 cm., 2 1/2 in. as stated above, and 1 1/2 in. for use in note-books. The protractors, graduated on boxwood, are manufactured by W. H. Harling, 47 Finsbury Pavement, London.

[4] Albert Johannsen: *A drawing-board with revolving disk for stereographic projection.* Jour. Geol.. XIX (1911), 752–755.

fies the operation of rotating a stereographic net. In it is combined, on a single dial, arcs covering all vertical and horizontal great and small circles (Fig. 34). Ordinarily, in stereographic nets, it is necessary to rotate the drawing above the net, and great care is necessary to keep the two accurately adjusted. In this protractor no centering is necessary, the net being accurately centered on a revolving disk. The base, which is a drawing-board, 33 by 43 cm. in size, carries a net 20.cm. in diameter. The latter is composed of two semi-nets, one half being a Wulff net, the other half drawn to show horizontal and vertical small circles. The figure shows divisions only to 10° although both halves of the actual net are divided to 2°. This drawing-board is inexpensive and is adapted to students' use. A sheet of tracing paper is laid above the net, and is fastened to the board by means of thumb-tacks.

Points are located, and angles and distances are measured by rotating the disk, curves being sketched free-hand where needed. Drawings made on semi-opaque tracing paper or cloth will readily reproduce by photo-engraving.

A similar drawing-board, constructed of pasteboard, was described later by Noll.[1]

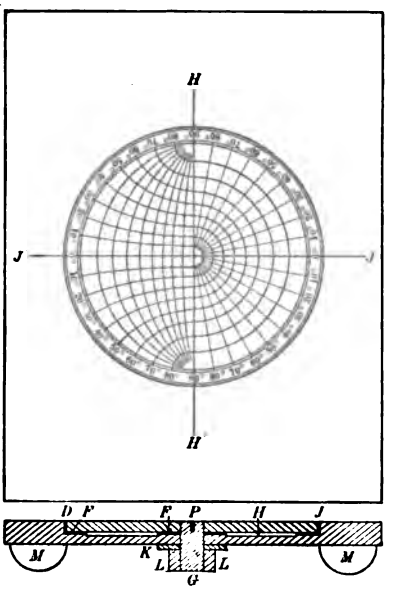

FIG. 34.—The Johannsen drawing-board for stereographic projection.

14. Calculating the Location of Points in Stereographic Projection.—If it is intended to make accurate drawings for reproduction, it is not sufficient to sketch the desired curves free-hand, but one must locate the centers of the circles in the projection. For a circle of small radius, the point may be determined by the methods shown in Figs. 20 and 21; for long radii a curved ruler may be used or the distance to the center may be computed. A scale giving the computed values for most of the projected circles may easily be constructed on cardboard. The process of computing the points is simple since a stereographically projected degree point, such as *b*, Fig. 35, is located at a distance (*Ob*) from the center equal to the radius multiplied by the natural tangent of half the angle measured on the arc (*ND*).

For example: let it be required to find the center of the projection of a vertical small circle having a radius of 10°. By geometry, the angle on the

[1] F. Noll. *Zeichenblock für stereographische Projektionen.* Centralbl. f. Min., etc., 1902, 380–381.

circumference of a circle includes between its lines an arc twice as great. In Fig. 35, the point b is the stereographic projection of the point D on the circumference, the latter point being 80° from N. Let this point represent the upper edge of the desired small circle on the sphere. The angle $OSb = 40°$; $\tan OSb = \frac{Ob}{OS}$; $Ob = OS \tan OSb$. Ob is the required distance to the inner line of the circle in the projection, OS the radius, and OSb half the angle ND. In the present case $Ob = \tan 40° \times 100$ mm. (the circle of reference being a Wulff net 20 cm. in diameter) $= 0.8391 \times 100 = 83.91$ mm., the required point b. Similarly $aSO = \frac{GON}{2} = 50°$; $aO = 1.1918 \times 100 = 119.18$ mm., the required distance to the outer rim of the circle. Upon a sphere 20 cm. in diameter, therefore, the diameter of a projected circle which includes an arc of 20° of the equator is $aO - bO = 119.18 - 83.91 = 35.27$ mm. The radius $\frac{ab}{2} = 17.635$ mm.

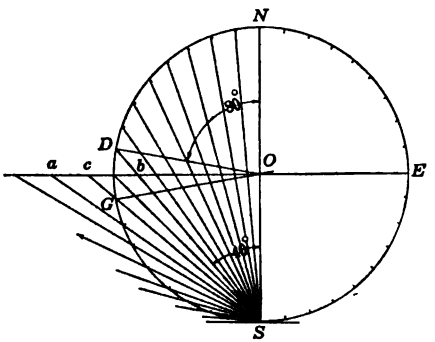

FIG. 35.—Tangent relations in stereographic projection.

I. TABLE GIVING THE CALCULATED POSITIONS OF CENTERS IN THE PROJECTION OF VERTICAL SMALL CIRCLES, MEASURED IN MM. FROM THE POINT WHERE IT CROSSES THE MERIDIAN WITHIN THE SPHERE

Formula: $2x = 100$ (tan 1/2 larger arc − tan 1/2 smaller arc.)

Radius of circle on sphere, degrees	Radius of projected circle, mm.	Radius of circle on sphere, degrees	Radius of projected circle, mm.	Radius of circle on sphere, degrees	Radius of projected circle, mm.
5	8.850	56	148.260	74	348.745
10	17.635	57	153.985	75	373.205
15	26.792	58	160.035	76	401.076
20	36.895	59	166.430	77	433.148
25	46.630	60	173.210	78	470.465
30	57.735	61	180.405	79	514.455
35	70.002	62	188.075	80	567.130
36	72.655	63	196.260	81	631.375
38	78.130	64	205.030	82	711.538
40	83.910	65	214.450	83	814.437
42	90.040	66	224.600	84	951.434
44	96.570	67	235.585	85	1143.007
46	103.555	68	247.510	86	1430.069
48	111.060	69	260.510	87	1908.115
50	119.175	70	274.750	88	2863.627
52	127.995	71	290.425	89	5728.999
54	137.640	72	307.770	90	∞
55	142.815	73	327.085		

For convenience of use with a net 20 cm. in diameter, the radii of vertical small circles at close intervals are computed above. The position of any center is found by laying off the proper distance outward from the point where the required circle cuts the meridian through its center.

II. TABLE OF CENTERS OF GREAT CIRCLES MEASURED FROM THE TRACE OF THE PROJECTED CIRCLES

Formula: $x = 100$ (tan 1/2 one arc + tan 1/2 other arc). Fig. 36, lettered same as Fig. 35.

Angle between pole (P) and N, or tilt of section from equator, degrees	Radius of projected great circles, mm.	Angle between pole (P) and N, degrees	Radius of projected great circles, mm.	Angle between pole (P) and N, degrees	Radius of projected great circles, mm.
5	100.38	56	178.83	74	362.79
10	101.54	57	183.60	75	386.37
15	103.52	58	188.70	76	413.35
20	106.41	59	194.16	77	444.54
25	110.34	60	200.00	78	480.97
30	115.47	61	206.26	79	524.08
35	122.08	62	213.00	80	575.88
36	123.60	63	220.27	81	639.74
38	126.90	64	228.12	82	718.53
40	130.54	65	236.62	83	820.55
42	134.56	66	245.86	84	956.67
44	139.02	67	255.93	85	1147.37
46	142.95	68	266.95	86	1433.56
48	149.20	69	279.04	87	1910.73
50	155.57	70	292.38	88	2865.37
52	162.42	71	307.15	89	5729.87
54	170.13	72	323.61	90	∞
55	174.34	73	342.03		

The following table gives the projected positions, as measured from the center, of degree points in the southern hemisphere. Their projections lie

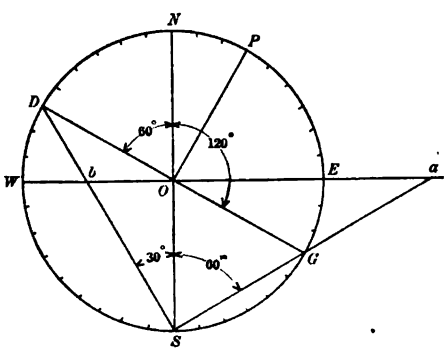

FIG. 36.—Section through sphere, showing a great circle ($D\,G$) tilted 30° from the equator.

beyond the limits of the net. Thus Oa, Fig. 36, is the projected position of a point 30° south of the equator. Formula for a circle 20 cm. in diameter; $x = 100$ times tan 1/2 the arc on circumference, measured from N.

III. TABLE OF DISTANCES IN MM. FROM THE CENTER OF THE SPHERE TO THE PROJECTED POSITIONS OF POINTS LYING BELOW THE EQUATOR

Point located y° from N, degrees	Distance from center of sphere, mm.	Point located y° from N, degrees	Distance from center of sphere, mm.	Point located y° from N, degrees	Distance from center of sphere, mm.
y=90	100.00	150	373.21	166	814.43
95	109.13	151	386.67	167	877.69
100	119.18	152	401.08	168	951.44
105	130.32	153	416.53	169	1,038.54
110	142.81	154	433.15	170	1,143.01
115	156.97	155	451.07	171	1,270.62
120	173.21	156	470.46	172	1,430.07
125	192.10	157	491.52	173	1,634.99
130	214.45	158	514.46	174	1,908.11
135	241.42	159	539.55	175	2,290.38
138	260.51	160	567.13	176	2,863.63
140	274.75	161	597.58	177	3,818.85
142	290.42	162	631.38	178	5,729.00
144	307.77	163	669.12	179	11,458.87
146	327.09	164	711.54	180	∞
148	348.74	165	759.58		

15. Accuracy of Stereographic Projection.—To test the accuracy of measurements made on a stereographically projected map, as compared with distances mathematically computed, Penfield[1] made a number of determinations. Using a circle 14 cm. in diameter, he found that in spherical triangles in which no side was of even degrees, the angles and arcs could be measured with an average error of about 5 minutes, the maximum error in twenty-one measurements being 15 minutes. On a map of the hemispheres, he plotted the locations of New York and New Orleans. The true distance between these two cities, as computed from their latitudes and longitudes, is 16° 52′ or 116 1/2 statute miles. With his protractor he found the distance to be 16° 53′ to 17° 8′, a maximum error of only 16 minutes or about 18 statute miles. The actual size of the map of the United States in the drawing which he used was 1 1/8 by 11/16 in. (28×17 1/2 mm.), and the distance between the two cities about 0.4 in.! Wulff[2] gives his errors of reading as averaging ± 28′ on a circle 20 cm. in diameter.

16. Problems solved by Means of a Stereographic Net.—The use of the Johannsen drawing-board, or any stereographic net, is illustrated best by several problems which are given to show the simplicity of this method, as compared with the solutions previously worked out.

(1) *Given a pole, to find the corresponding great circle.* Taking the problem given in Article 12, Case I, we have a pole located 30° above the horizon and 130° to the left front. For future orientation, upon a sheet of

[1] Samuel L. Penfield: *Op. cit.*, XI (1901), 131.

[2] Georg Wulff: *Op. cit.*, 17–18.

tracing paper, fastened to the drawing-board (not to the disk), draw vertical and horizontal lines through the center. Count, on the circumference, 130° from H to a (Fig 37), and 30°, as measured by horizontal small circles, to p. This is the stereographic projection of the pole. Count 90° from p to b, or 30° from M to b, and 90° from a to d, and from a to d'. Three points, d, b, and d', are now located on the great circle. Rotate the dial until the separation line between the two half nets (hereafter called the HH' line) falls on dMd' and the b point lies over the left half of the net (hereafter called the J' net). Through b sketch a circle following, or interpolated

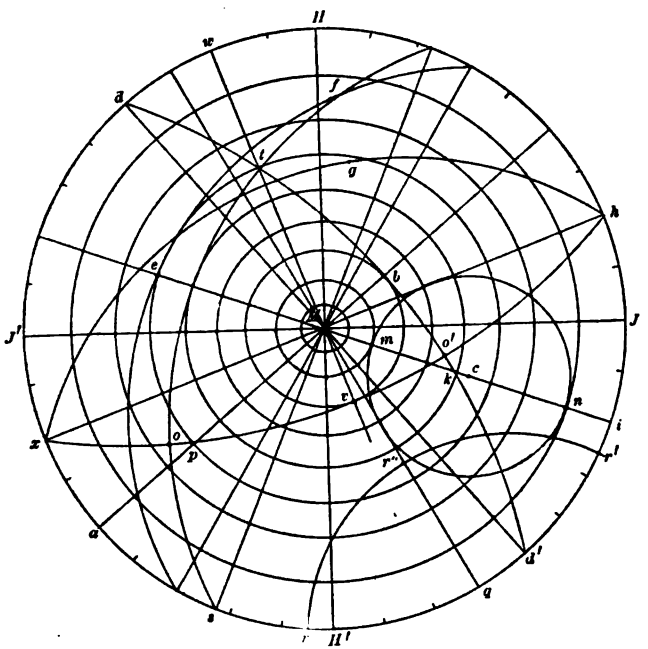

FIG. 37.—Stereographic projections of problems 1 to 9.

between, curves beneath. If a very accurate line is desired, use a curved ruler (Fig. 29), or find the center of the circle from Table II, Article 14, or by construction, and draw with a compass.

(2) *To pass a great circle through two points which fall within the equatorial line.* Let e and f, Fig. 37, be these points. Rotate the disk until a great circle of the J' net passes through, or is equally distant from, the two points. Sketch the curve.

(3) *To pass a great circle through two points, one of which is on the equator.* Let g and h (Fig. 37) be these points. Rotate the disk until the HH' line falls on h and the J' net lies beneath g. Sketch the great circle passing through this point.

(4) *To find the poles of a given great circle.* Rotate the disk until a great circle of the J' net coincides with the given curve. Count 90° from the curve on the JJ' line to locate the pole (curve *dbd'* and point *p*, Fig. 37).

(5) *To draw a small circle, given its size and the location of its center on the sphere.* Let the center (*k*) be located in the same position as in problem 5, Article 12, 40° above the equator and 110° to the right front, and let its radius be 30°. By means of the *J* net count 110° from *H* to *i*, and 40° *i* to *k* (Fig. 37). This is the projection of the pole. The required circle cuts the line *ik* at a distance of 30 stereographically projected degrees in either direction, namely, at *m* and *n*. The center of the projected circle is halfway between the two at *c*.

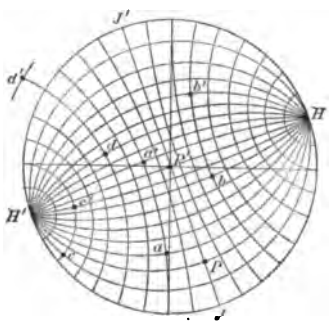

FIG. 38.—Method for rotating the plane of projection.

(6) *To draw a vertical small circle of given size.* Let the circle be one of 35° radius, with its center on a meridian 30° to the right of *H'*. Locate its center at *q* (Fig. 37) and, by means of the *J* net, locate three other points, *r* and *r'* on the equator and *r''* on the meridian, each 35° from *q*. Draw *rr''r'* by means of the *J'* net. Owing to the fact that vertical small circles are cut in half in the Johannsen drawing-board, the continuation of the arc must be located by the degree marks on the equator. If it is desired to sketch the complete circle, the disk may be rotated through 180°.

(7) *To measure a spherical triangle.* Let the triangle be *std'*, Fig. 37. On the *J* net count 90° on a meridian from *t* to *v*, and 90° to the right and left of *w* (the backward extension of the meridian) to *x* and *h* on the equator. By means of the *J'* net, draw a great circle through *hvx*. The part of this circle (*oo'*) cut off by *st* and *td'* may be measured by the vertical small circles of the *J'* net. It gives the value of the angle.

(8) *To measure angular distances between two points.* Use the method given in the latter part of the preceding problem.

(9) *To change the plane of the projection.*[1] For drawing maps in which a particular point is desired in the center, it is necessary change the plane of the projection. Let it be desired to move the point *P* (Fig. 38), which lies 160° to the right front and 20° above the equator, to the center of the projection (*P'*). Rotate the net until the JJ' line, which is the trace of a vertical plane, passes through *P*. In this position the line HH' represents the axis about which the sphere must be rotated to bring the point *P* to the center. During the rotation every other point upon the sphere, such as

[1] Georg Wulff: *Ueber die Vertauschung der Ebene der stereographischen Projection und deren Anwendung.* Zeitschr. f. Kryst., XXI (1892–3), 249–254.

a or *b*, will be turned through the same angle and about the same axis, consequently in vertical planes at right angles to it. Since vertical planes appear in stereographic projection as vertical small circles, it is only necessary, for the rotation of any point, to count along its vertical small circle the same number of degrees, as from *P* to *P'*, for example *a*–*a'*, *b*–*b'*, etc. Should a point lie such a short distance above the equator that the rotation will bring it below, its projection will appear beyond the periphery of the net, as *d* at *d'*.

17. Various Accessories used in Stereographic Projection.—When making drawings upon thick paper, it is not always convenient to transfer points by pricking through a von Fedorow tracing-paper net. For such transfers, a three-point

FIG. 39.—Von Fedorow's three-point compass. 1/2 natural size. (Fuess.)

compass[1] (Fig. 39) is useful. Two of the points are set to marks appearing on both drawing and net, and the third to the one which is to be transferred. A more rigid compass, designed by Hutchinson,[2] is shown in Fig. 40.

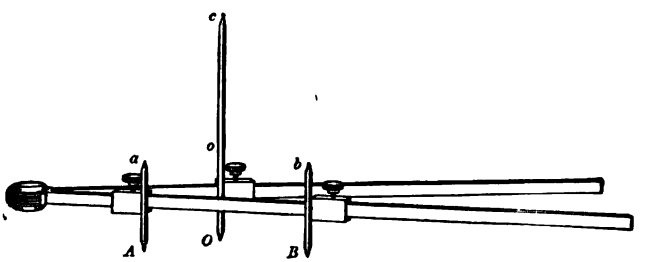

FIG. 40.—Hutchinson's three-point compass.

A simple protractor, valuable as an accessory to the drawing-board described above, may be constructed by students for their own use by drawing upon a sheet of cardboard the stereographically projected degrees beyond the equatorial circle of 20 cm. The values given in Table III, Article 14 may be used. On a scale 30 in. in length, the degrees up to 165 can be plotted. Beyond that, the distances rapidly increase.

For the accurate plotting of spherical triangles, a Nikitin[3] hemisphere (Fig. 41), with movable graduated circles, is valuable, although for teaching purposes, a

[1] E. von Fedorow: *Ueber die Anwendung des Dreispitzzirkels für krystallographische Zwecke.* Zeitschr. f. Kryst., XXXVII (1902–3), 138–142.

[2] A. Hutchinson: *On a protractor for use in constructing stereographic and gnomonic projections of the sphere.* Mineralog. Mag., XV (1908), 93–112.

[3] W. Nikitin: *Halbsphäroid zur graphischen Lösung bei Anwendung der Universalmethode.* Zeitschr. f. Kryst., XLVII (1910), 379–381.

wooden sphere, 12 to 16 in. in diameter and covered with blackboard paint, will answer. In addition, parallels and meridians, spaced 10° apart, should be shown by very narrow, white lines and the intermediate degrees by dots on the equator. A narrow, graduated, brass strip, attached by single screws at the north and south poles and bent to follow the contour of the globe, is an additional help.

Another convenient class-room accessory is a Wülfing[1] wall-chart for stereographic projection. It consists, in its latest form, of a plate of ground-glass overlying a Wulff net 70 cm. in diameter. The net is mounted on pastebcard and pro-

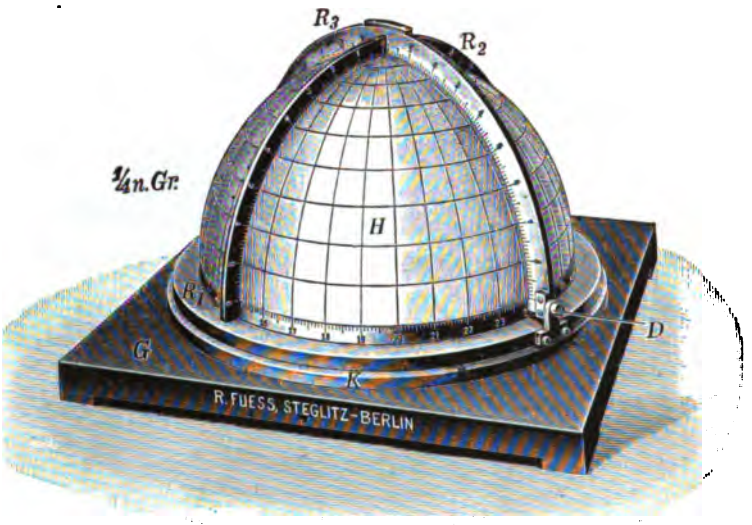

FIG. 41.—Nikitin's porcelain hemisphere for graphical representation of properties of crystals. 1/4 natural size. (Fuess.)

jects beyond the edges of the frame so that it may be rotated conveniently. The ground-glass is hinged at some distance below the net whereby, when tilted forward, crayon marks upon it will show clearly, but the net will not be seen (Fig. 42).

Figs. 43 and 44 represent models of another apparatus constructed by Professor Wülfing[2] and useful in showing first, the relation between a crystal and the projection sphere (Fig. 43), and second, the projection upon the plane of the points thus located on the sphere (Fig. 44). The method of using the model is clear from the illustrations.

[1] E. A. Wülfing: *Wandtafeln für stereographische Projektion.* Centralbl. f. Min., etc., 1911, 273–275.

[2] Idem: *Modell zur Erläuterung der stereographischen Projektion.* Centralbl. f. Min., etc., 1911, 749–752.

PROBLEMS

Let the equatorial plane be the plane of the projection.

(a) By means of a Wulff net or a Johannsen drawing-board, pass a great circle through two points, A and B, one lying on the 70th meridian (left side) and 30° above the equator (outer circle of net), the other on the 120th meridian (left side) and 60° above the equator.

(b) What is the angular distance between these two points?

(c) Measure the angle which the plane passing through these points makes with the equatorial plane.

FIG. 42.

FIG. 43.

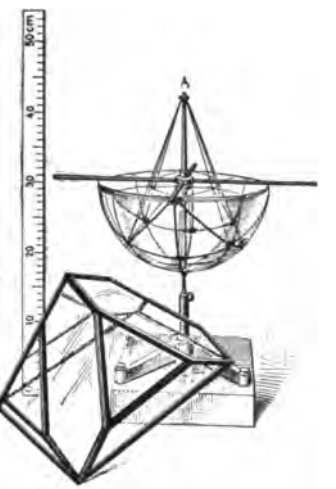

FIG. 44.

FIG. 42.—The Wülfing wall chart for stereographic projection. (Krantz.)

FIGS. 43 AND 44.—Wülfing projection model. (Krantz.)

(d) Measure the angular distance between this plane and the axis forming the center of the projection.

(e) What is the angular distance between the first point and the center of the projection?

(f) Draw another plane passing through the point A and through a point on the 60° meridian (right side) and 40° above the equator.

(g) What angle does this plane make with the equatorial plane?

(h) What angle does the great circle formed by the second plane make with the great circle of the first plane?

(i) Locate the poles of each of the two planes.

(j) What is the angle between the poles (that is, the angle between the two planes)?

REFERENCES

Besides the articles mentioned in the preceding chapter, the books and papers below may be of assistance.

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1871. E. Reusch: *Bezeichnung der Hemiëdrie bei Anwendung der stereographischen Projection.* Pogg. Ann., CXLII (1871), 46–54.

1872. Idem: *Zur Lehre von den Krystallzwillingen.* Pogg. Ann., CXLVII (1872), 569–589.

1881. Idem: *Die stereographische Projektion.* Leipzig, 1881.*

1873. F. A. Quenstedt: *Grundriss der Krystallographie.* Tübingen, 1873.*

1881. Th. Liebisch: *Geometrische Krystallographie*, Leipzig, 1881, 116–134.

1884. A. Brezina: *Methodik der Krystallbestimmung.* Wien, 1884.*

1887. V. Goldschmidt: *Ueber Projection und graphische Krystallberechnung.* Berlin, 1887.*

1893. B. Hecht: *Anleitung zur Krystallberechnung.* Leipzig, 1893.*

1897. E. Gelcich und F. Sauter: *Kartenkunde geschichtlich dargestellt.* Leipzig, 2 Aufl., 1897, 38–54.

1904. Rosenbusch und Wülfing: *Mikroskopische Physiographie.* Stuttgart, 4 Aufl., 1904, I–1.

1905. P. Groth: *Physikalische Krystallographie*, Leipzig, 1905, 314.

1911. H. E. Boeke: *Die Anwendung der stereographischen Projektion bei kristallographischen Untersuchungen.* Berlin, 1911.

CHAPTER III

A FEW PRINCIPLES OF OPTICS

18. The Nature of Light.—Before describing the petrographic microscope and the methods of its use, it will be necessary to discuss briefly a few elementary principles of optics.[1] Using the language of the elastic solid theory for descriptive purposes, without implying it to be an accepted theory of the actual physical facts, we may say that light consists of vibrations of some kind in an all-pervading medium which we call ether. What the exact nature of these vibrations is we do not know, although we do know that they follow the laws of wave-motion. It is quite probable that there actually is a rapid periodic change in the magnetic and electric condition of the ether. This electromagnetic theory, as it is called, is the most recent one, and in it, like in the former generally accepted undulatory theory of Huygens, the periodic oscillations take place at right angles to the direction of transmission of the ray.

19. Corpuscular or Emission Theory.—It was supposed by Newton that light consisted of innumerable small particles sent out with extreme rapidity by all luminous bodies. He thought that these small particles could pass freely through all transparent bodies and into the eye, where they produced the sensation of light by their impact upon the optic nerve. Bennett[2] reasoned that if such great numbers of particles, even though extremely minute, were actually sent out by a luminous body, they should have some effect of deflection upon a suspended body, yet he found that when light was concentrated by mirrors and lenses and was directed against a most delicate balance made of a fragment of straw suspended horizontally from a single spider web, not the slightest motion due to the impact of the light particles appeared.[3]

The shooting forth of light particles, under the corpuscular theory, was compared with the movement of a projectile, and refraction was supposed to be due to forces of attraction or repulsion in the medium into which the particles passed. If the medium were denser than the one from which the light came, the rays were supposed to be more attracted, consequently

[1] See General Bibliography at the end of the chapter.

[2] Rev. A. Bennett: *A new suspension of the magnetic needle, intended for the discovery of minute quantities of magnetic attraction, etc.* Phil. Trans. Roy. Soc., London, Pt. I, 1792, 81–98.

[3] In this connection see the modern measurements of light pressure by Lebedew and by Nichols and Hull which show that light does actually exert a pressure.

an oblique ray would be bent toward the normal. At the same time this greater attraction should have the effect of augmenting the velocity of the particles. It was shown experimentally by Foucault that this is not the case, but that the velocity of light in water is less than it is in air. It has been shown definitely that the velocity of light decreases as the index of refraction increases.

20. The Undulatory or Wave Theory of Huygens.—The Dutch astronomer and physicist Huygens[1] was the first to oppose the emission theory, although it was supported by such men as Laplace, Biot, and Brewster. He suggested that light is due to wave-motion, a theory which fell into disuse but was revived after nearly a century and gradually gained ground, especially through the work of Thomas Young[2] and Augustin Fresnel. Although Huygens had stated that light is due to a vibratory motion in the ether, yet on this theory he was unable to account for the phenomenon of polarization which he had discovered. Thirteen years before, Hooke[3] had defined light as due to quick and extremely short "vibratile" movements, and later[4] suggested that they might be transverse, although he did not prove it. It was not until Young[5] suggested and Fresnel[6] demonstrated that the vibrations actually take place in a direction transverse to the direction of transmission that the theory gained ground. According to the elastic-solid theory, the speed of propagation depends upon the elasticity and density of the medium through which it is passing, consequently the greater the elasticity and the less the density, the greater the velocity.

A more recent theory, although also dependent upon undulatory or wave-motion, is the electromagnetic theory.

21. The Electromagnetic Theory.—Recent researches in regard to electromagnetic waves seem to show, without doubt, that light is due to waves of the same character. This theory, fundamentally purely electrical,

[1] Christian Huygens: *Op. cit.* in General Bibliography at end of chapter.

[2] Thomas Young: *On the theory of light and colours.* Bakerian lecture, read Nov. 12, 1801. Phil. Trans. Roy. Soc., London, XCII (1802), 12–48.

Idem: *An account of some cases of the production of colours, not hitherto desvribed.* Read July 1, 1802. Ibidem, XCII (1802), 387–397.

Idem: *Experiments and calculations relative to physical optics.* Bakerian lecture, Nov. 24, 1803. Ibidem, XCIV (1804), 1–16.

The above three papers reprinted by Henry Crew in *The wave theory of light, Memoirs by Huygens, Young and Fresnel.* New York, (1900), 47–76.

[3] Hooke: *Micrographia,* 1665, 15.*

[4] Idem: *Lecture on Light.**

[5] Thomas Young: Jan. 12, 1817.*

[6] Augustin Fresnel: *Mémoire sur la double réfraction.* Mém. Acad. France, VII (1827) 45–176.

Idem: *Ueber die doppelte Strahlenbrechung.* (Translation of preceding.) Pogg. Ann., (1831), 372–434; 494–560.

was proposed by Maxwell,[1] who supposed that there is an intimate connection between the vibrations constituting light and electricity. He said: "The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws." Reflection and refraction of electromagnetic waves were first discussed by Lorentz,[2] and later by J. J. Thomson,[3] Fitzgerald,[4] Glazebrook,[5] and Lord Rayleigh.[6]

So far as we are concerned, in the explanation of the phenomena of light, it will be sufficient to regard it simply as wave-motion which transmits energy, but not matter, by means of oscillations taking place in the ether at right angles to the direction of propagation.

22. The Ether.—What the ether actually is, or what its properties are, we do not know. It is generally assumed to be a medium which occurs everywhere, filling intermolecular space as well as extending through interstellar regions.

WAVE-MOTION

23. The Movements of Oscillation.—Assuming that light is transmitted by wave-motion, it will be well to consider next what wave-motion is, and how the ether is affected by different waves and different combinations of waves.

If a particle moves in a certain direction from a point of equilibrium, it will move with gradually diminishing velocity until it reaches its position of maximum swing. It will pause there a moment, and then will return with gradually increasing velocity to its position of rest. Since it is moving without friction, it will pass beyond this point of rest with gradually decreasing velocity until it has reached a point, in the opposite direction, equal to its first swing. Here it will pause, will then return with increasing velocity, and so on. The retardation and acceleration of the motion is such as would be seen by viewing, in the plane of its rotation, a particle moving uniformly around a circle. Thus if the particle *a*, Fig. 45, moves uniformly around

[1] J. Clerk Maxwell: *A dynamical theory of the electromagnetic field.* Phil. Trans. Roy. Soc., London, CLV (1865), 459.*

[2] H. A. Lorentz: *Ueber die Theorie der Reflexion und Refraction des Lichtes.* Schlömilch's Zeitschr., XXII (1877), 1–30, 205–219.

[3] J. J. Thomson: *On Maxwell's theory of light.* Phil. Mag., IX (1880), 284–291.

[4] G. F. Fitzgerald: *On the electromagnetic theory of the reflection and refraction of light.* Phil. Trans. Roy. Soc., London for 1880, CLXXI (1881), 691–711.

[5] R. T. Glazebrook: *On some equations connected with the electromagnetic theory of light.* Read 1881. Proc. Cambridge Phil. Soc., IV (1883), 155–167.

[6] Lord Rayleigh: *On the electromagnetic theory of light.* Phil. Mag., XII (1881), 81–101.

the circle, it will take successively the positions b, c, d, e, f, . . . i . . k . . . a, equally distant from each other, and if it is viewed from a point in the plane of the paper, the particle will appear to vibrate along the diameter of the circle. After reaching the point e, the motion will appear backward on the diameter. Since the movement forward and backward occurs at regular intervals of time, it is said to be **periodic.** The circle abc . . $k.p$ is called the **circle of reference,** or the **auxiliary circle.**

The equation of displacement in a circle is as follows:

Let $\alpha = aOd$, $\sin \alpha = \frac{d'd}{Od}$ (Fig. 45).

But Od = radius = r, therefore $d'd$ (the displacement) = $r \sin \alpha$. Also let t = the time required for the particle to move one division on the circle.

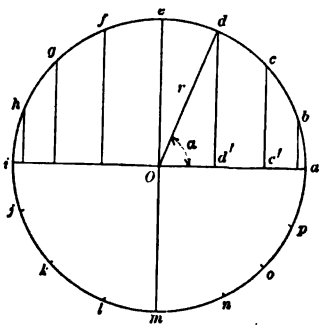


FIG. 45.—Movement of a particle around a circle.

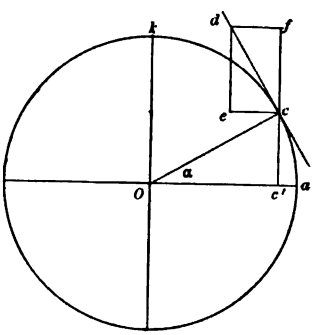


FIG. 46.—Velocity of a particle around a circle.

ω = the angle through which this particle moves in a unit of time. Then since $\sin \alpha = \frac{d'd}{Od}$, $d'd = r \sin \alpha$, and the arc $ad = \omega t$ = the value of the angle α, we have:

The displacement ($d'd$) = the amplitude ($Oa = r$) times the sine of ωt, or:

$$d = r \sin \omega t \qquad (1)$$

When ωt = 0° or 180°, the sin ωt = 0 and the displacement = 0.

When ωt = 90° or 270°, the sin ωt = 1 and the displacement = a.

The *equation of the velocity of any point in a circle* is derived as follows:

Let the particle be at c (Fig. 46) on the auxiliary circle, and let cd, tangent at c, represent the velocity. Resolve cd into two components cf and ce, parallel and at right angles to Ok. The two right-angled triangles $c'cO$ and ecd are similar, for $c'cO + Oce = 90°$ and $ecd + Oce = 90°$, therefore $c'cO = ecd$ and therefore, also, the other angles are equal, and $edc = \alpha$, $\cos \alpha = \frac{ed}{cd}$, and $ed = cd \cos \alpha$.

Substituting v = velocity at c, and v' = velocity projected on Ok, we have

$$v' = v \cos \alpha \qquad (2)$$

Since v is a constant, the velocity at any point on the diameter Ok is proportional to the cosine of the corresponding arc. Also (Fig. 48) the velocity at any

point, as B',C',D', etc., is proportional to the corresponding distance $A'B'$, $A'C'$ $A'D'$, etc.

When $\alpha = 0°$ or $180°$, $v' = v$. It is the maximum value.

When $\alpha = 90°$ or $270°$, the velocity equals zero.

Comparing (1) and (2) we see that when the displacement (1) is at its maximum, the speed (2) is zero, and *vice versa*.

24. Simple Harmonic Motion.—Simple harmonic motion is the name given to such motion as that which apparently takes place backward and forward along the diameter of a circle (*me*, Fig. 45) when looking in the plane of the circle at a steady motion around its periphery. Such motion is periodic, for it repeats itself at regular intervals. The distance from the position of rest of the particle to the limit of its movement is called the **amplitude** (*Oe*). The **period** is the interval of time which elapses between two successive passages of a particle through a certain point in a certain direction. In Fig. 45 the period is *O* to *e* to *m* to *O*. The **phase** is the fraction of a period which has elapsed since the particle last passed through the position of rest. When it is farthest from *O* on the positive side, it is said to be in its position of **maximum positive elongation**; when farthest from *O* on the negative side, of **maximum negative elongation.**

25. Isochronism and Angular Velocity.—When a particle moves in a circular path, its velocity of rotation may be measured by the distance traveled divided by the time, or it may be measured by the angle through which a particle at unit distance passes in a unit of time. The latter measurement is called the **angular velocity** and is indicated by ω.

Let T = the time of a complete oscillation.

2π = the circumference of a circle having a radius of unity.

$$\text{Then } \omega = \frac{2\pi}{T}. \tag{3}$$

This is the equation for the angular velocity.

If α is constant, T also must be constant. That is, in simple harmonic motion, the period is independent of the amplitude. In other words, the particle will perform its oscillations in equal periods of time irrespective of its amplitude. It will vibrate *isochronously*.

The angular velocity may be expressed in another way. The velocity of any other particle on the same radius, but at a distance of r', will be $r'\omega$. That is, $v = r'\omega$, $\omega = \frac{v}{r'}$. Expressed in words—the angular velocity of a particle is its velocity divided by its distance from the center.

We have further; at the end of any period of time t the particle c will have moved through the angle α (Fig. 47), therefore

$$\alpha = \frac{2\pi t}{T}. \tag{4}$$

Draw sc and cc' parallel to Oa and Ok, whereby

$$\sin \alpha = \frac{cc'}{r} = \frac{d}{r} \tag{5}$$

and

$$d = r \sin \alpha, \tag{6}$$

in which d is the distance through which the particle has moved from the position of rest.

Substitute the value of α from (4) in (6),

$$d = r \sin \left(\frac{2\pi}{T} t.\right) \tag{7}$$

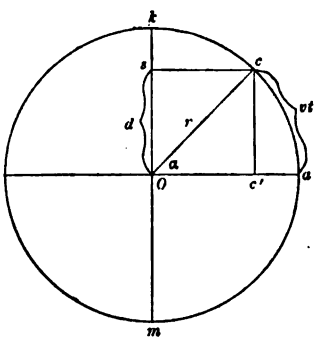


FIG. 47.—Angular velocity.

This equation gives the position of any point, whose displacement is d, in terms of the amplitude (r), the time of one period (T), and the time since the beginning of the movement (t).

If a certain interval of time (t_1) had already elapsed before the particle was set in motion, the remaining time equals $t-t_1$, and the equation becomes

$$d = r \sin \left[\frac{2\pi}{T}(t-t_1)\right]. \tag{8}$$

We also have

$$\cos \alpha = \frac{Oc'}{r}, \quad Oc' = r \cos \alpha = r \cos \left[\frac{2\pi}{T} t\right]. \tag{9}$$

But the cosine of an angle is equal to the sine of 90° plus the angle, therefore

$$Oc' = r \sin \left(\frac{\pi}{2} + \frac{2\pi}{T} t\right) = r \sin \frac{2\pi}{T} \left(\frac{T}{4} + t\right). \tag{10}$$

Equations (8) and (9) are the equations for the lateral displacement of the particle.

26. Harmonic Curves.—A particle may be subjected to two or more movements at the same time. If a particle, moving in simple harmonic motion along a line, be also subjected to a uniform motion of translation in a direction perpendicular to that line, the resultant curve is called the **harmonic curve.**

Let G and S, in the circle of reference (Fig. 48), be the positions of maximum elongation of a particle moving along the diameter. Let the distances AB, BC, CD, etc., on the circle, be equal. Draw horizontal lines through each of these points; the distances between them will thus represent the spaces passed over on the diameter in equal periods of time. Draw also a series of vertical lines equally spaced and at right angles to the first lines, representing the distances laterally passed over in equal time intervals. Suppose a particle to start at A'. At the end of the first instant of time the effort of the

simple harmonic motion would be to move the particle to B'. At the same time, the motion of translation would tend to move it to b. The resultant of the two movements will be to move it to b'. At the end of the second interval of time the resultant of the two forces drawing it to C and c will move it to c', at the end of the third interval to d', and so on to g' where it has reached its point of minimum velocity and maximum displacement. Beyond this point the velocity increases and the displacement decreases in the negative direction until the particle has reached the point s', the position of maximum negative elongation. The next movement is again a retrograde movement and the particle passes to the position of rest at y'. It has now com-

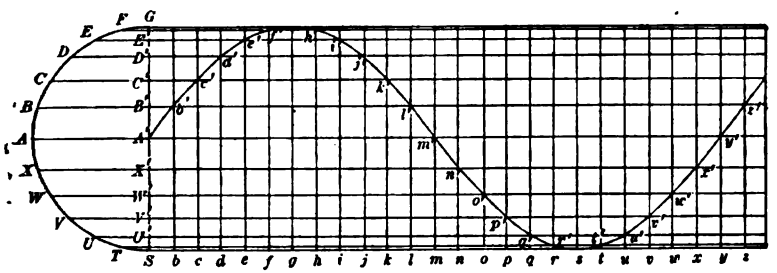


FIG. 48.—Harmonic curve.

pleted one cycle and has moved forward an equal number of spaces. If a line be drawn through the successive points, the resulting curve is a harmonic curve. The distance from a to y is called a **wave length**; the distance from A to G is the **amplitude.**

The equation of the harmonic curve may be obtained by combining the equations of simple harmonic motion and of uniform rectilinear motion. We found above[1] that

$$d = r \sin \omega t, \tag{7a}$$

where d is the ordinate or displacement.

Let l be the lateral displacement (Fig. 48), and v the velocity, then

$$l = vt. \tag{8a}$$

Solving for t and substituting in (1), we have $d = r \sin \omega \frac{l}{v}$.

But $\omega = \frac{2\pi}{T}$,[2] therefore

$$d = r \sin \frac{2\pi l}{vT}. \tag{9a}$$

Since T = time of a complete oscillation and v = the velocity, the abscissa of one wave length ($a'y'$) will be vT. Let this value be represented by λ,

$$vT = \lambda. \tag{10a}$$

Substituting in (9a), we have

[1] Eq. 1, Art. 23.

[2] Eq. 3, Art. 25.

$$d = r \sin \frac{2\pi l}{\lambda}. \tag{11}$$

If l is the distance along the abscissa $A'y'$, we have

When $l = 0$, $\frac{2\pi l}{\lambda} = 0$, $\sin \frac{2\pi l}{\lambda} = 0$, and $d = 0$.

When $l = \frac{\lambda}{4}$, $\sin \frac{2\pi l}{\lambda} = \sin 90° = 1$, and $d = r$.

When $l = \frac{\lambda}{2}$, $\sin \frac{2\pi l}{\lambda} = \sin \pi = \sin 180° = 0$, and $d = 0$.

When $l = \frac{3\lambda}{4}$, $\sin \frac{2\pi l}{\lambda} = \sin 270° = -1$, and $d = -r$.

When $l = \lambda$, $\sin \frac{2\pi l}{\lambda} = \sin 360° = 0$, and $d = 0$.

From these equations it may be seen that at the beginning of a wave, when there is no movement at right angles to the simple harmonic motion, the displacement of the particle is equal to zero. With an abscissa of one-fourth of a wave length, the particle has a displacement equal to the amplitude, and it is, consequently, at its maximum in a positive direction (Fig. 48). With an abscissa of one-half wave length, the displacement again equals zero. At three-fourths wave length it is again equal to the amplitude, but in the negative direction, and the movement has reached its maximum in the opposite direction. At the end of a complete wave length the particle has again reached the position of rest.

The equation for velocity at any moment in the harmonic curve may be obtained by combining the speed equation of simple harmonic motion (2) with the equation of uniform rectilinear motion.

Substituting values from equations (4), (8a), and (10a) in (2) we have:

$$v' = v \cos \alpha = v \cos \frac{2\pi t}{T} = v \cos \frac{2\pi l}{vT} = v \cos \frac{2\pi l}{\lambda}. \tag{12}$$

When $l = 0$, $\frac{2\pi l}{\lambda} = 0$, $\cos \frac{2\pi l}{\lambda} = 1$, $v' = v$.

When $l = \frac{\lambda}{4}$, $\frac{2\pi l}{\lambda} = 90°$, $\cos \frac{2\pi l}{\lambda} = 0$, $v' = 0$.

When $l = \frac{\lambda}{2}$, $\frac{2\pi l}{\lambda} = 180°$, $\cos \frac{2\pi l}{\lambda} = -1$, $v' = -v$.

When $l = \frac{3\lambda}{4}$, $\frac{2\pi l}{\lambda} = 270°$, $\cos \frac{2\pi l}{\lambda} = 0$, $v' = 0$.

When $l = \lambda$, $\frac{2\pi l}{\lambda} = 360°$, $\cos \frac{2\pi l}{\lambda} = 1$, $v' = v$.

That is, the speed of the particle at the beginning of the wave is at its maximum and is equal to the velocity on the circumference. At one-fourth wave length it is zero. It is again at its maximum, but in the negative direction, at one-half

wave length, zero at three-fourths wave length, and at its maximum in the positive direction at the completion of the wave. A curve (Fig. 49) constructed with these values is exactly like the harmonic curve, differing from it only in position by one-fourth wave length. If the maximum value at the circumference is represented by $r=v$, and the curves are shifted one-fourth wave length, it will be found that the two coincide exactly. From their form, these curves are known as sine curves.

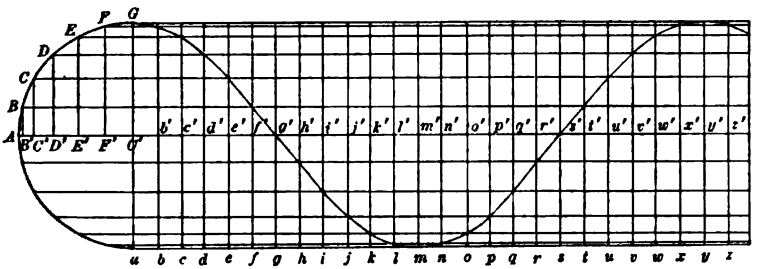


FIG. 49.—The velocity curve of the combination of simple harmonic motion and uniform rectilinear motion.

27. Combinations of Simple Harmonic Motions.—It was mentioned above that a particle might be subjected to two or more motions at the same time. If two such motions are simple harmonic motions, the resulting curve may be constructed graphically, or it may be calculated by combining the equations of each.

1. *Two simple harmonic motions, equal, along the same line, and in the same direction.* Let OA (Fig. 50) be the amplitude of a simple harmonic motion. The first movement would send the particle from O to F'. The first movement of a second simple harmonic motion of equal amplitude, acting along the same line and in the same direction, would send it the same distance, the two together, therefore, sending it to F''. ($OF''=OF'+OF'$.) The second movement of the first simple harmonic motion would move it a distance equal to $F'E'$, and the second movement of the second motion would send it a like distance. But the particle was already at F'', consequently the second movement of both simple harmonic motions would move the particle a distance of $2F'E'$ beyond F'', or to E'', and so on until the particle finally reaches a position at A'', distant $2OA$ from O. The resulting motion, therefore, will be itself a simple harmonic motion but with an amplitude of twice that of each of the original motions.

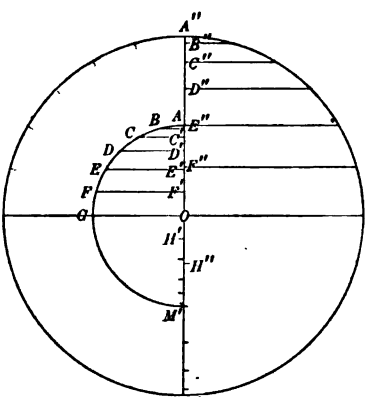


FIG. 50.—Combination of two simple harmonic motions along the same line.

II. *Two simple harmonic motions, equal, along the same line, but in opposite*

directions. The first movement of the first simple harmonic motion would tend to move the particle to F' (Fig. 50), the first movement of the second simple harmonic motion, to H'. Since these motions are equal and in opposite phase, the resultant will be the algebraic sum of the two, or zero. As these two movements neutralize each other, so will every other movement, and the final result will be absolute rest. The particle will remain at O.

III. *Two simple harmonic motions, equal but at right angles to each other, and moving in the same phase.* Let YY' and XX' (Fig. 51) be two simple harmonic motions at right angles to each other, and let $A'O'B'$ and $B'O''A$ be halves of their circles of reference. Let their motions be equal and in the same phase, that is, let both be either positive or negative. At the end of the first interval of time, the particle O, influenced only by the YY' movement, would have moved to b_1,

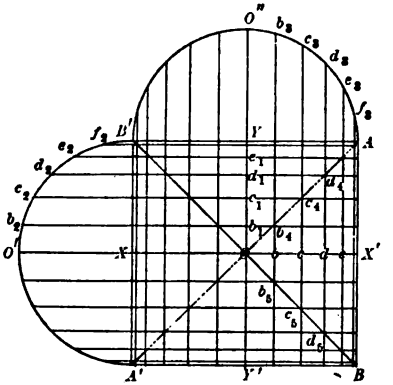


FIG. 51.—Two equal, simple harmonic motions acting at right angles to each other.

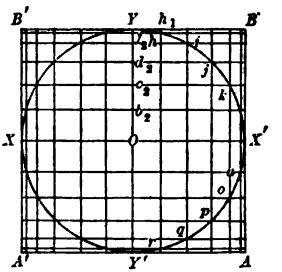


FIG. 52.—Two simple harmonic motions, equal, at right angles, and differing in phase by one-fourth of a period.

while if influenced only by the XX' movement it would have moved to b. The actual position of the particle will therefore be at b_4, at the end of the diagonal of a parallelogram of forces whose sides are Ob_1 and Ob. At the end of the second interval of time, the particle will be at c_4, at the end of the third at d_4, and so on, until it reaches A when it will return to O, and then move on to A', and so continue to oscillate between A and A' in a direction at 45° to YY' and XX'. Any projection of simple harmonic motion being simple harmonic motion, the resulting vibration along AA' is also simple harmonic motion.

IV. *Two simple harmonic motions, equal but at right angles to each other, and moving in opposite phases.* Let the vibration along YY' be in the negative direction and the vibration along XX' in the positive (Fig. 51). The resultant of the first motion of the two simple harmonic motions will be to move the particle to b_5, then to c_5, and so on to B when the particle will return and continue to oscillate between B and B' in a direction at 45° to YY' and XX' and at 90° to AA'.

V. *Two simple harmonic motions, equal, and at right angles to each other, but differing in phase by one-fourth of a period.* In Fig. 52 let the particle already have been moved by the YY' simple harmonic motion from O to Y when the OX' compo-

nent begins to act. That is, the OX' component is one-fourth of a period behind the other. Beginning then at Y, the first motion of YY' would tend to move the particle to f_2 while the horizontal movement would tend to move it to h_1, the resultant being a movement to h. The second motion will move the particle, in a like manner, to i, the third to j, and so on; the resultant being a uniform movement in a circle in the Y–h–i–j–k–X' direction. This clock-wise direction is called negative.

VI. *Two simple harmonic motions, equal, at right angles to each other, but differing in phase by three-fourths of a period.* Let the YY' component already have made oscillations from O to Y to O to Y' when the OX' component starts. The particle will move (Fig. 52), as a result of the two motions, along r–q–p–o–n–X'– – –Y, etc., in a counter clock-wise or positive direction.

VII. *Two simple harmonic motions, equal, at right angles to each other, and differing in phase by less than one-half a period but by some other fraction than one-fourth.* The amplitudes being equal, the circles of reference (Fig. 53) are equal.

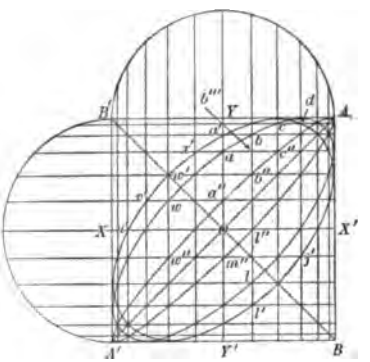


FIG. 53.—Two simple harmonic motions, equal, at right angles to each other, and differing in phase by less than one-half a period but by some other fraction than one-fourth.

(a) Let the YY' component be one-eighth of a period ahead of the XX' component. The particle will, consequently, be at a (3/24 of a period on YY') when the XX' motion begins. The first impulse along YY' would move the particle to a' while the first XX' movement would move it to b''', the resultant being a movement to b. The next impulse will move the particle to c, the third to d, and so on, with a resulting curve which is an ellipse. As the difference in phase between the two components becomes greater, the ellipses become broader (ellipse $j'l'$ . . . . . v', etc.) and finally reach the circle as a limiting value when the phase difference equals one-fourth of a period. As the difference in phase becomes less, the ellipses become narrower (ellipse $a''b''c''$ . . . . $l''m''$, etc.) and reach the limiting value of a straight line when the phase difference equals zero.

(b) If the difference in phase is between one-fourth and one-half of a period, the motion is negative, but the ellipse has BB' for its long diameter instead of AA'.

VIII. *Two simple harmonic motions, equal, at right angles to each other but differing in phase by some fraction of a period other than three-fourths, between one-half and a full period.* In this case the motion will be in the positive direction as in Case VI. (a) With a difference of phase between one-half and three-fourths of a period, the ellipse will be elongated on the BB' line; (b) with a difference between three-fourths and a whole period, along the AA' line.

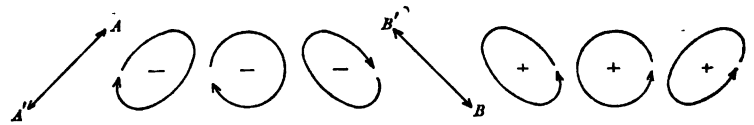
Eight combinations of two simple harmonic motions at right angles to each other have thus been considered.

1. The difference of phase is zero (Case III). Movement takes place in the straight line AA' (Fig. 54).

2. The difference of phase is less than one-fourth of a period (Case VII a). The movement is negative (−) around an ellipse elongated on AA' (Fig. 55).

3. The difference of phase is one-fourth of a period (Case V). The movement is negative (−) around a circle (Fig. 56).

4. The difference of phase is greater than one-fourth and less than one-half of a period (Case VII b). The movement is negative (−) around an ellipse elongated on BB' (Fig. 57).



FIGS. 54 TO 61.—Directions of movement in combinations of two simple harmonic motions.

5. The difference of phase is one-half a period (Case IV). The movement is in the straight line BB' (Fig. 58).

6. The difference of phase is greater than one-half and less than three-fourths of a period (Case VIII a). The movement is positive (+) around an ellipse elongated on BB' (Fig. 59).

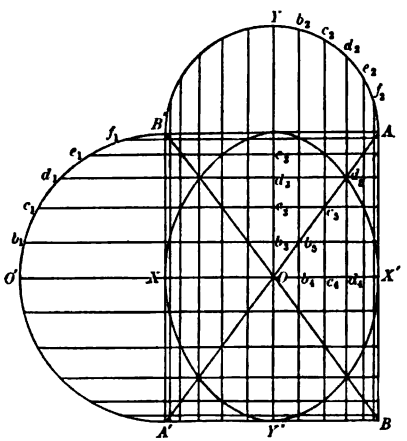


FIG. 62.—Two simple harmonic motions at right angles to each other and unequal in amplitude.

7. The difference of phase is three-fourths of a period (Case VI). The movement is positive (+) around a circle (Fig. 60).

8. The difference of phase is greater than three-fourths but less than a whole period (Case VIII b). The movement is positive (+) around an ellipse elongated on AA' (Fig. 61).

When the difference of phase is unity, the effect is the same as in No. 1. Of course if the XX' movement is in advance of the YY', the motions will be reversed.

From this summary it is clearly evident that compounding two equal simple harmonic motions at right angles to each other will produce elliptical motion in every case, limiting values being the straight line when the phasal difference is zero or one-half of a period, and the circle when the phasal difference is one-fourth or three-fourths of a period.

IX. *Two simple harmonic motions at right angles to each other, unequal in amplitude but in the same phase* (Cf. Case III). If the amplitudes are unequal the auxiliary circles will be of different size (Fig. 62). Let the two movements be positive. Being in the same phase, the first impulse of the YY' movement, acting alone, would move the particle from O to b_3, and the first impulse of the XX' movement, acting alone, move it to b_4. The resultant of the two movements would send

it to b_5. The resultant of all the impulses will be to move the particle to A, and it will oscillate between A and A' in a straight line.

X. *Two simple harmonic motions at right angles to each other, unequal in amplitude, and in opposite phase.* If the movements are in opposite phase, that is, if they differ by half a period, the particle will oscillate between B and B' (Fig. 62. Cf. Case IV).

Since the amplitudes are unequal in Cases IX and X, the movements along AA' and BB' will not be at right angles nor at 45° to XX' and YY'.

XI. *Two simple harmonic motions at right angles to each other, of different amplitudes, and differing in phase by one-fourth of a period* (Cf. Case V). The movement is in the negative direction as in Case V, but here, since the amplitudes of the two motions are different, the curve is an ellipse instead of a circle (Fig. 62).

XII. *Two simple harmonic motions at right angles to each other, of different amplitudes, and differing in phase by three-fourths of a period* (Cf. Case VI). The movement is in the positive direction around an ellipse.

XIII. *Two simple harmonic motions at right angles to each other, of different amplitudes, and differing in phase by some other fraction of a period than one-fourth but less than one-half of a period* (Cf. Case VII). The movement will take place in the negative direction around an ellipse.

XIV. *Two simple harmonic motions at right angles to each other, of different amplitudes, and differing in phase by some fraction of a period other than three-fourths, and between one-half and a full period* (Cf. Case VIII). The movement will take place around an ellipse in the positive direction.

28. Combinations of Harmonic Curves.—We have already seen that a simple harmonic motion may be combined with a uniform rectilinear motion to give a harmonic curve (Fig. 48). Two harmonic curves in the same plane may likewise be combined, and the resultant will be a different harmonic curve in the same plane.

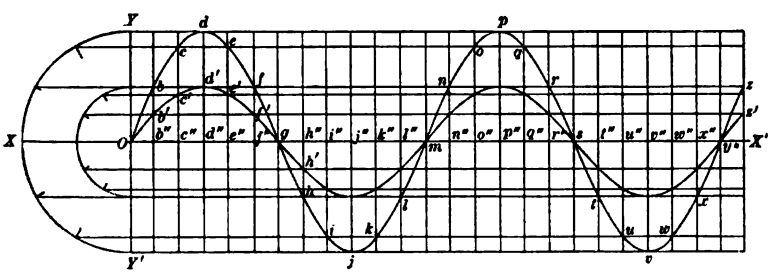


FIG. 63.—Combination of two harmonic curves having the same amplitudes and wave lengths and acting in the same phase.

I. *Two harmonic curves having the same amplitudes and wave lengths, and in the same phase.* Let $Ob'c'd' \ldots h' \ldots$ etc. (Fig. 63) be a harmonic curve. Let another harmonic curve, having the same amplitude and wave length, and acting in the same phase, also pass through O. It likewise will occupy the position $Ob'c'd' \ldots h' \ldots$ etc. If the two motions act together, the resultant will be a

harmonic curve having the same wave length but an amplitude which at any point is the algebraic sum of the two displacements at that point. Thus $b''b'+b''b'=b''b$, $c''c'+c''c'=c''c$, etc.

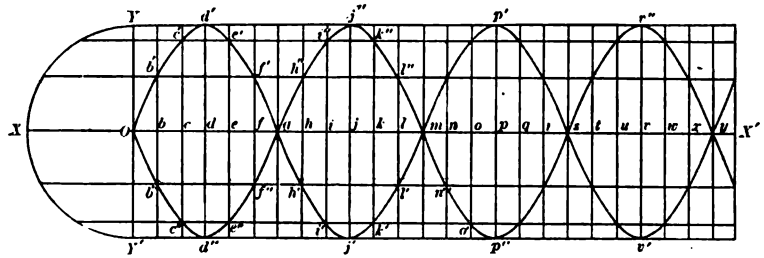


FIG. 64.—Combination of two harmonic curves of the same amplitudes and wave lengths but in opposite phases.

II. *Two harmonic curves having the same amplitudes and wave lengths but in opposite phase.* Two harmonic curves having the same amplitudes and wave

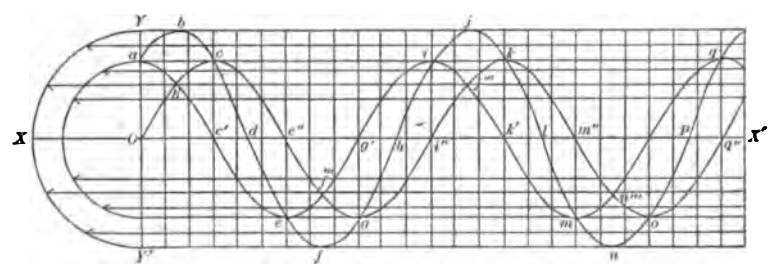


FIG. 65.—Combination of two harmonic curves of the same amplitudes and wave lengths but acting in different phases.

lengths but in opposite phase have equal opposite displacements at any point on the curve (Fig. 64). Being in opposite phase the two curves differ by half a period

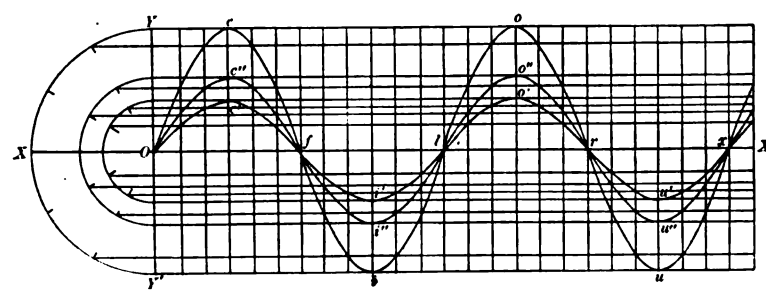


FIG. 66.—Combination of two harmonic curves of the same wave lengths and phase but differing in amplitudes.

$(1/2\lambda)$. The amplitude at any point will be the algebraic sum of the two displacements at that point. Thus at c the resultant of cc'' and cc' equals zero since cc' is equal to cc'' $(+cc'-cc''=0)$. The same result is obtained for every other point

on the curve, whereby the resultant of two harmonic curves of the same amplitudes and wave lengths, but differing by one-half of a period, is zero, or complete rest. The curve is a straight line.

III. *Two harmonic curves having the same amplitudes and wave lengths, but differing in phase by some fraction of a period other than one-half.* In this case (Fig. 65)

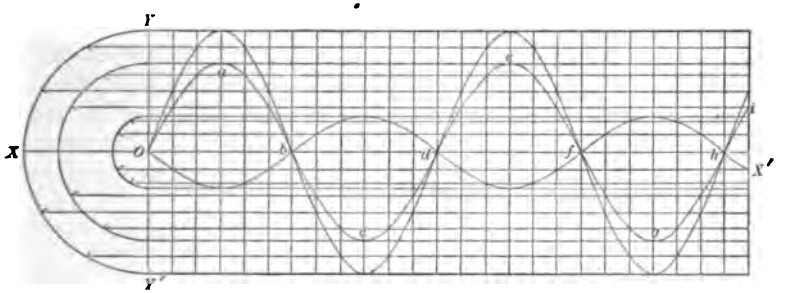


FIG. 67.—Combination of two harmonic curves of the same wave length but differing in amplitudes and opposite in phase.

the resulting harmonic curve will be of the same wave length as either component but differ from them in amplitude. Its amplitude will be less than that in Case I, and greater than that in Case II.

IV. *Two harmonic curves having the same wave lengths and in the same phase but differing in amplitudes.* In this case (Fig. 66) the result obtained by determining the algebraic sum at every point is a harmonic curve of the same wave length and phase as either component, but differing in amplitude.

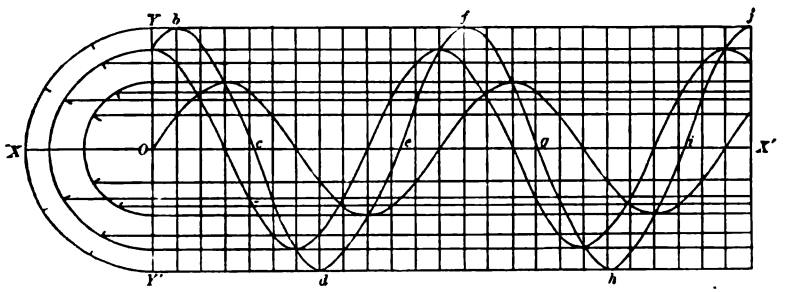


FIG. 68.—Combination of two harmonic curves of the same wave lengths but differing in phase by some fraction of a period other than one-half, and differing in amplitude.

V. *Two harmonic curves having the same wave lengths but different amplitudes and opposite phases.* The resultant (Fig. 67) is a harmonic curve of the same wave length but differing in amplitude from either component. The amplitude is the least possible of any combination of the original curves.

VI. *Two harmonic curves having the same wave lengths but differing in amplitude and differing in phase by some fraction of a period other than one-half.* In this case (Fig. 68) the resultant is of the same wave length as either component but it has an amplitude which is less than that in Case IV and greater than that in Case V.

From these six cases we see that no matter what the amplitude or what the phasal difference, if the original components have equal wave lengths, the resulting wave length is the same. The amplitude, however, decreases from a maximum of the sum of the two amplitudes, when there is no phasal difference, to a minimum when the components differ by half a wave length, this minimum being the algebraic sum of the two displacements, which, of course, is equal to zero when the amplitudes are the same.

VII. *Two harmonic curves of different wave lengths with equal or unequal amplitudes.* The resulting curve in the case of two harmonic curves of different wave lengths and with equal or unequal amplitudes is much more complex, and differs both in amplitude and wave length from either component. It was drawn in Fig. 69, as were all the preceding curves, by determining the algebraic sum of the displacements at different points.

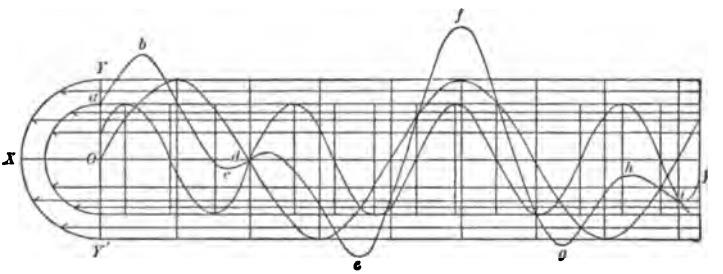


FIG. 69.—Combination of two harmonic curves of different wave lengths and of unequal amplitudes.

The amplitude of the resultant of two simple harmonic movements may be shown analytically as follows:

We have as the equation for the displacement of a particle at any time,[1]

$$d_1 = r_1 \sin \frac{2\pi(t-t_1)}{T},$$

and for a second vibration

$$d_2 = r_2 \sin \frac{2\pi(t-t_2)}{T}.$$

Since the resulting amplitude is the algebraic sum of the amplitudes of the separate components, we have

$$d = d_1 + d_2 = r_1 \sin \frac{2\pi(t-t_1)}{T} + r_2 \sin \frac{2\pi(t-t_2)}{T}$$

$$= \sin \frac{2\pi t}{T}\left(r_1 \cos \frac{2\pi t_1}{T} + r_2 \cos \frac{2\pi t_2}{T}\right) - \cos \frac{2\pi t}{T}\left(r_1 \sin \frac{2\pi t_1}{T} + r_2 \sin \frac{2\pi t_2}{T}\right).$$

If we let

$$A \cos \frac{2\pi t_2}{T} = r_1 \cos \frac{2\pi t_1}{T} + r_2 \cos \frac{2\pi t_2}{T}, \qquad (1)$$

[1] Eq. 8, Art. 25.

and

$$A \sin \frac{2\pi t_3}{T} = r_1 \sin \frac{2\pi t_1}{T} + r_2 \sin \frac{2\pi t_2}{T}, \tag{2}$$

we have

$$d = \sin \frac{2\pi t}{T}\left(A \cos \frac{2\pi t_3}{T}\right) - \cos \frac{2\pi t}{T}\left(A \sin \frac{2\pi t_3}{T}\right)$$

$$= A \sin \frac{2\pi t}{T} \cos \frac{2\pi t_3}{T} - A \cos \frac{2\pi t}{T} \sin \frac{2\pi t_3}{T},$$

whence

$$d = A \sin \left(\frac{2\pi t}{T} - \frac{2\pi t_3}{T}\right) = A \sin \frac{2\pi (t - t_3)}{T}. \tag{3}$$

Squaring (1) and (2), and adding, we have

$$A^2 \sin^2 \frac{2\pi t_3}{T} + \cos^2 \frac{2\pi t_3}{T} = r^2_1 + r^2_2 + 2r_1 r_2 \cos \frac{2\pi (t_2 - t_1)}{T},$$

and

$$A^2 = r^2_1 + r^2_2 + 2r_1 r_2 \cos \frac{2\pi (t_2 - t_1)}{T}, \tag{4}$$

which is the equation of the amplitudes of the resultant vibration of two harmonic motions.

We may arrive at the same equation geometrically as follows:

In Fig. 70 let

$Oa = r_1$, the amplitude of the first vibration,

$bc = Oa = r_1$,

$Ob = r_2$, the amplitude of the second vibration,

$ac = Ob = r_2$,

$Oc = A$, the amplitude of the resultant of r_1 and r_2,

$Oe = d_1$, the displacement of the first vibration,

$Of = d_2$, the displacement of the second vibration,

$Oj = d_3$, the displacement of the resultant.

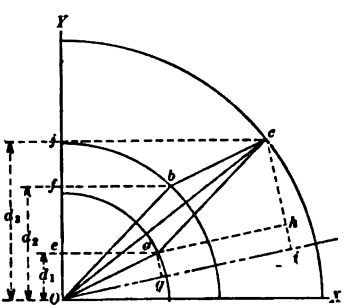


FIG. 70.—Amplitude of the resultant of two simple harmonic movements.

Draw Oi, so that $iOX = \alpha = \omega t = \frac{2\pi t}{T}$.

Then $aOg = \frac{2\pi t_1}{T}$, the angular displacement of the first vibration,

$bOg = \frac{2\pi t_2}{T}$, that of the second, and

$cOg = \frac{2\pi t_3}{T}$, that of the resultant.

Solving the triangles given in Fig. 70, we have:

$$(ah)^2 + (hc)^2 = (ac)^2;$$
$$(Og)^2 + (ag)^2 = (Oa)^2;$$
$$(Oi)^2 + (ic)^2 = (Oc)^2;$$

$$\cos aOg = \frac{Og}{Oa}; \cos cOg = \frac{Oi}{Oc},$$

$$\sin aOg = \frac{ag}{Oa}; \sin cOg = \frac{ic}{Oc}.$$

Substituting values from those given above,

$$A^2 = r^2_1 \cos^2 \frac{2\pi t_1}{T} + 2r_1 \cos \frac{2\pi t_1}{T} \cdot r_2 \cos \frac{2\pi t_2}{T} + r^2_2 \cos^2 \frac{2\pi t_2}{T}$$
$$+ r^2_1 \sin^2 \frac{2\pi t_1}{T} + 2r_1 \sin \frac{2\pi t_1}{T} \cdot r_2 \sin \frac{2\pi t_2}{T} + r^2_2 \sin^2 \frac{2\pi t_2}{T}$$
$$= r^2_2 \left(\sin^2 \frac{2\pi t_2}{T} + \cos^2 \frac{2\pi t_2}{T}\right) + r^2_1 \left(\sin^2 \frac{2\pi t_1}{T} + \cos^2 \frac{2\pi t_1}{T}\right)$$
$$+ 2r_1 r_2 \left(\cos \frac{2\pi t_1}{T} \cdot \cos \frac{2\pi t_2}{T} + \sin \frac{2\pi t_1}{T} \cdot \sin \frac{2\pi t_2}{T}\right).$$

But the sum of the squares of the sine and cosine of an angle is equal to unity,[1] and the sum of the product of the sines and cosines of two angles is equal to the cosine of their difference,[2] therefore

$$A^2 = r^2_2 + r^2_1 + 2r_1 r_2 \cos \frac{2\pi (t_2 - t_1)}{T}.$$

This is the same equation as equation (4) above.

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[1] Eq. 37, Appendix.
[2] Eq. 57, Appendix.

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CHAPTER IV

ISOTROPIC MEDIA

29. Definitions.—Substances in which the velocity of the transmission of light is independent of the direction of vibration are called **isotropic** (ἴσος, equal, and τροπή, a turning). They include amorphous substances, such as gases, liquids, and annealed glasses, and all unstrained crystals of the isometric system.

Substances in which the velocity of the transmission of light differs in different directions are called **anisotropic.**

30. Wave Motion in Isotropic Media.—We may now consider the movement of a series of particles equally spaced along a line in an isotropic medium in which the light travels with equal velocities in all directions. Let *a b c d e*

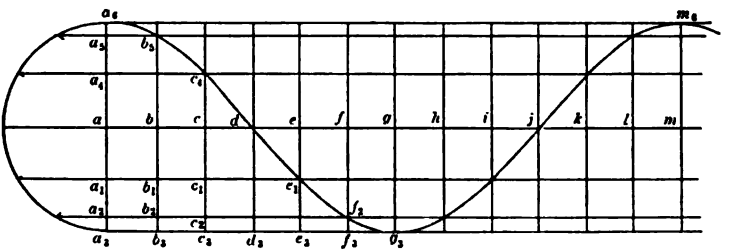


FIG. 71.—Wave motion transmitted along a series of particles in an isotropic medium.

. . . . . *m*, Fig. 71, represent such a series of particles in equilibrium. If some force displaced the particle *a*, for example, in the direction a_1, the equilibrium would be disturbed, and a pull would be exerted upon the particle *b* in the direction b_1. The movement of *b* would set up a movement in *c*, and so on. In the meantime the particle *a* would have moved on in a direction at right angles to the line *a* . . . . . *m*, for a distance which was governed by the impulse it originally received and the pull exerted by the other particles. It would move outward with gradually decreasing velocity until it had reached the limit of displacement at a_2. At the same time the particle *b* would have reached b_2, and *c*, c_1, while *d* would not yet have felt the pull. The particle *a* would now tend to return to its original position of rest, but would be carried by its momentum almost an equal distance on the other side to a_6. Meantime *b* also would have been carried backward, although a fraction behind *a*. While the particles first moved were thus moving to

the opposite side, the particles in advance would still be drawn down until each had reached the limit of its impulse, for example, g to g_3. The movement of all the particles thus vibrating will be that of a harmonic curve. It is the movement imparted to a rope, held at both ends, when shaken up and down. While each particle retains its relative position, a progressive wave seems to travel along the rope. If only enough energy is given to the line of particles to cause each to perform a single oscillation, only a single wave travels along the line, successive particles having energy imparted to them while the line behind the wave sinks to rest. If the energy originally imparted is great enough so that the particle does not stop at the end of a single oscillation, a succession of waves of gradually diminishing amplitude travels along the cord. If a continuous periodic force agitates the line, a succession of equal waves travels along it.

In a light wave the distance between two particles in the same position and moving in the same phase (a_6 and m_6, Fig. 71), is called the **wave length** and is represented by λ. The distance a to a_6, from the position of rest to the position of maximum displacement, is called the **amplitude.**

31. **Intensity of Light.**—The intensity of light in the physical sense, as contrasted with the physiological sense, depends upon the amplitude of its vibrations. That is, it depends upon the force of the original impulse: the greater the original displacement, the greater the intensity.

32. **Color of Light.**—The color of light depends, with certain limitations, upon its wave length. Strictly speaking, the rapidity of oscillation governs color, for a ray of a certain color, passing through different media, changes its velocity of propagation and proportionately its wave length, but the frequency of oscillation at the source remains constant, and therefore, likewise, the color. It is the number of waves of light which reach the eye in a given time that determines the color sensation.[9] White light contains waves of all colors reaching the eye simultaneously.

33. **Velocity and Wave Length of Light.**—The velocity of light of all colors *in vacuo* is the same, and is about 300,000 km. per second.

The wave length of red light (solar A) is 0.0007604 mm. and of violet (solar H, calcium), 0.0003968 mm. This gives about 395×10^{12} oscillations per second for the red and about 757×10^{12} oscillations for the violet.

34. **Wave Front and Wave Surface.**—A ray of light, traveling in an isotropic substance, will travel with equal ease in every direction, consequently, at the end of the same interval of time, a movement arising at O, Fig. 72, will have reached the points a, b, c, d, e, f, g; all equally distant from O. The **wave front,** as it is called, is a circle, and the **wave surface,** in space, is a sphere.

Again, consider each point on the circle as a new center of disturbance.

At the end of a unit of time a movement of the **ray front** *b* will have extended the motion to all points on the circle *b'b'*. Likewise the movement at *c* will reach the circle *c'c'*, and so on; since the new radii are equal, the new wave front will everywhere be parallel to the original wave front; that is, it also will be a circle, and the new wave surface will be a sphere.

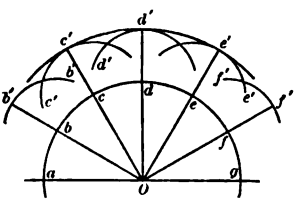


FIG. 72.—Wave front of light in an isotropic medium. Light originating in a point. (Huygens' construction.)

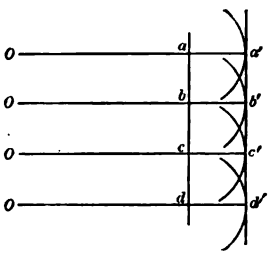


FIG. 73.—Wave front of light in an isotropic medium. Light originating at infinity. (Huygens' construction.)

If the source of light is at an infinite distance, the rays will be parallel (Fig. 73), and the points *a*, *b*, *c*, *d* will be equally distant from the source, consequently the line *abcd* will be at right angles to the direction of propagation of the ray. New impulses from these points, at the end of a unit of time, will lie in the circles *a'*, *b'*, etc., having equal radii, consequently the tangents to all of them will be a line parallel to *abcd*. In space the wave surface will be a plane.

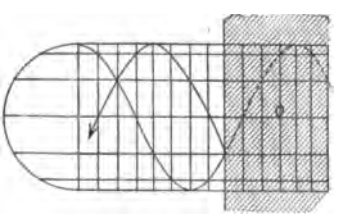


FIG. 74.—The effect of an obstacle in the path of a ray. The solid line indicates the wave as it appears, the dotted line as it would have appeared had it been free to continue in its original direction. *O* is the obstacle.

35. Reflection of Waves.—If a wave in its course meets an obstacle to its free movement, the particles act as if compressed; they rebound and a retrograde movement takes place exactly equal to the original in wave length, period, amplitude, and phase (Fig. 74). This reflected ray will appear in form, though not in direction, exactly as the original wave would have done had it been free to continue its course.

The obstacle may not entirely prevent the light from passing through, but a part may be reflected and a part transmitted. In this case the amplitude of the reflected wave will be less than the original; the wave length, however, will remain the same although transmitted in the opposite direction.

If a wave from an optically denser medium passes to a rarer, the expansion of the particle on emerging into the second medium has the same effect, and a reflected ray of less amplitude returns into the denser medium.

In the same manner, when a ray of light, the so-called **incident ray,** strikes a second surface at an angle, a certain amount passes through and a certain amount is reflected.

Let a bundle of rays of parallel light originate in an isotropic medium at O, O', and O'', Fig. 75. When the ray O has reached the point a, the ray O will have reached a', O'' will have reached a'', and the ray front $a\ a'\ a''$ will be at right angles to the direction of propagation.

The ray O'' will continue after reaching the point a'', and will soon reach c. At the same time the ray O will have been reflected at a. Traveling in the original medium, its velocity will be unchanged, and it will travel, in the time that the O'' ray travels from a'' to c, a distance from a equal to $a''c$, $(ac''=a''c)$. The wave front of the ac'' ray will be a sphere with a as its center and with a radius equal to $a''c$.

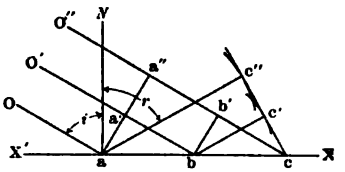


FIG. 75.—Huygens' construction showing the course of reflected rays in an isotropic medium.

In the same way, the ray O' reaches b when O'' is at b'. O' is reflected, and its wave front is a sphere with a radius equal to $b'c$ $(bc'=b'c)$. The wave front of all the rays between O and O'' will lie, when O'' has just reached c, on a line through c and tangent to all the circles representing the new wave fronts.

In Fig. 75 the angle $OaX'=a''ca$, since Oa and $O''c$ are parallel. The angle $ac''c$ is a right angle, since $c''c$ is tangent at c'' to the circle of which ac'' is the radius. In the two right angle triangles $ac''ca$ and $aa''ca$, the line ac is common to both, and $ac''=a''c$ by construction. Having two lines equal in a right angle triangle, the angles must be equal, and

$$a''ac=c''ca. \tag{1}$$

Since $OaX'=a''ca$, the complementary angles are equal and

$$OaN=a''ac. \tag{2}$$

Combining (1) and (2) we have

$$OaN=c''ca. \tag{3}$$

But $c''ca+c''ac=90°$, and $Nac''+c''ac=90°$, therefore $c''ca=Nac''$. Substituting in (3)

$$OaN=Nac''. \tag{4}$$

The angle OaN is called the **angle of incidence,** and the angle Nac'' the **angle of reflection.** Equation (4) therefore means that the *angle of reflection is always equal to the angle of incidence.*

As the angle of incidence increases, so does the intensity of the reflected light, depending also, of course, upon the nature of the reflecting medium.

36. Passage of Light into a Medium of Different Density.—The particles of which an optically dense medium is composed may be considered as being more closely spaced than those in one that is rarer, as is shown along the horizontal line $X'X$, Fig. 76. A wave traveling from X' to X arrives at m where it enters the optically denser medium. The particles in the second medium must necessarily move in unison with those in the first, therefore the period and the phase remain unchanged. Since the second medium is optically

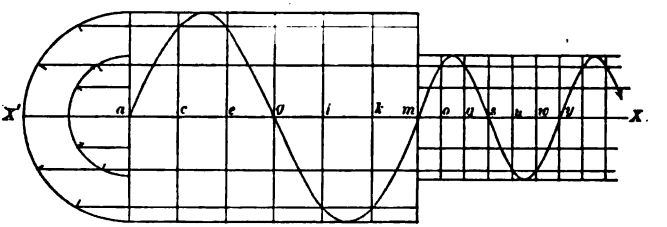


FIG. 76.—A wave passing from a rarer to a denser isotropic medium.

denser than the first, a wave cannot travel so rapidly in it, for in the time that a wave can travel from a to m in the rarer medium, it can only travel from m to y in the one which is denser, consequently the wave length must be less. The ease of vibration also is less in the second medium, whereby the amplitude of vibration will be less, a decrease made still greater by the fact that a certain amount of energy is expended in producing the reflected ray.

Upon passing into a rarer medium, the particles may be considered as being less closely spaced, and the reverse of the above takes place.

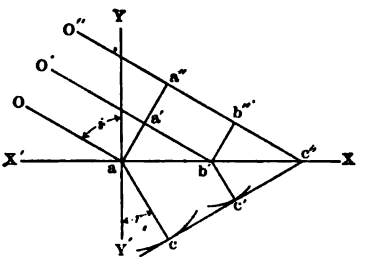


FIG. 77.—Huygens' construction showing the refraction of a ray of light upon passing into a medium of greater density.

37. Refraction of Light upon Passing into an Isotropic Medium of Different Density.—When light falls upon the surface of a transparent medium, a part of it is reflected back into the first medium and a part passes into the second, generally in a changed direction. The second part is said to be **refracted.**

Let the rays of light O, O', and O'', Fig. 77, pass from air into a denser isotropic medium $X'X$. At the instant that the ray O is at the point a, the ray O'' will be at a''. This second ray continues on to c''. Meanwhile the ray O has been partly reflected back into the first medium and partly refracted into the second. If the latter medium is denser than the first, the distance traveled in it by the ray, in a unit of time, will not be so great.

Let v = velocity of light in air,

v' = velocity of light in the second medium.

When the second medium is denser than the first, $v > v'$.

Let t = time of transmission of light from a'' to c'', or from a to c, then $a''c'' = vt$, and $ac = v't$,

whereby
$$\frac{a''c''}{ac} = \frac{vt}{v't} = \frac{v}{v'}. \tag{1}$$

The ray front of the ray O, at the instant that the ray O'' reaches c'', will be somewhere on the surface of a sphere having a as a center and a radius of ac.

Likewise a second ray, as O', will travel from b' to c' while the ray O'' travels from b'' to c'', and we have again:

$$\frac{b''c''}{b'c'} = \frac{vt'}{v't'} = \frac{v}{v'},$$

with a ray front somewhere on a sphere with b' as a center and $b'c'$ as a radius. The wave front of all rays between O and O'', at the instant the ray O'' reaches c'', will be a plane passing through c'' and tangent to all the spheres upon which the individual ray fronts lie. If, then, a line be drawn tangent to all these circles and passing through c'', it will represent the trace of the wave front. Since a tangent forms a right angle with a radius, the lines perpendicular to this tangent and passing through a, b', etc., will represent the direction of the individual rays. Further, in Fig. 77, $aa''c''$ is a right angle by construction, and acc'' is a right angle because it is formed by a radius and a tangent. Therefore

$$\sin a''ac'' = \frac{a''c''}{ac''}, \tag{2}$$

$$\sin ac''c = \frac{ac}{ac''}. \tag{3}$$

Combining (2) and (3) we have

$$\frac{\sin a''ac''}{\sin ac''c} = \frac{\dfrac{a''c''}{ac''}}{\dfrac{ac}{ac''}} = \frac{a''c''}{ac} \tag{4}$$

But by equation (1) we have

$$\frac{a''c''}{ac} = \frac{v}{v'} = n, \text{ a constant, therefore}$$

$$\frac{\sin a''ac''}{\sin ac''c} = \frac{v}{v'} = n. \tag{5}$$

Let the line YY' be normal to XX', then

$$a''ac'' + Yaa'' = 90°.$$

But $OaY+Yaa''=90°$

whereby $a''ac''=Oay=i$, *the angle of incidence.* (6)

We also have

$$c''ac+ac''c=90°$$
$$c''ac+Y'ac=90°,$$

whereby $ac''c=Y'ac=r$, *the angle of refraction.* (7)

Substituting (6) and (7) in (5),

$$\frac{\sin i}{\sin r}=\frac{v}{v'}=n. \tag{8}$$

That is, *the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant and bears the same ratio as the respective velocities of light in the two media.* This is known as **Snell's law,** having been discovered by Willebrod Snellius, professor of mathematics at Leyden, about 1621. It was first published by Descartes, Snell having died in 1626 without having made the statement in print.

38. Index of Refraction.—The definite ratio between the sines of the angles of incidence and of refraction of two substances[1] is called the index of refraction. It is necessary that some medium be chosen as a standard for comparison, air being the one generally used, and the ratio is then that of the sine of the angle of incidence in air to the sine of the angle of refraction in the other medium.

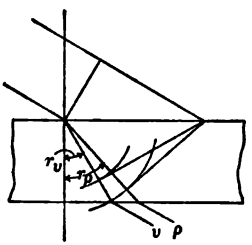


FIG. 78.—Dispersion of light in isotropic media.

In isotropic media, in which the velocity of light is the same in every direction, this index of refraction has a characteristic, constant value for every substance. In anisotropic media it varies with the direction of transmission and the character of the polarization, but it is constant for any definite direction.

For very accurate measurements it is necessary to use monochromatic light, since white light, which contains many constituent rays, is variously refracted, as may be seen in the spectrum. This difference in refraction depends upon the wave lengths of the rays, which, in turn, produce different colors. Thus, in glass, the index of refraction for blue is greater than for red ($n_v > n_p$), consequently blue is most refracted and red least, and the angle of refraction for blue is less than that for red. This difference in refraction is called the **dispersion of light** (Fig. 78).

39. Passage of Light into Different Isotropic Media.—By trigonometry we have (Fig. 79): $\sin A=\frac{a}{c}$. As the angle increases, the value of the sine increases. When $A=0$, $\sin A=0$; when $A=90°$, $\sin A=1$.

[1] Art. 37, *supra.*

The denser the medium, the less the velocity of the transmission of light within it. Consequently we may have three cases.

a. Media of the Same Densities (Fig. 80).—The velocity of the light in the two media is the same, therefore

$$v=v',\ vt=v't, \text{ and } a''c''=ac,$$

$$\frac{\sin i}{\sin r}=\frac{\frac{a''c''}{ac''}}{\frac{ac}{ac''}}=\frac{a''c''}{ac}=1.$$

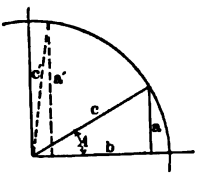


FIG. 79.—Trigonometric functions.

Therefore $\sin i=\sin r$, and $i=r$.

That is, when light passes from one medium into another of like density, there is no change in the direction of the ray.

b. Rarer to Denser Medium (Fig. 81). In this case $v>v'$, therefore $a''c''>ac$.

$$\frac{\sin i}{\sin r}=\frac{\frac{a''c''}{ac''}}{\frac{ac}{ac''}}=\frac{a''c''}{ac}>1.$$

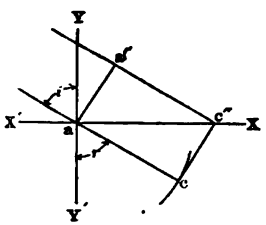


FIG. 80.

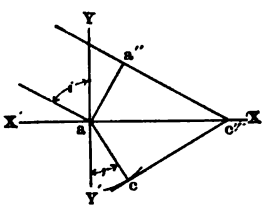


FIG. 81.

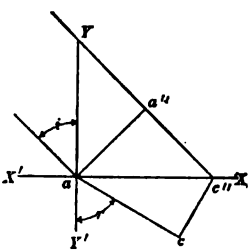


FIG. 82.

FIG. 80.—Refraction of light on passing into a medium of equal density.
FIG. 81.—Refraction of light on passing from a rarer to a denser medium.
FIG. 82.—Refraction of light on passing from a denser to a rarer medium.

Therefore $\sin i>\sin r$, and $i>r$.

That is, when light passes from a rarer to a denser medium, the ray is bent toward the normal.

c. Denser to rarer medium (Fig. 82).—In this case $v<v'$, therefore $a''c''<ac$.

$$\frac{\sin i}{\sin r}=\frac{a''c''}{ac''}<1.$$

Therefore $\sin i<\sin r$, and $i<r$.

That is, when light passes from a denser to a rarer medium, the ray is bent away from the normal.

40. Relation between Indices of Refraction and Velocity of Propagation of Light.—From equation (8), Article 37, we have

$$\frac{\sin i}{\sin r} = \frac{v}{v'} = n.$$

If we compare two substances whose indices are n' and n'', we have,

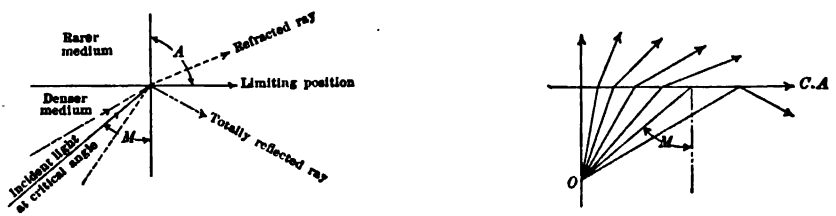
$$n' = \frac{v}{v'} \text{ and } n'' = \frac{v}{v''}.$$

Combining, we have

$$\frac{n'}{n''} = \frac{\frac{v}{v'}}{\frac{v}{v''}} = \frac{v''}{v'}.$$

That is, for a given color of light, *the indices of refraction of two media are inversely proportional to the relative velocities of the propagation of light within them.* If one medium be taken as a standard and considered unity, (n'' and $v'' = 1$) the equation becomes $n = \frac{1}{v}$.

41. Total Reflection and the Critical Angle.—We found[1] that $\frac{\sin i}{\sin r} = n$, where i is the angle of incidence in air. If we reverse the course of the light and let it pass from the denser to the rarer medium, and, to avoid confusion, substitute A for i and M for r, we have



FIGS. 83 AND 84.—The critical angle (M).

$$\frac{\sin A}{\sin M} = n, \text{ and } \frac{\sin M}{\sin A} = \frac{1}{n}.$$

That is, we now have M as the angle of the incident ray in the denser medium, and A as the angle at which it is refracted in air. If $A = 0°$, $\sin A = 0$, and

$$\frac{\sin M}{\sin A} = \frac{\sin M}{0} = \frac{1}{n}$$

whereby $\sin M = 0$ and $M = 0°$.

[1] Art. 37, *supra*.

Expressed in words, when the light falls normal to a second medium it passes through without change of direction.

If $A = 90°$, $\sin A = 1$, and $\sin M = \frac{1}{n}$ = a constant (Fig. 83). That is, at some angle, constant for the same substances, the refracted ray is parallel to the separating surface. In this position the angle of incidence is called *the critical angle* and *is that angle whose sine is the reciprocal of the index of refraction.*

From these two statements it is clear that rays of light which pass from a denser to a rarer medium at any angle of incidence between 0° and the critical angle, will be partly refracted into the second medium. Light falling upon the second medium at angles greater than the critical angle will be *totally reflected* (Figs. 83–84).

If the critical angle of a substance is measured, the index of refraction of that substance may be determined from the above formula. Thus for water the critical angle is 48° 31′. $\sin 48° 31' = 0.749 = \frac{1}{1.335} = \frac{1}{n}$, and $n = 1.335$. The critical angle for diamond is 24° 25′. $\sin 24° 25' = 0.413 = \frac{1}{2.419} = \frac{1}{n}$, and $n = 2.419$.

Since the brilliancy of a mineral depends upon the amount of light which is reflected from it, the smaller the critical angle, the more totally reflected light appears, and the greater is the brilliancy.

42. Polarization, and Light Polarized by Reflection.—We have said that, in general, in an isotropic medium light vibrates in all directions at right

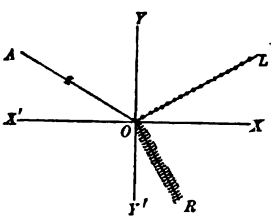


FIG. 85.—Section through a reflected and a refracted ray in an isotropic medium.

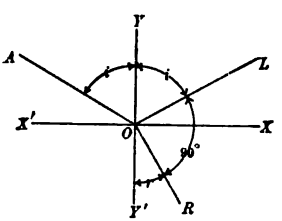


FIG. 86.—The angle of polarization.

angles to the direction of propagation, and that when it meets with another isotropic medium at an angle, part of the light is reflected and part refracted. It has been found that after reflection or refraction, the vibrations do not move with the same freedom in every direction as before, but that they are more or less limited to two planes, so that the ray A (Fig. 85), originally vibrating in all directions, vibrates, in the reflected ray L, parallel to the plane of the reflecting surface (in the figure, perpendicular to the plane of the paper),

and in the refracted ray R, in a plane at right angles to it. The light, in such cases, is said to be **plane polarized.**

The **plane of polarization,** for light polarized by reflection, is defined as the plane containing the incident and the reflected rays, the vibrations taking place at right angles to it. The plane of polarization of the refracted ray is the plane at right angles to the vibration direction, consequently at right angles to the plane of the incident and refracted rays.

43. Angle of Polarization.—It has been found that when the angle of incidence is such that the reflected and the refracted rays make an angle of 90° with each other (Fig. 86), polarization is at its maximum. This does not mean that all of the light is completely polarized, but that the amount decreases in either direction from a certain angle. According to M. Jamin, only those substances which have an index of refraction of about 1.46 completely polarize light by reflection. The angle of incidence at maximum polarization naturally differs with substances having different refractive indices but, for each substance, it possesses a definite value, called the **angle of polarization.**

In Fig. 86, since the angle of incidence equals the angle of reflection,

$$AOY = YOL = i,$$

$$YOL + LOX = 90°,\ i + LOX = 90°. \qquad (1)$$

Also $$LOX + ROX = 90°, \text{ and } Y'OR + ROX = 90°.$$

Combining, $$LOX = Y'OR = r.$$

Substitute this value in (1)

$$i + r = 90°. \qquad (2)$$

By trigonometry, in a right triangle (Fig. 87),

$$\sin i = \frac{a}{c},\ \sin r = \frac{b}{c},$$

therefore $$n = \frac{\sin i}{\sin r} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{c}.$$

But $\tan i = \frac{a}{c}$, therefore $n = \tan i$.

This is **Brewster's law** which may be stated: *The tangent of the angle of polarization is equal to the index of refraction of the reflecting substance.*

A few examples of the polarizing angles of different substances follow.

Crown glass, mean index $n=1.515$, $\tan i=1.515$, $i=56°35'$
Flint glass, $n=1.622$, $i=58°\ 21'$
Water, $n=1.335$, $i=53°\ 10'$
Diamond, $n=2.419$, $i=67°\ 32'$
Spinel, $n=1.718$, $i=53°\ 10'$

Since the refractive indices in a medium differ slightly for different colored rays, so also must the angles of polarization differ. If the index for any color in a given medium is known, the angle of polarization for that color may be computed from the formula.

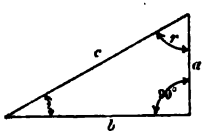


FIG. 87.—Relations between sine, cosine, and tangent.

If the incident light falls upon a plate at some angle other than the angle of polarization, only part of the light is polarized, the amount depending upon the angle; the nearer to the polarizing angle, the greater the amount. The remainder of the ray is reflected as ordinary light, but if it is reflected subsequently one or more times, the proportion polarized is increased. It is customary, in practice, in order to get a strong ray, to use ten or twelve parallel thin glass plates, termed a **pile of plates.**

That light is polarized when reflected may be shown experimentally by the use of two reflecting surfaces. A simple contrivance to demonstrate this is the Nörremberg polarizer, shown in Fig. 88.

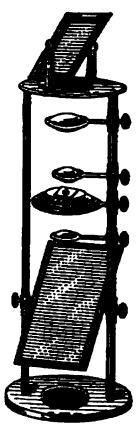


FIG. 88.—Nörremberg polarizer. (Steeg und Reuter.)

44. Variation in Intensity—Malus' Law.—Malus found that *the intensity of light, polarized by reflection from one mirror and reflected from a second, varies as the square of the cosine of the angle between the two planes of incidence.*

Let $\alpha=$ this angle,
$a=$ the value of the maximum intensity of light,
$I=$ the intensity at the angle α.
Then by Malus' law

$$I=a\cos^2\alpha$$

When $\alpha=0$ or $180°$, $\cos\alpha=1$, $\cos^2\alpha=1$ and $I=a$ or the maximum intensity.
When $\alpha=90°$ or $270°$, $\cos\alpha=0$, and $I=0$ or darkness.

45. Polarization by Refraction.—Not only is the reflected portion of the incident ray polarized, but the refracted portion is polarized as well. The plane of polarization of the refracted ray is at right angles to the plane of the incident and refracted rays (Fig. 89) and the vibrations take place within the latter plane.

The vibration direction of the refracted ray may be determined experi-

mentally by inspecting the emerging ray at E (Fig. 89) by means of an analyzer, such as a piece of tourmaline cut parallel to c, or a nicol prism such as will be described later.

46. Arago's Law.—As in the reflected ray, so also in the refracted ray it is only a part of the light which is polarized. Its amount depends upon the angle of incidence; the nearer this is to the polarizing angle, the greater is the amount. There is, however, a definite relation between the amount of light polarized in the reflected and the refracted rays. This is expressed in Arago's law: *The reflected and the refracted rays of light, polarized in planes at right angles to each other by reflection from and refraction through a transparent medium, each contain an equal amount of polarized light.*

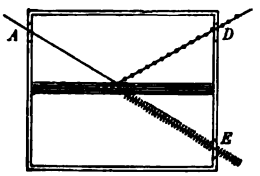


FIG. 89.—Apparatus for showing directions of vibration of the reflected and refracted rays.

CHAPTER V

ANISOTROPIC MEDIA

47. Single Refraction and Double Refraction.—We have seen that in isotropic media, light vibrates with equal ease in every direction, consequently the wave surface in such a medium, at the end of any interval of time, is a sphere through which light passes in a single direction, although changed from its original course. Isotropic substances, therefore, are said to be *singly refracting*.

We have seen also that there is another class of substances in which the rate of propagation differs in different directions. These substances are called anisotropic. If a beam of light, with equal vibrations in every direction, passes from an isotropic medium into one which is antisotropic, its vibrations no longer remain the same. If the second medium is denser than the first, the ease of vibration in it must everywhere be less, and one direction must be of greater ease and one of less, then all the others. It has been determined that the direction of least ease lies at right angles to that of maximum ease, and one would naturally suppose, since the wave, before entering vibrates in every direction, that light entering between these two positions would vibrate with an intermediate ease. This, however, is not the case. The intermediate entering wave is broken up into two waves, and no more, and these waves vibrate at right angles to each other in the principal sections. In all anisotropic crystals there is a third direction of vibration at right angles to the other two. Its value, in uniaxial crystals, is equal to either the maximum or minimum ease; in biaxial crystals it is intermediate between the other two, and is called the direction of intermediate ease although, in value, it is not necessarily actually the mean. These three **principal vibration axes** or **axes of the optical ellipsoid,** as they are called (formerly, axes of elasticity), form a system of rectangular coordinates, so that, in every anisotropic mineral section, there are two vibration directions at right angles to each other, one of which usually will be of greater ease than the other, although the greater ease in a section will not necessarily be the direction of greatest ease in the mineral. This property of anisotropic crystals of resolving light rays into two sets of vibrations is called **double refraction** or **birefringence.**

The axes of the optical ellipsoid have a definite direction in a given crystal, and the relative ease of vibration along any crystallographic axis is constant for that substance. That is, the vibrations take place in directions

which always bear the same definite relations to the crystallographic axes.

According to the positions of their vibration axes, crystals may be divided, as we shall see later, as follows:

Isotropic crystals		isometric		
Anisotropic crystals	uniaxial	tetragonal		extinction parallel.
		hexagonal	hexagonal	
			trigonal	
	biaxial	orthorhombic		extinction inclined.
		monoclinic		
		triclinic		

Before discussing further these subdivisions of the crystal systems, let us see what takes place when a ray of light passes through an anisotropic substance.

OPTICALLY UNIAXIAL CRYSTALS

48. Double Refraction in Calcite.[1]—The divergence of the two refracted rays, in a clear, transparent mineral with strong double refraction, is so great that an image viewed through it appears double (Figs. 90–91). This property

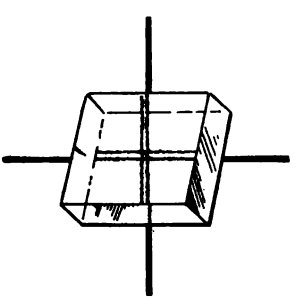


FIG. 90.

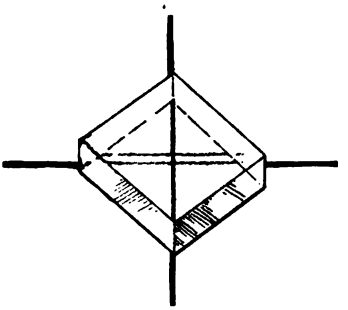


FIG. 91.

FIGS. 90 AND 91.—Double refraction in calcite.

was first discovered in Iceland spar by Erasmus Bartholinus in 1669, and can be well demonstrated by the apparatus[2] shown in Fig. 92.

The cleavage angle of a rhombohedron of calcite (Fig. 93) is 74° 56′, and the axis *c* connects the obtuse angles of the faces. If such a rhombohedron is placed with the short diagonal of one of its faces vertical, it will appear, in section, as shown in Fig. 94. In Fig. 92 two such rhombohedrons

[1] For a theoretical discussion see R. T. Glazebrook: *Double refraction and dispersion in Iceland spar; an experimental investigation with a comparison with Huyghen's construction for the extraordinary wave.* Phil. Trans. Roy. Soc., London, II (1880), 421–449. See also Charles S. Hastings: *On the law of double refraction in Iceland spar.* Amer. Jour. Sci., XXXV (1888), 60–73.

[2] C. Leiss: *Die optischen Instrumente der Firma R. Fuess.* Leipzig, 1899, 152.

of Iceland spar are shown, the one in the center (*Rh*) so mounted that it may be rotated in a plane at right angles to a ray of light passing through the screen at the left.

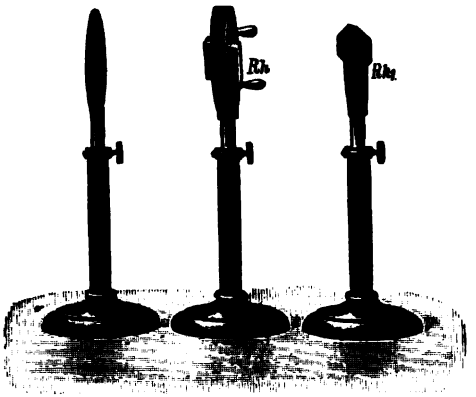


FIG. 92.—Apparatus for showing double refraction in calcite. 1/5 natural size. (Fuess.)

If, now, a ray of light (*P*, Fig. 94) passes through the aperture in the screen and falls upon the prism at right angles to its face, it will be found,

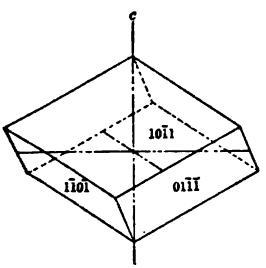


FIG. 93.—A rhombohedron of calcite.

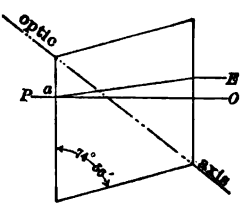


FIG. 94.—Separation of rays in calcite. Section cut parallel to the *c* axis.

when viewed from the back, that it has been broken up into two. Instead of the single spot of light which would have appeared through an isotropic

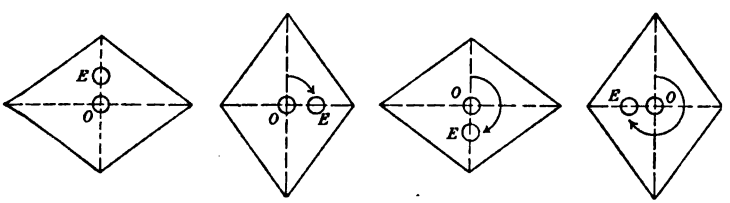


FIG. 95. FIG. 96. FIG. 97. FIG. 98.

FIGS. 95 TO 98.—Positions of the spots of light on rotating a rhombohedron of calcite.

medium, there will be two equally bright spots, the one vertically above the other (Fig. 95). If the rhombohedron *Rh* (Fig. 92) be rotated, it will

be found that one image (*O*, Figs. 95–98) remains stationary while the other moves around it. It is perfectly clear that the ray *O* has passed through without changing its direction, just as it would have done had the medium been isotropic. It is, therefore, called the **ordinary ray.** The ray *E*, however, acts in a different manner, for although the incident light falls normal to the surface of the rhombohedron, it is refracted, as shown in Fig. 94. It is called the **extraordinary ray.**

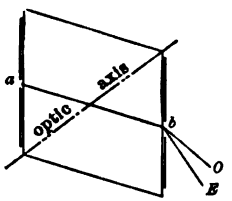


FIG. 99.—Experiment showing that light passes through a calcite crystal with two speeds along the same line.

If an opaque card, through which a very small hole has been punched, is placed in contact with the farther side of a calcite rhombohedron, and a second card is placed on the near side, it will be found that there are two positions of the eye where the image can be seen (Fig. 99). Evidently the ray must have traveled along the line *ab* with two different speeds, since they were differently refracted when they emerged in air. It will be found that the ray having the greater velocity within the crystal has the lesser index of refraction, and *vice versa*. For calcite, in which the velocity of the extraordinary ray is greater than that of the ordinary ($E>O$), the refractive index of the former is less than that of the latter ($\epsilon<\omega$),[1] consequently, since it passes from a denser to a rarer medium, the angle of refraction is greater.

If, in the first experiment, the rhombohedron Rh_1 (Fig. 92) be used instead of the *Rh* rhombohedron, and it is rotated until the apices of the obtuse angles between the faces lie in the horizontal line—that is, until crystallographic *c* is horizontal—it will be found that but a single spot of light appears, just as it would do were the medium isotropic. If the rhombohedron is now rotated on crystallographic *c* as an axis, it will be found that the position of the spot does not change. There is, here, no double refraction, and to this direction the name **optic axis, axis of isotropy,** or **axis of no double refraction** has been given. If the rhombohedron be rotated in any direction other than about this axis, the distance between the refracted images will be found to increase to a maximum and then again decrease until the crystal has been rotated 90° from its former position, that is, until crystallographic *c* is vertical. In this position again but one spot appears, and the crystal may be rotated about the vertical axis with no apparent change. Although there is no bending apart of the rays in this direction, a change has, however, taken place, and one ray has been greatly retarded, as we shall see presently.

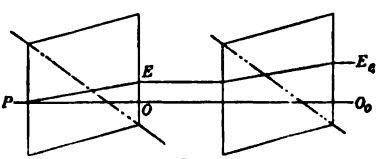


FIG. 100.—Double refraction through two calcite rhombohedrons of equal thickness and in the same orientation.

[1] Cf. Art. 40, *supra*.

If the second rhombohedron of calcite (Rh_1, Fig. 92) is of the same thickness as the first, and the two are so placed that their shorter diagonals are vertical, consequently having the orientation of crystallographic c the same, two bright spots of equal illumination, but separated by twice the former distance (Figs. 100 and 101), will again appear. If the second crystal is of a greater or less thickness than the first, the increase in the distance between the spots will be greater or less, the result, in every case, being as though the original rhombohedron had been thickened the amount of the second.

If, for convenience, we place the calcite rhomb Rh_1 (Fig. 92) between the one which can be rotated (Rh) and the screen, and in such a position that both of the rhombohedrons are parallel, and then rotate one (Rh) slightly either to the right or to the left, we shall find that four images, unequally illuminated, appear (Fig. 102). When the front calcite has been rotated 45°,

FIG. 101. FIG. 102. FIG. 103. FIG. 104. FIG. 105.

FIGS. 101 TO 105.—The bright spots as they appear upon rotating one of two calcite rhombohedrons

the four images will be equally illuminated (Fig. 103), there having been a gradual decrease in the brightness of the original spots, and a gradual increase in the other two. This decrease and increase in brightness continues as the second calcite is rotated still farther until, when the 90° position has been reached, the first two spots have disappeared and only the other two remain (Fig. 104). Upon farther rotation, the four spots again appear, and remain, with varying brightness, until the rhombohedron has been turned 180°, when but a single one is left (Fig. 105). Its brightness is double that of either of the first pair.

Upon an examination of the observed phenomenon, we see that two of the spots remain stationary, though with increasing or decreasing brightness, while around each as a center, another one revolves. From the alternate brightening and darkening of the spots, it is clearly evident that the light did not pass through the crystals as ordinary light, vibrating equally in every direction, for if it had, the illumination would have remained uniform. We can explain the phenomenon, however, if we consider the vibrations of the extraordinary and ordinary rays as taking place at right angles to each other.[1] Not only the appearances here seen, but all the phenomena of polarized light as well, may be explained by considering the vibrations of the extraordinary ray as taking place in the plane containing the ray of light and the shorter

[1] Examine the spots by means of a nicol prism held in the hand, and note their vibration directions.

diagonal of the face of the rhombohedron, and the vibrations of the ordinary ray as perpendicular to it, or *vice versa*. The former assumption, however, is the one commonly used. It is to be noted that this ambiguity is not separate and distinct for each kind of crystal. The relative polarizations can be determined, and if the direction of polarization is assumed to be in a certain direction in one crystal, it determines the direction in another.

Applying the knowledge of these vibration directions to the observed phenomenon, we have, in the case of two parallel rhombohedrons with the same orientation, vibrations taking place as shown in Fig. 106. The ray of light, originally vibrating in every direction (R), reaches the first rhombohedron and is polarized into two rays at right angles to each other (O and E). The plane of the paper, containing the ray of light and the shorter diagnonal of the rhombohedron, is the plane of vibration of the extraordinary ray—the vibrations being represented in the figure by the short parallel lines. The vibrations of the ordinary ray (O), taking place at right angles to the plane of the paper, are represented by dots.

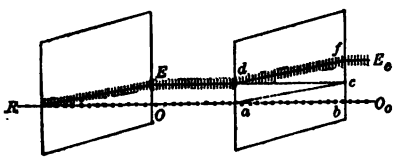


FIG. 106.—Vibration directions of light in passing through two parallel rhombohedrons of calcite in the same orientation.

After passing through the first calcite, the extraordinary ray is refracted back to its original course, and the two rays travel in parallel directions, but with vibrations perpendicular to each other, until they reach the second calcite (B). Consider first the ordinary ray. At the point a there is a tendency in the second rhombohedron to break the ray into two sets of vibrations perpendicular to each other. The calcite will permit a ray with horizontal vibrations to pass through in a direct line to O_o—the ordinary component of the first ordinary ray. It would also permit any vertical vibration to pass through in the refracted direction ac, but since no such component reaches a, only horizontal vibrations of O reach the eye.

A ray of light, on reaching d, tends to pass along the lines df and dc. Without refraction, only horizontal vibrations can pass through the crystals, but since there is no horizontal component in the ray which reaches d, no light passes to c. The extraordinary component of the extraordinary ray (E_e), however, can pass along the line df, for its vibrations take place in the plane of the paper. In consequence of the suppression of the extraordinary component of the ordinary ray (O_e) and the ordinary component of the extraordinary (E_o), only two spots reach the eye, one at O_o and one at E_e. In Figs. 107 to 118, vibration directions, lines of refraction, and spots of light are shown as they occur in the two crystals when looking through them at the hole in the screen from the right in Fig. 92. The solid lines are refraction directions, construction lines, etc., in the rhombohedron nearest

the eye (Rh_1); the dotted lines, in the rhombohedron which first refracts the light (Rh).

After passing through the first calcite only, the spots appear as in Fig. 107. (The short lines within the small circles indicate the vibration direction of the ray which produced that spot of light, and also show, by their number, the relative intensities of the illumination.) When the second cal-

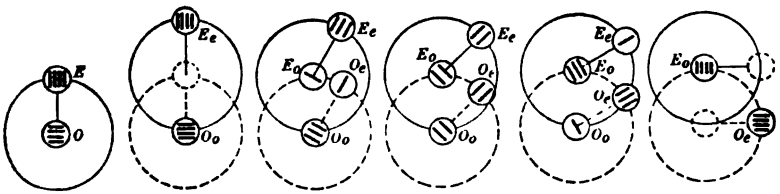


FIG. 107. FIG. 108. FIG. 109. FIG. 110. FIG. 111. FIG. 112.

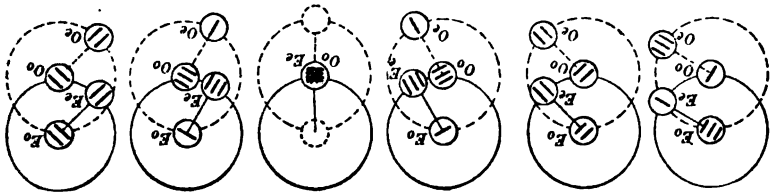


FIG. 113. FIG. 114. FIG. 115. FIG. 116. FIG. 117. FIG. 118.

FIGS. 107 TO 118.—Diagrams showing the double refraction in two calcite rhombohedrons, and the effect upon a spot of light when one rhombohedron is rotated.

cite rhombohedron is placed before it, the spots appear farther apart (Fig. 108), as explained above. When it is slightly rotated, four spots appear (Figs. 102 and 109), since each spot of light, upon reaching the second rhombohedron (Fig. 109), acts as a new center of disturbance and is doubly refracted, the amount of light passing through depending upon the angle which the vibration plane of the on-coming ray makes with the vibration planes of the second calcite. Let CO (Fig. 119) represent half the amplitude of the on-coming ordinary ray, and CE half the amplitude of the extraordinary ray. To pass through the second calcite, the light must vibrate along the new vibration planes CE_e and CO_o. To determine the amplitude of each of the new vibrations, we may resolve each of the original rays, by means of the usual parallelogram of forces, into two others acting at right angles to each other and along the new axes. Thus the ordinary ray, with half its amplitude of vibration equal to CO, will be broken up, in the second rhombohedron, into two rays with half amplitudes of CO_e and CO_o, and with vibrations parallel respectively to the

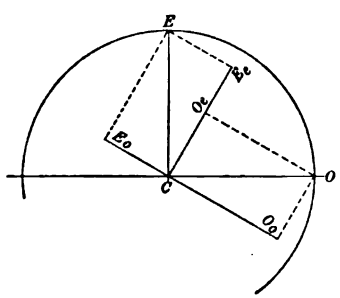


FIG. 119.—Resolution of vibrations.

short diagonal of the rhombohedron and at right angles to it. The extraordinary ray, having an original vibration of twice CE, will be resolved into vibrations having half amplitudes of CE_e and CE_o.

When the calcite rhombohedron has been rotated until its diagonals make angles of 45° with the diagonal of the first rhombohedron (Fig. 110), the

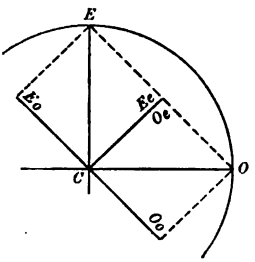


Fig. 120.

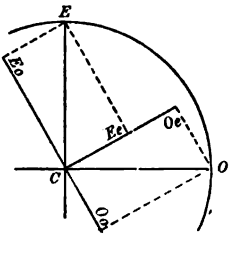


Fig. 121.

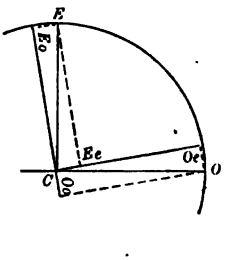


Fig. 122.

FIGS. 120 TO 122.—Amplitudes of resultant vibrations when the calcite rhombohedrons are rotated to various angles.

four spots will appear of uniform illumination if the vibration amplitudes of the original rays were equal, for the amplitudes of the resulting vibrations will also be equal (Fig. 120).

On still farther rotation, the spots which were brightest in the positions shown in Figs. 102 and 109 are dimmest (O_o and E_e, Fig. 111), and the spots dimmest in Fig. 109 are brightest in Fig. 111 (E_o and O_e). That such should be the case may be seen from the diagram of forces, Fig. 121.

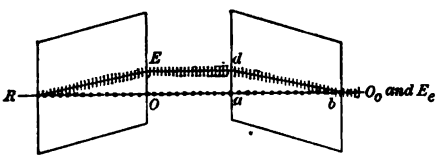


FIG. 123.—Vibration directions of light passing through two parallel rhombohedrons of calcite in opposite orientations.

When the revolution has reached almost 90°, the vibration amplitude of O_e has become nearly as great as O, and E_o nearly as great as E, while O_o and E_e have nearly become zero (Fig. 122). At 90° the vibration directions coincide, and the amplitudes of E_o and O_e are equal to O and E, while O_o and E_e have become zero, consequently but two spots appear (Figs. 104 and 112).

Continuing the rotation, the spots which were brightest previously become darker, and *vice versa* (Figs. 113–115), until at 180° the E_o and O_e components are zero and the other two are at their maxima. In this case, since the refraction of E_e is to a position above the original O, and O_o passes straight through at the same point, but one spot appears (Figs. 105 and 116). This is shown better in the cross-section (Fig. 123), where the vibrations are indicated by dots and lines in the same manner as they were in Fig. 106.

Experimentally the passage of the light in Figs. 107, 108, 112, and 116 may be shown by a model. In a wooden handle (Fig. 126) four spring-brass wires are set as shown at *b*. The amplitude of the vibration of the ordinary ray is represented by the distance between the wires *O* and *O*, and the amplitude of the vibration of the extraordinary ray, by *E* to *E*. Fig. 124 is a wooden block representing the first calcite rhombohedron. In it are bored, accurately parallel, two holes, *O* and *O*, perpendicular to the face of the rhombohedron, and two holes, *E* and *E*, at a conventional angle of 20° to 30°, representing the passage of the extraordinary ray; the

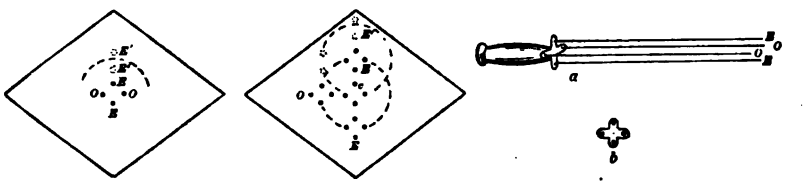


FIG. 124. FIG. 125. FIG. 126

FIGS. 124 TO 126.—Apparatus for demonstrating double refraction in two calcite rhombohedrons.

angle depending upon the thickness of the model, but so taken that the points of emergence of the holes on the lower side (*E'E'*, Fig. 124) will be well above the emergence of the *O* holes. Fig. 125 represents the second rhombohedron. In it are bored eight holes (*O*), perpendicular to the front face, and eight inclined holes (*E*), in the positions shown. A stiff wire through *C* (Figs. 124 and 125) serves as an axis of rotation, and, by keeping the rhombohedrons separated 2 or 3 in., the passage of the wires through the different sets of holes in parallel, 90° and 180° positions may be shown clearly, and the vibration directions demonstrated.

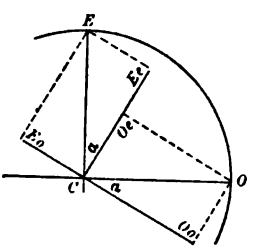


FIG. 127.—Intensity of emerging rays.

Another helpful model may be made by constructing two rhombohedrons of glass. Within the first, upon two other plates of glass fastened at right angles to each other, are painted the vibration directions and the directions of transmission of the ordinary and the extraordinary rays. In the second rhombohedron, other plates of glass, at right angles to each other, are fastened in such a way that painted lines upon them represent two sets of ordinary and extraordinary rays, the distance between them being so chosen that the two sets form the continuation of the rays from the first rhombohedron. By rotating the second rhombohedron through 0°, 90°, or 180°, the extinction of the proper spots will be brought out clearly.

The intensity of the light in any position of the rhombohedrons may be computed mathematically. It follows the same laws as light polarized by reflection (Art. 42), consequently Malus' law holds here also: The intensity varies as the square of the cosine (or sine) of the angle between the principal sections of the two rhombohedrons.

If, in Fig. 127, we represent the amplitude of CE by E, that of CE_o by E_o, etc., and consider the radius of the circle as unity ($CE = CO = 1$), then

$E_e = E \cos \alpha = \cos \alpha$,
$E_o = E \cos (90° - \alpha) = E \sin \alpha = \sin \alpha$,
$O_o = O \cos \alpha = \cos \alpha$,
$O_e = O \cos (90° - \alpha) = O \sin \alpha = \sin \alpha$.

But the intensity of light varies as the square of the amplitudes, therefore

Intensity of $E_e = \cos^2 \alpha$,
Intensity of $E_o = \sin^2 \alpha$,
Intensity of $O_o = \cos^2 \alpha$,
Intensity of $O_e = \sin^2 \alpha$.

From these equations it will be seen that, when the vibration amplitudes of O and E are equal, there will be but two light intensities for any angle, that is, the intensity of $E_e = O_o$ and of $E_o = O_e$.

49. Optic Axis in Uniaxial Crystals.—We saw, by rotating a calcite prism, that there is one direction, and one only, along which there is no double refraction.[1] This direction, or **optic axis,** as it is called, is parallel to crystallographic *c*. By examining various other minerals we would find that every crystal belonging to the tetragonal or the hexagonal (including trigonal) system, has, similarly, but one axis of isotropy, and that, in every case, it is parallel to crystallographic *c*. Crystals of these systems, consequently, are called **optically uniaxial.**

50. Principal Optic Section of a Uniaxial Crystal.—The principal optic section of a uniaxial crystal may be defined as the plane which contains the axes of greatest and least ease of vibration. Since the ease of vibration along crystallographic *c* must be either greater or less than along any other direction, the principal optic section must contain that axis. In the plane at right angles to crystallographic *c* the ease of vibration is the same in every direction, and its value is the minimum if *c* is the maximum, and *vice versa*. Since there may be any number of planes through crystallographic *c*, so also must there be innumerable principal optic sections.

51. Positive and Negative Uniaxial Crystals.—Biot[2] first recognized the fact that uniaxial crystals could be divided into two classes according to whether the index of refraction of the ordinary (ω) or of the extraordinary (ϵ) ray is the greater. For convenience of description, crystals in which the refractive index of the ordinary ray is the greater ($\omega > \epsilon$) are called **negative** (−), and crystals in which the reverse is the case ($\omega < \epsilon$) are called **positive** (+). Thus apatite, with $\omega = 1.638$ and $\epsilon = 1.634$, and calcite with $\omega = 1.658$ and $\epsilon = 1.486$ are negative, while quartz, with $\omega = 1.544$ and $\epsilon = 1.553$, is positive.

[1] Art. 48, page 61, *supra*.

[2] J. B. Biot: *Mémoire sur la découverte d'une propriété nouvelle dont jouissent les forces polarisantes de certains cristaux.* Mém. Acad. France, Année 1812, XIII (1814), pt. II, 19–30.

Since crystallographic c is always the direction of vibration of the extraordinary ray in uniaxial crystals, the rule may be stated, that if crystallographic c is the direction of vibration of the fastest ray, the crystal is negative; if it is the direction of the slowest ray, it is positive.

52. Velocity of Any Intermediate Ray in a Uniaxial Crystal.—If the maximum and minimum indices of refraction of a mineral, and, consequently, their wave velocities, are known, it is possible to compute the index of refraction and the ray and wave velocities in any other direction.

Since the velocity of the ordinary ray is the same in every direction, its

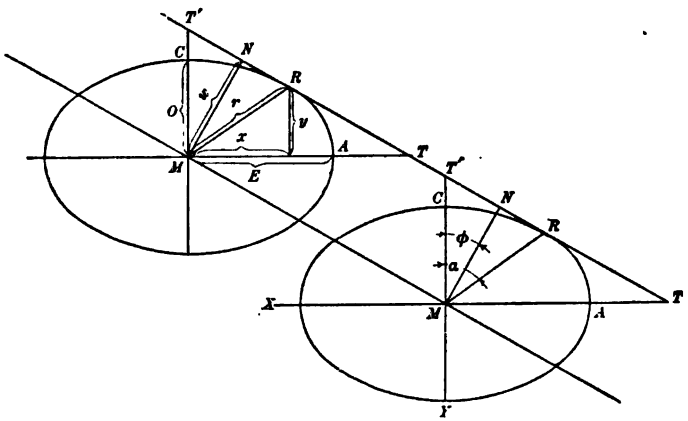


FIG. 128.—Ray and wave velocities.

index of refraction is likewise the same. The velocity of the extraordinary ray, however, differs in different directions, therefore the index of refraction of an intermediate ray will be different.

Let Fig. 128 represent a section through the extraordinary ray surface of a negative uniaxial crystal at the end of a unit of time (t).

Let r = the velocity of the desired ray (MR),

α = the angle which the desired ray makes with the c axis.

Then $MR = rt.$

But since t = unity, $MR = r.$

$CMR = \alpha$ = direction of propagation of the desired ray,

MC = direction of vibration of the ordinary ray,

MA = direction of vibration of the extraordinary ray,

$MC = Ot$, $MA = Et$.

But $t = 1$, therefore

$$MC = O, \quad MA = E.$$

From the equation of an ellipse[1] we have

$$O^2x^2 + E^2y^2 = O^2E^2. \tag{1}$$

[1] Eq. 83, Appendix.

We also have $\sin \alpha = \frac{x}{r}$, and $\cos \alpha = \frac{y}{r}$,

from which

$$x^2 = r^2 \sin^2 \alpha, \text{ and } y^2 = r^2 \cos^2 \alpha. \tag{2}$$

Substituting in (1)

$$O^2 r^2 \sin^2 \alpha + E^2 r^2 \cos^2 \alpha = O^2 E^2;$$

$$r^2 = \frac{O^2 E^2}{O^2 \sin^2 \alpha + E^2 \cos^2 \alpha} \tag{3}$$

This is an equation giving the velocity of a ray making an angle of α with the c axis of a uniaxial crystal. It is to be noted, however, that the index of refraction of the ray r is not $\frac{1}{r}$, as at first sight one might suppose, but is of a different value. This will be proved below.

53. Velocity of Any Intermediate Wave in a Uniaxial Crystal.—The velocity of the wave produced by the extraordinary ray is not the same as the velocity of the ray itself. Following Fresnel, one may consider a narrow cone of rays as aMc (Fig. 129). At the end of a unit of time, the light disturbance, arising at M, will have reached a, b, and c, and if the distance ac is small enough, the line abc will coincide with the tangent to the ellipse. In other words the ray front will be tangent to the ellipse.

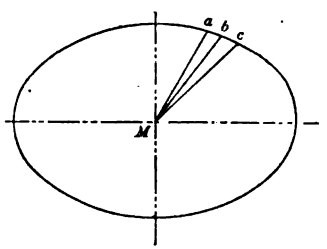


FIG. 129.—Fresnel's figure for showing that in a small cone of rays the ray front will be tangent to the ellipse.

If, now, instead of a single ray of light, we consider a series of parallel rays, MR, MR, Fig. 128, we will find that they will all reach the plane surface, of which $NRNR$ is the trace, at the same instant. The angle CMR will represent the direction of the refracted rays, and MR the distance traveled by them in a unit of time. The *wave front*, however, has only traveled from MM to NN, the normal MN representing the actual distance through which it has moved.

Let $MN = w$, the velocity of the *wave* produced by the rays r, r.

$\varphi = CMN$,

ϵ_1 = the index of refraction of the ray r.

Since the disturbance produced by the ray r results in forming a wave whose velocity is w, the index of refraction of this wave is the index of the ray producing this velocity, or

$$w = \frac{1}{\epsilon_1} \tag{4}$$

Substituting $E = a$ and $O = b$ in the general equation of the intercepts of the tangent to an ellipse, we have

$$MT = \frac{E^2}{x} \text{ and } MT' = \frac{O^2}{y}. \tag{5}$$

But

$$\sin \varphi = \frac{w}{MT} \text{ and } \cos \varphi = \frac{w}{MT'}. \tag{6}$$

Combining (5) and (6), we have

$$\sin\varphi=\frac{wx}{E^2} \text{ and } \cos\varphi=\frac{wy}{O^2},$$

or
$$x=\frac{E^2\sin\varphi}{w} \text{ and } y=\frac{O^2\cos\varphi}{w} \tag{7}$$

Substituting these values in (1), the equation of the ellipse, we obtain

$$\frac{O^2E^4\sin^2\varphi}{w^2}+\frac{E^2O^4\cos^2\varphi}{w^2}=O^2E^2 \tag{8}$$

or
$$E^2\sin^2\varphi+O^2\cos^2\varphi=w^2.$$

This is *the equation of the velocity of the wave front of the extraordinary ray of a uniaxial crystal in a medium having maximum and minimum velocities of E and O.*

Substituting in this equation

$$\sin\varphi=\frac{x'}{y},\ \cos\varphi=\frac{y'}{w},\ w^2=x'^2+y'^2 \tag{8a}$$

we obtain
$$E^2x'^2+O^2y'^2=(x'^2+y'^2)^2.$$

This is *the equation of the curve of the oval of the wave fronts of a uniaxial crystal.*

If we substitute in (8) the values

$$E=\frac{1}{\epsilon},\ O=\frac{1}{\omega},\ w=\frac{1}{\epsilon_1}.$$

we have
$$\frac{\sin^2\varphi}{\epsilon^2}+\frac{\cos^2\varphi}{\omega^2}=\frac{1}{\epsilon^2}. \tag{9}$$

or
$$\epsilon_1=\frac{\epsilon\omega}{\sqrt{\omega^2\sin^2\varphi+\epsilon^2\cos^2\varphi}}. \tag{9a}$$

This is *the equation of the index of refraction of a wave whose normal makes an angle φ with the c axis of a uniaxial crystal.* It is the equation of an ellipse which is the inverse of that of equation (8).

From Fig. 128 we have

$$\tan\alpha=\frac{x}{y}. \tag{10}$$

Substituting values from equations (7), we have

$$\tan\alpha=\frac{E^2\sin\varphi}{O^2\cos\varphi}=\frac{E^2}{O^2}\tan\varphi, \tag{11}$$

and, further, substituting the values $O=\frac{1}{\omega}$ and $E=\frac{1}{\epsilon}$, we have

$$\tan\alpha=\frac{\omega^2}{\epsilon^2}\tan\varphi. \tag{12}$$

This is an equation from which the angle φ may be obtained, knowing the maximum and minimum indices of refraction of the substance and the angle α.

54. Vibration Directions in Uniaxial Crystals.—Let Fig. 130 represent a principal section through a crystal of calcite, MY being parallel to crystallographic c. A ray of light P, entering the crystal at M, will be broken up

into two rays, an ordinary ray *MO*, and an extraordinary ray *ME*. At the end of a unit of time, the disturbance of the ordinary ray will have reached the point *O*, while at the same time that of the extraordinary ray will have reached *E*.

We have seen that the ordinary ray behaves as does ordinary light in an isotropic medium, consequently its vibrations will take place at right angles to the direction of propagation and, following Fresnel, perpendicular to a principal section. In Fig. 130, consequently, these vibrations are represented by the dots between *M* and *N*. The vibrations of the extraordinary ray take place at an angle to its line of propagation and in the plane of crystallo-

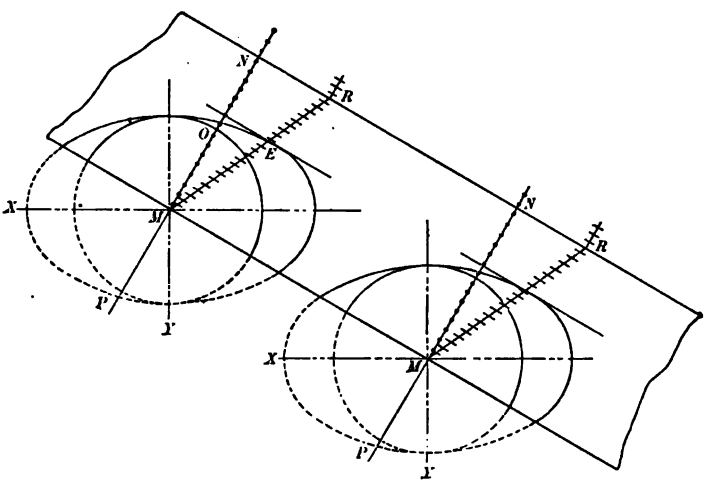


FIG. 130.—Vibration directions of light passing through an anisotropic medium.

graphic *c*, consequently they lie in the plane of the paper and are represented by the short lines. Whether the direction of vibration is at right angles to the ray (*ME*) itself, or to the normal (*MO*) to the wave front, is unknown. Fresnel[1] first assumed that it was perpendicular to the ray and therefore formed an angle with the wave front, but later[2] he decided that it was at right angles to the normal and thus formed an angle with the ray. The latter direction seems the more probable, and is the one assumed in the electromagnetic theory of light.[3]

[1] A. Fresnel: *Mémoire sur la double réfraction.* Read, Nov. 26, 1821. Mém. Acad. France, VII (1827), 45–176.

Idem: *Oeuvres complètes*, Paris, 1868, II, 287.

[2] Idem: *Ibidem*, II, 339. Read, Jan. 23, 1882.

[3] J. Clerk Maxwell: *Electricity and magnetism*, Oxford, 1881, II, 404.

R. T. Glazebrook: *On the application of Sir William Thomson's theory of a contractile æther to double refraction, dispersion, metallic reflexion, and other optical problems.* Phil. Mag., XXVI (1888), 521–540.

G. F. Fitzgerald: *Electromagnetic radiations.* Nature, XLII, 1890, 172–175.

55. Ray Surface and Wave Surface in Uniaxial Crystals.—If we compare equations (1) and (8*a*), we shall see that the former is the equation of an ellipse while the latter is that of an oval, differing slightly in form from the former and coinciding only along the diameters.

The *surface* reached by all *rays* at the end of a unit of time is known as the *ray surface*, that reached by the *waves*, the *wave surface*.

The form of the extraordinary ray surface, as we shall find, is an ellipsoid, which, in uniaxial crystals, is one of rotation, oblate for negative crystals, and prolate for positive.

The equation[1] of the curve of ray fronts $\frac{x^2}{E^2}+\frac{y^2}{O^2}=1$, is that of an ellipse. By making $x=\rho$, combining it with the functional equation of a surface of revolution ($\rho^2=x^2+y^2$), and changing the coordinates so that the Y axis extends from front to back and the Z vertical,

$$\frac{x^2}{E^2}+\frac{y^2}{E^2}+\frac{z^2}{O^2}=1. \tag{13}$$

This is *the equation of the ray surface of a uniaxial crystal.*

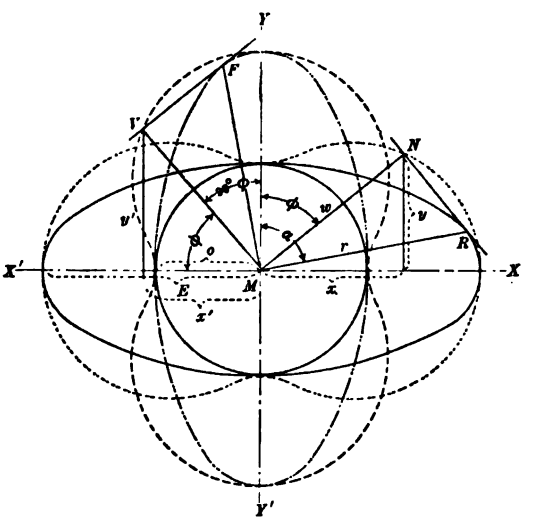


FIG. 131.—Ray surface (solid line), wave surface (dotted line), ease of vibration ellipsoid (broken line), and Fresnel ellipsoid (dot and dash line), in a negative, uniaxial crystal ($O<E$, $\omega>\epsilon$).

The wave surface of a uniaxial crystal is a surface of rotation. Fresnel considered it as developed by a system of plane waves starting at the same time from the center of a crystal, and traveling in different directions along the normals (*MN*, Fig. 131) with velocities depending upon the direction of propagation. At the end of any instant of time all of the waves will be tangent to it. The position of the point *N* of the dotted curve, which represents

[1] Eq. 1, Art. 52, *supra*.

the wave surface, corresponds to the point R of the curve representing the ray surface. At the points where x or $y = O$, that is, on the axes, the ray and normal coincide, consequently the two surfaces meet.

The equation of the wave fronts represents an oval of the form

$$E^2x^2+O^2y^2=(x^2+y^2)^2. \qquad \text{Eq. 8 Art. 53}$$

Combining with the functional equation of a surface of revolution, and changing the coordinates so that the Y axis extends from front to back and the Z axis is vertical, we have, after making $x=\rho$,

$$E^2(x^2+y^2)+O^2z^2=(x^2+y^2+z^2)^2, \qquad (14)$$

the *equation of the wave surface of a uniaxial crystal.*

Along each of the three axes the ray and wave surfaces coincide, for if we make, for example, y and z equal to zero, equations (13) and (14) alike become $E^2=x^2$.

The ray and wave surfaces of the ordinary ray coincide, and appear in section as a circle (Fig. 131). That they form a sphere in space may be proved by making the values of the ordinary and extraordinary rays equal in equations (13) and (14). The former becomes

$$x^2+y^2+z^2=E^2 \text{ or } O^2, \qquad (15)$$

and the latter

$$E^2(x^2+y^2+z^2)=(x^2+y^2+z^2)^2,$$

which equals

$$x^2+y^2+z^2=E^2 \text{ or } O^2. \qquad (16)$$

In each case, the equation is that of a sphere.

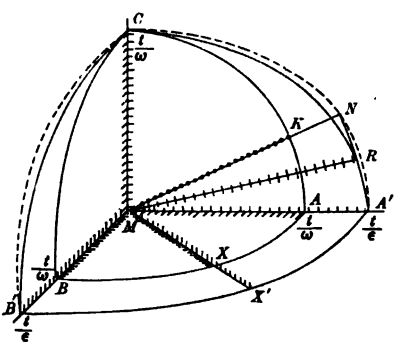


FIG. 132.—Propagation of light in a uniaxial crystal, forming uniaxial wave and ray surfaces.

56. Graphic Development of Ray and Wave Surfaces of a Uniaxial Crystal.—We may now develop the ray and wave surfaces graphically. Let MC, Fig. 132, be the c axis of a uniaxial negative crystal; all vibrations taking place parallel to the BMA plane, therefore, will be equal. Since the ease of vibration is a measure of the rate of propagation or velocity of a ray, we can determine the position of any ray front or wave front at the end of any instant of time.

Let O = velocity of the ordinary ray,
E = velocity of the extraordinary ray,
ω = index of refraction of the ordinary ray,
ϵ = index of refraction of the extraordinary ray.

We have already determined[1] that

[1] Art. 40, *supra.*

$$n=\frac{1}{v},\ O=\frac{1}{\omega}\text{ and }E=\frac{1}{\epsilon}.$$

If the mineral is negative, $\omega>\epsilon$, and $O<E$. $AM=MB=$direction of vibration of the ordinary ray (index $=\omega$). MC = direction of vibration of the extraordinary ray (index $=\epsilon$).

If a disturbance starts at M, there will be two rays of light passing from M to A. One, the ordinary ray, will vibrate parallel to MB. Its index of refraction $=\omega$, its velocity $=\frac{1}{\omega}$, and, at the end of any interval of time (t), it will have traveled along MA[1] a distance of $\frac{t}{\omega}$. In the same period of time, the other vibration, the extraordinary ray, vibrating parallel to MC with an index $=\epsilon$, will travel a distance of $\frac{t}{\epsilon}$. The vibrations are shown in the figure by the short vertical lines parallel to MC and along MA. Since $\omega>\epsilon$, $\frac{t}{\omega}<\frac{t}{\epsilon}$.

There will be, also, two rays traveling along MB; the ordinary ray, vibrating parallel to MA with an index of ω and traveling a distance of $\frac{t}{\omega}$, and the extraordinary ray, vibrating parallel to MC, with an index of ϵ, and traveling a distance of $\frac{t}{\epsilon}$.

In a similar manner there will be two rays propagated along MC, one ray with an index of ω, vibrating parallel to MA, and traveling a distance of $\frac{t}{\omega}$; and another with an index of ω, vibrating parallel to MB, also traveling a distance of $\frac{t}{\omega}$. That is, in this direction both rays will reach the eye at the same time, a fact which we had already ascertained by our examination of the calcite rhombohedrons.

So far we have considered the two rays vibrating along each of the three coordinate axes. As we have already seen,[2] along these axes the light ray and the normal to the wave front coincide since the tangents to an ellipse at the end of the axes lie at right angles to these axes; the tangents representing the directions of vibration, and the axes, the normals to the wave front and also the lines of propagation of the rays.

Consider now the plane AMB. Since the crystal under examination is uniaxial, all vibrations in this plane are equal, and any ray, as MX and MX', will reach distances of $\frac{t}{\omega}$ and $\frac{t}{\epsilon}$ at the end of the time t, whereby the ray

[1] In Fig. 130, to avoid confusion, the vibrations are shown, not bisected by the line MA, etc., but on one side only.

[2] Art. 55, *supra*.

fronts of all of the ordinary rays in the plane AMB will lie along AXB, a circle, and those of the extraordinary ray along $A'X'B'$, also a circle. Now a tangent to a circle is perpendicular to a radius, consequently the vibrations of all rays, both extraordinary and ordinary, act at right angles to the direction of transmission of the ray. Since this is the case, the normal to the wave front coincides in direction with the direction of propagation of the ray, whereby the curves of the ray and wave fronts are the same.[1]

Let us see what this means. If the normal to the wave front of the extraordinary ray coincides with the direction of propagation of the ordinary ray, the two rays must be propagated along the same line so that, if we were to look through a uniaxial crystal along any line in the plane AMB, that is, along any line perpendicular to the c axis, we should see but a single image. That such is actually the case we have already seen in the case of calcite. That the two rays do not reach the eye at the same time, however, and thus differ from the rays along crystallographic c, we can determine by a measurement of the retardation—a measurement, as we shall find later, which can be made under the microscope by means of polarized light.

In no other direction, however, do the two curves coincide. For example, a ray from M, traveling in the plane CMA, will be broken up into two rays, the ordinary ray MK with vibrations parallel to MB and at right angles to the direction of propagation, and the extraordinary ray MR with vibrations parallel to the tangent to its ray front, the ellipse CRA', at R.

The distance traveled in any direction in this plane by the ordinary ray in the time t will be $\frac{t}{\omega}$. Since the ray and the normal to the tangent lie along the same line, the ray front and wave front coincide. The extraordinary ray, however, travels a distance of t times the value of equation 3, Art. 52, and its surface is the ellipse given by this equation. Its major and minor axes are shown by MA' and MC, Fig. 132. The wave front (MN) travels a distance equal to $\frac{1}{\epsilon}$; its curve is given by equation (8) and is shown graphically by the broken line CNA', Fig. 132.

In the plane BMC, both ray front and wave front of the rays which have their vibrations at right angles to that plane (the ordinary rays) will lie on the circle CB. The rays whose vibrations lie within the BMC plane (the extraordinary rays) have for their ray front the curve shown by the solid line between C and B', an ellipse, while their wave front is shown by the dotted curve between the same points, an oval.

From this construction we can see that the two rays, the ordinary and the extraordinary, may be considered as forming two double surfaces. The *ray surface of the ordinary ray* is a sphere (whose equation is given by equation 15) of which $CABC$ (Fig. 132) is the part appearing in the upper front right-

[1] Art. 55, *supra*.

hand octant. This form of surface was to have been expected, since the ordinary ray acts like a ray of light in an isotropic substance, in which the vibrations are equal in all directions. The *ray surface of the extraordinary ray* has for its section in AMB, a circle, while the sections in CMB and CMA are similar ellipses. In any other plane, as CMX, the vertical section of the extraordinary ray is also a similar ellipse, consequently the ray surface is an ellipsoid of rotation (proved by equation 13). The *wave surface of the extraordinary ray* has for its section in AMB the same circle as the ray surface, therefore the two rotation surfaces coincide along this line. In the planes CMB or CMA, however, or in any intermediate plane, the vertical section of the wave front of the extraordinary ray is an oval, consequently the wave surface is a spheroid of rotation (proved by equation 14).

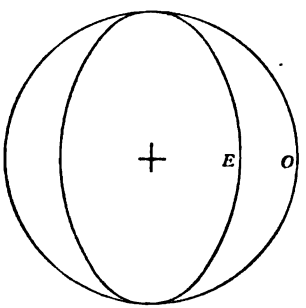


FIG. 133.

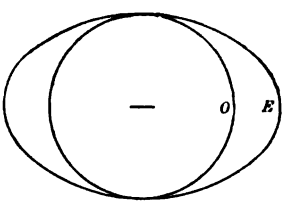


FIG. 134.

FIGS. 133 TO 134.—Wave surfaces of positive and negative uniaxial crystals. O = ordinary ray, E = extraordinary ray.

In the case considered, the value of ω was taken as greater than that of ϵ, and in the surfaces developed the sphere lies within the ellipsoid or spheroid, which is oblate (Fig. 134). In positive crystals, with $\omega < \epsilon$, a prolate ellipsoid or spheroid lies within the sphere (Fig. 133).

57. Curve of Ease of Vibration (Fresnel's Curve of Elasticities).—The distance which a ray or a wave travels depends upon the ease of vibration in a direction at right angles to the normal, that is, the greater the ease of vibration in a certain direction, the greater the velocity of the wave advancing at right angles to this direction. Thus in Fig. 131, a wave advancing from M to N and a ray from M to R will have their vibrations in the direction of MV.

$$VMY = 90^\circ - \varphi.$$

Using this value for the angle in the equation to the curve of wave fronts (8), we have

$$E^2 \sin^2 (90^\circ - \varphi) + O^2 \cos^2 (90^\circ - \varphi) = w^2,$$

or

$$E^2 \cos^2 \varphi + O^2 \sin^2 \varphi = w^2.$$

But

$$\sin \varphi = \frac{x_1}{w}, \quad \cos \varphi = \frac{y_1}{w}, \quad x_1^2 + y_1^2 = w^2$$

which, substituted in (17), gives

$$E^2\frac{y_1^2}{w^2}+O^2\frac{x_1^2}{w^2}=w^2$$

or
$$E^2y_1^2+O^2x_1^2=(x_1^2+y_1^2)^2. \tag{18}$$

This is the equation of an oval exactly like the oval of wave fronts but with its long axis at right angles to the long axis of the latter. It is shown in Fig. 131 by the broken line. *This curve gives both the ease and the direction of vibration anywhere within a crystal.*

Combining (18) with the functional equation of a surface of revolution, and using z for the vertical axis, we obtain a surface like that given in (14) except that it has its major and minor axes at right angles to the major and minor axes of the latter. Its axis of rotation is the axis extending from left to right. Since $\rho=x$

$$O^2(x_1^2+y_1^2)+E^2z_1^2=(x_1^2+y_1^2+z_1^2)^2. \tag{19}$$

It is the equation of a spheroid and is the surface of ease of vibration.

58. Fresnel's Ellipsoid.—The Fresnel ellipsoid is a simple ellipsoid of one surface which has for its three axes the maximum, minimum, and mean ease of vibration. In uniaxial crystals the mean ease coincides with one of the other values, and the figure is an ellipsoid of rotation. Along the axes, this ellipsoid coincides with the single surface ease of vibration spheroid (equation 19). It bears the same relation to the ray surface as does the ease of vibration spheroid to the wave surface, and is shown in Fig. 131 by the solid line *YFY'*. The line *MF* does *not* represent the vibration direction of the ray *MR*.

The equation of the Fresnel ellipsoid may be obtained as follows:
The inverse of the equation of the ray front curve (1) is

$$\frac{x^2}{O^2}+\frac{y^2}{E^2}=1$$

which, changed to coordinates with the z axis vertical, and combined with the functional equation of a surface of revolution, becomes

$$\frac{x^2}{O^2}+\frac{y^2}{O^2}+\frac{z^2}{E^2}=1. \tag{20}$$

It is the equation of the Fresnel ellipsoid.

59. The Optical Indicatrix.—Another method of representing the optical characters of crystals is by means of a figure based upon its indices of refraction. This surface of reference was called the ellipsoid of indices by MacCullagh, ellipsoïde des élasticitiés by Fresnel, indexellipsoid by Liebisch, and optical indicatrix, or simply indicatrix, by Fletcher.[1]

In Fig. 135 let *Mr* represent the direction of propagation and the velocity of a ray of light.

[1] L. Fletcher: *The optical indicatrix and the transmission of light in crystals.* London, 1892, 20.

MC and MA, and Mr and MR are two pairs of conjugate radii.

Draw RN perpendicular to Mr.

From the property of an ellipse, we have: The area of a parallelogram circumscribing an ellipse in which the sides are tangent to the ellipse at the vertices of a pair of conjugate diameters, is constant and equal to the rectangle constructed on the axes.

Therefore $MA \cdot MC = Mr \cdot RN = k$, a constant;

hence $$Mr = \frac{MA \cdot MC}{RN} = \frac{k}{RN}, \text{ or } RN = \frac{k}{Mr}.$$

That is, RN and Mr are inversely proportional to each other, no matter what the direction of the ray Mr.

Now $$n = \frac{1}{v},$$

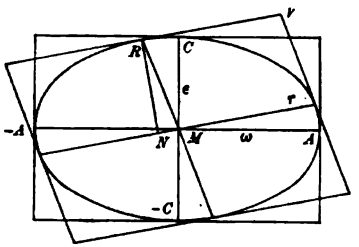


FIG. 135.—Geometrical relations in an ellipse.

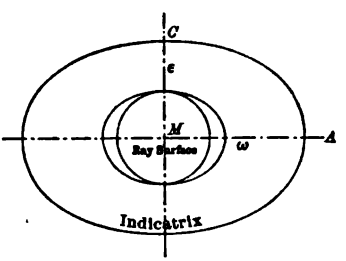


FIG. 136.—Relation between indicatrix and ray surface.

therefore, if Mr represents the distance traveled by the light in the time k,

$$n = \frac{k}{Mr}.$$

But $$n = \epsilon \text{ or } \omega, \text{ and } Mr = E \text{ or } O = \frac{k}{\epsilon \text{ or } \omega}.$$

If the crystal under consideration is uniaxial and negative, $E > O$ and $\epsilon < \omega$, consequently

$$Mr \text{ (the major axis)} = E, \text{ and } RN = O;$$

also $$MA = E = \frac{k}{\epsilon} \quad \text{and} \quad MC = O = \frac{k}{\omega}.$$

That is, if MA (or Mr) represents the velocity of a ray of light, the normal from the vertex of its conjugate CM (or RN) will represent its index of refraction multiplied by a constant.

The indicatrix, then, is an ellipsoid of rotation whose major radius is equal to its maximum index of refraction, and whose minor axis is equal to its minimum index of refraction.

For example (Fig. 136), if $\omega = 1.8 = MA$, and $\epsilon = 1.2 = MC$, at the end of a unit of time

$$O=\frac{1}{1.8},\quad E=\frac{1}{1.2},$$

as is shown by the inner ellipse. In the ordinary ray $\omega=\epsilon=1.2$ and $O=E=\frac{1}{1.2}$, as shown by the inner circle.

The indicatrix for a positive crystal will be prolate instead of oblate.

Analytically the equation of the index surface may be obtained as follows:

The values of the major and minor axes of the curve of indices in a negative crystal are ω for the major and ϵ for the minor axis. Substituting these values in the equation of an ellipse we have

$$\epsilon^2x^2+\omega^2y^2=\epsilon^2\omega^2,$$

or

$$\frac{x^2}{\omega^2}+\frac{y^2}{\epsilon^2}=1,$$

which, combined with the functional equation of a surface of rotation gives, after changing the vertical axis to z,

$$\frac{x^2}{\omega^2}+\frac{y^2}{\omega^2}+\frac{z^2}{\epsilon^2}=1.$$

This is the equation of the optical indicatrix of a negative uniaxial crystal.

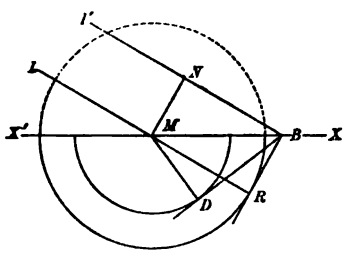


FIG. 137.—Refraction of light in an isotropic medium.

60. Huygens' Construction for Double Refraction in Uniaxial Crystals.—Refraction of light in an isotropic medium may be shown in another manner:

In Fig. 137 let I and I' be two parallel rays of light meeting the surface $X'X$ at M and B. When the ray I is at M, I' is at N. If I were free to go on without change of velocity it would reach the point R when I' reached B.

Draw a circle with M as a center and NB as a radius. The wave front RB is evidently tangent to it at R. Now draw, with M as a center, a circle having a radius of MD, equal to the distance traveled by the ray in the denser medium in the same length of time that is required to travel from N to B in air. The line DB, drawn through the point B and tangent to the circle at D, will evidently give the wave front of the ray in the denser medium, and the radius MD, perpendicular to DB, the direction of propagation of the ray.

In an anisotropic uniaxial crystal, as we have seen, light is separated into two rays. We may have several cases:

I. THE OPTIC AXIS IS PERPENDICULAR TO THE PLANE OF INCIDENCE

a. *The crystal is negative.* Let Fig. 138 represent a crystal of calcite with its c axis perpendicular to the plane of the paper. In this case the

sections through the ray surfaces will appear as two circles, as shown in the drawing, the radii being proportional to the velocities of the ordinary and extraordinary rays, that is, inversely proportional to their indices. The wave fronts of the two rays, I and I', after passing into the second medium, will lie on lines passing through B and the point of tangency of their respec-

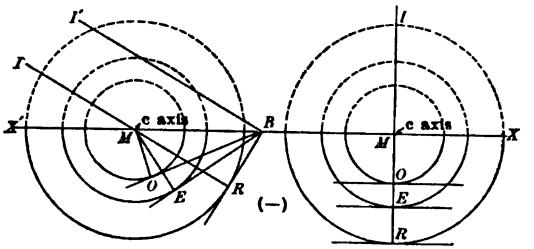


FIG. 138. FIG. 139.

FIG. 138.—Refraction of light in a negative, uniaxial crystal with the plane of incidence perpendicular to the optic axis.

FIG. 139.—Refraction of light in a negative, uniaxial crystal with the plane of incidence perpendicular to the optic axis and with the incident ray perpendicular to the reflecting surface.

tive circles. Since in calcite $E>O$ and $\epsilon<\omega$, the shorter radius MO (O being the point of tangency) will be the direction of propagation of the ordinary ray, and the longer radius ME will be the direction of propagation of the extraordinary ray.

If the incident ray is normal to the reflecting surface (Fig. 139), the tangents to the three circles are parallel to each other and at right angles to the ray.

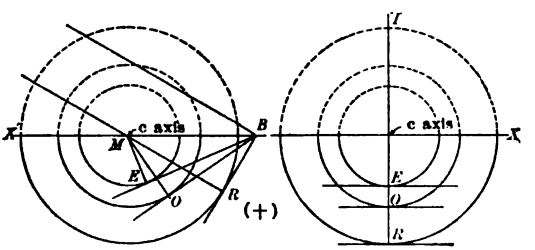


FIG. 140. FIG. 141.

FIG. 140.—Refraction of light in a positive, uniaxial crystal with the plane of incidence perpendicular to the optic axis.

FIG. 141.—Refraction of light in a positive, uniaxial crystal with the plane of incidence perpendicular to the optic axis and with the incident ray perpendicular to the reflecting surface.

From these two cases we see that the ordinary ray is more deflected than the extraordinary ray, in negative crystals, when the plane of incidence is perpendicular to the optic axis. An exception occurs in the case of normal incidence where neither ray is deflected, although the extraordinary ray travels faster than the ordinary.

b. *The crystal is positive.* In positive crystals, with $O > E$ and $\omega < \epsilon$ the length of the radius of the ordinary ray surface is greater than that of the extraordinary (Fig. 140), consequently, by the same construction as above, it is seen that the extraordinary ray is more deflected than the ordinary ray. When the incident ray is perpendicular to the reflecting surface there is no

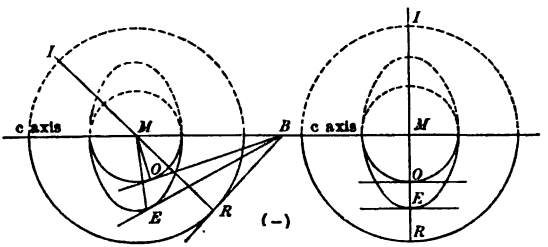


FIG. 142. FIG. 143.

FIG. 142.—Refraction of light in a negative, uniaxial crystal with the optic axis parallel to both the plane of incidence and the reflecting surface.

FIG. 143.—Refraction of light in a negative, uniaxial crystal orientated as in the preceding figure but with the incident ray perpendicular to the reflecting surface.

deflection (Fig. 141), but the ordinary ray reaches the eye sooner than the extraordinary.

II. THE OPTIC AXIS LIES IN THE PLANE OF INCIDENCE AND IS PARALLEL TO THE REFLECTING SURFACE

a. *Negative Crystals.*—In a negative crystal, when the optic axis lies in the plane of incidence and is parallel to the reflecting surface, the extraordi-

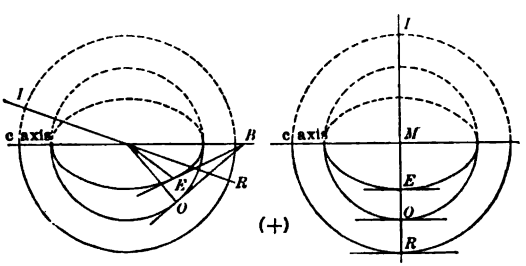


FIG. 144. FIG. 145.

FIG. 144.—Refraction of light in a positive, uniaxial crystal with the optic axis parallel to both the plane of incidence and the reflecting surface.

FIG. 145.—Refraction of light in a positive, uniaxial crystal orientated as in the preceding figure but with the incident ray perpendicular to the reflecting surface.

nary ray is deflected more than the ordinary, as may be seen from Fig. 142. When the incident ray is normal to the reflecting surface neither ray is deflected, but the extraordinary ray reaches the eye before the ordinary (Fig. 143).

b. *Positive Crystals.*—In a positive crystal the ordinary ray is more deflected than the extraordinary (Fig. 144). When the incident ray is normal to the reflecting surface neither ray is deflected, but the ordinary ray reaches the eye before the extraordinary (Fig. 145).

III. The Optic Axis Lies in the Plane of Incidence and is Perpendicular to the Reflecting Surface

a. Negative Crystals.—In a negative crystal, when the optic axis lies in the plane of incidence but is perpendicular to the reflecting surface, the ordinary ray is deflected more than the extraordinary (Fig. 146). When the incident ray is normal to the reflecting surface, there is neither deflection nor retardation, both rays reaching the eye at the same time (Fig. 147).

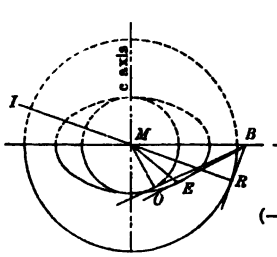


FIG. 146.

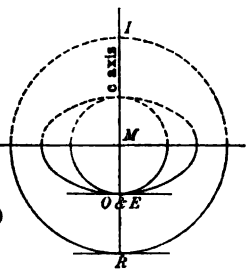


FIG. 147.

(−)

FIG. 146.—Negative, uniaxial crystal, optic axis vertical.
FIG. 147.—Negative, uniaxial crystal, optic axis vertical, incidence normal.

b. *Positive Crystals.*—In a positive crystal with its optic axis perpendicular to the reflecting surface, the extraordinary ray is deflected more than the ordinary (Fig. 148). When the incident ray is normal to the reflecting surface, there is neither deflection nor retardation, both rays reaching the eye at the same time (Fig. 149).

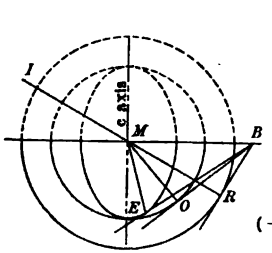


FIG. 148.

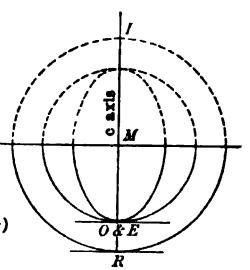


FIG. 149.

(+)

FIG. 148.—Positive, uniaxial crystal, optic axis vertical.
FIG. 149.—Positive, uniaxial crystal, optic axis vertical, incident ray normal to reflecting plane.

IV. THE OPTIC AXIS LIES IN THE PLANE OF INCIDENCE BUT FORMS AN ANGLE WITH THE REFLECTING SURFACE

When the optic axis is inclined to the reflecting surface, the relative refraction of the ordinary and the extraordinary rays varies with the inclination of the axis. This may be seen by an inspection of Figs. 150 and 151, which show the refraction of various rays in a negative uniaxial crystal.

A comparison of the variations found when the optic axis is parallel, perpendicular, or inclined to the reflecting surface is shown in Fig. 152.

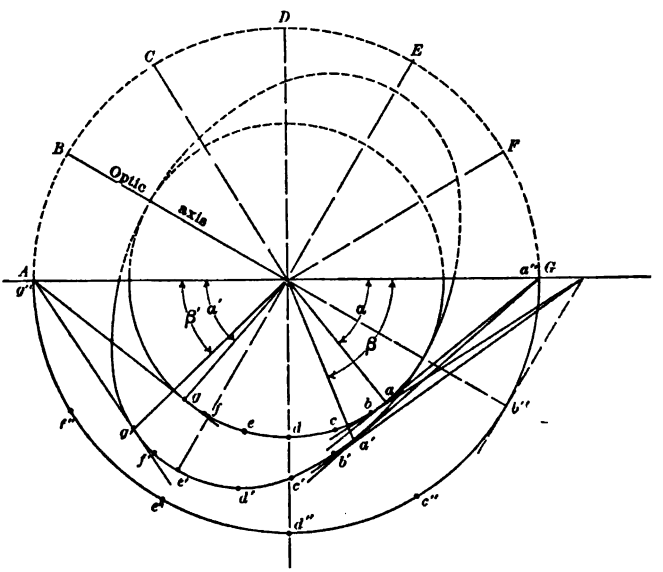


FIG. 150.—Negative, uniaxial crystal, optic axis inclined and lying in the plane of incidence which is perpendicular to the reflecting surface. The optic axis lies beyond the critical angle. α and α', complements of the critical angle for the ordinary ray; β and β', complements of the critical angle for the extraordinary ray.

Points from *A* to *G* represent the rays *A* to *G* (Figs. 150 and 151), and horizontal distances, the deflection to the right or left from the direction of the incident ray measured on any circle. Deflection in the direction of the movement of the hands of a watch (negative deflection) is shown to the left of the vertical line, while deflection in the opposite direction (positive deflection) is shown to the right (Fig. 152). The vertical line represents the direction of the incident ray.

A comparison of Figs. 150 and 151 with 152 will show the following facts:

1. THE ORDINARY RAY.—No matter how the crystal is oriented, the deflection of the ordinary ray (curve *OO*, Fig. 152) remains the same. Light

entering between A and G (Figs. 150–151) will be refracted within a space equal to double the critical angle, that is, a space measured on either side of the normal (Dd) by the critical angle. Rays between 0° and 90° (A to D) will be deflected in the negative direction, the 90° ray (D) will pass straight through, and rays between 90° and 180° (D to G) will be deflected in the positive direction. The critical angle for the ordinary is the same on either side of the normal ($AO=GO$, Fig. 152) and the curve is a straight line, crossing the vertical at D (90°) where there is no deflection.

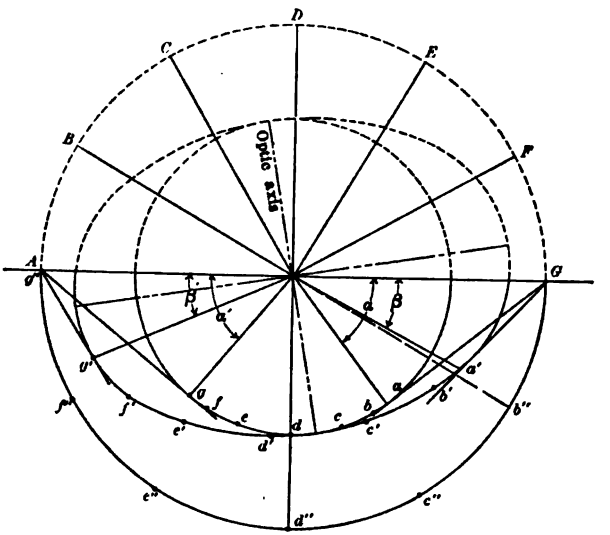


FIG. 151.—Negative, uniaxial crystal, optic axis inclined and lying in the plane of incidence which is perpendicular to the reflecting surface. The optic axis falls within the critical angle. α, α', β, and β' represent the same angles as in the preceding figure.

II. THE EXTRAORDINARY RAY.—1. *The optic axis lies in the plane of incidence and is parallel to the reflecting surface* (Fig. 142 and line E-142, Fig. 152). The extraordinary ray is symmetrically deflected when the optic axis is parallel to the reflecting surface. The maximum deflection on either side is at the critical angle, all other rays being deflected between these two points. For rays entering between 0° and 90° (A to D) the deflection is negative, the 90° ray passes straight through, and between 90° and 180° the rays are deflected in a positive direction. In every case the extraordinary ray is more deflected than the ordinary ray, as we have already seen (Fig. 142).

2. *The optic axis lies in the plane of incidence and is perpendicular to the reflecting surface* (Fig. 146 and line E–146, Fig. 152). When the optic axis is perpendicular to the reflecting surface, the light also is symmetrically deflected. The maximum value is at the critical angle. The 90° ray is not

deflected. Between 0° and 90° the deflection is negative, between 90° and 180°, positive. In every case the extraordinary ray is less deflected than the ordinary (cf. Fig. 146).

3. *The optic axis lies in the plane of incidence but forms an angle with the reflecting surface.* When the optic axis is inclined to the normal at more than the critical angle (Figs. 150 and lines *E*–150, E_{nw}, and E_{ne}, Fig. 152), the extraordinary ray is more or less deflected than the ordinary, depending upon the position of the optic axis.

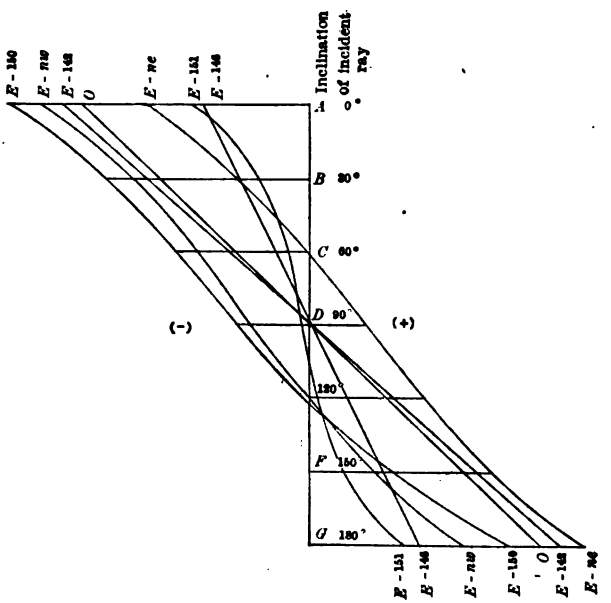


FIG. 152.—Diagram showing deflection to the right or left of the incident rays of Figs. 142, 146, 150 and 151. The ordinary ray is uniformly deflected, irrespective of the position of the crystal. E_{ne} and E_{nw} show deflections in the crystal when it is placed with its optic axis respectively at 45° to the normal to the reflecting surface in the upper right quadrant and in the upper left quadrant. Deflection is considered negative when in a counter-clock-wise direction.

When the optic axis is inclined to the normal at less than the critical angle (Fig. 151 and line *E*–151, Fig. 152) there will be some point where both the ordinary and the extraordinary rays are equally deflected. This position is reached when the refracted ray lies along the optic axis. In this position there is no double refraction, although the ray is deflected except when the incident ray is at 90°. The position of no double refraction is shown in Fig. 152 where the *O* and *E*–151 curves cross; the relative amounts of deflection are indicated by the distances to the left (or right) of the vertical.

It is to be noted that wherever an "*E*" curve crosses the vertical line,

Fig. 152, there is no deflection of the extraordinary ray, but it passes straight through the crystal.

Besides the deflection curves given above, many others may be constructed for positions of the optic axis when it does not lie in the plane of incidence.

For positive crystals the phenomena above described are reversed, since in such crystals the velocity of the ordinary ray is greater than that of the extraordinary.

61. Summary of the Optical Properties of Uniaxial Crystals.—From our examination of the effect of uniaxial crystals upon the passage of light through them, we have derived the following data:

1. There are certain substances which differ from isotropic media in that the light, in passing through them, is broken up into two rays vibrating at right angles to each other. Such substances are called **anisotropic** (Arts. 47 and 48).

2. Of the two rays of light resulting from the double refraction of a ray passing through a uniaxial crystal, one vibrates with equal ease in every direction. This is called the **ordinary ray** and it is represented by the letter O. Its ray surface is a sphere and its index of refraction is denoted by ω.

3. The second ray is called the **extraordinary ray.** It is indicated by the letter E and its index of refraction by ϵ. It passes through the crystal with different velocities in different directions, agreeing only in one direction with the ordinary ray. In this direction, consequently, there will be no double refraction, since the values of ω and ϵ are equal; in other words, this is an **axis of isotropy.** In uniaxial crystals this axis always coincides with crystallographic c.

4. Crystals belonging to the tetragonal and hexagonal systems are uniaxial. In them the axis of greatest or least ease of vibration coincides with crystallographic c, the optic axis. All rays forming the same angle with this axis have equal velocities, consequently any section of the ray surface at right angles to this axis is a circle, while every other section is an ellipse.

5. The ray surface consists of two sheets or shells representing the velocities of the ordinary and the extraordinary rays. The former is a sphere, the latter an ellipsoid of rotation (Art. 55).

6. The wave surface consists of two sheets or shells representing the velocity of transmission of the waves produced by the ordinary and the extraordinary rays. The direction of propagation of a wave is represented by the normal to a tangent to the ray front, while the direction of propagation of a ray is represented by a radius (Art. 53). These two directions coincide for the ordinary ray, since the ray surface is a sphere and the normal to the tangent at the surface is the radius. The directions of

propagation being the same, the wave and ray surfaces likewise coincide. The direction of the normal in the ellipsoid of the ray surface is not the same as the radius at the point of tangency, consequently the extraordinary ray surface is not the same as its wave surface. The former is an ellipsoid of rotation, the latter a spheroid of rotation differing slightly from an ellipsoid (Art. 55).

7. There are two kinds of uniaxial crystals. In one the extraordinary ray has its greatest velocity along the principal axis; in the other this is the direction of least velocity. The form of the ray surfaces are as shown in Figs. 133 and 134, depending upon whether the velocity of $O > E$ ($\omega < \epsilon$), or $O < E$ ($\omega > \epsilon$). The former are called positive (+) crystals, the latter negative (−) (Art. 51).

8. A principal optic section is one containing the axes of greatest and least ease of vibration. In uniaxial crystals any section parallel to crystallographic axis c is a principal section (Art. 50).

CHAPTER VI

ANISOTROPIC MEDIA (Continued)

Optically Biaxial Crystals

62. Vibration Axes.—We have seen, in uniaxial crystals, that light vibrates in one plane with equal ease in every direction, and that at right angles to this direction is the position of maximum or minimum ease of vibration. With a system of three coordinates we would have, then, $x=y \lesseqgtr z$, where x and y are the horizontal components and z the vertical.

There is another class of crystals in which the ease of vibration, and consequently the indices of refraction, differ in different directions, therefore $x \lesseqgtr y \lesseqgtr z$. In a given direction in a crystal this ease of vibration is constant, and the three chief vibration axes, always at right angles to each other (Art. 47), are indicated by the German letters $\mathfrak{a}$, $\mathfrak{b}$, and $\mathfrak{c}$;[1] $\mathfrak{a}$ being considered the direction of greatest ease, $\mathfrak{b}$ the direction of intermediate ease, and $\mathfrak{c}$ the direction of least ease of vibration ($\mathfrak{a}>\mathfrak{b}>\mathfrak{c}$). The velocities of light corresponding to these axes vary inversely as their respective indices of refraction, as has been shown above (Art. 40), and since a ray is propagated in a direction at right angles to the direction of its vibrations, along these axes it will advance fastest when it is vibrating parallel to $\mathfrak{a}$ and slowest when parallel to $\mathfrak{c}$. The planes, always at right angles to each other, in which these three principal vibration axes intersect, are called the **principal optic sections** of the biaxial crystal.

The index of refraction of the ray with vibrations parallel to $\mathfrak{a}$, and advancing at right angles to it, is represented, in biaxial crystals, by α. Taking the velocity in vacuum as unity, its velocity is equal to $\frac{1}{\alpha}$. The index of re-

[1] X, Y, and Z were substituted for $\mathfrak{a}$, $\mathfrak{b}$, and $\mathfrak{c}$ by Iddings, but this causes confusion when writing equations in which these letters are used also for coordinate axes, as in Art. 288. Wright (*The index ellipsoid [optical indicatrix] in petrographic microscopic work*, Amer. Jour. Sci., XXXV (1913), 133–138) suggests abandoning the "elasticity ellipsoid" and the symbols for the "axes of elasticity" in the explanation of the phenomena of light in crystals. He would use instead only the indicatrix and the symbols for the refractive indices, regarding the use of other symbols as bewildering to the student. The writer's experience has been that students can grasp, much more readily, the idea of an ease (or difficulty) of vibration in a certain direction in a crystal, and a corresponding rate of movement at right angles to it, than they can the inverse relation of the refractive indices. The writer long ago abandoned the terms "axes of elasticity" and substituted for them "ease of vibration axes." In his *Determination of rock-forming minerals*, 1908 (pages 5, 6, 7, 9, 11, 12, 13, 19, 21, 22, 24, etc., etc., except page 8 where the former term was inserted by an oversight) he invariably used the latter.

fraction of the ray with vibrations parallel to b and advancing at right angles to it, is represented by β. Its velocity is equal to $\frac{1}{\beta}$. The index of refraction of the ray with vibrations parallel to c and advancing at right angles to it, is represented by γ. Its velocity equals $\frac{1}{\gamma}$.

The positions of the vibration axes vary in different crystals. In orthorhombic crystals the vibration axes coincide with the crystallographic axes. In monoclinic crystals one vibration axis coincides with crystallographic *b*, the other two lie in the plane of *a* and *c* but are inclined to these axes except in one special case where a principal vibration axis coincides with *a* or *c*. In triclinic crystals, in general, no vibration axis coincides with a crystallographic axis, although in special cases one axis of vibration may coincide with one crystallographic axis.

63. Fletcher's Indicatrix.[1]—As in uniaxial crystals, so in biaxial, we may represent the ease of vibration,[2] the refractive indices, and the ray and wave surfaces by geometrical figures. Of these the indicatrix and the ray and wave surfaces are the most important.

Since the indices of refraction, in biaxial crystals, differ in different directions, and the directions of maximum and minimum ease of vibration lie at right angles to each other (therefore their indices of refraction likewise), and a direction of intermediate ease lies at right angles to the other two, we may represent the indices of refraction in any direction in a crystal by a triaxial ellipsoid. Thus we may construct an ellipsoid (Fig. 153) such that $MA = \alpha$, $MB = \beta$, and $MC = \gamma$.[3] In such a figure there are only three planes of symmetry, namely the $CBC'B'$, the $A'CAC'$, and the $A'BAB'$ planes, and these are the principal optic sections of the biaxial crystal.

In a triaxial ellipsoid every section is an ellipse, consequently $A'CAC'$ (Figs. 153 and 154) is an ellipse, and between C and A there will be all values of radii vectores between α and γ, the two semi-axes. Since $a > b > c$ always, it is necessarily also true that $\alpha < \beta < \gamma$, for the values of the velocities and the indices of refraction are inversely proportional. There will be, for the radius vector MP, some value between MA (α) and MC (γ) equal to β (intermediate between α and γ). We will thus have, as a section of the triaxial ellipsoid, an ellipse whose semi-axes are $BM = \beta$ and $MP = \beta$, that is, a circle (Fig. 153). Likewise, between A' and C, there will be a radius vector equal to β and, consequently, a radius of another circular section.

[1] L. Fletcher: *The optical indicatrix and the transmission of light in crystals*, London, 1892, 20.

[2] Ease of vibration spheroid, Fresnel's surface d'élasticité (not Fresnel's ellipsoid). This is a single sheet, triaxial ellipsoid whose equation is $(x^2+y^2+z^2)-(a^2x^2+b^2y^2+c^2)=0$.

[3] It is to be noted that the directions of α, β, and γ are not necessarily parallel respectively to the crystallographic axes *a*, *b*, and *c*. α may be in the direction of *a*, or *b*, or *c*, and the others likewise.

No other section besides these two can be circular, for any section at a greater or less angle than that of the MP section will have a greater or less value for its radius vector in the MCA plane, while in the $BCB'C'$ plane there can be no radius vector equal to the axis MA $(=\alpha)$, which lies at right angles to the plane, for the value of the latter is less than the minor axis of the $BCB'C'$ ellipse.

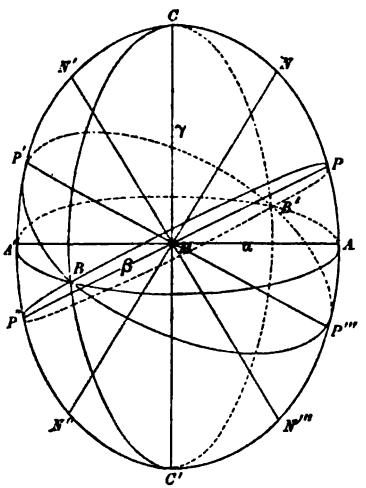


FIG. 153.—A triaxial ellipsoid (indicatrix).

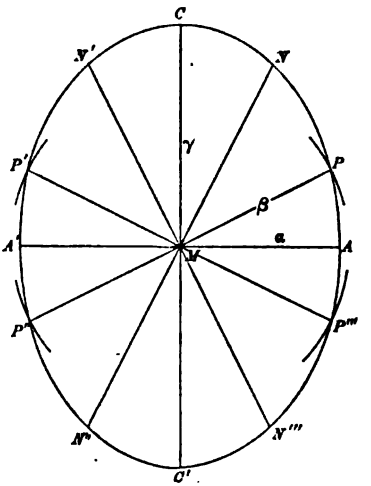


FIG. 154.—Section through the indicatrix.

We can have, then, in a triaxial ellipsoid, two sections, and two only, which are circular. They both contain the intermediate axis $(MB=\beta)$ and lie symmetrically with respect to the other two.

Every cross-section of the ellipsoid represents the indices of refraction of the rays whose vibrations are parallel to it and whose wave fronts advance normal to it. In the circular sections the difference between the refractive indices in different directions is zero, consequently in these sections the velocities of the waves will be the same, and they will suffer no double refraction. The normals NN'' and $N'N'''$ (Figs. 153–154) are called the **primary optic axes** or **optic binormals.** All crystals of the orthorhombic, monoclinic, and triclinic systems have two primary optic axes, and are, consequently, called **biaxial.**

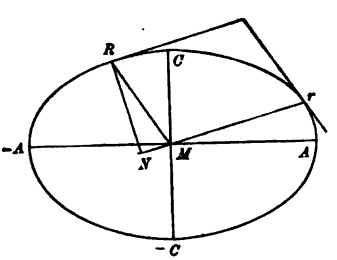


FIG. 155.—Section through the optical indicatrix of a biaxial crystal.

The index surfaces previously described, namely the sphere and the ellipsoid of rotation, are but special cases of the triaxial ellipsoid, for if $\beta=\alpha$ or $\beta=\gamma$, the figure will become one of rotation, and if $\alpha=\beta=\gamma$, it will become a sphere.

Just as in the indicatrix of uniaxial crystals, so also in that of biaxial crystals does Mr (Fig. 155) represent the direction of propagation of the ray, and $\frac{1}{RN}$ its velocity; consequently $Mr=\frac{1}{RN}$.

Equation of the indicatrix. Analytically we have the general equation of a triaxial ellipsoid whose semi-axes $MA=\alpha$, $MB=\beta$, and $MC=\gamma$:

$$\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=1 \tag{1}$$

or, substituting $a=\frac{1}{\alpha}$, $b=\frac{1}{\beta}$, $c=\frac{1}{\gamma}$, we have

$$a^2x^2+b^2y^2+c^2z^2=1. \tag{2}$$

64. The Ray Surface of a Biaxial Crystal.—We can develop the ray surface of a biaxial crystal in the same way that we developed the ray surface of a uniaxial crystal.

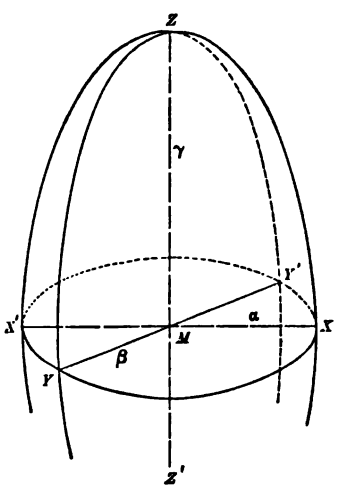


FIG. 156.

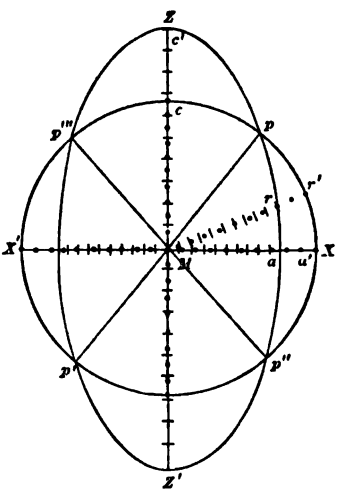


FIG. 157.

FIGS. 156 AND 157.—The indicatrix and a section through the ray surface along the MZX plane. Scale of indicatrix one-sixth of the scale of the ray surface.

Let Fig. 156 represent part of the indicatrix of a biaxial crystal in which $\gamma>\beta>\alpha$. For convenience of drawing let $MZ=\gamma=4$, $MY=\beta=3$, and $MX=\alpha=2$. These are the indices of refraction along the three principal axes and are called the **principal indices of refraction.**

Let us consider first the rays vibrating in the plane MZX, Fig. 157. From the point M two rays will travel in the direction of X, one of which has its vibration direction parallel to MZ. Now the index of refraction of the ray which vibrates parallel to MZ is γ (Fig. 156), consequently the velocity

of the ray advancing in the direction of X will be $\frac{1}{\gamma}$ because $v=\frac{1}{n}$. In a unit of time, with the values assumed above, since $vt=\frac{t}{n}$, the distance traveled by the ray will be $Ma=1/4$. (In Fig. 157, 6 cm. is taken as unity, consequently $Ma=1/4=15$ mm.) The other ray advances from M toward X with vibrations parallel to MY, with an index of refraction equal to β, and therefore a velocity of $\frac{1}{\beta}$, and travels a distance equal to $\frac{t}{\beta}$. When $t=1$, the distance equals $\frac{1}{\beta}$ or 20 mm.

At the same time that the rays of light are traveling from M to X, other rays travel from M toward Z. One ray, with vibrations parallel to MX, will have an index of refraction of α and will advance a distance of $Mc'=\frac{1}{\alpha}$ in the direction of Z. $Mc'=\frac{1}{\alpha}=1/2=30$ mm. The second ray, in the same direction, will have its vibrations parallel to MY. Its index of refraction being β, the distance traveled in a unit of time will be $\frac{1}{\beta}=1/3=20$ mm. (Mc).

In the directions MX' and MZ' the two rays will advance the same amounts but in opposite directions from MX and MZ.

In any intermediate direction in the plane MXZ, such as Mrr', one ray will have its vibrations constantly parallel to MY, irrespective of the direction of its transmission. Its velocity will be uniformly $\frac{1}{\beta}$ and the distance traveled in a unit of time will be $\frac{1}{\beta}=1/3=20$ mm. Since the velocity is the same in every direction in the plane MXZ, the ray front will be a circle. The other ray, whose vibrations lie in the plane MXZ, will advance with a velocity varying as the radii vectores of the ellipse whose major and minor semi-axes are γ and α. The vibrations, being always parallel to the tangent to the ellipse at the extremity of the ray (cf. Art. 54) will be at right angles to the ray only along the axes MX and MZ.

Completing the ray front for the plane $XZX'Z'$ we will have two curves; a circle, representing the front of the rays whose vibrations are at right angles to the plane of reference, and an ellipse, representing the front of the rays whose vibrations lie within that plane. The vibrations in the latter are, in general, not at right angles to the ray.

In like manner in the plane MYZ (Fig. 156), which is represented as rotated into the plane of the paper in Fig. 158, there will be two rays from M advancing toward Y. One, with vibrations parallel to MZ and an index of refraction of γ, will advance with a velocity of $\frac{1}{\gamma}$, equal, in the case cited, in a unit of time, to a distance of $Mb=1/4=15$ mm. The other, with vibra-

tions parallel to MX and an index equal to α, will advance with a velocity equal to $\frac{1}{\alpha} = 1/2$, or a distance, in a unit of time, of $Mb' = 1/2 = 30$ mm.

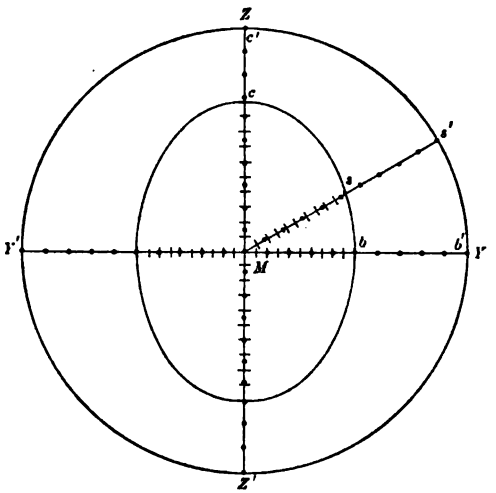


FIG. 158.

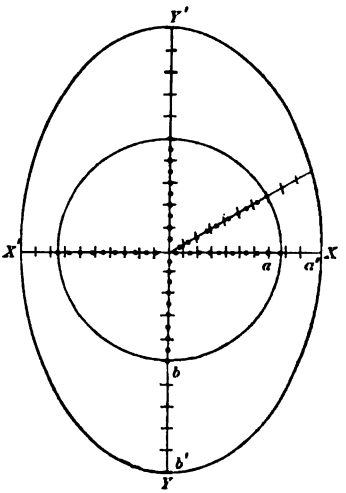


FIG. 159.

FIGS. 158 AND 159.—Sections through the ray surface along the MYZ and MXY planes. Scale same as Fig. 157.

In the direction of MZ one ray will have its vibrations parallel to MY, its index of refraction equal to β, its velocity $\frac{1}{\beta}$, and its distance of transmission

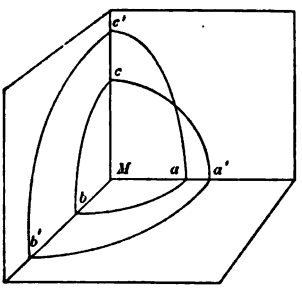


FIG. 160.

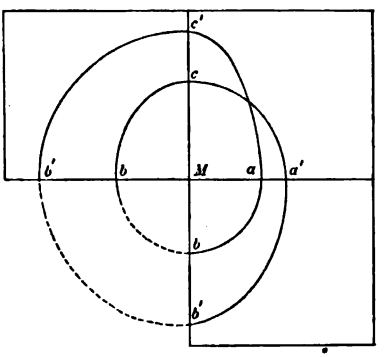


FIG. 161.

FIGS. 160 AND 161.—Form of the ray surface developed on a hinged blackboard.

$\frac{t}{\beta}$, which equals $1/3$, in a unit of time, or 20 mm. (Mc) on the scale adopted in the figures. The other ray vibrates parallel to MX, has an index of α, a

velocity of $\frac{1}{\alpha}$, and travels a distance of 1/2 or 30 mm. (*Mc'*). In any intermediate direction, as *Mss'*, there will likewise be two rays. The one with vibrations perpendicular to the plane *MYZ* will advance with an index of α and a velocity of $\frac{1}{\alpha}$, and this velocity will be uniform in every direction in the section, since its vibrations remain parallel to the same line, consequently its ray front will be a circle. The other ray, with vibrations lying in the plane *MZY*, will have velocities varying as the radii vectores of the ellipse having axes of γ and β.

In the horizontal plane, *MYX* of Fig. 156, which is represented as rotated into the plane of the paper in Fig. 159, all of the rays having vibrations parallel to *MZ* will advance, in a similar manner, a distance equal to $\frac{t}{\gamma} = 1/4 = 15$ mm. (*Ma* and *Mb*). The ray front, consequently, is a circle. The rays whose vibrations lie within the *XY* plane will advance with different velocities, and, consequently, different distances in different directions in the plane. The ray advancing along *MX* will have vibrations parallel to *MY*, a refractive index equal to β, a velocity of $\frac{1}{\beta}$, and, in the time $\frac{t}{\beta}$, will travel a distance of $\frac{t}{\beta}$, equal, in a unit of time, to $\frac{1}{\beta} = 1/3 = 20$ mm. (*Ma'*). Intermediate rays, as before, will reach the ellipse whose semi-diameters are α and β.

The form of the ray surface may be made clearer by working it out on three planes at right angles to each other, as shown in perspective in Figs. 160 and 161. A blackboard, hinged at the joints, or even part of a cigar box, may be used. Vibration directions, perpendicular to the plane, are shown by pins, while vibration directions lying in the plane are shown by marks on the board.

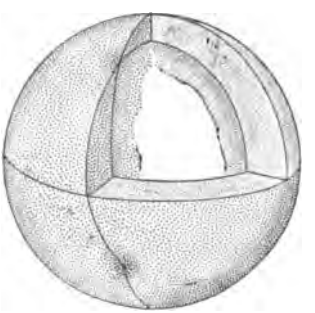


FIG. 162.—Plaster model of a biaxial ray surface.

If, now, we consider the form of the solid which has been developed, we will see that it differs decidedly from the symmetrical ellipsoid of rotation of uniaxial crystals. It is a warped surface such as is shown in Fig. 162, symmetrical along the three principal axes and having four depressions lying in a single plane.

The equation of the ray surface of a biaxial crystal.[1]

The form of the ray surface may be expressed by the equation

$$\frac{x^2}{r^2 - a^2} + \frac{y^2}{r^2 - b^2} + \frac{z^2}{r^2 - c^2} = 1, \qquad (3)$$

[1] L. Fletcher: *The optical indicatrix, etc.*, p. 37, §7.

or, substituting $r^2=x^2+y^2+z^2$, we have

$$\frac{x^2}{x^2+y^2+z^2-a^2}+\frac{y^2}{x^2+y^2+z^2-b^2}+\frac{z^2}{x^2+y^2+z^2-c^2}=1. \qquad (4)$$

Equation of the velocity of any ray in a biaxial crystal.[1]

The equation of the velocity of any ray (r, Fig. 155) whose normal NR intersects the indicatrix at a point represented by x_1, y_1, and z_1 is

$$x^2+y^2+z^2=r^2=a^4x^2_1+b^4y^2_1+c^2z^2_1. \qquad (5)$$

This is the equation of any radius vector of the ray surface corresponding in direction with the line Mr of the indicatrix.

65. The Wave Surface of a Biaxial Crystal.—We saw, in the development of the wave surface of a uniaxial crystal, that it differed slightly from the ray surface. In the former the surface is not an ellipsoid of rotation, and in the latter it is. By the same construction we may develop the wave surface of a biaxial crystal.

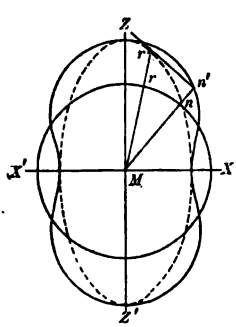


FIG. 163.

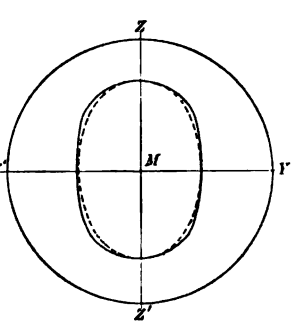


FIG. 164.

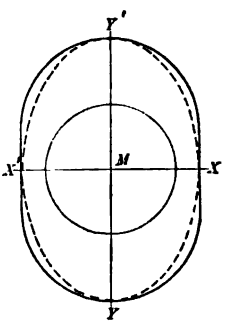


FIG. 165.

FIGS. 163 TO 165.—Principal sections through the wave surface.

Let the dotted lines, Fig. 163, represent a principal section through the *ray* surface along the MXZ plane. The ray Mr', with vibrations within the plane, will reach the ray surface at r'. The *wave* front of all rays progressing parallel to Mr' will lie along the tangent $r'n'$ (cf. Art. 52), consequently the intersection of the normal Mn' with the tangent at n' will be a point on the wave surface. The whole curve of wave fronts, shown as a solid line in Fig. 163, may thus be constructed. For any ray Mr, whose vibrations take place at right angles to the MXZ plane, the direction of the ray and the normal to the tangent coincide (cf. Art. 55), and the wave front for those rays is a circle.

Likewise, in the planes MYZ and MXY, the curve of wave fronts may be constructed as shown in Figs. 164 and 165. The solid resulting from all of

[1] Idem: *Ibidem*, p. 36, §4.

the wave fronts is similar in form to the ray surface, but does not coincide with it.[1]

The equation of the wave surface of a biaxial crystal.[2] Analytically the ray surface may be expressed by the equation

$$\frac{x^2}{r^2-a^2}+\frac{y^2}{r^2-b^2}+\frac{z^2}{r^2-c^2}=0, \tag{6}$$

or, substituting $x^2+y^2+z^2=r^2$, we have,

$$\frac{x^2}{x^2+y^2+z^2-a^2}+\frac{y^2}{x^2+y^2+z^2-b^2}+\frac{z^2}{x^2+y^2+z^2-c^2}=0. \tag{7}$$

66. Optic Biradials or Secondary Optic Axes.—Let us consider a little more fully both ray and wave surfaces. If we examine Fig. 157 we shall see that

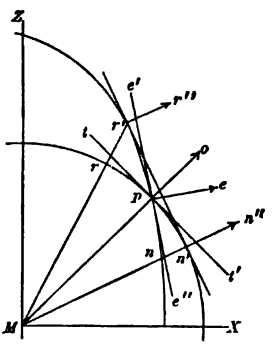


FIG. 166.—Section through one-fourth of the ray surface.

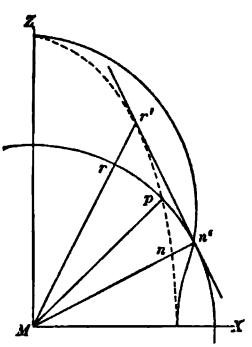


FIG. 167.—Section through one-fourth of the wave surface.

there are four points, p, p', p'', and p''', where the two ray fronts intersect. Since these curves represent the velocities of the rays, obviously along the lines $p'Mp$ and $p''Mp'''$ (Fig. 157), within the crystal, the rays will travel together without double refraction. Upon emerging, however, the waves advance normal to the tangents to the wave surfaces. Now the tangent to the circle at p (Fig. 166) is tt', while the tangent to the ellipse is $e'e''$, and the two waves, not having a common front, are doubly refracted upon emerging, consequently two light waves advance in the directions po and pe. The lines Mp, Mp', etc., along which the two rays advance with equal velocity, are called the **secondary optic axes, optic biradials,**[3] or **lines of single ray velocity.**[4]

[1] An illustration of a plaster model of the wave surface is given in Rosenbusch-Wülfing, *Mikroskopische Physiographie*, I–1, 93.

[2] L. Fletcher: *Op. cit.*, 60, §39.

[3] Idem: *Op. cit.*, 43–44.

[4] Sir William Hamilton: *Third supplement to an essay on the theory of systems of rays.* Read Jan. 23 and Oct. 22, 1832. Trans. Roy. Irish Acad., Dublin, XVII (1837), 1–144, in particular, 132.

67. Optic Binormals or Primary Optic Axes.—We have already seen that in the optical indicatrix there occur two sections which are circular (Art. 63), and for each ray having vibrations parallel to it a wave will advance in a direction at right angles to it. Identical in direction with the normals to the circular sections are certain lines in the wave surface. If we examine the *MXZ* section of the wave surface (Fig. 167), we will see that the only place where two waves, with vibrations at right angles to each other, coincide, is where the oval crosses the circle at *n'*. This point is located by the normal to the wave front which is produced by the ray advancing in the direction of *r*, and therefore represents the direction of transmission of the wave produced by that ray. The particular ray with which we are concerned is the one whose front is tangent to the circle as well as to the ellipse. In other words it is the ray which causes a wave to advance along the same line, and with the same velocity, as the wave produced by another ray whose vibrations take place at right angles to the section.

But a plane, tangent to the ray fronts at *r'* and *n'* (Fig. 167), will also be tangent to the ray surface (Fig. 162) in other points, namely, in a continuous circle[1] having a diameter of *r'n'*. Since the wave normal *Mn'* forms a right angle with the tangent *r'n'*, the latter being the trace of the base of the cone *r'Mn'* whose apex is at *M*, it follows that *all rays refracted from M to r' or to any other point lying in the periphery of the base of the cone, must be represented by a wave advancing along the common normal Mn'*. The line *Mn'*, consequently, represents the only line along which more that one wave advances, and is, therefore, at right angles to the circular section of the indicatrix. The normal to the circular section and the normal to the wave fronts coincide. It is the **primary optic axis,** also called the **optic binormal,**[2] **line of single normal velocity,**[3] or **axis of single wave velocity.**

In all the preceding figures showing indices of refraction or velocities of light, the differences in different directions have been greatly exaggerated over those which occur in nature. As a matter of fact, the ellipses actually do not depart greatly from circles, consequently the difference between the ray surface and the wave surface of a crystal is not great. Likewise the primary and secondary optic axes, represented by *Mn* and *Mp*, Fig. 167, nearly coincide, the difference between their directions being rarely over one degree. When simply optic axes are mentioned, the primary optic axes are usually understood.

68. Interior Conical Refraction.—From the sections of the ray surface cut by the three principal planes, Figs. 157, 158, and 159, we saw that each consists of an ellipse and a circle having the same center, but that in only one, the *XZ* plane (Fig. 157) do the two intersect in four points, *p*, *p'*, *p''*, *p'''*.

[1] See Article 68.

[2] L. Fletcher: *Op. cit.*, 62–63.

[3] Sir William Hamilton: *Op. cit.*, 132.

Above each of these points a tangent to both the circle and the ellipse may be drawn, as shown in one quadrant in Fig. 166, $r'n'$. Now these tangents are the traces of planes which not only touch the wave surface at the points r' and n', but in a continuous line, which was shown by Sir William Hamilton[1] to be a circle.

We will thus have formed an oblique cone having a circular base whose diameter is $r'n'$, and an altitude of $n'M$. Not only is $n'M$ the altitude, but it also forms one of the lines of the cone extending from the apex to the circumference of the base. Since $n'M$ not only represents the direction of transmission of the wave produced by the ray Mr', but also of those produced by all other rays progressing from M to any point on the ray surface where this is touched by the tangent plane (contact a circle), the sum of all these rays will represent the curved surface of a cone whose base is a circle with a diameter of $r'n'$ and whose altitude is $n'M$. If, then, a section be cut from a biaxial crystal so that the two faces are at right angles to the line $n'M$ and a beam of light be made to enter at M, it will pass through the crystal as the cone $r'Mn'$ and emerge as a cylinder with circular cross section $r'r''$, $n'n''$. If, on the other hand, the beam has a diameter of $r'n'$, and enters from without, it will converge to the apex at M. This property of biaxial crystals is called **interior** (or **internal**) **conical refraction.**

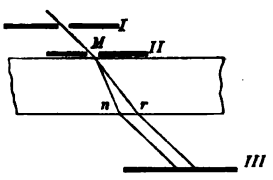


FIG. 168.—Lloyd's experiment to show internal conical refraction.

This cone of light was shown experimentally by Lloyd[2], who passed a fine beam of light along the optic axis (Mn, Fig. 168) of a plate of aragonite. Two thin metal screens, I and II, pierced by small holes, one screen at a little distance and the other in contact with the plate, were used to regulate a narrow beam of light, and the emerging ray was examined on the screen III. When the angle of incidence was other than that required to refract the one ray along the optic axis Mn, two spots of light were seen upon the screen. The crystal was moved very slowly, and the instant the light fell

[1] Sir William Hamilton: *Op. cit.*

See also Th. Liebisch: *Physikalische Krystallographie*, Leipzig, 1891, 341–345.

[2] Rev. Humphrey Lloyd: *On the phænomena presented by light in its passage along the axes of biaxial crystals.* Phil. Mag., II (1833), 112–120.

Idem: *Ueber die Erscheinungen beim Durchgange des Lichts durch zweiaxige Krystalle längs deren Axen.* Translation of preceding. Pogg. Ann., XXVIII (1833), 91–104.

Idem: *Further experiments on the phænomena presented by light in its passage along the axes of biaxial crystals.* Phil. Mag., II (1833), 207–209.

Idem: *Fernere Versuche über die Erscheinungen beim Durchgange des Lichts durch zweiaxige Krystalle längs deren Axen.* Translation of preceding. Pogg. Ann., XXVIII (1833), 104–108.

Idem: *On the phenomena presented by light in its passage along the axes of biaxial crystals.* Read Jan. 28, 1833. Trans. Roy. Irish Acad., Dublin, XVII (1837), 145–157.

at the right angle of incidence, the two spots immediately united and formed a continuous ring of light. Upon varying the distance of screen *III* from the crystal, no enlargement of the ring was observed, showing the cylindrical form of the emerging beam.

69. Exterior Conical Refraction.—If a section be cut from a biaxial crystal so that the two parallel faces are normal to the line Mp (Fig. 166), and a ray of light be passed along the line Mp, it will emerge in the cone formed by the perpendiculars to the planes tt' and $e'e''$. Conversely, a cone of light ope, entering along the secondary optic axis, will pass through along the single line pM. This phenomenon is called **exterior** (or **external**) **conical refraction** and also was shown experimentally by Lloyd.

70. Optic Axial Angle, True and Apparent.—As we have already seen (Fig. 154), if a circle, having a radius of β, is drawn with M as a center, the

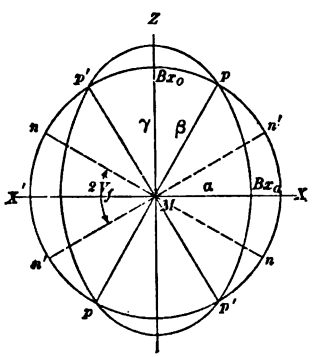


FIG. 169.—Optic axial angle and bisectrices.

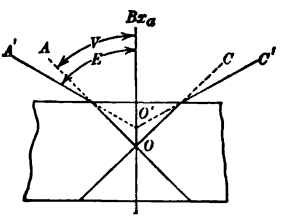


FIG. 170.—True and apparent optic axial angle.

lines connecting M with the point where the circle cuts the ellipse, represent the traces of the two circular sections. Perpendiculars, MN and MN', erected to this plane, represent the optic axes of the crystal.

Let Fig. 169 represent a section through the indicatrix of a biaxial crystal, and let $MZ=\gamma$, $MX=\alpha$, and $Mp=\beta$. Then Mp and Mp' will represent the traces of the circular sections, and Mn and Mn', normal to these planes, the primary optic axes. One optic axis coincides, in direction, with the line Mn' of Figs. 166 and 167.

The angle $n'Mn$ (Fig. 169) between the two optic axes is called the **optic angle** or **axial angle.** It is indicated by the symbol $2V$, and is the **true optic angle** in contradistinction to the **apparent optic angle** in air, which is indicated by the symbol $2E$. The relation between the two is shown in Fig. 170 in which AOC is $2V$, and $A'O'C'$, the apparent angle in air, is $2E$. Sometimes the optic angle is determined by immersion in water, oil, etc. In this case the angle is indicated by $2H$.

71. Equations Expressing the Value of the True Axial Angle.—Analytically the optic angle may be computed if the values of the refractive indices are known.

The equation of the indicatrix (Eq. 1, Art. 63) is

$$\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=1. \tag{1}$$

For any point in the elliptical section, as p, Fig. 169, $y=0$, and

$$\frac{x^2}{\alpha^2}+\frac{z^2}{\gamma^2}=1. \tag{8}$$

The equation of the circle whose radius $Mp=\beta$ is

$$x^2+z^2=\beta^2, \text{ or } x^2=\beta^2-z^2. \tag{9}$$

Combining (8) and (9) we have,

$$\frac{\beta^2-z^2}{\alpha^2}+\frac{z^2}{\gamma^2}=1, \text{ or } z^2=\frac{\gamma^2(\beta^2-\alpha^2)}{\gamma^2-\alpha^2}. \tag{10}$$

The coordinates of p being x and z, we have

$$\sin pMX=\frac{z}{\beta}, \text{ or } z=\beta \sin pMX. \tag{11}$$

Substitute in (10),

$$\beta^2 \sin^2 pMX=\frac{\gamma^2(\beta^2-\alpha^2)}{\gamma^2-\alpha^2}$$

or

$$\sin^2 pMX=\frac{\gamma^2(\beta^2-\alpha^2)}{\beta^2(\gamma^2-\alpha^2)}. \tag{12}$$

But $pMX=90^\circ-nMX=90^\circ-V_f$, and

$$\sin^2 pMX=\sin^2(90^\circ-V_f)=\cos^2 V_f=\frac{\gamma^2(\beta^2-\alpha^2)}{\beta^2(\gamma^2-\alpha^2)}. \tag{13}$$

It is to be noted that the value here given for V_f is one-half the acute optic angle for a negative crystal; in other words, it is the angle between one optic axis and the fast vibration direction (α). Had the other vibration axis (γ) been chosen, the ormula would have been:

$$\cos^2 V_s=\frac{\alpha^2(\gamma^2-\beta^2)}{\beta^2(\gamma^2-\alpha^2)}. \tag{13a}$$

The equation of the sine may be similarly expressed. Equation (9) may be written

$$z^2=\beta^2-x^2, \tag{14}$$

which, substituted in (8), gives

$$\frac{x^2}{\alpha^2}+\frac{\beta^2-x^2}{\gamma^2}=1, \text{ or } x^2=\frac{\alpha^2(\gamma^2-\beta^2)}{\gamma^2-\alpha^2}. \tag{15}$$

But $\cos pMX = \frac{x}{\beta}$, from which $x = \beta \cos (90° - V_f) = \beta \sin V_f$. (16)

Substitute in (15)

$$\beta^2 \sin^2 V_f = \frac{\alpha^2(\gamma^2 - \beta^2)}{\gamma^2 - \alpha^2}$$

or

$$\sin^2 V_f = \frac{\alpha^2(\gamma^2 - \beta^2)}{\beta^2(\gamma^2 - \alpha^2)}. \quad (17)$$

As above, the equation for the angle between the optic axis and the slow vibration direction is

$$\sin^2 V_s = \frac{\gamma^2(\beta^2 - \alpha^2)}{\beta^2(\gamma^2 - \alpha^2)}. \quad (17a)$$

The tangent relation may be obtained from the equations

$$\tan V_f = \frac{\sin V_f}{\cos V_f}, \text{ and } \tan^2 V_f = \frac{\sin^2 V_f}{\cos^2 V_f}. \quad (18)$$

Substituting equations (13) and (14) in (18), we have

$$\tan^2 V_f = \frac{\frac{\alpha^2(\gamma^2 - \beta^2)}{\beta^2(\gamma^2 - \alpha^2)}}{\frac{\gamma^2(\beta^2 - \alpha^2)}{\beta^2(\gamma^2 - \alpha^2)}} = \frac{\alpha^2(\gamma^2 - \beta^2)}{\gamma^2(\beta^2 - \alpha^2)}. \quad (19)$$

$$\text{Likewise } \tan^2 V_s = \frac{\gamma^2(\beta^2 - \alpha^2)}{\alpha^2(\gamma^2 - \beta^2)}. \quad (19a)$$

72. Relation between the True and Apparent Axial Angles.—In Fig. 171 let
$V = n'Mb = Mn'c = i$ (the angle of incidence),
$E = n'm'b = m'n'd = en'f = r$ (the angle of refraction).

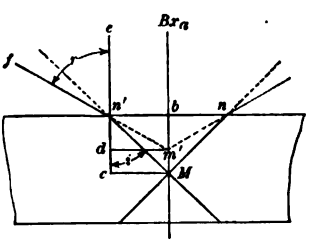


FIG. 171.—Relation between true and apparent axial angle.

In passing from a denser to a rarer medium we have (Art. 41),

$$\frac{\sin i}{\sin r} = \frac{1}{n}.$$

Substitute β, the mean refractive index, for n, and we have

$$\frac{\sin V}{\sin E} = \frac{1}{\beta}, \text{ or } \sin E = \beta \sin V. \quad (20)$$

That is, *the sine of the true optic angle of the mineral multiplied by the intermediate index of refraction will give the sine of the apparent axial angle in air.*

73. Plane of the Optic Axes.—It is obvious, from the statements made in Articles 63 and 67, that the optic axes must always lie in the plane of maximum and minimum indices of refraction (γ and α), resp. velocities (a and c), consequently the rule follows that *the plane of the optic axes is the plane containing* γ *and* α, *resp.* c *and* a.

74. Bisectrices.—The lines which bisect the angles between the optic axes are known as the bisectrices. That bisecting the acute angle is called the **acute bisectrix,** while that bisecting the obtuse angle is called the **obtuse bisectrix.** They are expressed by the symbols Bx_a and Bx_o. The bisectrices always lie at right angles to each other, and always coincide with the axes of minimum and maximum ease of vibration.

75. Positive and Negative Biaxial Crystals.—We found, in uniaxial crystals, that as the *c* axis coincided with the slowest or fastest ray, the crystals were called positive or negative. Biaxial crystals are considered positive when the acute bisectrix coincides with the direction of vibration of the slow ray (c), and negative when it coincides with the vibration direction of the fast ray (a).

There are certain crystals in which the acute bisectrix, for example, is the vibration direction of the fast ray, and the crystal is, consequently, negative. By a progressive change in chemical composition there may be formed other minerals of the same group. Coincident with this change in composition, the acute optic angle may become larger and larger until it reaches 90°, beyond which point the acute bisectrix lies in the direction of the vibrations of the slow ray, and the mineral is positive. Such a change, for example, takes place in the hypersthene-enstatite group. Hypersthene, $(Mg,Fe)SiO_3$, with an axial angle of about 80° and the fast vibrations in the direction of the acute bisectrix, is negative, while enstatite, $MgSiO_3$, with an axial angle of 70° and the slow vibrations in the direction of the acute bisectrix, is positive. That is, with the decrease in the percentage of iron, the angle has changed from 80° to 110°. Intermediate between these two there are other orthorhombic pyroxenes with varying proportions of iron, consequently having axial angles which lie between 80° negative and 70° positive. At some point between the two the axial angle is 90°, but 90° only for a certain color of light. The effect of different colored light is well shown in danburite, between the optic axes of which there is an angle of 89° 14′ by green light, and 90° 24′ by blue. That is, the mineral is negative for green and positive for blue.

When $2V=90°$, $V=45°$, and $\tan^2 V=1$, equation (19) becomes

$$\frac{\alpha^2(\gamma^2-\beta^2)}{\gamma^2(\beta^2-\alpha^2)}=\tan^2 V=1,$$

and

$$\beta=\sqrt{\frac{2\alpha^2\gamma^2}{\gamma^2+\alpha^2}}.$$

For any other value of β the mineral will be either positive or negative. Since the wave surface is actually nearly a sphere, one may say, with approximate truth, that *the nearer β approaches α, the nearer the optic axes will lie to the vibration direction of* $\mathfrak{c}$, *and vice versa*, consequently for all values except close to $V=45°$, if $\gamma-\beta>\beta-\alpha$ the mineral is positive, and if $\gamma-\beta<\beta-\alpha$ the mineral is negative.

FIG. 172.

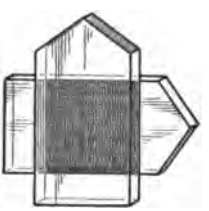


FIG. 173.

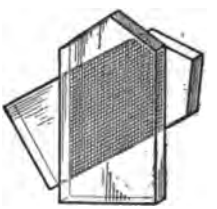


FIG. 174.

FIGS. 172 TO 174.—Tourmaline crystals in parallel position permitting light to pass through (172), crossed and with light extinguished (173), and at some angle between 0° and 90°, permitting some of the light to pass through (174).

76. Polarization by Double Refraction.—The statement was made that when a ray of light passes into an anisotropic medium, it is resolved into two rays; that is, it is polarized by double refraction into two sets of waves vibrating in planes at right angles to each other. This can be shown very readily in the case of certain doubly refracting crystals which naturally absorb one set of vibrations. For example, if a transparent crystal of tourmaline is cut parallel to crystallographic *c*, and is placed over another crystal of the same mineral similarly cut, it will be found that when the two *c* axes are parallel, the light will pass through (Fig. 172), but when they lie at right angles, it will be totally extinguished. It is as though the crystals were made up of gratings of parallel bars and the polarized light was a sheet of cardboard. When the two crystals are in parallel position the sheet slips through without hindrance, but when they are crossed the passage is closed (Fig. 173). If instead of placing the two crystals at right angles, they are turned to some other angle, an amount of light proportional to the rotation passes through.

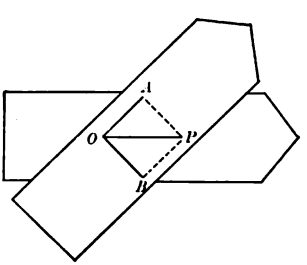


FIG. 175.—Resolution of polarized light upon passing through two crystals of tourmaline.

In Fig. 175 let *OP* represent the amplitude of the vibrations of the light passing through the horizontal crystal. When it enters the inclined crystal it is resolved into two rays, one of which vibrates parallel, and the other at right angles to crystallographic *c* (*OA* and *OB*). Of these two rays only the one parallel to crystallographic *c* (*OA*) can pass through tourmaline, the other (*OB*) being absorbed, and the amplitude of the vibrations which reach the eye is less than that with which it arrived at the under side of the upper

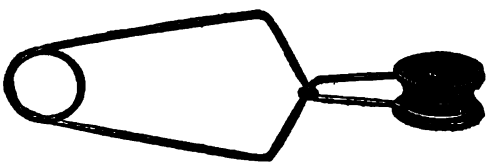


FIG. 176.—Tourmaline tongs (Central Scientific Co., Chicago).

crystal. As the upper crystal is rotated more and more toward a position at 90° to the horizontal crystal, the *OA* component becomes smaller and smaller, therefore less and less light reaches the eye until, when the 90° position is reached, there is complete darkness.

The **tourmaline tongs** (Fig. 176), which are used to determine whether a mineral is singly or doubly refracting, are based on this principle. They were formerly much used in the determination of gems.

77. Circular and Elliptical Polarization.—We have spoken, so far, only of light polarized in planes, consequently called **plane polarized,** but it may be polarized, also, in circles and ellipses.

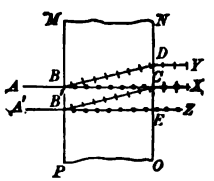


FIG. 177.

Let *A* and *A'* (Fig. 177) represent two rays of light which meet the doubly refracting crystal *MNOP* at *B* and *B'*. Each ray is broken into two components (*BC*, *BD*, and *B'C*, *B'E*) with vibrations at right angles to each other. Let the two rays *A* and *A'* be so selected that the extraordinary component of one, and the ordinary component of the other, emerge at the same point *C*. On entering the crystal at *B* and *B'*, the two rays were in the same phase, but since the ray *B'C* has traveled a longer path than the ray *BC*, they are no longer so on emergence. We thus have two rays emerging at *C*, with vibrations in planes at right angles to each other and in slightly different phase. As we have already seen (Art. 27), an elliptical motion is set up in such a case. Added to this, however, is the forward movement from *C* to *X*, and as a result the complete motion is in the form of a helix of elliptical cross-section. Light of this kind is said to have **elliptical polarization.**

If the two amplitudes are equal and the phasal difference is $\frac{\lambda}{4}$ or $\frac{3\lambda}{4}$ (Art. 27), the resulting motion is circular, and we have **circular polarization.**[1]

[1] Care should be taken not to confuse circular polarization with rotary polarization (Art. 78), which is something quite different.

78. Rotary Polarization.—In 1811 Arago[1] discovered that if a thick plate of quartz, with parallel faces cut at right angles to the optic axis, be examined in plane polarized light between crossed nicols, it does not appear dark, as one would expect of a uniaxial crystal, but shows an interference tint. If the section be rotated on the stage of the microscope, this color does not change, and, if monochromatic light be used, it is only by rotating one or the other nicol through a certain angle that darkness can be produced. It is perfectly clear, therefore, that the emerging light is plane polarized but not in the direction in which it entered the crystal. In other words, the plane of polarization has been rotated.

The only other natural mineral known to possess the property of rotary polarization is cinnabar. The effect is rather weak in each case. In the former it is about 1/200 as great as the double refraction in a direction at right angles to the axis, in the latter about thirteen times as great as in quartz. Rotary polarization is found, however, in many organic substances which crystallize in enantiomorphous forms.

All substances which possess the power of rotating the plane of polarization are called **active,** the others **inactive.**

While rotary polarization is of interest, it is of no great use in petrography except that advantage is taken of it in the construction of certain sensitive plates, such as that of Biot, Klein, etc.

The amount of rotation depends upon the color of the light used and the thickness of the section, as was shown by Biot.[2] In the table on the following page the values in millimeters for the rotation, given in the fourth column, were determined experimentally by Soret and Sarasin.[3] They may be calculated by Lommel's[4] formula,

[1] F. J. Arago: *Mémoire sur une modification remarquable qu'éprovent les rayons lumineux dans leur passage à travers certains corps diaphanes et sur quelques autres nouveaux phénomènes d'optique.* Mém. Acad. France, Année 1811, Pt. 1, XII (1812), 93–134.

Idem: *Ueber eine eigenthümliche Modification, welche die Lichtstrahlen beim Durchgehen durch gewisse durchsichtige Körper erleiden, und über einige andere neue optische Erscheinungen.* (Translation of the preceding by Gilbert.) Gilbert's Ann., XL (1812), 145–161.

Idem: *Mémoire sur la polarisation colorée. Oeuvres complètes,* Paris, X (n.d.), 36–74, especially 54–55.

[2] J. B. Biot: *Mémoire sur un nouveaux genre d'oscillation que les molécules de la lumière éprouvent en traversant certains cristaux.* Lu à l'Institute, 31 mai, 1813, et 3 nov., 1813. Mém. Acad. France, Année 1812, Paris. XIII (1814), Pt. 1, 1–371, especially 218–314.

[3] J. L. Soret et Ed. Sarasin: *Sur la polarisation rotatoire du quartz.* Comptes Rendus, XCV (1882), 635–638.

Idem: Arch. soc. phys. et nat. de Genève, LIV (1875) 253, VIII (1882), 5, 97, 201.*

[4] E. Lommel: *Theorie der Drehung der Polarisationsebene.* Wiedem. Ann., XIV (1881), 523–533.

Idem: *Das Gesetz der Rotationsdispersion.* Wiedem. Ann., XX (1883), 578–592, in particular 592.

$$\Delta = \frac{a}{\lambda^2\left(1-\frac{\lambda^2_0}{\lambda^2}\right)^2}$$

in which a and λ^2_0 are constants such that $\log a = 0.8555912$ and $\log \lambda^2_0 = 7.9351257 - 10$. The result is in thousandths of millimeters (μ).

ROTATION OF THE PLANE OF POLARIZATION OF QUARTZ

Color extinguished	Fraunhofer line	Wave length λ in $\mu\mu$ Angström	Rotation per mm. at 20° C.	Thickness necessary to rotate 90°	Thickness necessary to rotate 180°	Interference color between crossed nicols
Red.....	*A*	760.4	12.65°	7.15	14.30	Green.
	a	718.36	14.30	6.29	12.58	Blue.
	B	686.71	15.75	5.71	11.42	
Orange....	*C*	656.21	17.31	5.19	10.39	
Yellow....	D_2	589.513	21.69	4.14	8.29	Violet.
	D_1	588.912	21.725	4.11	8.23	
Green.....	*E*	526.913	27.54	3.26	6.53	Red.
Blue.....	*F*	486.074	32.76	2.75	5.49	Orange.
Indigo.....	*G*	430.725	42.59	2.10	4.20	Brownish yellow.
Violet.....	*h*	410.12	47.49	1.89	3.79	
	H	396.81	51.19	1.75	3.51	Yellow.

The effect of this power of rotation upon the interference colors may be seen from Fig. 178, which represents the directions of vibration of the red *A*, orange *C*, yellow D_1, green *E*, and blue *F* rays through a section of quartz 8.23 mm. in thickness, or just thick enough to rotate the plane of polarization for yellow D_1 through 180°. The consequence is, when white light is used, that the yellow, vibrating at right angles to the plane of the analyzer, is extinguished, and the plate shows a color composed of the resultants of the remaining rays, namely, the sensitive violet. If the analyzer is rotated, each color in succession, as it becomes perpendicular to the vibration plane, is extinguished, and a different interference color appears. If the thickness of the section had been but 4.12 mm., the amount of rotation of the D_1 line would have been but 90°, consequently the sensitive violet would have appeared when the nicols were in parallel position.

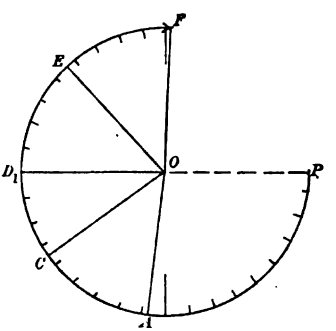


FIG. 178.—Diagram showing amount of rotary polarization of different rays in passing through quartz.

The direction of rotation of the plane of polarization is to the right in some crystals and to the left in others, as might be expected from their enan-

tiomorphous forms.[1] The former are called dextrogyrate or right handed, the latter levogyrate or left handed, and these optical directions are the same as the crystallographic rotation directions. Crystals left handed optically are left handed crystallographically, and *vice versa*. The rotation directions may be readily recognized, under the microscope, by noticing that when the analyzer is turned to the right (clock-wise) the colors pass from green through blue, purple, and red to green, while in a left-handed quartz the reverse order appears.[2]

79. Summary of Optical Principles.—We have learned that light consists of vibratory motion of the ether, and that it is transmitted by means of vibrations taking place at right angles to the direction of transmission (Art. 18). If we consider a single particle in motion, its movements of acceleration and retardation are comparable with the motion of a pendulum, or the projection of the motion of a particle around a circle. When a particle receives two impulses the resulting motion will depend upon the direction, amount, and phase of the components (Arts. 27–28).

There are two kinds of media in which light may travel; isotropic, in which the ease of vibration is the same in all directions, and anisotropic, in which the ease of vibration differs in different directions (Art. 29). To the former class belong amorphous substances and unstrained crystals of the isometric system; to the latter, crystals of the tetragonal, hexagonal, orthorhombic, monoclinic, and triclinic systems.

The intensity of light depends upon the amplitude of the vibrations, the color upon the rate of oscillation (Arts. 31–32).

The wave front of light, in isotropic substances, lies at right angles to the direction of transmission of the rays (Art. 34). When a ray meets with an obstacle to its free movement, it is partially reflected and partially refracted. The angle formed by the reflected ray with the normal to the reflecting surface is equal to the angle formed by the incident ray (Art. 35), but the relation between the incident and refracted rays is such that the ratio of the sine of the angle of incidence to the sine of the angle of refraction has a constant value, known as the index of refraction (Arts. 37–38). The index of refraction of a ray is inversely proportional to its velocity (Art. 40). When light passes from a rarer to a denser medium, the refracted ray is bent toward the normal, when from a denser to a rarer, away from it (Art. 39).

At some angle of incidence, constant for the same substances, the refracted ray is parallel to the separating surface. This angle is known as the critical

[1] J. F. W. Herschel: *On the rotation impressed by plates of rock crystal on the planes of polarization of the rays of light, as connected with certain peculiarities in its crystallization.* Read April 17, 1820. Trans. Cambridge Phil. Soc., I (1821), 43–52.

Anon: *Abstract of Mr. Herschel's experiments on circular polarization.* Edinburgh Phil. Jour. IV (1821), 371–373.

Anon: *Mr. Herschel's experiments on plagiedral quartz.* Ibid. VI (1822) 379.

[2] For further references to rotary polarization see the bibliography given at the end of this chapter.

angle, and is the angle whose sine is the reciprocal of the index of refraction (Art. 41).

While in isotropic media, in general, light vibrates with equal ease in every direction, it has been found that after reflection and refraction the rays do not so vibrate but are polarized so that the vibrations of the reflected ray take place at right angles to the plane containing the incident and reflected rays, while the refracted ray vibrates in the plane of the incident and refracted rays (Art. 42).

It has been found that in anisotropic media the directions of maximum and minimum ease of vibration lie at right angles to each other, and at right angles to these two is a third axis of intermediate, though not necessarily mean, ease of vibration (Art. 47). In tetragonal and hexagonal crystals the vibrations taking place in the basal plane are equal in every direction. The direction of crystallographic c is the direction of maximum or minimum ease of vibration. In orthorhombic, monoclinic, and triclinic crystals, the ease differs in different directions.

When light enters an anisotropic crystal it is broken up into two rays vibrating at right angles to each other. One ray vibrates perpendicular to the plane of the incident ray and the direction of propagation, and is called the ordinary ray, and one vibrates in the planes of the incident and the refracted rays, and is called the extraordinary ray. The index of refraction of the former is represented by the letter ω, that of the latter by ϵ (Art. 48).

In tetragonal and hexagonal crystals there is but one direction in which there is no double refraction, a direction at right angles to the plane of equal ease of vibration, consequently parallel to crystallographic c. These crystals are called uniaxial, and may be divided into two classes: those in which the c axis is the direction of maximum ease of vibration, called negative crystals, and those in which the c axis is the direction of minimum ease of vibration, called positive (Arts. 49–50). In the former $\omega > \epsilon$ and in the latter $\omega < \epsilon$.

The principal optic section of a uniaxial crystal is one containing the axes of greatest and least ease of vibration, consequently any section containing crystallographic c is a principal section (Art. 50).

The direction of vibration of the ordinary ray is at right angles to the direction of transmission and also at right angles to the plane of the incident and refracted rays; the direction of vibration of the extraordinary ray is parallel to the tangent to the ellipse of ray fronts, and lies in the plane of the incident and refracted rays (Art. 54).

The indicatrix is an ellipsoid whose axes are the principal indices of refraction of any crystal (Art. 59).

The ray surface and the wave surface do not coincide since waves of light are not transmitted in the direction of the rays except when parallel to the axes of the indicatrix. The movement of a wave is measured by the normal to the tangent at the end of a ray (Art. 55).

Crystals belonging to the orthorhombic, monoclinic, or triclinic systems have two directions along which the light waves advance with equal velocities and, from analogy with uniaxial crystals, they are called biaxial. The optic axes are of two kinds: primary axes or binormals, and secondary axes or biradials. The two differ very slightly in position in a crystal, and when optic axes are spoken of, the primary axes are usually meant (Arts. 66–67).

The optic or axial angle is the angle between the optic axes. Its true value is indicated by $2V$, and its apparent angle in air by $2E$. If measured in oil, etc., it is shown by $2H$ (Art. 70).

The maximum ease of vibration in biaxial crystals is indicated by a, the minimum ease by c, and the intermediate by b ($a > b > c$). The corresponding refractive indices are shown by α, β, and γ ($\alpha < \beta < \gamma$) (Art. 62).

The plane of the optic axes, in biaxial crystals, is always the plane containing a and c (Art. 73).

A bisectrix is the line bisecting the optic angle. In the acute angle it is called the acute bisectrix; in the obtuse angle, the obtuse bisectrix. The bisectrices always coincide with a and c (Art. 74).

When c is the acute bisectrix, the mineral is considered positive; when a is the acute bisectrix, it is negative (Art. 75).

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CHAPTER VII

LENSES

80. Definitions.—Of primary importance in a microscope are the lenses.[1] A lens is an instrument consisting of one or more pieces of transparent material, usually glass, and having curved surfaces which may be spherical, elliptical, parabolical, or cylindrical, the first being the most common. There are two types of simple lenses; **thin-edged, convex,** or **converging** (Figs. 179–181), and **thick-edged, concave,** or **diverging** lenses (Figs. 182–184). If the

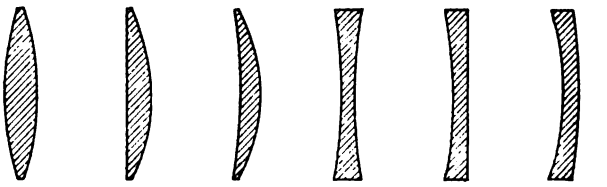

FIG. 179. FIG. 180. FIG. 181. FIG. 182. FIG. 183. FIG. 184.

FIGS. 179 TO 184.—Forms of lenses.

centers of curvature of the two faces are on opposite sides of the lens and the sum of the lengths of the two radii is greater than the distance between the centers, the lens is **double-** or **bi-convex** (Fig. 179). If the radius on one side is infinity, the lens is **plano-convex** (Fig. 180). If the centers of curvature are on opposite sides, and the sum of the radii is less than the distance between the centers, the lens is **double-** or **bi-concave** (Fig. 182). If the radius on one side is infinity, the lens is **plano-concave** (Fig. 183). If the radii are both on the same side, two cases may occur, depending upon the relation between the radii of the two curves and their centers. Such lenses are **meniscus** lenses, either converging (Fig. 181) or diverging (Fig. 184).

81. Axis, Vertices, and Thickness of a Lens.—The axis of a lens (MM', Fig. 185) is the line joining the centers of the two surfaces. The vertices (V, V') are the points at which the axis intersects the surfaces. The thickness (VV') is the distance between the vertices.

82. Optical Center.—The optical center[2] of a lens is such a point that any ray passing through it emerges in a direction parallel to that in which it

[1] See the General Bibliography at the end of this chapter.

[2] See also W. F. Durand: *A practical method of finding the optical center of an objective and its focal length.* Amer. Mon. Microsc. Jour., VI (1885), 141–145.

entered. In Figs. 185 and 186 let MP and $M'P'$ be two parallel radii of a biconvex and a biconcave lens. Then evidently tt' and $t''t'''$, at right angles to these lines, will be tangents to the surfaces of the lens. Connect P and P'. Obviously a ray of light IP, whose direction within the lens is PP', will emerge ($P'R$) in a direction parallel to IP since the bounding surfaces, tt' and $t''t'''$, are parallel. The point Oc, where the line PP' cuts the line MM',

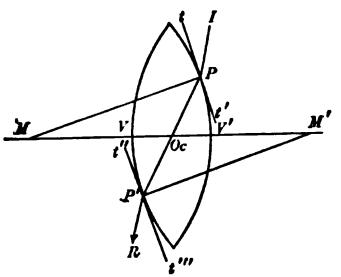

FIG. 185.

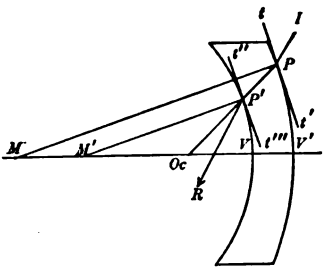

FIG. 186.

FIGS. 185 AND 186.—Optical center (Oc), thickness (VV'), and centers (M, M') of biconvex and concavo-convex lenses.

is the optical center of the lens. In biconvex and biconcave lenses this center falls within the lens, in plano-convex and plano-concave, it is at the vertex of the curved surface, and in a concavo-convex lens, it falls outside.

83. Principal Focal Point.—The principal focal point of a convex lens is the point to which all parallel rays sent through it converge (F, Fig. 187). It

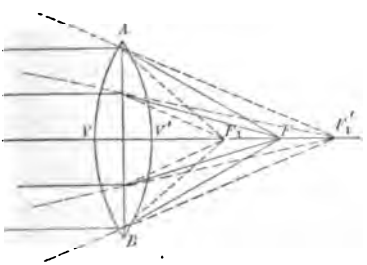

FIG. 187.—Real and virtual focus in a biconvex lens.

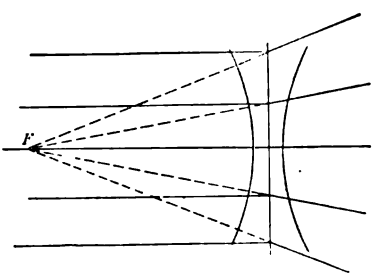

FIG. 188.—Virtual focus in a biconcave lens.

is a **real focus.** If the point (F) from which the light enters the lens is nearer than the principal focal point, the emerging rays will diverge and appear to come from a point on the same side of the lens but farther from it. This point (F'_1) is called the **virtual focus.**

In a concave lens the principal focus is the point from which the rays appear to diverge (F, Fig. 188). It is a virtual focus.

84. Conjugate Foci of Convex Lenses.—If a beam of light emanates from a point nearer a convex lens than infinity, the rays will converge at some point farther from the lens than the principal focus. Thus a beam of light, emanating from F_1, Fig. 189, will converge at F'_1, consequently a beam of light diverging from F'_1 will converge at F_1. The two points F_1 and F'_1 are called **conjugate foci.**

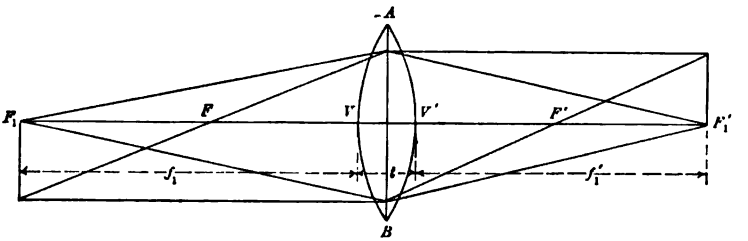

FIG. 189.—Conjugate foci of a biconvex lens.

85. Refraction through Simple Lenses.—A lens may be considered as having an infinite number of plane faces, represented by planes tangent at every point of its surface, and we may thus determine the passage of light through it by means of the laws of refraction in isotropic substances.

In order that our formulæ may be applicable to all kinds of lenses, concave as well as convex, the following points must be observed: All distances measured to

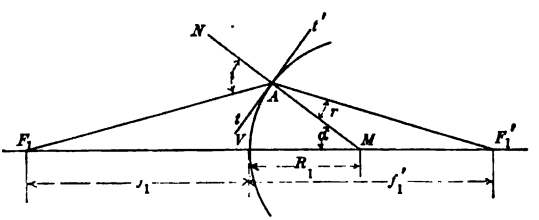

FIG. 190.—Refraction at a curved surface.

the left of the lens or below the axis are written with a minus sign, and all distances to the right or above, with a plus sign. Also, the radius of curvature of the face of a lens is considered positive if its center is to the right of the vertex, and negative if it is to the left.

Let the glass of a lens first be considered to extend indefinitely to the right from the curve AV, Fig. 190, and let a ray of light enter it at A, along the line F_1A, tt' being the tangent, and AN the normal to it. Let $AMV=\alpha$, $F_1AN=i=$ the angle of incidence, $F'_1AM=r=$ the angle of refraction, and $R_1=AM=$ the radius of curvature of the lens. Since in any triangle the sides are proportional to the sines of the opposite angles, we have, in the triangle MAF_1M:

$$\frac{\sin F_1AM}{\sin \alpha} = \frac{\sin(180°-i)}{\sin \alpha} = \frac{\sin i}{\sin \alpha} = \frac{F_1M}{F_1A} = \frac{-f_1+R_1}{-F_1A}, \qquad (1)$$

and in the triangle MAF'_1M

$$\frac{\sin r}{\sin AMF'_1} = \frac{\sin r}{\sin(180°-\alpha)} = \frac{\sin r}{\sin \alpha} = \frac{MF'_1}{AF'_1} = \frac{f'_1 - R_1}{AF'_1}. \tag{2}$$

Dividing (1) by (2) we obtain

$$\frac{\sin i}{\sin r} = \frac{(-f_1+R_1)\cdot AF'_1}{(f'_1-R_1)\cdot AF_1} = n, \tag{3}$$

in which n = the index of refraction of the material of the lens, usually glass, and R_1 = the radius of curvature of the lens.

On decreasing the angle AF_1M, and, consequently, the angle of incidence i, the minimum value will be reached when the ray coincides with the axis $F_1VF'_1$, and AF_1 will equal VF_1, and AF'_1 will equal VF'_1. But $VF_1 = -f_1$, and $VF'_1 = f'_1$. Substituting these values in equation (3), gives

$$n = \frac{-f'_1 f_1 + f'_1 R_1}{-f'_1 f_1 + f_1 R_1},$$

Transposing, we have

$$-\frac{1}{f_1} + \frac{n}{f'_1} = \frac{1}{R_1}(n-1). \tag{4}$$

This is the general equation of conjugate foci in two media.

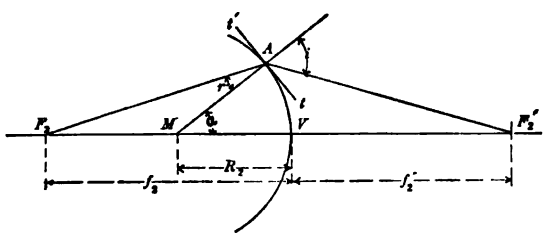

FIG. 191.—Refraction from glass through a curved surface into air.

In a manner similar to the above, we may trace the passage of light from glass into air, and we have (Fig. 191)

$$\frac{1}{f'_2} - \frac{n}{f_2} = -\frac{1}{R_2}(n-1), \tag{5}$$

where n is the index of refraction of the substance of the lens, f_2 the distance of the conjugate focus from the vertex of the lens and lying within it, f'_2 the distance of the second conjugate focus on the other side of the lens, and R_2 the curvature of the second face of the lens.

The effect of the passage of light through a lens may now be determined. If we consider the thickness of the lens as infinitely small, t (Fig. 192) will equal zero, and the radii of curvature, R_1 and R_2, will meet at V.

Adding equations (4) and (5), we have, since $f'_1 = f_2$,

$$-\frac{1}{f_1} + \frac{1}{f'_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{6}$$

This is the general equation of the conjugate foci for all lenses, disregarding thickness, in terms of the radii of curvature and the index of refraction of the material of the lens.

If $f_1 = \infty$, f'_2 will become the distance (f) of the principal focus (F) from the center, and equation (6) becomes,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \tag{7}$$

or

$$f = \frac{R_1 R_2}{(n-1)(R_2 - R_1)}. \tag{8}$$

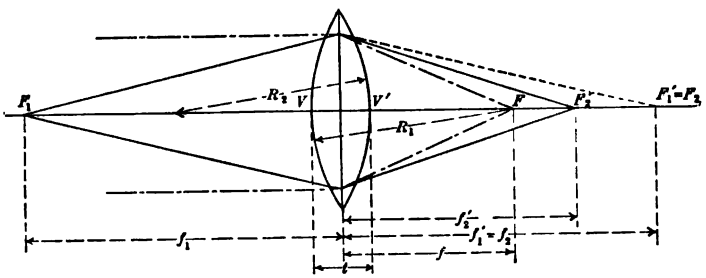

FIG. 192.—Passage of light through a lens.

This is the equation of the principal focus of any lens in terms of the curvatures and the index of refraction of the lens, thickness being disregarded.

By combining (6) and (7) we have

$$\frac{1}{f} = -\frac{1}{f_1} + \frac{1}{f'_2}. \tag{9}$$

This is the general equation of the principal focus for all lenses, thickness disregarded, in terms of the conjugate foci.

86. Focus of Combined Lenses.—In Fig. 193 let F'_1 be the point where the parallel rays I, I', and I'' would converge after passing through the lens $A'AA'$, and let f'_1 represent the distance of this point, the principal focus of the lens AA', from A. If now a second lens $B'BB'$ is inserted at a distance of $AB = h$ from the first lens, and in the path of the rays coming through it, the light falling upon the second lens, being now converging, will fall at F, at a distance of BF, and nearer the second lens than its principal focal point F'_2. The formula for combined lenses may be obtained from this diagram.

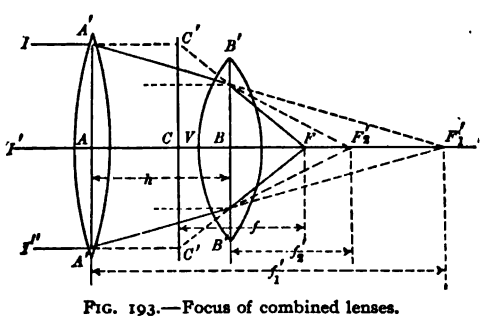

FIG. 193.—Focus of combined lenses.

The rays of light $A'B'$ pass through the second lens, converge, and have their virtual focus at F, which is the real focus of the combination as well as the conjugate focus, in the second lens, of the point F'_1. The true focal distance ($CF = f$) of the combination may be determined by equation (9).

Now the distance $F'_1B = f'_1 - h$, and this is equal to $f_1 - h$. Substituting $f_1 - h$ for f_1 in equation (9), we have

$$\frac{1}{f} = -\frac{1}{f_1 - h} + \frac{1}{f'_2}, \tag{10}$$

in which f = the principal focal distance of the combination, f_1 = the principal focal distance on the object side of the lens A, and f'_2 = the principal focal distance on the image side of the lens B.

This is the equation of the principal focus of combined lenses, thickness disregarded, in terms of the principal focus of each lens.

If the lenses are in contact, $h = 0$, and equation (10) becomes

$$\frac{1}{f} = -\frac{1}{f_1} + \frac{1}{f'_2}$$

which is the same as equation (9), as it should be.

87. Gauss' Method.—In the preceding discussion the thickness of lenses was disregarded. If this is introduced, the computations are much more complicated, though the problem is greatly simplified by a method devised by Gauss[1] and supplemented by Listing.[2] The method is applicable to all

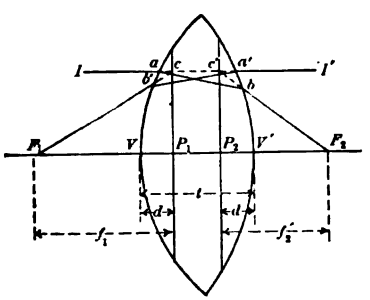

FIG. 194.—The Gauss points of a simple lens.

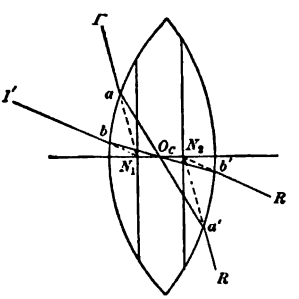

FIG. 195.—Location of nodal points and optic center of a lens.

rays which make a small angle with the optic axis of the lens combination. It consists of reducing a lens system to certain location points. If the system is well centered, it may be reduced to three pairs of location or **cardinal points** along the axis, called the **focal, nodal** and **principal points,** and by these points the image of any object may be determined for any system of lenses, at least with approximate accuracy. The essential features are as follows.

Focal Points and Planes.—The focal points (F_1 and F_2, Fig. 194) are the points to which all rays parallel to the axis are refracted, or, conversely,

[1] K. F. Gauss: *Dioptrische Untersuchungen.* Göttingen, 1841.

G. Govi: Rend. R. Accad. Lincei, IV(1888), 665–660.* (Review in Jour. Roy. Microsc. Soc., 1891, 122–126). Gives a system somewhat different from that of Gauss.

[2] Johann Benedikt Listing: *Beiträg aus physiologischen Optik.* Göttingen, 1845. Reprinted in Ostwald's Klassiker der Exakten Wissenschaften, Nr. 147. Leipzig, 1905.

all rays emanating from these points are refracted parallel to the axis. They are, consequently, the principal foci of the lens. F_1 is called the **first principal focal point,** and F_2 the **second principal focal point.** The planes through these points, and at right angles to the axis of the lens system, are called the **focal planes.**

Principal Points and Planes.—Suppose a ray F_1b', Fig. 194, emanating from F_1, falls upon the lens. It will emerge parallel to the axis along the line $a'I'$. Another ray, Ia, parallel to the axis, will be refracted along bF_2. The image of an object at c', when viewed from F_1, appears at c, and an object at c, when viewed from F_2, appears at c'. So with all points on the lines cP_1 and $c'P_2$, at right angles to the axis of the lens. The intersections of these lines with the axis at P_1 and P_2 are called the **principal points,** and the planes through these points and perpendicular to the axis, the **principal** or **Gauss planes.**

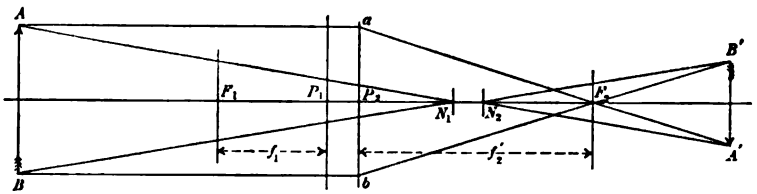

FIG. 196.—Focal, principal, and nodal points in a lens system, and the application of these points to the location of a refracted image.

Nodal Points.[1]—In Fig. 195 let the incident ray Ia, and the refracted ray $a'R$, be parallel. The points where the two rays extended cut the axis are called the **nodal points** (N_1 and N_2).

Focal Distance.—The distances between the focal points and the principal points are called the principal focal distances. $F_1P_1=f_1$ (Fig. 194) is the first principal focal distance, and $P_2F_2=f'_2$, the second. They are true focal distances.

The points F_1, P_1, P_2, N_1, N_2, and F_2 all lie along the axis of the lens system and bear definite relations to each other. Thus the distance between the first focal point and the first nodal point is equal to the distance between the second principal point and the second focal point, and the first principal focal distance is equal to the distance between the second nodal point and the second focal point. Consequently $F_1N_1=P_2F_2$ and $P_1P_2=N_1N_2$, Fig. 196, and $f'_2-f_1=P_2N_2$ or P_1N_1. Also $f_1 : n'=f'_2 : n''$ (n' and n'' being the indices of refraction of the media on either side of the lens). If the media on either side are the same, $n'=n''$ and $f_1=f'_2$. That is, *the two principal focal distances are the same when the media on either side of the lens are the same, and* furthermore, since $f_1=f'_2$, *the principal points and the nodal points coincide.*

[1] *Knotenpunkte*, introduced by Listing. *Op. cit.*

88. Application of Gauss' Cardinal Points to the Determination of the Image Formed by a Lens.—Let it be required to find the position of the image of the arrow produced by the lens system whose cardinal points are shown in Fig. 196. The ray Aa, parallel to the axis, will be refracted through the focal point F_2 to some point on the line aF_2A'. The ray AN_1, through the first nodal point, will be refracted along the line N_2A', parallel to AN_1. Where the two lines intersect at A' is the required point of the image.

In the same manner the point B has its image in B', at the intersection of the lines bF_2 and N_2B'.

EQUATIONS FOR THE DETERMINATION OF THE CARDINAL POINTS OF ANY LENS SYSTEM

SIMPLE LENS

89. Lateral Magnification.—Let OO_1, Fig. 197, be an object at a distance of $-x$ from the principal focus F, which itself is at a distance of $-f$ from P, the intersection of the axis of the system with the lens $P'P''$, which here is assumed to have no thickness.

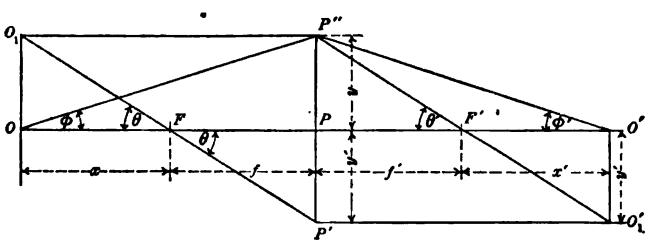

FIG. 197.

To determine the image point of O_1, two rays are passed through it. Rays parallel to the axis and coming from the left have their focus at F', consequently a parallel ray through O_1 will cut the plane $P'P''$ at P'' and be projected along the line $P''F'$ toward O'_1. A second ray through O_1, passing through F, will reach the lens at P' and, since it passes through the principal focus, will be projected along $P'O'_1$, parallel to the axis. The intersection of the two rays at O'_1 is the position of the image O_1.

Let x' represent the distance of this image to the right of F', which itself is at a distance of f' from P.

From the similar triangles $PP'F$ and OO_1F we have

$$\frac{-y'}{-f} = \frac{y}{-x}, \text{ or } \frac{-y'}{y} = \frac{-f}{-x}, \tag{11}$$

and from the similar triangles $P''PF'$ and $O'O'_1F'$

$$\frac{y}{f'} = \frac{-y'}{x'}, \text{ or } \frac{-y'}{y} = \frac{x'}{f'}. \tag{12}$$

The ratio of the size of the image (y') to the size of the object (y) is called the **lateral magnification of a lens** and is represented by β. From equations (11) and (12) we have

$$\beta=\frac{-y'}{y}=\frac{-f}{-x}=\frac{x'}{f'} \tag{13}$$

or

$$-ff'=-xx'. \tag{14}$$

90. Convergence of a Lens.—From any two conjugate foci, as O and O' (Fig. 197), draw rays OP'' and $O'P''$, making angles of φ and φ' with the axis, and forming the angular aperture for the object and the angular aperture for the image. The convergence (γ) of the lens is expressed by the ratio of the tangents of these angles.

$$\gamma=\frac{\tan\varphi'}{\tan\varphi}, \tag{15}$$

or

$$\gamma=\frac{\frac{y}{f'+x'}}{\frac{y}{-f-x}}=\frac{-f-x}{f'+x'}. \tag{16}$$

Adding fx' to each member of equation (14) we have

$$-ff'+fx'=-xx'+fx', \tag{17}$$

or

$$\frac{-f-x}{f'+x'}=\frac{-f}{x'}. \tag{18}$$

Substituting in equation (16) we have

$$\gamma=\frac{-f}{x'} \tag{19}$$

which is *the equation of the convergence of a lens.*

91. Formation of Images by Lenses.—Applying these formulæ to the determination of the size of images we have, from the equation for the lateral magnification of a biconvex lens (13), disregarding thickness and remembering that f and x are negative:

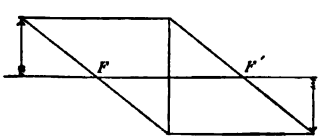

FIG. 198.

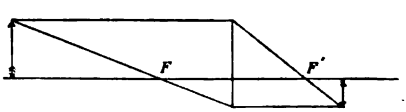

FIG. 199.

$$\beta=\frac{-y'}{y}=\frac{-(-f)}{-(-x)}=\frac{f}{x}. \tag{13}$$

If the object is at a distance just twice the focal length from the lens, $-f=-x$ (Fig. 198. Cf. also Fig. 197), therefore

$$\frac{-y'}{y}=1, \text{ and } -y'=y.$$

That is, *when the object is twice the focal distance from the lens, the image will be the same size as the object* and will be real and inverted.

If the object is at a distance greater than twice the focal length of the lens, $-x > -f$ (Fig. 199), therefore equation (13) becomes

$$\frac{-y'}{y} = \frac{f}{x} < 1, \text{ and } -y' < y.$$

That is, *when the object is farther from the lens than twice its focal length, the image will be smaller than the object* and will be real and inverted.

If the object is at a distance greater than the focal length but less than two times that distance, $-x < -f$ (Fig. 200) and

$$\frac{-y'}{y} = \frac{f}{x} > 1, \text{ and } -y' > y.$$

That is, *when the object is at a distance greater than the focal length but less than*

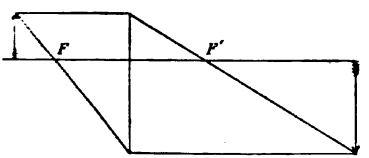

FIG. 200.

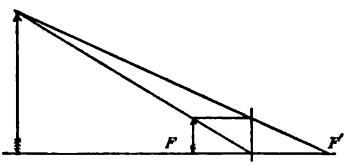

FIG. 201.

twice this distance from the lens, the image will be larger than the object and will be real and inverted.

If the object is at a distance less than the focal distance from the lens, $+x < -f$ (Fig. 201) and

$$\frac{-y'}{y} = \frac{-f}{x} > 1, \frac{y'}{y} = \frac{f}{x}, \text{ and } y' > y.$$

That is, *when the object is nearer the lens than its focal length, the image will be larger than the object* and will be virtual and erect.

92. System of Two Faces.—Let Fig. 202 represent a lens of two faces. The computation may be simplified if we consider the two faces as independent systems, the light first passing from air into the lens, reaching its focus, and then passing beyond, through the lens, to the second surface and back into air. The two faces will be treated as independent systems, and will be spoken of as the first and second systems.

In Fig. 202 let

F_1 = the principal focus on the object side of the first system,
f_1 = the focal distance of F_1,
F'_1 = the principal focus on the image side of the first system,
f'_1 = the focal distance of F'_1,
F_2 = the principal focus on the object side of the second system,

f_2 = the focal distance of F_2 from the vertex of the second system,
F'_2 = the principal focus on the image side of the second system,
f'_2 = the focal distance of F'_2 from the vertex of the second system,
F = the principal focus on the object side of the compound system,
f = the principal focal distance from the principal plane on the object side,
F' = the principal focus on the image side of the compound system,
f' = the principal focal distance from the principal plane, on the image side of the compound system.

It will be seen that a parallel ray of light, passing from left to right, will have its principal focus on the image side of the first system at F'_1. The ray will now pass on into the second system, no longer as parallel light, but converging. While F'_2 is the focus of rays entering the second system parallel to the axis, being the principal

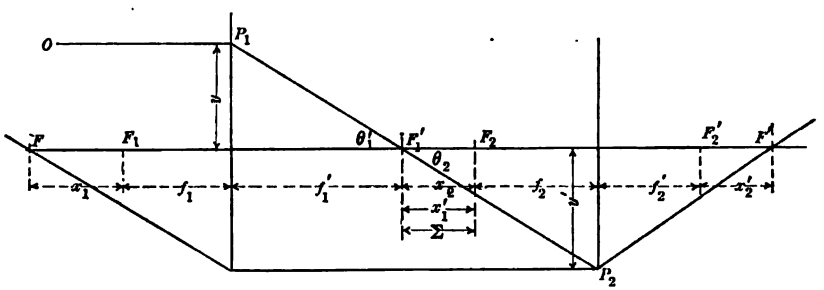

FIG. 202.—Passage of light through a system of two faces.

focus on the image side of the second system, it will not be the focus of the ray $OP_1F'_1$ which is now not parallel to the axis, although it was so originally. Instead of being refracted to F'_2 by the second system, therefore, the ray will be refracted to F', a point which is conjugate with F'_1 in the second system. That is, the image-side principal focus of a compound system of lenses is at a point which, in the second system, is the conjugate of the image-side principal focus of the first system.

From equation (14) we have, consequently, for the two systems

$$-f_1f'_1 = -x_1x'_1 \tag{20}$$

or

$$-x_1 = \frac{-f_1f'_1}{x'_1}; \tag{21}$$

and

$$-f_2f'_2 = -x_2x'_2, \tag{22}$$

or

$$x'_2 = \frac{-f_2f'_2}{x_2}. \tag{23}$$

If we represent by Σ the distance between adjacent focal planes, we will have, in the first system, Σ = the distance between F'_1 and $F_2 = x'_1$, whereby, in the compound system shown in Fig. 202, equation (21) will become

$$-x_1 = \frac{-f_1f'_1}{\Sigma}. \tag{24}$$

This is the equation of the distance, on the object side, between the principal focus of the combined system and the principal focus of the first system.

In like manner, equation (23) becomes, in the second system,

$$x'_2=\frac{-f_2f'_2}{\Sigma}. \tag{25}$$

This is the equation of the distance, on the image side, between the principal focus of the second system and the principal focus of the combined system.

In Fig. 197 let $\theta=PFP'$,

$$\tan\theta=\frac{-y'}{-f}, \text{ and } -f=\frac{-y'}{\tan\theta}. \tag{26}$$

Likewise

$$f'=\frac{f}{\tan\theta'}. \tag{27}$$

These are Gauss' equations for the focal lengths (f and f') of a compound system.

In the compound system, Fig. 202,

$$\tan\theta'_1=\frac{y}{f'_1}=\tan\theta_2 \text{ (of the second system)}. \tag{28}$$

Also from (15) and (19) we have

$$\gamma=\frac{\tan\theta'_2}{\tan\theta_2}=\frac{-f_2}{x'_2}.$$

but from equation (14) we have

$$-f_2f'_2=-x_2x'_2, \text{ or } \frac{-f_2}{x'_2}=\frac{-x_2}{f'_2}.$$

Since $F_2F'_2=x_2=\Sigma$, we have

$$\frac{\tan\theta'_2}{\tan\theta_2}=\frac{-x_2}{f'_2}=\frac{-\Sigma}{f'_2}$$

and

$$\tan\theta'_2=-\frac{\Sigma}{f'_2}(\tan\theta_2). \tag{29}$$

θ_2 of the second system corresponds to θ'_1 of the first (Fig. 202), and θ'_2 to θ' (Fig. 197), therefore

$$\tan\theta'=-\frac{\Sigma}{f'_2}(\tan\theta'_1). \tag{30}$$

But $f'=\frac{y}{\tan\theta'}$ (by equation 27), and $f'_1=\frac{y}{\tan\theta'_1}$, therefore

$$\tan\theta'=\frac{y}{f'}, \text{ and } y=f'_1(\tan\theta'_1).$$

Substitute these values in equation (30),

$$f'=-\frac{yf'_2}{\Sigma\tan\theta'_1}=-\frac{(f'_1\tan\theta'_1)f'_2}{\Sigma\tan\theta'_1}=-\frac{f'_1f'_2}{\Sigma}. \tag{31}$$

A ray coming from the image side would give

$$f=\frac{f_1f_2}{\Sigma}. \tag{32}$$

These values of f and f' are the values of the principal focal distances of the compound system in terms of the values of the conjugate foci.

By comparison with the refraction through a simple lens (equation 5), we know that if the rays of light fall upon a curved surface and emerge parallel to the axis, $f'_2 = \infty$, whereby equation (5) becomes

$$\frac{1}{\infty} - \frac{n}{f_2} = -\frac{1}{R_2}(n-1),$$

and

$$f_2 = \frac{nR_2}{n-1}. \tag{33}$$

If $f_2 = \infty$,

$$f'_2 = -\frac{R_2}{n-1}. \tag{34}$$

The general equation of the first system is similar to (5), and we have, similarly, f'_1 corresponding to f_2 of the second system:

$$-\frac{1}{f_1} + \frac{n}{f'_1} = \frac{1}{R_1}(n-1).$$

If $\frac{1}{f} = \infty$,

$$f'_1 = \frac{nR_1}{n-1}. \tag{35}$$

If $\frac{1}{f'_1} = \infty$,

$$f_1 = -\frac{R_1}{n-1}. \tag{36}$$

These are the values of the conjugate focal distances, f_1 and f'_1, of a simple lens in terms of the index of refraction of the material of the lens and its radii of curvature.

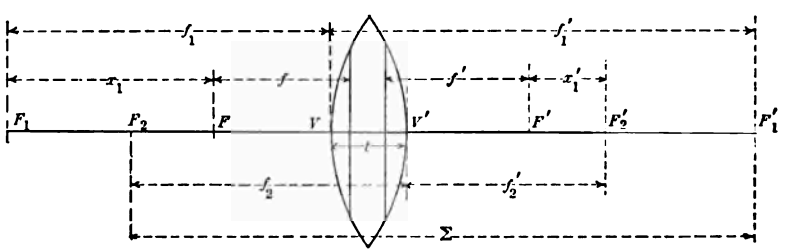

FIG. 203.

As before, let F'_1F_2 (Fig. 203) $= \Sigma$, then

$$\Sigma = F'_1F_2 = -f_2 + f'_1 + t.$$

Substitute for f_2 and f'_1 their values from equations (33) and (35)

$$\Sigma = -\frac{nR_2}{n-1} + \frac{nR_1}{n-1} + t, \tag{37}$$

$$-\Sigma(n-1) = n(R_2 - R_1) - t(n-1), \tag{38}$$

$$t = \frac{n(R_2 - R_1) + \Sigma(n-1)}{n-1}. \tag{39}$$

This is the equation for the thickness of any lens.

Substitute the values of f_1, f_2 and Σ from equations (36), (33), and (37) in equation (32), and we have

$$f=\frac{-\frac{R_1}{n-1}\cdot\frac{nR_2}{n-1}}{-\frac{nR_2-nR_1-t(n-1)}{n-1}}=\frac{nR_1R_2}{(n-1)[n(R_2-R_1)]-t(n-1)}. \tag{40}$$

This is the equation of the principal focal length of any lens.

Equation (39) may be changed to the form

$$\frac{1}{f}=(n-1)\frac{n(R_2-R_1)-t(n-1)}{nR_1R_2} \tag{41}$$

$$=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)-\frac{t(n-1)^2}{nR_1R_2}. \tag{42}$$

If the lens is infinitely thin, $t=0$, and equation (42) becomes

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

which is the same as equation (7), as it should be.

Substituting Σ for x'_1 in equation (21) we have

$$-x_1=\frac{-f_1f'_1}{\Sigma}.$$

Substitute values from equations (35) and (36) we have

$$-x_1=\frac{-\frac{-nR_1^2}{(n-1)^2}}{\Sigma}=\frac{nR_1^2}{(n-1)^2\Sigma}. \tag{43}$$

Now VF (Fig. 203) $=f_1-x_1$.

Substitute in this equation values from (43) and (36)

$$VF=\frac{nR_1^2}{(n-1)^2\Sigma}-\frac{R_1}{n-1}=\frac{-R_1(n-1)\Sigma+nR_1^2}{\Sigma(n-1)^2}. \tag{44}$$

This is the equation for the distance of the focus of any lens from the vertex on the object side.

In a similar manner the second principal focus of a biconvex lens may be determined. From Fig. 203 and equation (23)

$$V'F'=f'_2-x'_2=f'_2-\frac{f_2f'_2}{\Sigma}.$$

Substituting values as before, from equations (34) and (33),

$$V'F'=-\frac{R_2}{n-1}-\frac{\frac{-R_2}{n-1}\cdot\frac{nR_2}{n-1}}{\Sigma}=\frac{-R_2(n-1)\Sigma+nR_2^2}{(n-1)^2\Sigma}. \tag{45}$$

This is the equation for the distance of the focus of any lens from the vertex on the image side.

Let the distance between the principal focus and the principal plane be represented by f, and let d be the distance between the vertex of the lens and the principal plane. From Fig. 204 we have,

$$d = -f - VF.$$

Substituting values from equations (32) and (44), we have

$$d = -\frac{f_1 f_2}{\Sigma} = \frac{-R_1(n-1)\Sigma + nR_1^2}{\Sigma(n-1)^2}. \tag{46}$$

Substitute values from equations (42) and (39)

$$d = -\left(-\frac{nR_1R_2}{(n-1)^2\Sigma}\right) - \frac{-R_1(n-1)\Sigma + nR_1^2}{\Sigma(n-1)^2}$$

$$= \frac{nR_1R_2 + R_1(n-1)\Sigma - nR_1^2}{\Sigma(n-1)^2}$$

$$= \frac{R_1[n(R_2-R_1) + \Sigma(n-1)]}{(n-1)\Sigma(n-1)}. \tag{47}$$

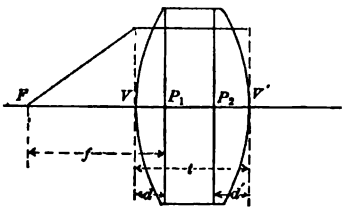

FIG. 204.

But from equation (39)

$$t = \frac{n(R_2-R_1) + \Sigma(n-1)}{n-1}.$$

Substituting this value in (47), we have

$$d = \frac{R_1 t}{(n-1)\Sigma}. \tag{48}$$

Likewise,

$$d' = \frac{R_2 t}{(n-1)\Sigma}. \tag{49}$$

These are the equations for the distances from the vertices to the principal planes.

From Fig. 204 we have

$$t = -d + P_1P_2 + d'.$$

$$P_1P_2 = t + d - d'.$$

Substitute values from equations (48) and (49),

$$P_1P_2 = t + \frac{tR_1}{(n-1)\Sigma} - \frac{tR_2}{(n-1)\Sigma} = \frac{t(n-1)\Sigma + tR_1 - tR_2}{(n-1)\Sigma} = \frac{t[(n-1)\Sigma + R_1 - R_2]}{n-1}.$$

But according to equation (37)

$$\Sigma = -\frac{n(R_2-R_1)-t(n-1)}{n-1}$$

therefore

$$P_1P_2 = -\frac{(R_2-R_1)+t(n-1)+(R_1-R_2)}{(n-1)\Sigma}$$

$$= -\frac{t(n-1)(R_2-R_1+t)}{(n-1)\Sigma}. \qquad (50)$$

This is the general equation for the distance between the principal planes of any lens.

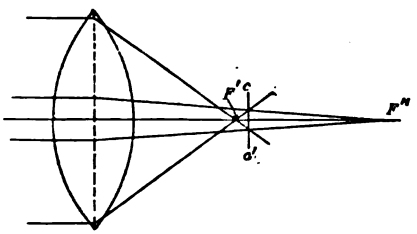

FIG. 205.—Under-corrected spherical aberration in a lens.

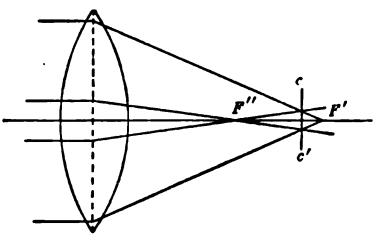

FIG. 206.—Over-corrected spherical aberration.

93. Aberration.—The location of the principal points by Gauss' theory, as has been pointed out, is accurate only when the pencil of light differs but slightly from the axis of the system. In practice it has been found that uncorrected lenses give images which are poorly defined, blurred, or distorted, an effect which is spoken of as the aberration of lenses.[1]

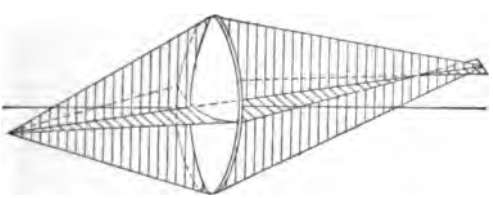

FIG. 207.—Astigmatism in a lens. (After Wright.)

Aberration is of two kinds, spherical and chromatic.

Spherical Aberration.—Parallel rays of monochromatic light, falling upon a spherical lens, will be found to be refracted to different points upon the axis. Thus in the spherical biconvex lens in Fig. 205, the marginal rays are refracted to F' while rays near the center converge at F''. This distance ($F'F''$) is

[1] For methods for determining the aberration of lenses see Reginald S. Clay: *Treatise on Practical Light.* London, 1911, 211–243, 381–383.

For methods for correcting aberration see W. Zschokke: *Anschauliche Darstellung der Entstehung und Hebung der sphärischen und astigmatischen Bildfehler.* Deutsche Mech. Zeitung, 1910, 81–87, 93–97.

called the **longitudinal spherical aberration,** and the diameter of the smallest **circle of confusion** (cc′) is known as the **lateral spherical aberration.**

If to a lens, such as that shown in Fig. 205, there is joined another lens whose marginal rays fall exactly the same distance beyond F'' as they fall within it in the first, the resultant is zero. Lenses in which the focus of the marginal rays falls within the focus of the central rays are said to be **under-corrected** (Fig. 205), and those in which the reverse is true, **over-corrected** (Fig. 206). Lenses corrected for spherical aberration are said to be **aplanatic** (α, privative; $\pi\lambda\acute{\alpha}\nu\eta$, to wander).

Another correction which must be made in lenses is for **astigmatism.** A ray of light falling obliquely upon a lens (Fig. 207) will not come to a sharp focus, for while the lens is symmetrical to the horizontal ray it is not so to the vertical. The marginal rays of the horizontal beam will intersect at the same distance from the lens no matter what the inclination of the ray may be, but there will be an increasing difference in the length of the upper and lower rays

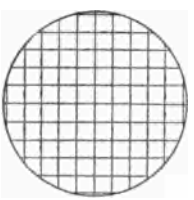

FIG. 208.

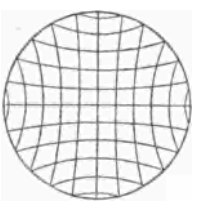

FIG. 209.

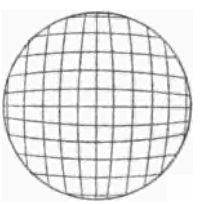

FIG. 210.

FIGS. 208 TO 210.—Images as viewed through an orthoscopic lens and through those which are uncorrected.

with increasing inclination. As a consequence there will be a displacement of the latter and the two will not come to a focus at the same point.

Lenses corrected for astigmatism are said to be **anastigmatic** ($\grave{\alpha}\nu\acute{\alpha}$, throughout, $\sigma\tau\acute{\iota}\gamma\mu\alpha$, a point) or **stigmatic.**

Another distortion for which correction must be made is the distortion of the image whereby rectilinear lines appear as shown in Figs. 209 or 210. Lenses corrected for this distortion are said to be **orthoscopic** ($\grave{o}\rho\theta\acute{o}\varsigma$, straight, $\sigma\kappa o\pi\epsilon\hat{\iota}\nu$, to look).

Chromatic Aberration.—If ordinary white light be used instead of monochromatic, another form of aberration results. Since different colored rays have different indices of refraction, images of different colors will be formed at different distances from an uncorrected lens. For example, in a biconvex lens (Fig. 211) violet light is more refracted than red, and will, consequently, come to a focus nearer the lens. Such a lens is said to be **chromatically under-corrected.** A lens whose violet rays converge beyond the red is **chromatically over-corrected.** If chromatic aberration is corrected, the lens is **achromatic** (α, privative, $\chi\rho\hat{\omega}\mu\alpha$, color). When a lens is corrected for the chromatic aberration of two colors, it does not necessarily follow that the

remaining colors are also corrected, and intermediate colors may come to a focus within or beyond the corrected focus. By using combinations of lenses made of different glasses or minerals, lenses may be chromatically corrected for three colors and spherically corrected for two. Such systems may consist of ten or twelve separate lenses. They have been called by Abbe, **apochromatic** (ἀπό, from; and χρῶμα, color).

94. Angular Aperture.—The angle which the rays of light passing through the extreme edges of the opening of a lens make at the focal point is called the angular aperture (*PFP*, Fig. 212).[1]

95. Numerical Aperture.—If, instead of air, the lens be immersed in a fluid of a different refractive index, it is clearly evident that the value of F will be changed according to the ratio $n = \frac{\sin i}{\sin r}$, or $\sin i = n \sin r$, where i is

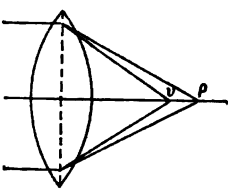

FIG. 211.—Chromatic aberration. Lens under-corrected.

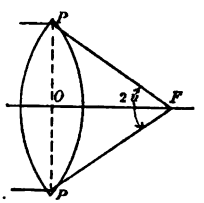

FIG. 212.—Angular aperture of a lens.

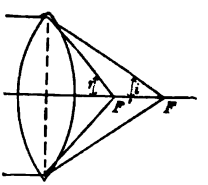

FIG. 213.—Change of focus due to an immersion fluid. F = focus in air, F' = focus in oil.

the angle made by the ray in air with the axis of the lens, and r the angle made by the ray in the refracting medium (Fig. 213). If we substitute u for half the angular aperture, the equation becomes

$$\sin i = n \sin u.$$

It may be written

$$N.A. = n \sin u.$$

To this value the name numerical aperture was given by Abbe,[2] and by it the apertures of lenses are commonly designated. In a dry system, where the

[1] See W. Blackburn: *The theory of aperture in the microscope.* Northern Microsc., II (1882), 325–334.

[2] E. Abbe: *On the estimation of aperture in the microscope.* Jour. Roy. Microsc. Soc., N. S., I (1881), 388–423.

Idem: *The relation of aperture and power in the microscope.* Ibidem, II (1882), 300–309, 460–473, III (1883), 790–812.

See also W. Blackburn: *Op. cit.*

Mr. Shadbolt: *Further remarks in the apertures of microscope objectives.* Jour. Roy. Microsc. Soc., N. S., I (1881), 376–380.

Charles Hockin: *On the estimation of aperture in the microscope.* Ibidem, IV (1884), 337–347.

lens acts through air, $n=1$ and

$$N.A. = \sin u.$$

In an immersion system

$$N.A. = n \sin u.$$

where n is the index of refraction of the medium lying between the cover-glass of the object and the lower lens of the objective.

96. TABLE OF NUMERICAL APERTURES FOR VARIOUS ANGULAR APERTURES
Formula, $N.A. = n \sin u$.

Angular aperture	$n=1.00$ (air)	$n=1.33$ (water)	$n=1.52$ (cedar oil)
$2u=10°$	0.087	0.116	0.132
15°	0.131	0.174	0.199
20°	0.174	0.231	0.264
25°	0.216	0.287	0.328
30°	0.259	0.344	0.394
35°	0.301	0.400	0.458
40°	0.342	0.455	0.520
45°	0.383	0.509	0.582
50°	0.423	0.563	0.643
55°	0.462	0.614	0.702
60°	0.500	0.665	0.760
65°	0.537	0.714	0.816
70°	0.574	0.763	0.872
75°	0.609	0.810	0.926
80°	0.643	0.855	0.977
85°	0.676	0.900	1.028
90°	0.707	0.940	1.075
95°	0.737	0.976	1.120
100°	0.766	1.019	1.164
105°	0.793	1.055	1.205
110°	0.819	1.089	1.245
115°	0.843	1.121	1.281
120°	0.866	1.152	1.316
125°	0.887	1.180	1.348
130°	0.906	1.205	1.378
135°	0.924	1.229	1.404
140°	0.940	1.250	1.429

As the angular aperture, and consequently the amount of light entering the lens, increases, so also does the numerical aperture, and as a result, other things being equal, the lens having the largest angular aperture is the most desirable.

97. Apertometer.—In practice the numerical aperture of an objective may be measured by means of an instrument called an apertometer, devised by Abbe[1] in 1876. The instrument consists of a semi-circular glass plate to

[1] Carl Zeiss: *Description of Professor Abbe's apertometer.* Jour. Roy. Microsc. Soc., I (1878), 19–22.

E. Abbe: *Some remarks on the apertometer.* Ibidem, III (1880), 20–31.

Ibidem: *Gesammelte Abhandlungen*, I, 113–118, 227–243.

E M. Nelson: *A simplification of the method of using Professor Abbe's apertometer.* Jour. Roy. Microsc. Soc., 1896, 592–594.

See also *Directions for using the Abbe apertometer.* Zeiss' circular, Mikro., 114.

which are attached two movable black metal slides, called **cursors** (*b*, Fig. 214), which may be seen through the opening *a* by reflection from the beveled surface below it. The instrument is placed on the stage of the microscope and the edges of the narrow slit at *a* are brought into sharp focus. Upon removing the eyepiece, or inserting the Bertrand lens or an auxiliary objective furnished with the instrument, the circular edge of the flat glass plate will appear, by reflection, as though it were directly along the axis of the microscope. The cursors are now moved until their points just appear at the edges of the field. In this position the scales are read, the mean of the two values indicated being taken as the angular aperture in air. Knowing the index of refraction of the apertometer glass, the numerical aperture in air may be calculated, or it may be read directly from a second scale engraved upon the glass.

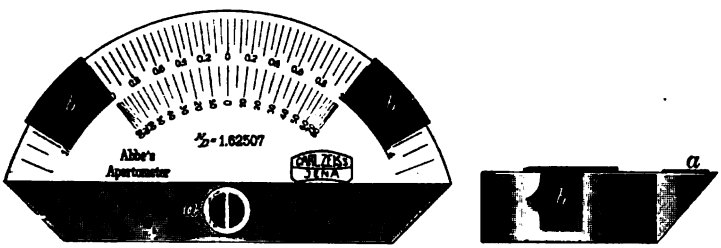

FIG. 214.—The Abbe apertometer. (Zeiss.)

98. Magnifying Power.—The amount of magnification[1] of a lens is the ratio of the size of the image to the size of the object, but the image will appear to the observer to be at a distance dependent upon the eye itself. A normal eye can see a small object most clearly when it is ten or twelve inches away. This distance is called the *distance of most distinct vision* and is conventionally taken at 250 mm. (10 in.). When an object is examined through a lens by a normal eye, in order to see clearly, the lens must be so placed, that is *focussed*, with reference to the object, that the virtual image produced will not tire the eye. In lenses of short focal distance this results when the object is practically at the principal focus. The image will appear to be at the distance of distinct vision or 250 mm.

In Fig. 215 the image AB subtends, in the eye E, an angle α whose tangent $= \frac{AB}{EB}$. If the object ab were situated at the same distance, it would subtend an angle α' $(a'EB)$ whose tangent $= \frac{a'B}{EB}$. The number of diameters which

[1] E. Abbe: *Note on the proper definition of the amplifying power of a lens or lens-system.* Jour. Roy. Microsc. Soc., 2 ser., IV (1884), 348–351.

E. Giltay: *Remarks on Prof. Abbe's Note on the proper definition of the amplifying power of a lens or lens-system.* Ibidem, V (1885), 960–967.

E. M. Nelson: *Virtual images and initial magnifying power.* Ibidem, 1892, 180–185.

the lens magnifies is naturally the ratio $\frac{\tan \alpha}{\tan \alpha} = \frac{\frac{AB}{EB}}{\frac{a'B}{EB}} = \frac{AB}{a'B}$. But in the similar triangles ACB and aCb, $AB:ab = CB:Cb = CB:F$, therefore the number of diameters magnified (N) would be

$$N = \frac{AB}{a'B} = \frac{AB}{ab} = \frac{CB}{F}.$$

When the angles α and aCb are small and the eye is placed near the lens, $CB = EB = 250$ mm., and the equation becomes

$$N = \frac{250}{F}.$$

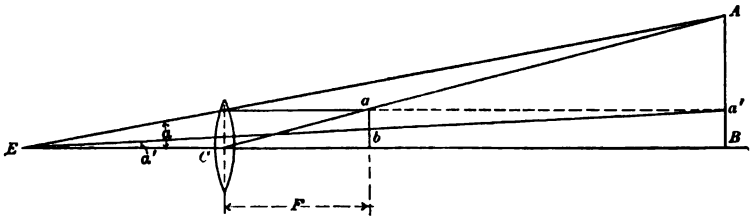

FIG. 215.—Magnifying power of a lens.

That is, the number of diameters which a lens of short focal length will magnify is equal, practically, to the number of times its focal distance is contained in 250. Thus a lens of 1 inch (25 mm.) focus will conventionally have a magnification of 10 diameters, but while this will give the apparent size of the magnified object to a normal eye, it will not be the apparent size to a person who is short-sighted. If, to him, the distance of distinct vision is only 5 in. (125 mm.), N becomes $\frac{125}{F}$ and the lens of one inch focus will give an apparent magnification of 5 diameters.[1]

Sometimes the magnifying power of a microscope is expressed in terms of areas. Thus a lens increasing the size of an object to ten diameters will magnify its area 100 times; a magnification of 50 diameters is equal to 2500 times, and so on.

[1] M. C. Montigny: Bull. Acad. Roy. Belgique, XLIX (1880), 670–678.* Review of preceding. *Difference in the appreciation of the apparent size of microscopical images by different observers.* Jour. Roy. Microsc. Soc., N. S., I (1881), 829–930.

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1895. Th. Marsson: *Beiträge zur Theorie und Technik des Mikroskops.* Zeitschr. f. angew. Mikrosk., I (1895), 33–35, 65–69.

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1906. P. Drude: *Lehrbuch der Optik.* Leipzig, 2te Aufl., 1906. An English translation by Mann and Millikan, London, 1902.

1907. Sir A. E. Wright: *Principles of Microscopy*, New York, 1907.

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1907. Lummer-Pfaundler-Müller-Pouillet: *Lehrbuch der Physik*, II, Pt. I, *Optik*, 10te Aufl., 1907.

1909. Arthur Schuster: *Theory of optics.* London, 2nd ed., 1909.

1910. S. O. Eppenstein: *Aberration.* Encyclopedia Britannica, 11th ed., I, 1910, 54–61.

1911. Otto Henker: *Lens.* Ibidem, XVI, 421–427.

1911. Idem: *Microscope.* Ibidem, XVIII, 392–407.

1911. Fred Eugene Wright. *The methods of petrographic-microscopic research.* Carnegie Institution Publication No. 158. Washington, 1911, 14–56.

CHAPTER VIII

THE MICROSCOPE

Simple Microscope

99. Hand Lenses.—A simple microscope (μικρός, small; and σκοπεῖν, to view) is one which consists of but a single lens or of a system acting as a single lens, and gives a virtual and erect image larger than the object. The simplest form is that of a perfect sphere, the primitive lens being a hollow glass globe filled with water. In a spherical lens the distortion (spherical aberration) produced by the outer parts is extremely great. To overcome this, Wollaston inserted a diaphragm between two hemispheres of glass, thus

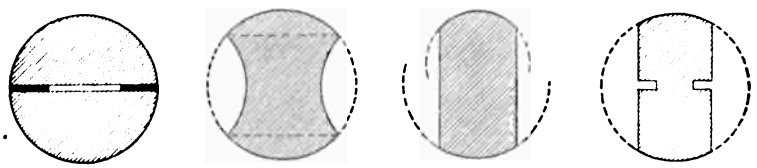

Fig. 216. Fig. 217. Fig. 218. Fig. 219.

Figs. 216 to 219.—Various forms of simple lenses. Fig. 216, Wollaston; Fig. 217, Brewster; Fig 218, Stanhope (Brewster); Fig. 219, Coddington.

cutting off the greater portion of the distortion (Fig. 216). Brewster's lens consists of a sphere with part of the equator cut away, as in Fig. 217. He also invented the so-called Stanhope lens (Fig. 218) which consists of a cylinder of glass whose two ends are spherical surfaces, the lower one being of greater radius of curvature than the upper. Somewhat similar, but con-

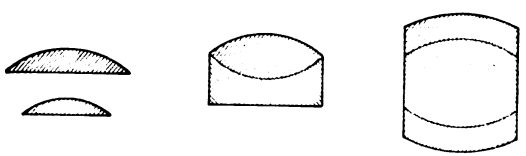

Fig. 220. Fig. 221. Fig. 222.

Figs. 220 to 222.—Various forms of doublets and triplets. Fig. 220, Wollaston's doublet; Fig. 221, Achromatic doublet; Fig. 222, Steinheil triplet.

sisting of a cylinder cut from a sphere and having a groove around the waist to serve as a diaphragm, is the lens devised by Coddington (Fig. 219).

More successful in overcoming aberration are lenses made of various combinations of two or three lenses, and called **doublets** or **triplets.** Wollas-

ton's doublet (Fig. 220) consisted of two plano-convex lenses of different sizes with the plane surfaces toward the object. An achromatic doublet is

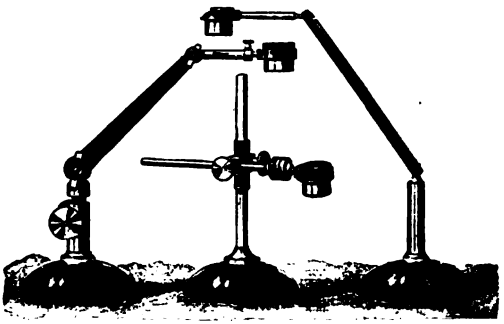

FIG. 223.—Lens stands. (Leitz.)

shown in Fig. 221; it consists of one flint-glass and one crown-glass lens. The Steinheil lens consists of a central crown glass between two of flint (Fig. 222), and gives corrections for both spherical and chromatic aberration.

FIG. 224.—Steinheil aplanatic lens. (Leitz.)

FIG. 225.—Coddington magnifier with handle for laboratory use. (Bausch and Lomb.)

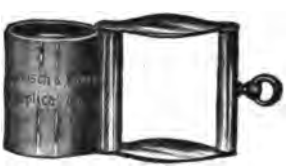

FIG. 226.—Coddington magnifier with folding case. (Bausch and Lomb.)

Lenses are variously mounted, depending upon the purposes for which they are intended. Fig. 224 shows a lens mounted for use in a lens holder

FIG. 227.—Hastings aplanatic triplet. (Bausch and Lomb.)

FIG. 228.—Steinheil aplanatic triplet. (Leitz.)

such as is shown in Fig. 223; Fig. 225 has a handle for laboratory use, and Figs. 226 to 228 are mounted in folding cases for field use.

There are several points to be considered in choosing a hand lens for petrographic work. Two of the most important are flatness of field[1] and depth of focus.[2] Since hand specimens of rocks have surfaces which are generally quite rough, it is desirable that the lens should be able to show clearly points which are in slightly different planes. A focal length of about 1 in. (25 mm.) is perhaps the best for petrographic work, since it is short enough to permit of considerable magnification and long enough to allow plenty of light to fall upon the specimen even when one is examining a mineral within a small depression. The Hastings Aplanatic Triplet with 1 in. focus and magnifying ten diameters (Fig. 227) is excellent but rather expensive. Leitz's Aplanatic (Figs. 224 and 228) is recommended as a moderate priced lens, while the Coddington magnifier (Figs. 225–226) is fairly satisfactory, and cheap.

In regard to the care of a pocket lens, much that is said[3] in regard to the care of lenses in general will apply. Carried in the pocket, it is likely to become dusty, and care should be taken not to scratch the surface in cleaning it. If kept in a small purse it will not only be protected but will be prevented from readily slipping out of the pocket.

In using a hand lens it is to be remembered that the nearer it is held to the eye, the greater will be the field of view.

COMPOUND MICROSCOPE

100. Formation of the Image.—The compound microscope[4] usually consists of two systems, themselves compound. The light is reflected from the mirror (*Sp*, Fig. 231) and passes through the diaphragm (*CD*, Fig. 229) which is placed at the point where the rays cross, a point known as the **entrance pupil** of the microscope. It then passes through the **objective** or **object-lens** and produces a real, enlarged image of an object placed at O_1. This image would normally fall at O_2, but owing to the passage of the rays through the lower or **field lens** of the **eyepiece** or **ocular,** they converge, and the image appears at O_3; that is, O_2 and O_3 are conjugate foci of the field lens. The eye is placed above the upper or **eye lens** of the ocular at the point *EP* where the rays cross, a point known as the **Ramsden disk, pupil of the eyepiece,** or **exit pupil.** The image O_3, which lies in the focal plane (F_2) of the eye lens, acts as the object for this lens and appears as a virtual, enlarged image (O_4) at the distance of distinct vision (C) 250 mm. from the eye.

101. Optical and Mechanical Tube Lengths.—The plane in which the rays cross at F_1, Fig. 229, is known as the **posterior focal plane** of the objec-

[1] Art. 145, *infra.*

[2] Art. 144, *infra.*

[3] Art. 198, *infra.*

[4] For a history of the microscope see R. J. Petri: *Das Mikroskop*, 248 pp., 191 figs.

Also E. M. Nelson: *Development of the compound microscope.* Trans. Middlesex Nat. Hist. and Sci. Soc., 1886–7, 103–111.* Review in Jour. Roy. Microsc. Soc., 1888, 136–7.

tive; where they converge at F_2, the **anterior focal plane of the ocular.** The distance between these two planes is called the **optical tube length**[1] and is expressed by Δ. The **mechanical tube length** (L) is the distance between the upper end of the body tube and the shoulder of the objective screw. It

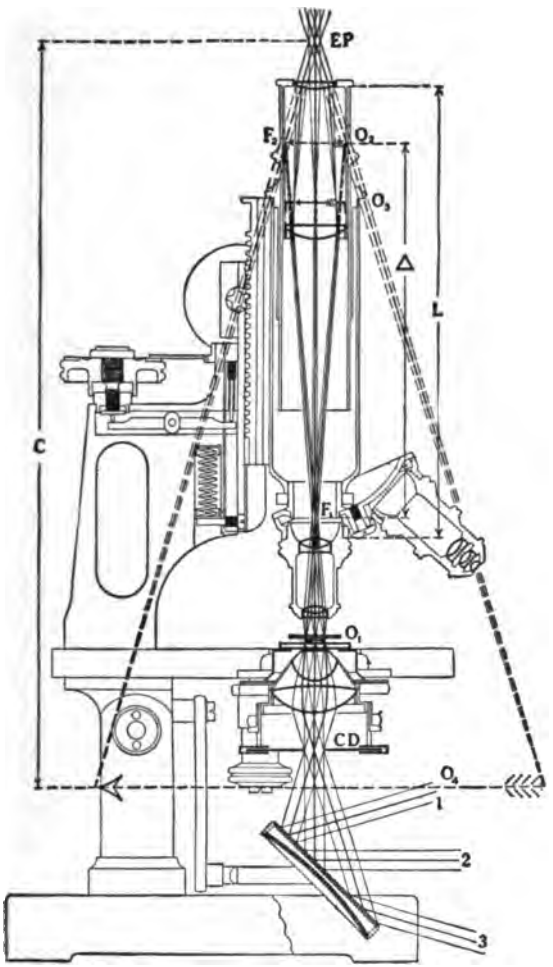

FIG. 229.—Passage of light through a compound microscope.

varies somewhat in the instruments of different makers, being between 160 and 170 mm. in German microscopes and between 200 and 250 mm. in those of English makers. A number of years ago the American Society of Micros-

[1] See Frank Crisp: *On optical tube length, an unconsidered element in the theory of the microscope.* Jour. Roy. Microsc. Soc., 2 ser., III (1883), 816–820.

copists[1] adopted two standards, one of 160 mm. (6.3 in.) for short, and one of 216 mm. (8.5 in.) for long tubes. There seems to be no special advantage of one over the other except that the short tube is often more convenient. Objectives made for a certain tube length should be used for that length and for no other, if the best results of which it is capable are to be obtained. Many instruments are so arranged that the tube is capable of adjustment to different lengths.

102. Focal Length.—The focal length of the compound microscope is expressed by the equation

$$F = \frac{F_1 F_2}{\Delta}$$

where F_1 equals the focal length of the objective, F_2 the focal length of the ocular, and Δ the optical tube length.

103. Magnifying Power.—It was shown[2] that the magnifying power of a simple microscope is expressed by the equation

$$N = \frac{250}{F}.$$

The same equation expresses the magnification of the compound microscope, for

$$N = \frac{\tan \alpha_1}{\tan \alpha} = \frac{\frac{y'}{F}}{\frac{y'}{250}} = \frac{250}{F}$$

where α = half the angle subtended by the object when viewed at a distance of 250 mm. without the microscope, and α_1 = half the angle formed by the image seen through the microscope. That is, the magnifying power of a compound microscope is the ratio of the angle subtended at the eye by the image at the distance of distinct vision, to that subtended by the object at the same distance. As with a simple lens, the actual magnification in the compound microscope differs for different observers, depending upon the power of accommodation of the eye.

104. Field of View.—The field of view decreases with the magnifying power. Roughly, the reciprocal of the magnifying power in diameters, multiplied by five, will give, in fractions of an inch, the size of the field when a Huygens eyepiece is used. Thus with a microscope magnifying 10 diame-

[1] *Report of the committee of the American Society of Microscopists on uniformity of tube length.* Microscope, X (1890), 297.

The tube lengths used by various makers are as follows: Bausch & Lomb, Beck, Reichert, and Zeiss 160 mm.; Fuess, Leitz, and Seibert, 170 mm.; Swift, and Nachet, 200 mm.

[2] See also Articles 98 and 149.

ters, an object 1/2 in. in diameter may be seen; with 20 diameters, an object 1/4 in. in diameter; with 100 diameters an object 1/20 in. in diameter; with 500 diameters, an object 1/100 in. in diameter, and so on.

THE PETROGRAPHIC MICROSCOPE

105. Description.—A petrographic microscope is much more complicated than a biologic microscope, for although the magnifying powers used

FIG. 230. FIG. 231.

FIGS. 230 AND 231.—Mineralogical microscope. Large stand AM. (Leitz.)

are not so great, yet there are attached to it appliances for special examinations which require careful adjustment.

Essentially, a petrographic microscope consists of a stand having a heavy foot (*F*, Fig. 231) and an upright pillar (*St*) to which are attached a revolving stage, and the arm or limb (*OT*) carrying an adjustable tube (*T*). Below the stage is a mirror (*Sp*) and a polarizer (*P*). The tube carries the ocular or eyepiece (*HO*), the objective (*O*), and the analyzer (*A*).

The Mechanical Parts of a Petrographic Microscope

106. Foot.—The foot (*F*, Fig. 231) of the microscope, in American and German instruments, is usually made in the form of a horseshoe; in English instruments, in the form of a tripod (Figs. 320–321). When in the form of a horseshoe, the intermediate parts should be so cut away that the instrument rests only upon three points, which should be covered with leather or felt. Of whatever form, it should be of sufficient weight to counter-balance the instrument when it is tilted backward as far as possible.

107. Pillar or Post.—The pillar or post (*St*) is the upright attached to the foot, and carries, at its upper end, a hinge (*G*) which may be loosened or tightened by means of a bar or a spanner. The hinge should work smoothly and yet be tight enough to prevent the upper part of the instrument from running backward by its own weight and, by the increased momentum, overturning the instrument. It is safer never to allow the microscope to remain in a tilted position when not in use. The habit of always placing it upright when leaving the table may some time save it from an expensive fall.

108. Limb or Arm.—The limb or arm (*OT*) is fastened by the hinge (*G*) below, and carries, at its upper end, fine (*fE*) and coarse (*gE*) adjustment screws for the draw-tube, and at the lower end a support for the stage. The arm is sometimes made with an unnecessary handle, which only makes more weight above the hinge.

109. Stages, Simple and Mechanical.—The stage is the table, usually circular, upon which the thin section is placed for examination. In almost all microscopes it is so arranged that it may be rotated and the amount of rotation determined by means of a graduated scale and vernier, reading to minutes. In its simplest form (Fig. 307) the stage is a circular disk with an opening in the center through which the light passes. The thin section usually is held in place by spring **object clips,** such as are shown in the figure. The objection to such clips is that they are likely to catch under the edges of the cover-glass, and either break or strip it from the mount. It is also difficult to move the slide a very small distance, such as may be necessary in centering a mineral for use with a high-power objective. One is likely to move the slide too far, first in one direction and then in another. To overcome this, various types of **mechanical stages** have been devised.

Among mechanical stages, one of the simplest and most satisfactory is that designed by Hirschwald[1] (Fig. 232). This stage possesses the advantage of holding a slide firmly in place with no interference by spring clips. It has two motions, excellently adapted to a rapid inspection of every part of a

[1] J. Hirschwald: *Ueber ein neues Mikroskopmodell, etc.* Centralbl. f. Min., etc., 1904, 629–630.

thin section, one produced by sliding the plate S within the grooves over the depressed portion A, the other by sliding the thin section itself between the bars b and b_1, one of which holds the slide firmly by means of the spring f. The entire plate S may be removed and a plain plate inserted, making, thus, a flat stage with which the usual spring clips may be used. Johannsen[1] suggested several improvements, among others the addition of graduations

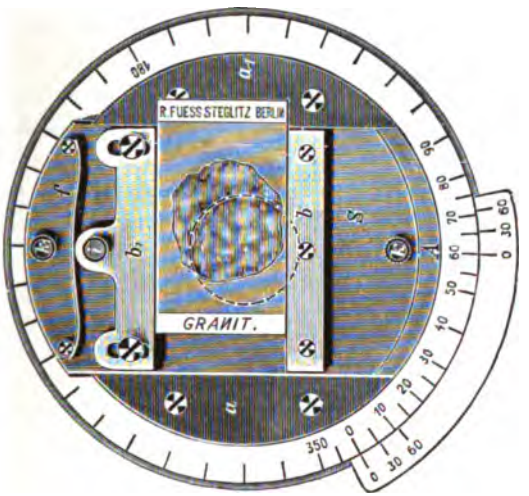

FIG. 232.—The Hirschwald stage. (Fuess.)

along the top bar and sides of the sliding plate whereby any desired point in a thin section may readily be found again.

Another mechanical stage[2] is shown in Fig. 233. This stage carries, besides the usual division of the circumference into a scale reading to minutes by means of verniers, two sliding portions controlled by the screws s and s', one of which has micrometer divisions to 0.01 mm., the other a screw with strong pitch which serves as a rapid finder with low magnifications. Its movement is read to 0.5 mm. by the scale on the stage. By means of these screws it is possible to measure, sufficiently accurately, by a much more rapid method than by a micrometer ocular, the dimensions and proportions of the minerals which occur in a slide. By their means, also, it is possible to find any mineral once determined, the position of the slide on the stage being fixed by the guide strip w.

There are several objections to this stage: its thickness, its liability to

[1] Albert Johannsen: *Some simple improvements for a petrographical microscope.* Amer. Jour. Sci., XXIX (1910), 438.

[2] R. Fuess: *Ueber Mikroskope für krystallographische und petrographische Untersuchungen.* Neues Jahrb., B.B., CII (1891), 55–89, in particular 57–58.

C. Leiss: *Die optischen Instrumente der Firma R. Fuess*, Leipzig, 1889, 185.

lost motion, and the interference, in certain positions, of the micrometer screws with reading the stage vernier.

Another type of mechanical stage is that shown in Figs. 230 and 231.

FIG. 233.—Mechanical stage. 1/2 natural size. (Fuess.)

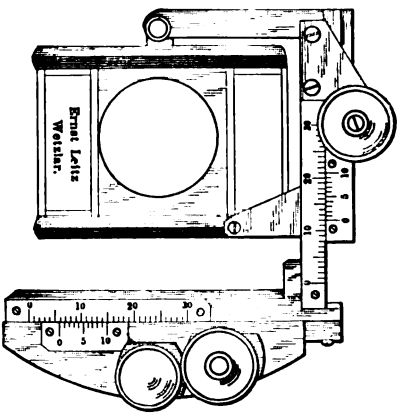

FIG. 234.—Attachable mechanical micrometer stage. (Leitz.)

It possesses the advantage of being thin and concealed within the stage, the amount of movement being read through openings in the latter. It is not, however, adapted to fine readings, which must be made by means of a micrometer ocular.

FIG. 235.—Attachable mechanical stage. (Bausch and Lomb.)

An attachable mechanical stage is shown in Fig. 234. This may be read to tenths of millimeters and is detachable, so that ordinarily it will not be in the way. It is fastened to the stage by means of a thumb screw.

Somewhat similar is the stage shown in Fig. 235.

110. Verniers.—In connection with the fine adjustment screw, and also attached to the sides of the stage of most microscopes, there are certain auxiliary scales called verniers, after the inventor, Pierre Vernier of Burgundy (1631). These verniers may be of various kinds, but in all of them the divisions are a little longer or a little shorter than the divisions of the main scale.

If, for example, in Fig. 236, nine parts of the main scale are equal to ten parts of the vernier, then each division of the latter is one-tenth of a space shorter than a space on the former. As the vernier is made to coincide with successive divisions of the larger scale, each successive coincidence indicates an advance of 1/10 of a division. In Fig. 236 each division of the scale is 0.1 in. therefore each successive mark on the vernier indicates a movement of 0.1 of 0.1 or 0.01 in.; and in a scale so marked, the number of whatever line on the vernier coincides with some line on the scale will indicate the number of hundredths of an inch of movement.

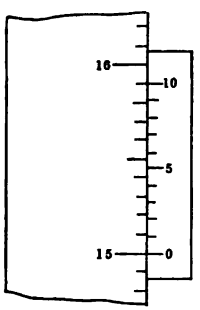

FIG. 236.—A direct vernier.

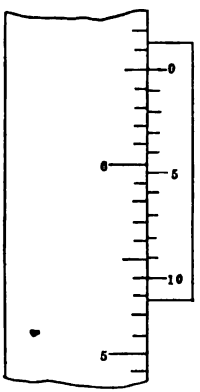

FIG. 237.—A retrograde vernier.

There are two classes of verniers, direct and retrograde. Scales such as that just described, in which the spaces on the vernier are shorter than those on the main scale and the numbering and the lines successively coinciding are in a forward direction, are called **direct.** A **retrograde** vernier has spaces larger than those on the main scale (Fig. 237), and the numbering and the successive coincidences are in the reverse direction. In both kinds, the difference between a division on the vernier and the scale is called the **least count,** and is the measure of its smallest reading.

The verniers on circular scales usually read, not to tenths, but to minutes or seconds. In Fig. 238 the least count is one minute. The main scale is divided to half degrees and 29 of its parts coincide with 30 of the vernier.

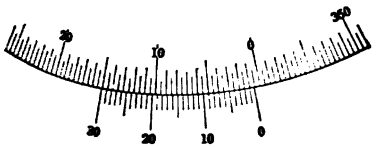

FIG. 238.—Vernier on circular scale.

In verniers where half divisions are the basis of subdivision, one must use great care not to neglect a part of the reading, such as the half degree in Fig. 238, which reads 0° 33′. If one is in doubt whether a division of the vernier coincides with a division on the scale, one should notice that those on either side differ an equal amount. Where several divisions seem to coincide, read the middle one.

111. **Body Tube.**—The body tube (*T*, Fig. 231) is the hollow cylinder to which most of the optical parts are attached. It sometimes consists of but a single tube (Fig. 311), but generally it carried within it a second cylinder called the **draw tube** (*TA*, Fig. 231). The latter has engraved upon its side

a millimeter scale which shows the mechanical tube length. The draw tube, in some microscopes (Figs. 307 and 318), may be extended by simply pulling it out, and it is held in place by friction, but in the more expensive instruments it is moved by means of a rack and pinion (*Oc*E, Fig. 231).

The methods of attaching the optical parts to the tube are quite different in different instruments, and will be described in greater detail in the section dealing with the various makes of microscopes. The optical parts themselves are described below, and consist of eyepiece, objective, polarizer, and analyzer and, in some instruments also, Bertrand lens, and quartz wedge and mica plate.

Besides the optical parts, there are also attached to the tube an objective clutch or a revolving nose-piece, and a centering device for the objective. The thickness of objective clutch or nose-piece, when used, must be added to the figures engraved on the side of the instrument to give the true mechanical tube length.

112. Objective Holders.—Objectives may be attached to the tube in one of four ways. In old microscopes it is necessary to screw each objective into place as it is needed. For such instruments the objective should be held between the thumb and first finger of the left hand and supported at the side, in line with the tube, by the bent second finger. It should then be placed against the threads of the tube, and gently turned *backward* with the fingers of the right hand, until the beginning of the thread drops into position, after which it should be turned gently into place. This method of attaching objectives takes much time, and there is great danger of cross-threading the screw, so that it is now almost universally abandoned in favor of some rapid-changing device. These are of three types, objective clutch, sliding objective changer, and revolving nose-piece.

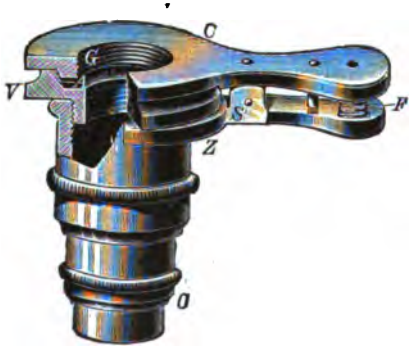

FIG. 239.—Objective clutch. Natural size. (Fuess.)

Most instrument makers manufacture objective clutches of a form similar to that shown in Fig. 239.[1] This is one of the most satisfactory appliances for changing objectives rapidly. It consists of a tongs *C*, made of steel, which is screwed to the tube at *G*. The under side of the upper jaw is beveled (*A*), and accurately fits the collar *V* which is screwed to the objective. To insert or remove an objective, it is only necessary to press the end of the tongs with the left hand and insert the objective between the jaws *C* and *Z* with the

[1] C. Leiss: *Die optischen Instrumente*, Leipzig, 1899, 187.
See also *Nachet's objective-carrier*, Jour. Roy. Microsc. Soc., 1881, 661–662.

right. The objective usually, automatically, slides into proper position of perfect centering, where it is held by the lower jaw. It is well to give the objective a half turn with the right hand, however, to be certain that it has dropped into its proper position.

A similar clutch (Fig. 308), recently put on the market by Leitz but not yet advertised, is so arranged that the clip presses against a slanting bar on the objective collar, whereby an objective, once centered, will always return to the proper position. A centering device is attached to each objective collar, the adjustment being made by means of a watch key. When once adjusted, the centering remains perfect for all objectives.

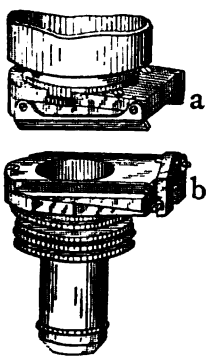

FIG. 240.

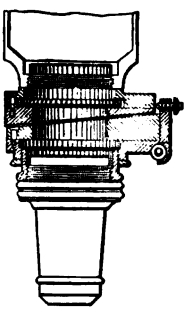

FIG. 241.

FIGS. 240 AND 241.—Sliding objective changer. (Zeiss.)

FIG. 242.—Double revolving nose-piece. (Bausch and Lomb.)

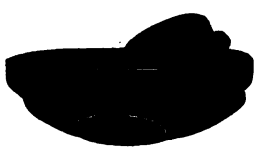

FIG. 243.—Triple revolving nose-piece. (Bausch and Lomb.)

The sliding objective changer,[1] shown in Figs. 240–241, consists of two slides, one of which is screwed to the end of the tube, the other to the objective. The latter can be adjusted by means of two set-screws so that the objective is accurately centered. This holder, like the objective clutch last described, possesses the advantage that the objective is always inserted with the same part to the front, and the centering is perfect.

A revolving nose-piece (Figs. 242–243) has a distinct advantage in supplying the most rapid method of changing objectives. It not only saves time, but is a safeguard against dropping objectives with injury to themselves or to the thin section. As now generally made, a nose-piece automatically centers and practically focusses all objectives, greater or less length of collar making up for differences in focal distances. Revolving nose-pieces are made double (Fig. 242), triple (Fig. 243), and quadruple, thus fitting a microscope with all the objectives ordinarily needed. The objection to them on petrographic microscopes is that usually all of the objectives are not absolutely

[1] Anon. *Zeiss's objective-changer with slide and centering adjustment.* Jour. Roy. Microsc. Soc., 1887, 646-647.

Anon. *Directions for using the sliding objective changer.* Zeiss' circular, pp. 4.

centered. This may be due to want of rigidity in the attachment to the tube, or to wear in the device itself. On a microscope with simultaneously rotating nicols, they are a great convenience, the small amount which an objective may be out of center being of no importance.

113. Slot for Accessories.—Connecting the tube with the objective holder is a short collar containing a slot for the insertion of a quartz wedge, or a gypsum or mica plate, a convenience due to the suggestion of Klein.[1] Ordinarily a great deal of time is lost in picking up these accessories and, in using the quartz wedge, in finding the proper end to insert first. To overcome this, Johannsen[2] suggested a permanently attached combination wedge which slides in and out in a manner similar to the Bertrand lens. It is described below.[3] Where no such permanently attached accessory is used, a sliding plate, in most microscopes, covers, or is supposed to cover, the slot when not in use.

114. Centering Device for Objective.—The center of rotation of the stage should lie on the axis of the tube, that is, it should coincide with the intersection of the cross-hairs of the ocular. For example, suppose an object to be placed upon the stage of the microscope and the latter turned. If the center of rotation appears at o' (Fig. 244) instead of at o, every other point in the slide will rotate about o' as a center. To make the center of rotation coincide with the axis of the microscope, use is made of two centering screws which, in different microscopes, are parallel or diagonal to the cross-hairs, as shown, respectively, in Figs. 312 and 231, *OC*. The former method is most satisfactory since the adjustment is easier to make. To make the correction shown in Fig. 244, the center o' must be moved, by means of the two screws a and b, successively through the distances $o'x$ and xo.

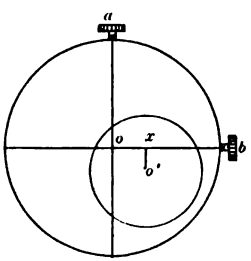

FIG. 244.—Method of centering the cross-hairs.

In some microscopes the stage is centered instead of the objective. This is objectionable since it displaces the entire axis of the instrument.

If the centering device is attached to the objective collar, as in Figs. 240–241 and 308, each objective should be centered.

[1] C. Klein: *Ueber das Arbeiten mit dem in ein Polarisationsinstrument umgewandelten Polarisationsmikroskop und über eine dabei in Betracht kommende, vereinfachte Methode zur Bestimmung des Charakters der Doppelbrechung.* Sitzb. Akad. Wiss., Berlin, 1893 (I), 241.

[2] Albert Johannsen: *Some simple improvements for a petrographial microscope.* Amer. Jour. Sci., XXIX (1910), 436. The wedge may be obtained from Fuess or Leitz. In ordering it should be stated whether the slot for the accessories lies parallel or at 45° to the principal sections of the nicols.

[3] Art. 298, *infra*.

115. Coarse and Fine Adjustment.—A microscope is focussed by raising or lowering the tube with its optical parts attached. In general there are two movements, one coarse (*gE*, Fig. 231) and one fine (*fE*). In the instruments of most makers, the coarse adjustment[1] is effected by a rack and pinion (shown to the right in Fig. 249), while the fine adjustment screws[2] are variously made. The latter adjustment is the most important, for it is often necessary to determine accurately the amount of motion; for example, in determining the thickness of a section.

One of the simplest forms of fine adjustment is shown in Fig. 245. The fixed upright contains within it a spiral spring, sufficiently strong to carry upward against the head, the weight of the tube and its attachments. The upper part of this spring rests against a cap which, in its turn, is kept down by a steel pin attached to the micrometer screw head. The object of the pin is to transmit the downward pressure without setting up a tendency to rotary motion. The pitch of the micrometer screw is 1/2 mm., and it has a total movement of about 5 mm. The head, being divided into 50 parts, will consequently read to 0.01 mm.

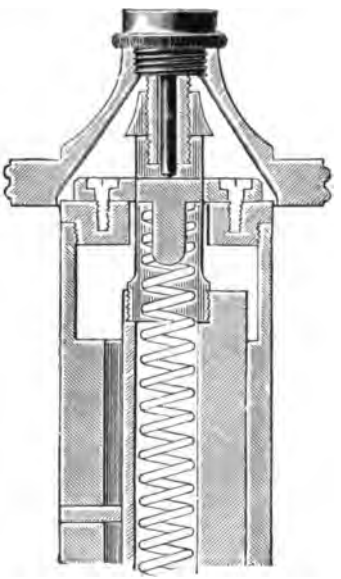

FIG. 245.—Simple fine adjustment screw. 2/3 natural size. (Leitz.)

A more accurate fine adjustment screw[3] is shown in Fig. 246. A worm spindle moves a heart-shaped cam, upon which rests a steel roller carrying the weight of the tube and its attachments. Since the periphery of the cam is formed by symmetrical Archimedian spirals, equal angular displacements produce equal linear displacements of the crest of the cam (Fig. 247), consequently equal elevations or depressions of the tube. The amount of vertical movement is indicated by the micrometer drum (Fig. 246) which is divided into 100 parts, each of which indicates a movement of 0.001 mm. or

[1] Edward M. Nelson: *The trackwork coarse adjustment.* Jour. Roy. Microsc. Soc., 1899, 256–262. (Gives a history of the development of the coarse adjustment.)

[2] Idem: *On the evolution of the fine adjustment.* Ibidem, 1899, 366–375. "M" (G. Marpmann): *Die feine Einstellung der Mikroskope.* Zeitschr. f. angew. Mikrosk., IV (1898–9), 86–90.

[3] Gabriel Lincio: *Das neue Leitz'sche mineralogische Mikroskopmodell A.* Neues. Jahrb., B.B., XXIII (1907), 163–186.

E. Leitz: *Ein neues Mikroskop-stativ und seine feine Einstellung.* Zeitschr. f. Instrum., XXIII (1903), 79–81.

Carl Metz: *Neuere Vervollkommnungen der Leitz'schen Mikroskop-Stative.* Zeitschr. f. wiss. Mikrosk., XXIII (1906), 430–439.

1 μ, about 1/2500 in. The total displacement of the cam is 3 mm., and continuous turning of the screw (*fE*, Fig. 229) produces an up and down movement of that amount. Before measuring thicknesses by means of this micrometer screw, the index mark at the side of the tube should be set halfway between the marks *SS* (Fig. 231) which show the limits of its motion.

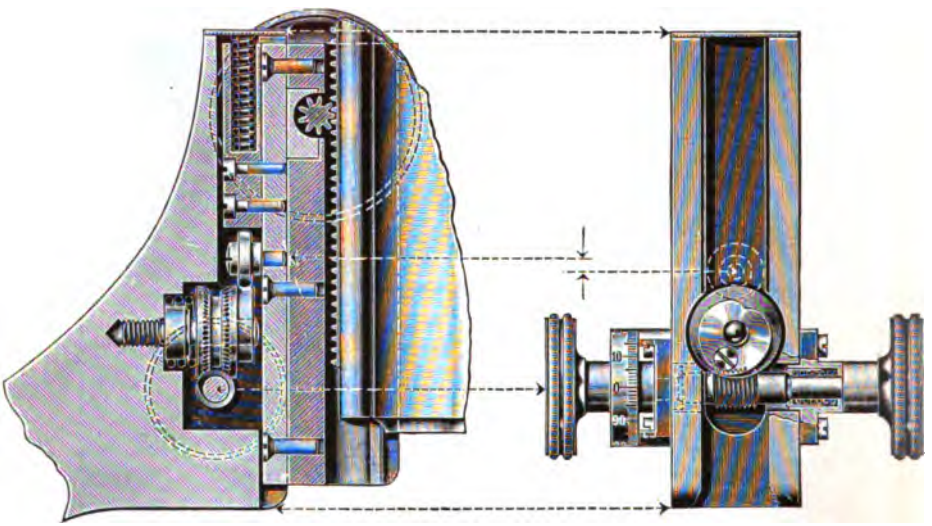

FIG. 246.—Fine adjustment screw. (Leitz.)

This fine adjustment has the distinct advantage of being extremely accurate and of having no lost motion. If the objective should happen to be screwed down upon the cover-glass, it cannot be forced through, but will rest upon it simply by the weight of the tube, which is not great enough to break it.

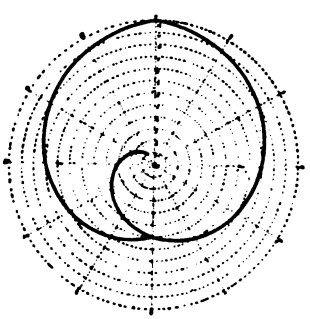

FIG. 247.—Spiral upon which the cam of the preceding fine adjustment screw is cut.

Somewhat similar, but having a cam-like inclined plane (*E*) to transmit the motion, is the fine adjustment shown in Fig. 248.

A fourth type (Fig. 249) depends upon a lever for elevating the tube, the pressure being produced by means of a micrometer screw. In this fine adjustment, as in the last, no pressure beyond the weight of the tube and its accessories can be exerted upon the cover-glass.

116. Sub-stage.—The sub-stage is the ring beneath the revolving stage. It carries the polarizer and, in some microscopes, the condensing system. It is described below[1] in connection with the condensing system.

[1] Art. 118, *infra*.

117. Diaphragms.—A petrographic microscope should contain two diaphragms (besides those in the ocular and objective). One, which is found in most microscopes and called the lower diaphragm, is placed above or below the analyzer, and is used to regulate the amount of light admitted to the eye. The lever controlling it is shown in Fig. 231, *JZ*. For some observations, especially in examining colorless objects, too much light hides the structure. Adjacent colorless minerals, also, may only be distinguishable by means of their slightly different refractive indices.

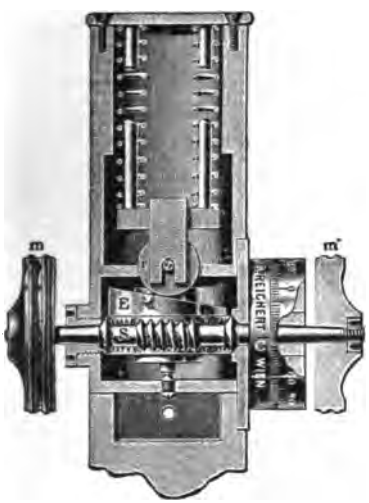

FIG. 248.—Fine adjustmentscrew. (Reichert.)

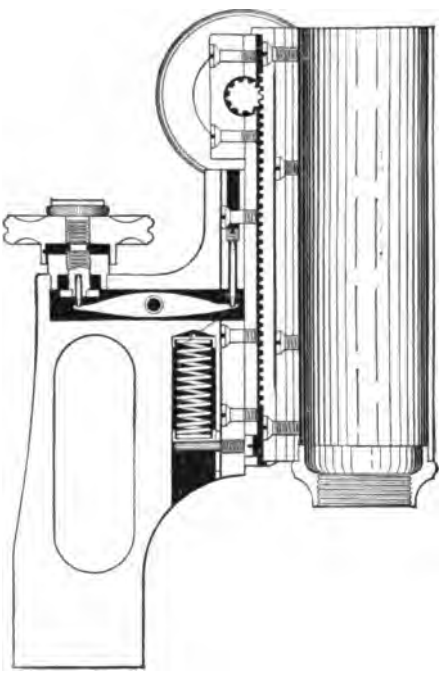

FIG. 249.—Lever fine adjustment. (Bausch and Lomb.)

The other, or upper diaphragm, is placed above or below the Bertrand lens and is used to cut out, from the field of view, interfering minerals when examining the interference figure of a small grain. The lever controlling it is shown at *J*, Fig. 311a. This diaphragm is not found in old microscopes nor in many modern ones, but is extremely useful and should always be provided. It need not be elaborate; a simple perforated slide being sufficient if one of the openings is extremely small.

Two types of diaphragms are used in microscopes: (*a*) with openings of fixed size, (*b*) with a changeable opening. There are various forms of the first type. Among the older kinds used were caps, called **cap-** or

cylinder-diaphragms[1] (Fig. 250), having various sized openings. The objection to one of this style is that it takes considerable time to insert it. In some forms of microscope, it is even necessary to remove the thin section in order to put it in place.

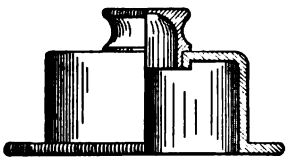

FIG. 250.—Cap- or cylinder-diaphragm. (Zeiss.)

Another form is a simple slide[2] perforated with holes of different sizes. A special variety,[3] with revolving stops, is shown in Fig. 251. This diaphragm is attached to the lower part of the polarizer by means of three clips on the disk *LS*, and is held by the screw *S*. The first opening (1) is of the size of that of the polarizer. The second (2) is centered when the pin of the catch (*St*) drops into the indentation I, while all of those on the rotating disk are centered when the pin engages II.

For oblique illumination, such as is used in the Schroeder van der Kolk

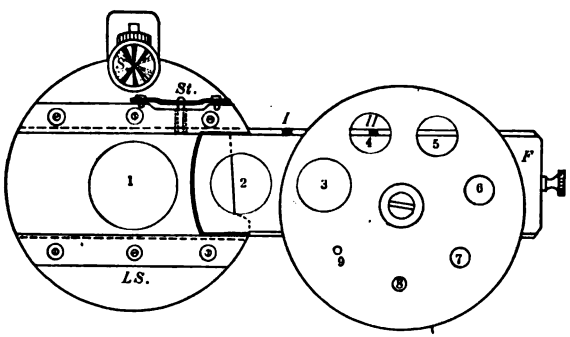

FIG. 251.—Sliding diaphragm with revolving stops. (Leitz.)

method of determining relative refractive indices, it is only necessary to displace the slide a little to one side of the central opening.

The first sliding diaphragm was made by Griffith[4] in 1886. It is shown

[1] Anon: *German "cylinder-diaphragms" and condensers.* Jour. Roy. Microsc. Soc., 2d series, III (1883), 426–427.

[2] J. Anthony: *Sliding stage diaphragms.* Ibidem, 1881, 520 (Describes cardboard or vellum slips, perforated with holes of various sizes, to be placed on the stage and under the glass.)

[3] Anon: *Bousfield's rotating diaphragm-plate.* Ibidem, 1881, 523–524. (Describes a rotating disk without slider, to be placed under the object.)

[4] E. H. Griffith: *Some new and improved apparatus.* Proc. Amer. Microsc. Soc., 8th Ann. meeting, Cleveland, VII (1885), 112–114. Review in Jour. Roy. Microsc. Soc., VI (1886), 130.

Griffith invented another form of lower diaphragm, consisting of circular disks of metal with various shaped holes, or glass disks covered with asphaltum except openings of various shapes. Idem: *On several new microscopical accessories.* Proc. Amer. Microsc. Soc., 9th meeting, VIII (1886), 150–152.

in Fig. 252. As may be seen from the illustration, it contains holes of various sizes and shapes which may be used centered, or, for oblique illumination, out of center. It is attached by a rotating collar below the nicol so that the slot-shaped openings may be turned in any direction. A similar sliding diaphragm with circular openings only (Fig. 253) was described by Wright[1] in 1901.

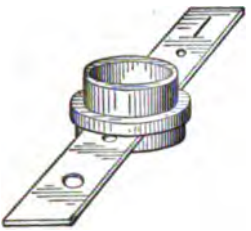

FIG. 252.—Griffith's sliding diaphragm.

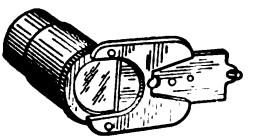

FIG. 253.—Wright's sliding diaphragm.

Sommerfeldt[2] made use of a sliding diaphragm with lath-shaped openings and placed above the Bertrand lens, to observe the interference figures of very small lath-shaped crystals. With circular stops, interference figures of such crystals are confused by those of surrounding minerals, but with small lath-shaped slits of various sizes, such an opening may be selected that all of the surrounding minerals are cut out. Such a diaphragm must, however, be used with a microscope having simultaneously rotating nicols.

FIG. 255.—Polarizer casing with nicol, iris diaphragm, and lower lens of condenser. 3/4 natural size. (Fuess.)

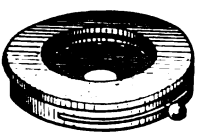

FIG. 254.—Iris diaphragm.

The second type of diaphragms is the iris[3] (Fig. 254), which consists of a number of overlapping leaves attached by pins to the rim of a casing, and having a central opening which is enlarged or diminished by moving a lever. This form is the most convenient to use, although, owing to its construction, the opening cannot be made so small as those in the other types.

[1] Fred. Eugene Wright: *Die foyaitische-theralitischen Eruptivgestein der Insel Cabo Frio, Rio de Janeiro, Brasilien.* T. M. P. M., XX (1901), 239, footnote.

[2] E. Sommerfeldt: *Die mikroskopische Achsenwinkel bestimmung bei sehr kleinen Kristallpräparaten.* Zeitschr. f. wiss. Mikrosk., XXII (1905), 361.

[3] Anon: *The iris diaphragm an old invention.* Amer. Jour. Microsc., V (1880), 136. A description is quoted from Nicholson's Journal for 1804, giving the construction of an iris diaphragm. Neither author nor page reference is given. In a hurried search through Nicholson's Journal, the original description could not be found.

CHAPTER IX

THE MICROSCOPE (Continued)

The Optical Parts of a Petrographic Microscope

118. Illuminating Apparatus.[1]—The illuminating apparatus of a microscope is attached to the sub-stage, and consists of a mirror, a diaphragm, and a system of condensing lenses. In some of the older forms of microscopes the thin section had to be removed, when changing from parallel to convergent light, in order to insert the condensing lens, thus making necessary a relocation of the mineral under examination. In most modern microscopes the change can be performed from beneath the stage, usually by means of a lever which throws in all, or part, of the condensing system; in some instruments with, and in some without, the polarizer. Each maker, almost, attaches the condenser to the microscope in a different manner. The object to be attained is the possibility of rapidly inserting or removing it with as little disturbance as possible to other attachments and to the thin section.[2]

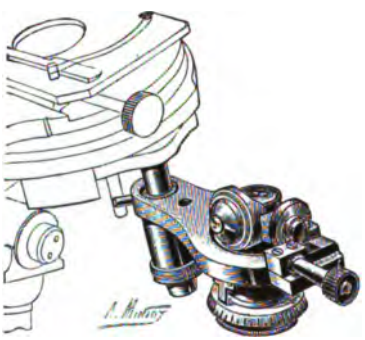

FIG. 256.—Swing-out condensing system and polarizer. (Nachet.)

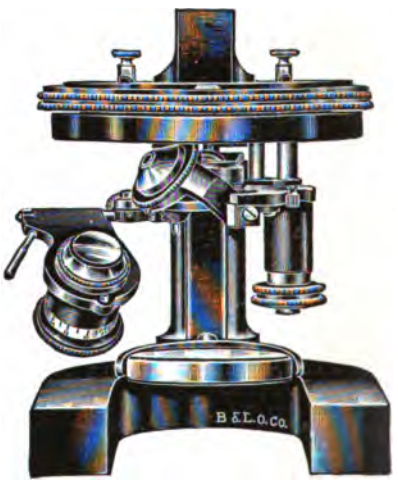

FIG. 257.—Condensing system and swing-out polarizer. (Bausch and Lomb.)

[1] The theory of illumination is fully given by Sir A. E. Wright: *Principles of Microscopy.* New York, 1907, 168–190.

See also H. E. Fripp: *On the theory of illuminating apparatus employed with the microscope.* Jour. Roy. Microsc. Soc., II (1879), 503–529.

Idem: *On daylight illumination with the plane mirror.* Ibidem. III (1880), 742–749.

[2] See E. A. Wülfing: *Ueber eine Vorrichtung zum raschen Wechsel der Beleuchtung am Mikroskop.* Neues Jahrb., 1889 (II), 199–202.

The upper condenser of the Fuess microscopes is shown at *b*, Fig. 233, and can also be seen through the opening in the center of the stage in Fig. 311. This condenser can be removed or inserted by means of a lever (*b'*, Fig. 233) at the side of the rotating stage. Since this lever does not always remain in the same position, rotating as it does with the stage, it is not as convenient as though it were beneath it. The lower condensing lens is attached to the polarizer casing (Fig. 255) and remains in position even when low power objectives are used. Beneath this lens, but above the nicol prism, is the iris diaphragm

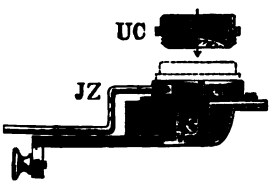

FIG. 258.

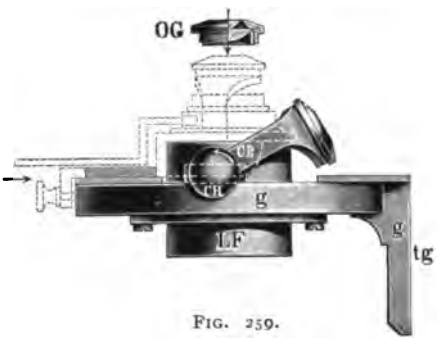

FIG. 259.

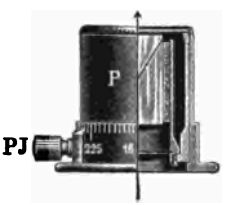

FIG. 260.

FIGS. 258 TO 260.—Condensing system, iris diaphragm, and polarizer. (Leitz.)

(*i*), which is regulated by the lever *Z*. In the microscope shown in Fig. 311 there is no appliance for swinging this nicol and condenser aside; it simply fits snugly in a collar, from which it must be withdrawn by hand.

Certain condensing systems may be swung entirely aside, as shown in Fig. 256. The screw beneath the pin upon which the whole attachment turns serves to elevate or depress the entire system. Above the nicol is the lower, weak condensing lens, which is used even in observations by parallel light. It is transformed into a powerful condenser by swinging above it a second lens by means of the milled head shown. Below the nicol is an iris diaphragm. The nicol itself may be rotated within the casing.

Similar in construction are the condensing systems shown in Figs. 257 and 307.

The condensing system in Leitz's large microscope[1] (Figs. 230 and 231),

[1] Gabriel Lincio: *Das neue Leitz'sche mineralogische Mikroskopmodell A*. Neues Jahrb., B. B., XXIII (1907), 163–186.

shown in detail in Figs. 258-260, may be raised or lowered by means of the milled head below the stage at *BT*, Fig. 231. Above the polarizer (*LF*, Fig. 259) is the iris diaphragm (*JZ*, Fig. 258) and the lower condenser (*UC*), both of which are carried by the sliding arm *S*, and may be swung aside. The upper condenser (*OC*, Fig. 259) is mounted on a fork (*CB*) and may be tilted aside by rotating the button *CH*.

The **mirror**[1] is used to reflect the light from the source to the object, and is attached to the sub-stage by means of the mirror bar. It may be inclined in any direction by means of its various pivots, and may be rotated so that either side is uppermost. A plane mirror forms one side and a concave the other, the former being used with low magnifications, where a comparatively weak light is sufficient, while the latter is used for higher magnifications concentrating the light by converging the rays included within an angular aperture of about 40°. For very high magnifications and for phenomena to be observed in convergent light, the condensing lens, already described, is used.

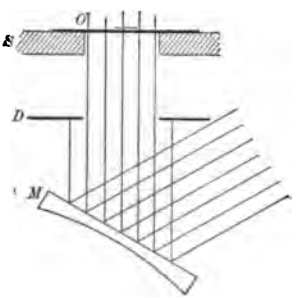

FIG. 261.—Parallel rays of light, plane mirror, no condenser.

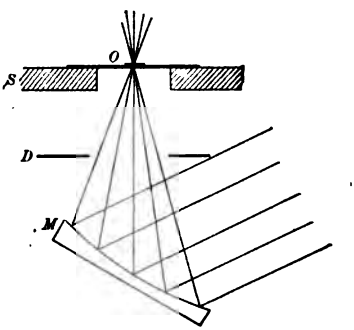

FIG. 262.—Parallel rays, concave mirror, no condenser.

When parallel rays, such as those reflected from the sky, are used, they are reflected from the plane mirror with a slight loss of intensity (Fig. 261). From the concave mirror they are reflected with increased intensity, the rays coming together at the focal point (Fig. 262). For any source of light nearer the instrument, the focal distance is greater (Fig. 263) until it may be twice as great when the source is quite near. To adjust this distance, the mirror is attached to the mirror-bar by means of a sliding sleeve (Figs. 307 to 324).

The condensing lens, being constructed so that its focus is some distance above its upper surface, should always have the plane mirror (Fig. 264) used in connection with it. The concave mirror causes the rays to converge too low down (Fig. 265) to give the best results, which are, clearly, to be obtained when the condenser is in focus. This may be accomplished by placing upon

[1] See Edward M. Nelson: *Construction of silvered lens mirrors.* Jour. Roy. Microsc. Soc., 1894, 254–260.

the stage a thin section and focussing upon it with a low power objective. The plane mirror is then turned until some moderately distant object, such as a window bar, tree, etc., appears in the field, after which the condenser is racked up or down until the image of this object becomes perfectly sharp. After the condenser is in focus, the image by which it was focussed should be removed from the field of the microscope by a slight rotation of the mirror.

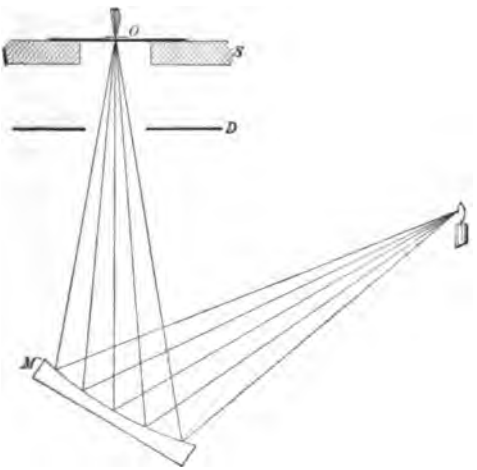

FIG. 263.—Divergent rays, concave mirror, no condenser.

Since focussing the condenser is the process of adjusting its elevation in relation to the surface of the slide, it is not necessary, ordinarily, to refocus it for different power objectives, but if the glasses upon which the various sections are mounted are greatly different in thickness, it may be necessary

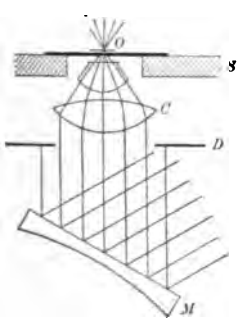

FIG. 264.—Correct method of illumination. Parallel rays, plane mirror, and condenser.

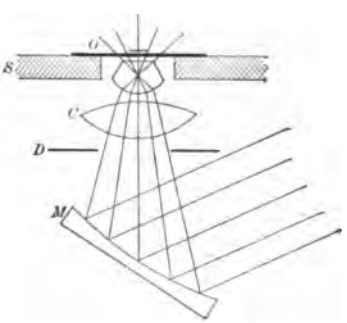

FIG. 265.—Incorrect method of illumination. Parallel light, concave mirror, and condenser.

to do so. In some microscopes there is not much leeway in regard to focussing, the image being sharp only when the condenser rests against the bottom of the mount.

POLARIZING PRISMS

119. Introduction.—Many of the most important characteristics of crystals are determined by polarized light, and the appliance for producing it forms an important adjunct to the petrographic microscope. We have already seen[1] what polarized light is, and have learned that it may be produced by reflection, single refraction, absorption, or double refraction. In a few simple microscopes (Figs. 723 and 724) light is polarized by reflection, but in the usual petrographic microscopes, some form of calcite (Iceland spar) prism is used.

120. Nicol Prism (1828).—The first polarizing prism of Iceland spar was invented by W. Nicol[2] in 1828, and, after him, such prisms are called nicol prisms or simply nicols.

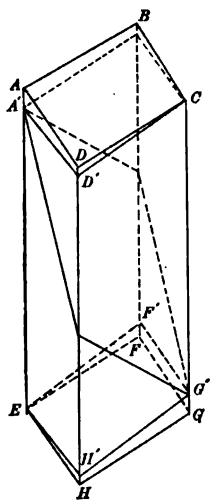

FIG. 266.—A cleavage rhombohedron of Iceland spar.

In constructing this prism, a cleavage rhombohedron of calcite, about three times as long as it is broad, is used. It is cut diagonally across in a plane parallel to the long diagonal of the end faces (*BD*, Fig. 266), along the line represented by *A'G'* in Fig. 267, making an angle of 22° with the edge *AE*. The natural faces *ABCD* and *EFGH*, Fig. 266, make angles (*CAE* and *ACG*, Fig. 267) of 109° 7′ and 70° 53′ with the edges *AE* and *CG*, and these faces are ground down, at *A* and *G*, until they form angles of 112° and 68° (*CA'E* and *A'CG*, Fig. 267) with *AE* and *CG*, making the angle $GA'C = 90°$. The two pieces, after being polished upon the cut faces, are cemented in their original position by Canada balsam, blackened on the faces which are vertical in Fig. 266, and set in a cork and metal casing.

If, now, a ray of light falls upon one end of the nicol prism, it will be broken up, by the double refraction, into two rays, one of which will be more refracted than the other. If the light falls upon the lower surface (Fig. 268) at such an angle that the extraordinary ray travels in a direction parallel to the long edges, it will

[1] Art. 42, *supra*.

[2] William Nicol: *On a Method of so far increasing the Divergence of the two Rays in Calcareous-spar that only one image may be seen at a time.* Edinb. New Phil. Jour., VI (1828–9), 83–94.

Idem: *Notice concerning an Improvement in the Construction of the Single Vision Prism of Calcareous Spar.* Ibidem, XXVII (1839), 332–333.

E. Sang: *Investigation of the action of Nicol's polarizing eye-piece.* Read Feb. 20, 1837. Published in Proc. Roy. Soc. Edinburgh, XXXIII (1890–1), 323–336.

Prof. Tait: *Note on Dr. Sang's paper.* Ibidem, 337–340.

M. Spassky: *Note über das Nicol'sche Prisma.* Pogg. Ann., XLIV (1838), 168–176.

K. Fuessner: *Ueber die Prismen zur Polarization des Lichtes.* Zeitschr. f. Instrum., IV (1884), 41–50.

make an angle of 63° 44′ with the c axis of the crystal until it strikes the film of balsam. The direction of vibration of the extraordinary ray in a crystal of calcite, as we have already seen,[1] is in the plane containing crystallographic c and the ray, consequently in the plane $ACGE$, Fig. 266,

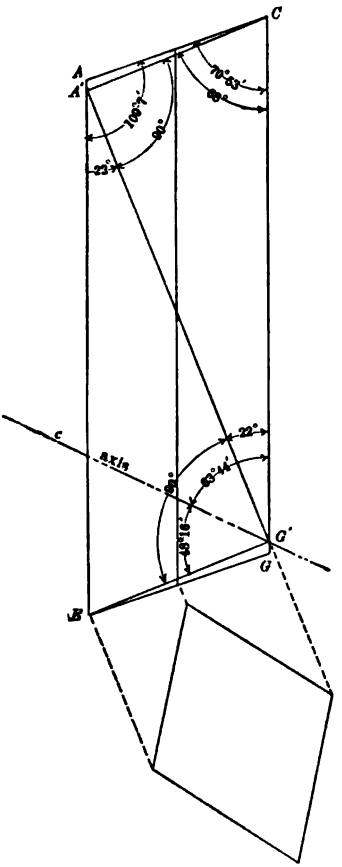

FIG. 267.—The form of a nicol prism.

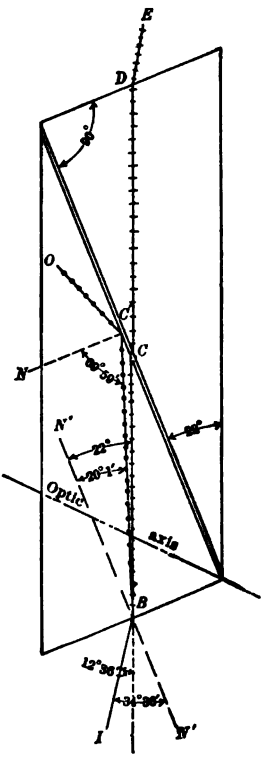

FIG. 268.—Passage of light through a nicol prism.

and may be represented by the short lines along the ray in Fig. 268. The index of refraction of this ray, by calculation, is found to be 1.5159.

From equation (9), Art. 53, we can obtain the index of refraction of the ray in question.

$$\frac{1}{\epsilon_1^2} = \frac{\sin^2\varphi}{\epsilon^2} + \frac{\cos^2\varphi}{\omega^2},$$

where ϵ_1 is the index of refraction of the wave whose normal makes an angle of φ with the c axis. Substituting, in this equation, the values of the principal indices

[1] Art. 48, *supra*.

of refraction of calcite, $\epsilon = 1.4864$, $\omega = 1.6585$, and the value of $\varphi = 63° 44'$, and solving for ϵ_1 we have

$$\epsilon_1 = 1.5159, \tag{1}$$

the value of the index of refraction of the ray.

It was assumed above that the ray of light, falling upon the lower face of the nicol, made such an angle that the extraordinary ray passed through it parallel to its long sides. In order to do this, as will be shown, the light, coming from below, must make an angle of 34° 36′ with the normal to the lower face. At this angle of incidence, the ordinary ray will make an angle of 20° 1′ with the normal, or 69° 59′ with the balsam film.

From equation (8) Art. 37, we know that

$$\frac{\sin i}{\sin r_\epsilon} = \epsilon \tag{2}$$

where i is the angle of incidence and r_ϵ, the angle of refraction of the extraordinary ray. Now since the extraordinary ray assumed above passes straight through the crystal, it forms an angle of 22° with the balsam film, and since the base of the crystal is at right angles to this film, the extraordinary ray makes the same angle with the normal to the base. That is, 22° is also the angle of refraction of the extraordinary ray ($r_\epsilon = 22°$, Fig. 268).

Substituting in equation (2) this value and the value for ϵ_1 (= 1.5159) found above, we find

$$\frac{\sin i}{\sin 22°} = 1.5159,$$

$$\sin i = .5678, \tag{3}$$

$$i = 34° 36', \tag{4}$$

which is the angle made by the incident light with the normal to the lower face of the prism.

The angle which the ordinary ray makes after passing into the crystal is obtained from the equation

$$\frac{\sin i}{\sin r_\omega} = \omega, \tag{5}$$

where i is the angle of incidence, and r_ω, the angle of refraction of the ordinary ray.

The index of refraction of the ordinary ray is the same in every direction, therefore $\omega = 1.6585$. Substituting this value and the value for sin i from equation (3) in equation (5), we have:

$$\sin r = \frac{0.5678}{1.6585} = 0.3423,$$

$$r_\omega = 20° 1', \tag{6}$$

which is the angle of refraction of the ordinary ray. The normal being parallel to the balsam film, this is also the angle between the ordinary ray and the balsam, which makes the angle of incidence at this film

$$90° - 20° 1' = 69° 59'. \tag{7}$$

Let us now see what takes place when the extraordinary and ordinary rays strike the balsam film.

The index of refraction of old balsam is approximately 1.54. That of the extraordinary ray is 1.5159 (Eq. 1); consequently, passing from a rarer to a denser medium, the ray will be transmitted, and, upon entering the second half of the prism, will resume its original course along a line parallel to the long sides of the prism and pass out at D.

The ordinary ray, with an index of 1.6585, strikes a medium with an index of 1.54 at an angle of 69° 59′. Now the critical angle for the ordinary ray, in passing from calcite to balsam, is 68° 13′, and the ray in question, falling upon the separating plane at a greater angle, is totally reflected, passes aside at D', and is absorbed by the black paint with which the sides are coated.

The critical angle of the ordinary ray (index of refraction = 1.6585) upon striking the balsam (index of refraction = 1.54) is obtained as follows:

In calcite we have

$$\frac{\sin i}{\sin r'} = n' = 1.6585,$$

and in balsam

$$\frac{\sin i}{\sin r''} = n'' = 1.54.$$

Dividing the former by the latter, we have

$$\frac{\frac{\sin i}{\sin r'}}{\frac{\sin i}{\sin r''}} = \frac{\sin r''}{\sin r'} = \frac{1.6585}{1.54}.$$

The critical angle between the two will be reached when $r'' = 90°$ (Art. 41) whereby the sine of r'' would equal 1.

Our equation therefore becomes

$$\frac{1}{\sin r'} = \frac{1.6585}{1.54},$$

$$\sin r' = 0.9286, \tag{8}$$

$$r = 68° 13',$$

which is the value of the critical angle between the two media.

We see, from this demonstration, that a ray of light, falling upon the base of a nicol prism and making an angle of 12° 36′ (34° 36′−22°) with its long direction, will be broken up into two rays, one of which, the extraordinary, will pass through it in a straight line (except for a slight displacement by the film of balsam), while the other, the ordinary, will be totally reflected. The extraordinary ray, at the same time, is polarized, and, upon emerging, has its vibrations in a plane at right angles to the film of balsam.

If one looks through a nicol prism at a bright surface, such as a white cloud, it will be seen that beyond a certain distance from the center, and

separated from it by a blue-black band, the field is dark. This is due to the fact that beyond that distance the extraordinary ray also falls on the balsam film at an angle greater than its critical angle, consequently it also is totally reflected. On the other side of the prism there is an area of great brightness, separated from the central part by a narrow series of colored bands. This area represents the region where the ordinary as well as the extraordinary

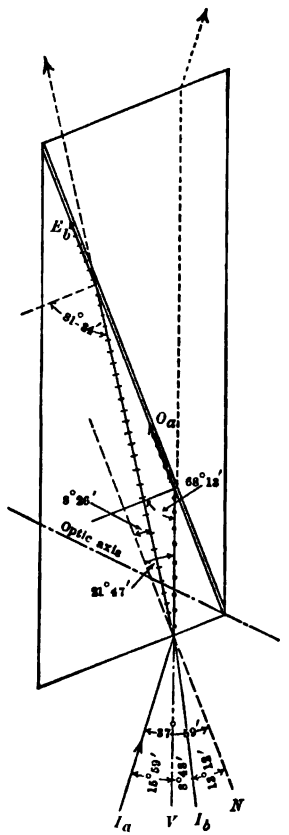

FIG. 269.—Nicol prism showing the limiting rays beyond which the light is not plane polarized.

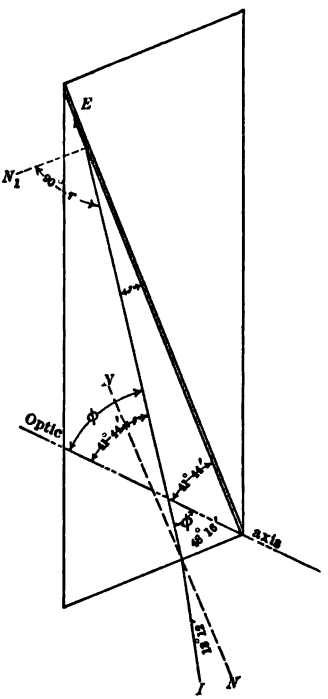

FIG. 270.—Nicol prism.

ray passes through. The internal angle between the rays forming these limits is about 14°, which is increased to 25° by refraction as they pass into the air. The latter angle is called the useful **opening angle of the nicol.**

To determine the opening angle of the nicol, the problem becomes that of finding the angles at which the light must enter the prism so that the ordinary and the extraordinary rays may reach the balsam film at their respective critical angles.

The critical angle for the ordinary ray is 68° 13′, and the angle which it makes with the normal to the base of the prism (Fig. 269) is equal to

$$90°-68°\ 13'=21°\ 47'=r,$$

whereby

$$\frac{\sin i}{\sin 21°\ 47'}=1.6585,$$

$$\sin i=1.6585\times 0.3711=0.61547,$$

$$i=37°59',$$

which is the angle of incidence at the lower surface when the ordinary ray is at the critical angle.

Since the normal lies at an angle of 22° with the line parallel to the long sides of the crystal, this ray will form an angle of $37°\ 59'-22°=15°\ 59'$ with it. This angle is the limiting value of the nicol in this direction, since the ordinary as well as the extraordinary ray of light, entering from beyond it, will pass through the balsam instead of being reflected.

The determination of the incident angle for the limiting value in the other direction is more complicated, since the critical angle must be determined for a ray whose index of refraction changes with its direction of transmission (Fig. 270).

Let i = the angle of incidence of the limiting extraordinary ray.
r = its angle of refraction.
$90°-r$ = the critical angle against the balsam film.
$\epsilon_{//}$ = its index of refraction.
$\varphi = 41°\ 44'+r$ = the angle between crystallographic c and the normal to the wave front of the ray.

Determining the critical angle as was done in equation (8), we have:

$$\sin(90°-r)=\cos r=\frac{1.54}{\epsilon_{//}},\ \text{or}\ \epsilon_{//}=\frac{1.54}{\cos r}. \tag{1}$$

Also

$$\frac{\sin i}{\sin r}=\epsilon_{//}. \tag{2}$$

and equation (9)

$$\frac{1}{\epsilon^2_{//}}=\frac{\sin^2\varphi}{\epsilon^2}+\frac{\cos^2\varphi}{\omega^2}. \tag{3}$$

Combining equations (1) and (3)

$$\frac{(1.54)^2}{\cos^2 r}=\frac{\omega^2\epsilon^2}{\omega^2\sin^2\varphi+\epsilon^2\cos^2\varphi}.$$

Reducing

$$\omega^2\epsilon^2\cos^2 r=(1.54)^2\omega^2\sin^2\varphi+(1.54)^2\epsilon^2\cos^2\varphi.$$

Substituting $\sin^2\varphi=1-\cos^2\varphi$, and the values of ϵ, ω, and φ,

$$1.216\cos^2 r=1.294-.255\cos^2(41°\ 44'+r),$$

from which

$$r = 8^\circ\ 26'. \tag{4}$$

Substituting this value in (1)

$$\epsilon_a = \frac{1.54}{\cos 8^\circ\ 26'} = 1.5568. \tag{5}$$

Substituting values from (4) and (5) in (2),

$$\sin i = 1.5568 \sin 8^\circ 26',$$

$$i = 13^\circ\ 12'. \tag{6}$$

The critical angle, therefore, is

$$90^\circ - r = 90^\circ - 13^\circ 12' = 81^\circ 34'. \tag{7}$$

The inclination of the incident ray to the vertical may be seen in Fig. 269 to be

$$22^\circ - 13^\circ\ 12' = 8^\circ\ 48'. \tag{8}$$

The total serviceable opening of the nicol is the angle between the entrance angles of the rays which produce the limiting rays, upon one side where the ordinary reaches its critical angle, and upon the other where the extraordinary reaches its critical angle. As shown above, this angle is equal to

$$(37^\circ\ 59' - 22^\circ) + (22^\circ - 13^\circ\ 12') = 24^\circ\ 47'.$$

For certain purposes, such as analyzers of microscopes, the sloping ends of the nicol prism are objectionable, since they both diminish the light by reflection, and displace the object. In certain cases it may be desirable to use a prism having a lesser length in relation to its breadth, or having a larger opening angle than that of the nicol. For these reasons many modifications have been proposed, among which the more important are the following:

121. Sang Prism (1837).—In a paper read before the Royal Society, Edinburgh, in 1837, but not published until 1891, Sang[1] discussed the principle of the Nicol prism, and calculated the most effective angle at which it can be cut. He suggested that a prism might be constructed so that the ordinary ray would pass and the extraordinary be reflected. To make this possible, a plate of calcite must be placed between two wedges of a medium having a higher index of refraction than that mineral. Such a prism is shown in Fig. 271, in which *AB* is a plate of Iceland spar cut at right angles to the optic axis, and *ABD* and *ABC*, two wedges of glass having a refractive index of 1.655. The angle *ABD* must be 26° 10′ 19″ in order that the extraordinary ray within the prism may suffer total reflection, whereby only the ordinary ray of all light entering between *A* and *D*, will pass through. Between *D* and *B*, both rays will pass. The angle *BDA* is 52° 50′ 35″ and *BAD*, 100° 59′ 06″.

[1] E. Sang: *Investigation of the action of Nicol's polarizing eye-piece.* Read Feb. 20, 1837, published in Proc. Roy. Soc. Edinburgh, XVIII (1890–1), 323–336.

Prof. Tait: *Note on Dr. Sang's paper.* Ibidem, 337–340.

122. Foucault Prism (1857).—The prism constructed by Foucault[1] consists of a cleavage rhombohedron of Iceland spar with natural faces, and so cut that the section makes angles of 51° and 58° 7′ with these faces (Fig. 272). The two parts are not cemented but are separated by a film of air. This has the effect of increasing the difference between the indices of refraction between the calcite and the film, consequently of decreasing the critical angle. The smaller the critical angle, the shorter will be the required prism; here it bears a ratio of about 1.5 to 1, length to width.

The ordinary ray reaches the film of air at an angle greater than its critical angle (C.A. = 37° 14′) and is totally reflected. The extraordinary ray is

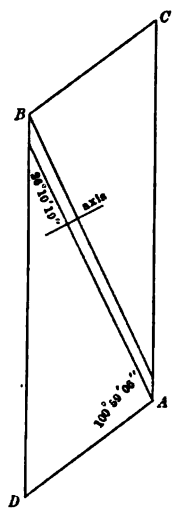

FIG. 271.—Sang prism.

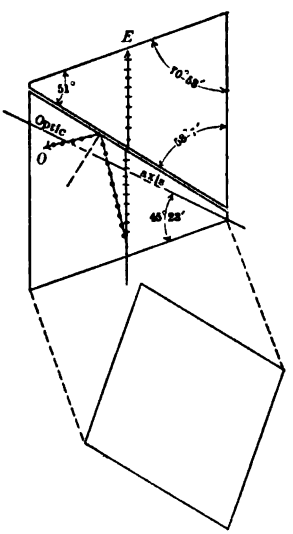

FIG. 272.—Foucault prism.

transmitted in part, but owing to the reflecting surfaces at the film, about 10 per cent. is lost. The critical angle of the extraordinary ray is 42° 23′; there is, therefore, an angle of only 5° 9′ between these limiting values, equal to an angle of about 8° upon the emergence of the rays in air.

The advantage of the Foucault prism is its economy in the use of Iceland spar. Its disadvantage is the loss of transmitted light and its small opening angle, which prevents its use for convergent light.

123. Hartnack-Prazmowski Prism (1866).—The prism of Hartnack-

[1] Léon Foucault: *Nouveau polariseur en spath d'Island.* Comptes Rendus, XLV (1857), 238–241.

Anon: *Neuer Polarisator von Kalkspath.* (Review of preceding.) Pogg. Ann., CII (1857), 642–3.

Prazmowski[1] gives a wide opening angle, varying between 28° and 42° with the kind of cementing material and the angle at which the ends are cut. As originally described the prism was rectangular (Fig. 273) and was so cut that the optic axis of the crystal lay at right angles to the plane joining the two parts. The angles which this intersecting plane made with the end faces for various cementing media are given in the following table.

Cementing material	Value of n	Angle between end faces and film	Field	Length	Interior angle
Canada balsam.....	1.549	79°	33°	5.2	20° 54′
Copaiva balsam....	1.507	76° 30′	35°	3.7	24° 42′
Linseed oil.....	1.485	73° 30′	35°	3.4	26° 24′
Poppy oil.....	1.463	71°	28°	3.0	17°

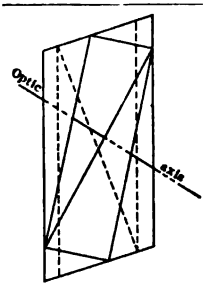

FIG. 273.—Hartnack-Prazmowski prism, compared with an ordinary nicol.

This prism was re-calculated by Fuessner[2] who found that its most advantageous form was produced when the end faces formed an angle of 76° 5′ with the film of linseed oil. Such a prism gives a field in air of 41° 54′ and the ratio of its length to width is 4.02. When the extraordinary ray is reflected parallel to the film, the ordinary forms an angle of 26° 22′ with it, r_ϵ being 13° 55′ and r_ω, 12° 27′. If a balsam film is used $r_\epsilon = 10° 18'$ and $r_\omega = 11° 30'$. The prism may be considerably shortened, at the expense of the field of view, by decreasing the angle between the film and the end faces.

Field	Angle between end faces and film	Ratio length to width	
41° 54′	76° 5′	4.04	All prisms with linseed oil films.
35°	74° 5′	3.51	
30°	72° 37′	3.19	
20°	69° 39′	2.70	

The advantages of this prism are its square ends and its high opening angle, which throws the blue fringe far to one side. Its disadvantages are its wastefulness of spar, its great length compared with its width, and the fact

[1] Hartnack et Prazmowski: *Prisme polarisateur.* Ann. Chim. et Phys., 4 ser., VII (1866), 181–189.

Deleuil: *Prisme polarisateur de MM. Hartnack et Prazmowski.* Comptes Rendus, LXII (1866), 149–150.

Review of preceding: *Polarisationsprisma von Hartnack und Prazmowski.* Pogg. Ann., CXXVII (1866), 494–496.

[2] K. Fuessner: *Ueber die Prismen zur Polarisation des Lichtes.* Zeitschr. f. Instrum., IV (1884), 41–50.

that linseed oil dries in time, causing its index of refraction to increase and producing bubbles in the film.

124. Talbot Prism (1872).—In order to reduce the amount of Iceland spar necessary to make a nicol prism, Talbot,[1] in 1872, constructed one in which one-half was replaced by a prism of glass. No further description was given of it except that "either end could be held foremost," probably meaning when used as an analyzer.

125. Glan Prism (1880).—The Glan[2] prism is much shorter than any of the preceding, the ratio of length to breadth being theoretically 0.831, though in practice it is customary to let the two pieces project beyond the cut surface as shown in Fig. 274, making the ratio 0.924 to 1.141.

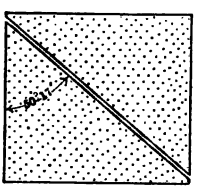

FIG. 274.—Glan prism in section.

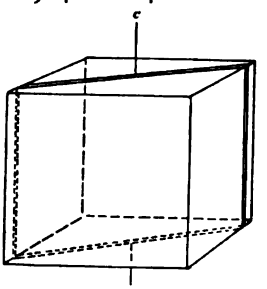

FIG. 275.—Glan prism in perspective.

The prism differs from those described above in that its optic axis lies in the plane of separation and at right angles to the side faces; consequently parallel to the end faces. The separating film, which is of air 1/2 mm. thick, forms an angle of 50° 17′ with the sides. While this prism has the advantage of shortness, it has the disadvantage of having an opening angle of only approximately 8°, and likewise of causing considerable loss of light on account of the separating air film.

This prism is sometimes called Glan-Foucault since it embodies some of the principles of the Foucault prism described above.

126. Thompson Prisms (1881 and 1886).—In Professor Thompson's 1881 prism[3] the opening angle is about 35°. The external form is the same as that of the nicol prism, but crystallographic *c* lies at right angles to the axis of the prism and in the balsam film (Fig. 276). By this means the blue fringe is removed from the field. Thompson suggested cutting the end faces more oblique, which would reflect the ordinary ray farther and increase

[1] H. F. Talbot: *On the nicol prism.* Proc. Roy. Soc. Edinburgh, VII (1872), 468–470. Compare the prisms of Leiss (1897) and of Lommel (1898), described below.

[2] P. Glan: *Ueber einen Polarisator.* Carl's Repertorium, XVI (1880), 570–73.
Idem: *Nachtrag zum Polarisator.* Ibidem, XVII (1881), 195.

[3] Silvanus P. Thompson: *On a new polarizing prism.* Phil. Mag. 5 ser. XII (1881), 349–351.
Idem: Same title. Rept. Brit. Asso. Adv. Sci. 1881, 563–564.

the opening angle. It would, however, decrease the amount of light by reflection, and increase the distortion of the field. There is much waste in cutting this prism.

In 1886, Professor Thompson[1] suggested another kind of prism which he called a "reversed Nicol," and which possesses certain advantages and does not add much to the cost. The broken line in Fig. 277 represents a

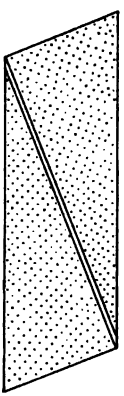

FIG. 276.—Thompson's earlier prism.

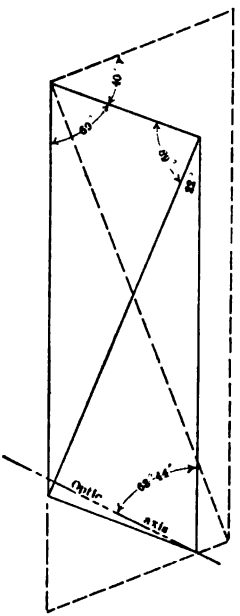

FIG. 277.—Thompson's reversed nicol.

nicol prism as usually cut, the solid lines, Thompson's modification. Each end is first ground down about 40° from the natural faces, leaving an angle of 69° with the long edges. It is then cut across at an angle of 22° with the same edges and cemented. The result is a shortened and reversed nicol which possesses the advantage of having the crystallographic axis nearly at right angles to the direction of transmission of the light and nearly at right angles to the balsam film, with the result that the blue fringe is thrown farther back, giving a prism which is shorter, and with a field equally wide or wider than the ordinary nicol.

127. Fuessner Prisms (1884).—Fuessner,[2] in 1884, invented a number of

[1] Idem: *Notes on some new polarizing prisms.* Phil. Mag., 5 ser., XXI (1886), 476–480.

[2] K. Fuessner: *Ueber die Prismen zur Polarisation des Lichtes.* Zeitschr. f. Instrum., IV (1884), 47–49.

Review of preceding in Jour. Roy. Microsc. Soc., IV (1884), 456–462.

See also Ph. Sleeman: *Dr. Fuessner's new polarizing prism.* Nature, XXIX (1884), 514–517.

new polarizing prisms designed to give a large field and at the same time be less expensive than the ordinary nicols. His prisms are similar to those suggested by Sang[1] in 1837 though not published until 1891.

Fuessner described a prism of glass cut diagonally across and reunited after the insertion of a thin plate of calcite. The cement used must have the same index of refraction as the glass, and both must equal the greatest index of the calcite. The directions of greatest and least ease of vibration must lie in a plane normal to the cut section of the glass. Since calcite is uniaxial, any section may be so placed, and cleavage pieces can be obtained easily. A calcite prism, 4.25 times as long as it is wide, when made on this principle, has an opening angle of 44°.

But other crystals than calcite may be used. All that is necessary is that they be colorless and transparent. If the difference in the indices in the two directions is greater than that of calcite, the field will be larger and the prism shorter. A prism constructed of glass and a plate of sodium nitrate, which has indices $\omega = 1.587$ and $\epsilon = 1.336$, gave a field of 56° with a ratio of length to breadth of 3.34. In this prism a cement of Damar resin in monobromnaphthalene was used. Damar resin consists of two resins, one of which is soluble in alcohol. The residue is very brittle and colorless, and has an index of refraction of 1.549. If one-third of its volume of monobromnaphthalene be added, a viscid cement with an index of 1.58 is produced. No satisfactory cement, with an index as high as that of calcite, was found, although tolu balsam in monobromnaphthalene gives an index of refraction of 1.62. This, however, on account of its lower refractive index, cut the field down to 34°. If the prism is fitted into a glass tube, a liquid film of monobromnaphthalene, which has an index exactly equal to ω of calcite, may be used.

128. Bertrand Prisms (1884–1885).—Bertrand[2] described a number of prisms quite similar to those of Fuessner. His flint glass prism, with a refractive index of 1.658, is cut in a plane inclined 76° 43′ 8″ to the end faces, and recemented after having had inserted between the two pieces a cleavage plate of calcite. It differs from that of Fuessner in the orientation of the calcite, which has its optic axis parallel to the end faces (Fig. 278). The ordinary ray, consequently, will be the one which passes through. The cement must have an index of refraction of 1.658 or greater. The resulting prism has a length equal to the Hartnack-Prazmowski but a field of 44° 46′ 20″.

[1] *Op. cit.*, Art. 121.

[2] Emile Bertrand: *Sur un nouveau polarisateur.* Comptes Rendus, XCIX (1884), 538–540.

Idem: *Sur différents prismes polarisateurs.* Bull. Soc. Min. France, VII (1884), 339–345.

Idem: *Ueber verschiedene Polarisationsprismen.* Beiblätter zu Wiedem. Ann., IX (1885), 428–430.

The advantage of this prism is its cheapness, since but a very small amount of calcite is used.

Another prism is made of flint glass, with an index of 1.586, cut at 74° with the end faces, and having a thin plate of sodium nitrate inserted with its optic axis parallel to the end faces. The cement must have an index of 1.568 or more. The field of view is 53°.

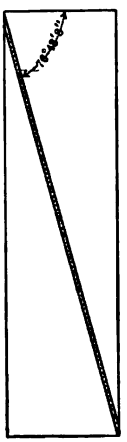

FIG. 278.

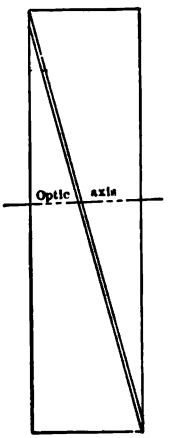

FIG. 279.

FIGS. 278 AND 279.—Bertrand prisms. Fig. 278, Flint glass with calcite lamella; Fig. 279, calcite with glass lamella.

A third prism (Fig. 279) consists of calcite, with the optic axis nearly parallel to the end faces, and cut on a plane making an angle of 76° to 77° with these ends. It is cemented with Canada balsam after having had inserted a thin glass plate with an index of 1.483. This prism is like the Hartnack-Prazmowski in its action without having the bad properties of the linseed oil film of the latter.

Bertrand also suggested that the field of view of all the earlier prisms may be considerably enlarged if a second cut is made normal to the first and lying above or even intersecting it. The objection to the latter method is that it produces a line across the center of the field. The angle between the cut and the end faces is considerably less than in the older forms, and the prisms, consequently, are shorter. The proportions in the following table, which is inserted for comparison, are those given by Bertrand. They differ somewhat in the lengths of the Nicol and the Hartnack-Prazmowski from the values given in Article 138.

Name	Prisms of one cut		Prisms of two cuts	
	Ratio of length to width	Field (in air)	Ratio of length to width	Field (in air)
Nicol prism	5.42 (sic.)	31° 16′	2.62	65° 34′
Hartnack-Prazmowski (linseed oil film)	4.27	39° 34′	2.02	82° 28′
Calcite prism with glass plate	4.27	39° 34′	2.02	82° 28′
Flint glass with calcite plate	4.27	44° 23′	2.02	96° 30′
Flint glass with sodium nitrate plate	3.416	52° 54′	1.56	117° 29′

129. Ahrens' Prisms (1884).—Ahrens' 1884 prism[1] consists of three

[1] C. D. Ahrens: *On a new form of polarizing prism.* Jour. Roy. Microc. Soc., 2 ser., IV (1884), 533–534.

Idem: *On a new form of polarizing prism.* Phil. Mag., 5 ser., XIX (1885), 69–70.

wedges of spar cemented together by Canada balsam. The optic axes of the two outer wedges are parallel to the refracting plane, the axis of the middle one is perpendicular to it. The ends are rectangular, and nearly in contact with one of them is a prism of dense glass, which serves to deflect one of the rays still farther (Fig. 280).

A second form, in which the glass wedge is cemented to the calcite, is shown in Fig. 281. This prism, although not having square ends, seems, on the whole, to be better than the one first described. It is of less length, having a ratio of about 2 to 1, length to breadth, and has a wider opening angle. A ray of light, entering parallel to the long axis, is divided into two rays, one of which emerges parallel to the incident ray; the other is deflected about 59° 30′. The latter ray is strongly colored and distorted, but this is of no consequence since the deviation is so great that it does not interfere.

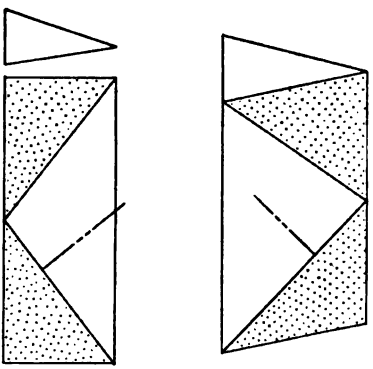

FIG. 280. FIG. 281.

FIGS. 280 AND 281.—Ahrens prisms (1885).

130. Madan Prism (1884).—Madan[1] suggested that if a film of air, as in the Foucault prism, be placed between two Iceland spar prisms (*a*, *b*, Fig. 282), the ordinary ray will be totally reflected. The transmitted extraordinary ray, however, is deflected and over-corrected for color, but both deviation and dispersion are practically corrected by passing them through a prism of crown glass (*c*) and one of very dense flint glass (*d*). The opening angle in this prism is about the same as that in the ordinary nicol (28°) and much greater than that of the Foucault (8°). While it is not quite free from chromatic aberration and distortion, this is not great enough to interfere with its use as a polarizer.

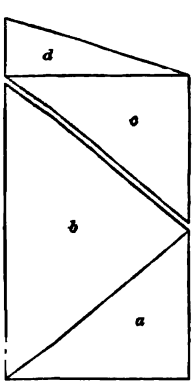

FIG. 282.—Madan prism.

131. Ahrens' Prism (1886).—Ahrens'[2] 1886 prism differs from most of those previously described in having two

[1] H. G. Madan: *On a modification of Foucault's and Ahrens's prisms.* Nature, XXXI (1884–5), 371–372.

[2] C. D. Ahrens: *New polarizing prisms.* Read April 14, 1886. Jour. Roy. Microsc. Soc., 1886, 397–398.

Silvanus P. Thompson: *Notes on some new polarizing prisms.* Phil. Mag., 5 ser., XXI (1886), 476–478.

Hugo Schröder: *Ahrens' neues Polarisationsprisma.* Zeitschr. f. Instrum., VI (1886), 310–311.

section planes cut through it. It differs from the Bertrand prism, and from Ahrens' 1885 prism, in the orientation of these sections. The prism is rectangular, has square ends, and a ratio of about 1.8 to 1 between the long and the short sides. Crystallographic *c* is at right angles to the long sides and passes through the cut sections (Figs. 283-284), although a few prisms were made with crystallographic *c* parallel to the section plane. The two oblique cuts meet in a line passing through the center of one of the square faces, this line being turned toward the source of light. Cut as this prism is, the field is symmetrically divided, and the ordinary ray is reflected to both sides, leaving an available polarized field of about 28° in one direction and about 100° in the other.

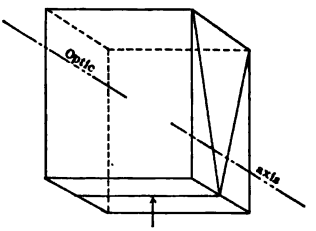

FIG. 283.

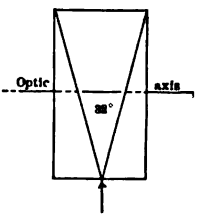

FIG. 284.

FIGS. 283 AND 284.—Ahrens prism (1886). Fig. 283, in perspective; Fig. 284, in section.

This prism, while having about the same opening angle as the Nicol or Thompson prisms, is much shorter. It has square ends and consequently but little light is lost by reflection from them. There is very little distortion, and it requires less Iceland spar than the Nicol, Hartnack-Prazmowski, Glan, or Thompson. Its chief disadvantage is the presence of the section line across the field, although recently the maker has cemented a thin cover-glass to the bottom with Canada balsam, thus making the line almost invisible. The prism is excellent as a polarizer, since it can be made of considerable size with comparatively little spar. As an analyzer it is likely to produce a little distortion at the section line.

132. Grosse Double-slit Air Prism (1890).—Grosse[1] suggested a prism, useful as a polarizer, with two diagonal intersecting slits, the parts not cemented, but separated by a film of air (Fig. 285).

133. Leiss Prism (1897).—Apparently without knowledge of Talbot's prism,[2] Leiss[3] constructed one on the same plan, one half being made of Iceland spar and the other of glass, the latter with a refractive index as nearly

[1] W. Grosse: *Ueber Polarisationsprismen.* Zeitschr. f. Instrum., X (1890), 445-446.

[2] See Art. 124, *supra.*

[3] C. Leiss: *Ueber ein neues, aus Kalkspath und Glas zusammengesetztes Nicol'sches Prisma.* Sitzb. Akad. Wiss. Berlin, 1897, 901-904.

J. Beckenkamp: Review of above. Zeitschr. f. Kryst., XXXIII (1900), 112.

as possible the same as that of the extraordinary ray in the first half. No glass was found having exactly the proper index, wherefore, on account of the displacement of the image on rotating the prism, it could be used only as a polarizer and not as an analyzer.

134. Von Lommel Prism (1898).—Independently of Talbot and Leiss, von Lommel[1] had constructed, in 1895, a similar prism. Owing to its faulty character he did not publish it until 1898. He found that the image became distorted, being shortened parallel to the principal section of the nicol. On

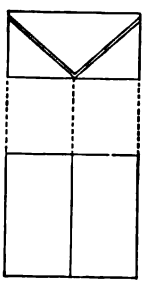

FIG. 285.—Grosse double slit air prism.

FIG. 286.—Von Fedorow's polarizer. (Fuess.)

looking through it at lines crossing at right angles, for example at window bars, they were found to appear at this angle only when they were parallel to the vibration planes of the prisms. When rotated to any other angle the lines crossed at acute (and obtuse) angles.

135. Von Fedorow's Polarizer (1901).—In order to obtain light, plane polarized as completely as possible, for use with his rotating apparatus, von Fedorow[2] constructed a polarizing prism built on entirely new lines. Instead of using a doubly refracting crystal cut on a plane, he made use of a hemisphere of calcite (*C*, Fig. 286) cut with the optic axis parallel to the flat surface, and set in a hemispherical recess in a piece of flint glass (*G*) whose refrac-

[1] E. von Lommel: *Ueber aus Kalkspath und Glas zusammengesetzte Nicol'sche Prismen.* Sitzb. Akad. Wiss. München, XXVIII (1898), 111–116.

P. Groth: Review of above. Zeitschr. f. Kryst., XXXIII (1900), 489–490.

[2] E. von Fedorow: Article in Russian with a French resumé in Annuaire géol. et minér. d. Russie, IV (1900), 142–149. Reviewed by V. von Worobieff: *Einige Hülfsapparate für das Polarisationsmikroskop.* Zeitschr. f. Kryst., XXXVII (1902–3), 413–414.

Idem: Article in Ibidem, V (1902), 217–221. Reviewed by P. Groth: *Optische Vorrichtungen, die auf der Anwendung der Glasplättchenpackete beruhen.* Zeitschr. f. Kryst., XL (1904–5), 297–298.

P. Groth: *Physikalische Krystallographie.* 4te Aufl., Leipzig, 1905, 768.

tive index lies between that of the two rays of the calcite. The extraordinary rays, passing into a glass of higher index, are strongly refracted, and are absorbed by the black enclosing ring. The ordinary rays, passing into a medium of little less density, change their direction but slightly. To prevent the central extraordinary ray from passing through the apparatus, a small black plate *m* is cemented over the center to a glass plate *a*.

As first described, the instrument was available only for use with monochromatic light. To make it available for white light also, there are placed

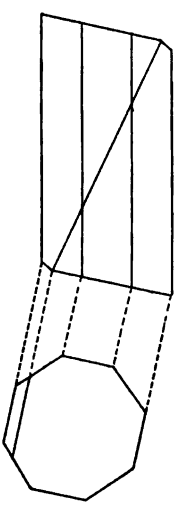

FIG. 287.

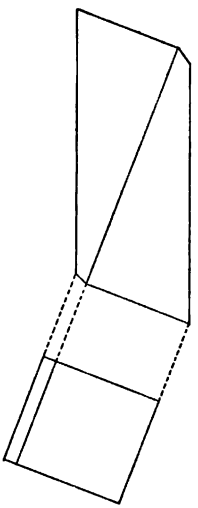

FIG. 288.

FIGS. 287 AND 288.—Halle prisms.

beneath it two bundles of thin glass plates (*I* and *II*) of the thickness of coverglasses, ground on the flat surfaces and cemented together, the balsam making a film from 0.5 to 0.75 mm. between each. The emerging light, when it reaches the calcite hemisphere, is already nearly plane parallel, and the finally emerging ray has a divergence of no more than 1°.

136. Halle Prisms (1908).—Halle[1] designed two modified nicol prisms giving opening angles respectively of 17° to 19° (Fig. 287), and 25° (Fig. 288). From 126 c.c. of calcite, a prism of the first form 37 mm. by 67 mm. could be cut, or one of the second 22 mm. by 60 mm.

137. Glass Polarizing Prisms.—Stolze,[2] in 1895, described a polarizing

[1] Bernhard Halle: *Ueber Polarisationsprismen.* Deutsche Mechan. Zeitung, 1908, 6–7, 16–19.

[2] Stolze. Atelier d. Photographen, 1895, 140.*

prism made entirely of glass. The angles of the faces *FE* and *BC* (Fig. 289) are so chosen that the ray of light which enters and leaves the prism perpendicularly to *AB* and *ED*, is totally polarized by the face *BC*, the face *FE* being silvered to prevent loss of light by reflection.

Owing to the lateral displacement of the polarized ray, and to its incomplete polarization if the glass is strained, this prism has been little used. A better form, proposed by Schulz,[1] is shown in Fig. 290. The emerging ray, being polarized outside the glass, is not affected by strain in the glass, nor is there any displacement of the light ray. While the intensity of the emerging

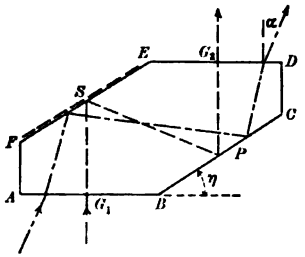

FIG. 289.—Stolze glass polarizing prism.

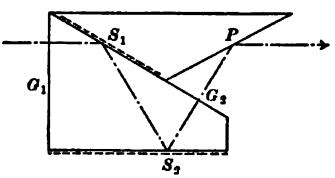

FIG. 290.—Schulz glass polarizing prism.

light is but 10 per cent. of that entering, and in a nicol prism it is from 25 per cent. to 40 per cent., this is no great disadvantage, since a glass prism may be made of any desired size.

138. Summary of Properties of Polarizing Prisms.—The various polarizing prisms described above are compared in the following table, taken, in part, from Fuessner.[2]

Name	Vibration direction of ray passing through in relation to the separating film	Approximate opening angle	Inclination of cut plane to vertical axis of prism	Inclination of balsam film to the *c* axis	Ratio of length to breadth
Nicol	N*	29°	22°	41° 44′	3.28
Foucault	N	8°	40°	5° 37′	1.528
Hartnack-Prazmowski	N	35°	13° 54′	90°	3.51
Hartnack, oil film	N	42°	13° 54′	90°	4.04
Glan	P	7° 56.5′	50° 18′	0°	0.831
Thompson	P	35°	22°	0°	3.28 and up.
Thompson reversed nicol	N	27°	22°	94° 16′	2.5
Fuessner, calcite plate	N	44°	13° 12′	90°	4.26
Fuessner, calcite plate	N	30°	17° 24′	90°	3.19
Fuessner, calcite plate	N	20°	20° 18′	90°	2.70
Fuessner, sodium nitrate	N	54°	16° 42′	90°	3.53
Fuessner, sodium nitrate	N	30°	24°	90°	2.25
Fuessner, sodium nitrate	N	20°	27°	90°	1.96
Bertrand, calcite plate	N	44° 23′	13° 15′	0°	4.27
Bertrand, glass plate	N	30°	13° 30′	90°	4.27
Ahrens (1886)	N	28° × 100°	16°	74°	1.75

*N = normal to the balsam, air, etc., film.
P = parallel to the film.

[1] H. Schulz: *Polarisationsprismen aus Glas.* Zeitschr. f. Instrum., XXXI (1911), 180–182.

[2] K. Fuessner: *Ueber die Prismen zur Polarisation des Lichtes.* Zeitschr. f. Instrum., IV (1884), 49.

Grosse[1] gives the following comparative table of the values of various prisms. The numbers 1 to 5 indicate the value of the prism in regard to the purpose specified in the first column. The last column gives the most advantageous forms for each of these purposes, namely, those given a score of 4 or 5.

	Nicol group			Dove	Abbe	Air prisms		Double slit prisms		Double slit air prism	Prism containing plate of doubly refracting medium	Most advantageous form
	Nicol	Hartnack	Thompson			Glan	Foucault	Bertrand	Ahrens			
1. Plane polarized field.	3	4	5	2	2	2	2	3	3(1)	2	1	T. H.
2. Field of view.....	3	3	3	2	2	1	1	4	4	1	5	Plate, B. Ah.
3. Loss of light.....	4	5	5	5	3	2	2	3	5	2	1	D. H. T. Ah.
4. Displacement of ray.	2	5	5	5(1)	5	3	1	3	5	3	4	H. T. Ah. (D). Ab.
5. Ratio of length to breadth	1	1	1	3	3	4	4	1	3	5	2	G. F. Air (double slit).
6. Waste of material...	4(3)	2	1	4	4	2	4	3	3	4	5	D. Ab. Air, Plate, F.
Total points.....	16(+1)	20	20	17(+4)	19	14	14	17	21(+2)	17	18	

139. Polarizer and Analyzer.—In a petrographic microscope there are two polarizing prisms, generally called nicols, although one usually is, and both may be, prisms of a different type.

The lower "nicol" is placed below the stage, and is called the polarizer. Any prism described above may be used as a polarizer since the effect of displacement of light or of cross-cutting lines does not reach the eye.

The fittings by which polarizers are attached to the sub-stages are different in different microscopes. Some have been described above,[2] and are shown in Figs. 255–257. Others may be seen in Figs. 307–324. The fittings should be so made that the nicol may be easily rotated, and yet fall into its position at 0° accurately. The amount of rotation should be determinable, the nicol should be readily removable, and it should be so arranged that it can be elevated or depressed.

The analyzer, in most recent microscopes (*A*, Fig. 231; *N*, Figs. 310, 311, 311a, etc.) is made to slide in a slot in the tube, and may be rotated through 90° (Figs. 309, 311a, 316), the amount of rotation being indicated on a scale. Analyzers generally are flat-ended prisms, so made to avoid displacing the image. For ordinary purposes this analyzer is sufficient, but when certain accessories, to be described later, are used, it is necessary to use a nicol which fits over the eyepiece, and which is called a **cap nicol** (A_2, Fig. 309). Cap nicols only were used as analyzers in the older types of microscopes, and their removal and replacement was a most awkward proceeding. With their use, too, the field of view of the microscope is much reduced.

The insertion of the analyzer in the path of the rays causes a displacement of the focus and it is necessary, consequently, that there should be attached

[1] W. Grosse: *Op. cit.*

[2] Article 118, *supra.*

above it, in the analyzer carriage, a long focus lens so made that it is not necessary to re-focus when the analyzer is inserted. This lens must be very accurately adapted to the nicol used, for the image must be perfectly sharp under all conditions. A very slight change in focus is extremely tiring to the eye.

For certain purposes, such as measuring small extinction angles, determining weak pleochroism, locating the points of emergence of the optic axes and so on, it is desirable to rotate both nicols at the same time instead of rotating the stage. This is made possible, in certain microscopes, by attaching the nicols to geared wheels[1] (Figs. 310, 320 and 322), or by connecting them by a rigid bar (Figs. 291, 312 and 313). The latter method was first used by Dick[2] in 1888 in his own instrument, but when put on the market by Swift & Son, the rotation was produced by geared wheels.[3] The first illustration of a microscope with rigid bar connection appears to be that of de Souza-Brandão,[4] in 1903. This microscope was put on the market later, as the Fuess *Ib* (Fig. 312), and is described below.[5] In this microscope the connection between the nicols is a simple rigid bar so arranged that it may be detached and the upper nicol moved in or out independently, as in the usual microscopes. A year later Sommerfeldt[6] described and illustrated a microscope of the same type which he used in high temperature work. The analyzer, in this case, was a cap nicol, and the connecting bar, telescopic. In 1905 he[7] described

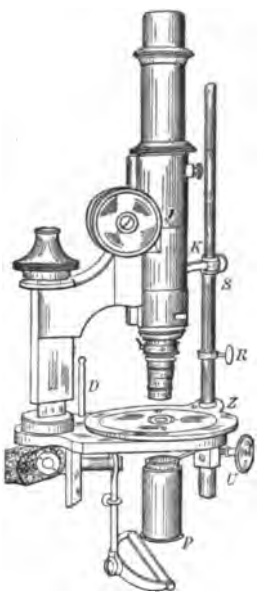

FIG. 291.—Sommerfeldt's microscope with simultaneously rotating nicols.

[1] C. Leiss: *Ueber neuere Instrumente und Vorrichtungen für petrographische und krystallographische Untersuchungen.* Neues Jahrb. B.B., X (1895–6), 412–420.

Idem: *Ueber Neuconstructionen von Instrumenten für krystallographische und petrographische Untersuchungen.* Ibidem, 179–183.

[2] Allan Dick: *A new form of microscope.* Mineralog. Mag., VIII (1888), 160–163.

[3] Anon: *Dick and Swift's patent petrological microscope.* Jour. Roy. Microsc. Soc., 1889, 432–436.

Anon: *Messrs. Swift and Son's improved Dick petrological microscope.* Ibidem, 1895, 97.

[4] V. de Souza-Brandão: *O novo microscopio da commissão do serviço geologico.* Communicadões da Commissão do Serviço Geologico de Portugal, V (1903–1904), 118–250.

[5] See Art. 172, *infra.*

[6] Ernst Sommerfeldt: *Ein für mineralogische Untersuchungen bei hoher Temperatur geeignetes Mikroskop.* Zeitschr. f. wiss. Mikrosk., XXI (1904), 181–185.

[7] Idem: *Die mikroskopische Achsenwinkel bestimmung bei sehr kleinen Kristallpräparaten.* Ibidem, XXII (1905), 356–362.

a similar bar connection (Fig. 291) for observing interference figures in very small minerals. Here, however, the polarizer was connected with an analyzer in the inner tube, and the amount of rotation was read from the graduated circle of the stage. The same principle was used by Wright[1] in 1910. The method of connecting or disconnecting the bar differs from the preceding in being by means of a hinged fork instead of a telescopic rod. This microscope is illustrated in Fig. 313.

After continued usage, lost motion is more likely to develop with the geared wheel than with the rigid bar connection. The latter, too, is less clumsy.

140. Determination of the Vibration Directions in the Nicol Prisms.—It is important, for some determinations, to know the vibration directions of the nicol prisms; for example in the determination of the directions of absorption in a crystal. If the form of polarizing prism is known, its vibration direction may be found from the table in Article 138, the separating plane being easily seen upon looking through the prism at an angle. If the kind of prism is unknown, its vibration direction may be determined by examining plates of certain minerals through it. For example, a section of biotite, cut at right angles to its cleavage, has its greatest absorption, consequently it is darkest, when its cleavage direction is parallel to the plane of vibration of the polarizer. Tourmaline, on the other hand, extinguishes vibrations at right angles to the optic axis, that is, it absorbs the ordinary ray, and only the light rays vibrating parallel to crystallographic *c* emerge. It is, therefore, dark when the elongation (*c*) is at right angles to the vibration plane of the polarizer.

Another method is to remove the polarizer from the microscope and examine, through it, light reflected by a horizontal polished surface, such as a plate of glass or a varnished table top. Knowing that light is polarized in a plane parallel to the reflecting surface,[2] it is clear that the polarizing plane of the "nicol" lies at right angles to the reflecting surface when the latter appears dark.

141. Bertrand Lens.—In the upper part of the tube of most modern microscopes, there is a slot for the insertion of a Bertrand lens which, preferably, should be fastened in a sliding carrier and permanently attached to the tube (*BL*, Fig. 230; *B*, Fig. 309; *g*, Fig. 311a; *B*, Fig. 313, etc.). This lens, in connection with the ocular, acts as a small microscope and magnifies

[1] Fred. Eugene Wright: *A new petrographic microscope.* Amer. Jour. Sci., XXIX (1910), 407–426.

Idem: *The methods of petrographic-microscopic research.* Carnegie Publication No. 158, Washington, 1911, 58.

C. Leiss: *Mikroskop mit gemeinsamer Nicoldrehung in vereinfachter Form.* Zeitschr. f. Kryst., XLVII (1909), 377–378.

[2] Art. 42, *supra.*

the interference figure to be described later. In order to bring the image to a sharp focus with different oculars, the Bertrand lens must be fastened in a sliding collar, ordinarily held in place by friction, though in some microscopes it is moved by means of a rack and pinion.

The Bertrand lens will be discussed in greater detail in connection with the observation of interference figures. The method of its adjustment in the tube of the microscope is given in Article 201.

CHAPTER X

THE MICROSCOPE (Continued)

The Objective

142. Introductory.—The objective is the lower lens system of a microscope, and is the one that first receives the light from the object itself. It brings the light to a focus in the principal focal plane of the ocular (F_2, Fig. 229) and produces there a real and magnified image. A great deal of care and skill are necessary in making an objective, and one of high power is, consequently, the most expensive part of the microscope.

Objectives may be classified according to their magnification. Roughly one may consider an objective as of low power when its focal length is above 13 mm. and its magnification $\left(\frac{\Delta}{F}\right)$ less than 15 diameters; of medium power when its focal length is between 12 and 5 mm. and its magnification up to 40 diameters; and of high power when its focal length is less than 4.5 mm. and its magnification over 40 diameters.

143. Definition.—The greater the correction for chromatic and spherical aberration, the greater the definition. Owing to the impossibility of entirely correcting all aberration, every point of the object will be represented in the image, not by a point, but by a small circle of aberration, the size of the circles depending upon the type of objective. While this feature is one that must be taken into account in biologic work, and caution taken to distinguish between phantom structures, which do not exist in the object, and real structures, yet it is one that may be practically disregarded in working with the comparatively low powers used in petrographic work.

144. Depth of Definition (Depth of Focus), or Penetration.[1]—With low-power objectives it is quite possible to see sharply, at the same time, objects lying in slightly different planes. With high-power lenses this is not possible to so great an extent, for the depth of focus diminishes inversely as the numer-

[1] Anon: *Penetration of wide-angled objectives.* Jour. Roy. Microsc. Soc., II (1879), 322–323.

George E. Blackham: *Penetration. A contribution to the study of the subject.* Amer. Jour. Microsc., V (1880), 145–150.

E. Abbe: *Conditions of microstereoscopic vision. Penetration.* Jour. Roy. Miscrosc. Soc., III (1880), 207.

C. M. Vorce: *Penetration in objectives—is it a defect or an advantage.* Amer. Mon. Microsc. Jour., I (1880), 170–171.

Anon: *Penetrating power of objectives.* Jour. Roy. Microsc. Soc., N. S., I (1881), 831–832.

Edward M. Nelson: *The penetrating power of the microscope.* Ibidem, 1892, 331–341.

ical aperture. A thin section may be considered as being made up of a series of superimposed planes, only one of which may be seen for one adjustment of focus. The penetrating powers for various values of numerical aperture are given in the table in Article 154.

145. Flatness of Field.—An objective is said to have a flat field when all portions of a flat object observed through it appear equally sharp at the same time. Usually the image around the edges of the field appears blurred, and it is necessary, with high powers, to change the focus between center and sides. This indistinctness is due to what is known as the curvature of the image, that is, the image produced does not lie in a true plane but on a more or less curved surface. It has not yet been possible to correct this entirely in high-power objectives, consequently one showing such blurring is not necessarily to be condemned as defective.

146. Illuminating Power.—The illuminating power or brightness of image, for a given magnification, other things being equal, increases as the square of the numerical aperture.

147. Resolving Power.—The resolving power of an objective is that property by virtue of which one is enabled to see the finer details of an object. It is a fixed quantity of an objective, not necessarily increased with its magnifying power, but depending upon the numerical aperture and the correction for spherical and chromatic aberration.[1] The resolving power increases with the number and obliquity of the rays coming from the object, that is, it increases directly as the numerical aperture, consequently an immersion fluid, by increasing the number of rays brought to the object, increases it. All lenses having the same numerical aperture, however, may not have the same resolving power.

Lenses may be able to resolve an object into details too fine to be seen by the eye, which can only distinguish about 250 lines to the inch. That there is a limit to the smallness of an object which one can see is due to the fact that the nerve fibers of the eye have a definite size, and when the angle in the eye, formed by the rays from two points, is smaller than about thirty seconds of arc, these points appear as one. Expressed in distances, it may be said that when two points are removed from the eye 6876 times the distance separating them, they will appear as a single point. Thus *pleurosigma angulatum*, with about 50,000 lines to the inch, can be resolved by a 1/2 in. objective so as to be clearly seen with a 3/4 in. ocular, but not with a 1 1/2 in. A much smaller line may be seen than an interval between two

[1] See Edward M. Nelson: *On the limits of resolving power for the microscope and telescope.* Jour. Roy. Microsc. Soc., 1906, 521–531.

Sir A. E. Wright: *Principles of Microscopy*, 1906, 231.

See also Article 154, *infra.*

lines. Jurin[1] could see a single pin when the angle at the eye was between two and three minutes of arc, but the interval between two pins disappeared when the angle was thirty minutes.

148. Working Distance.—The working distance[2] of an objective is the distance between its front lens and an object in focus without cover-glass. The actual working distance is lessened by the cover-glass and, usually, by a rim of metal projecting beyond the lens.

149. Magnifying Power.—The magnifying power of an objective, called its initial magnification, depends upon the tube length of the microscope. It can be increased by extending the draw tube. It should be measured from the equivalent plane of the objective to the equivalent plane of the ocular, but since this varies with different oculars, the magnifying power of objectives is not always given by the makers. It is usually easier to measure the magnification of a combination of objective and ocular direct. According to Abbe the power of an objective is obtained by the formula $\frac{250}{F_o}$,[3] where F_o is the focal length of the objective. This is the value used by Zeiss, Reichert, Swift, and others, while Bausch & Lomb, Leitz, and others use $\frac{\Delta}{F_o}$, where Δ is the optical tube length. These values are shown, for various objectives, in the table in Article 153.

From the same table the increase in the magnifying power $\left(\frac{\Delta}{F}\right)$ of an objective, produced by extending the draw tube, may be computed. For example, a Leitz No. 5 objective used with a mechanical tube length of 180 mm. magnifies 33.3 times. Increasing the tube length by 50 mm. increases the optical tube length practically the same amount, and the value becomes, $N=\frac{205}{5\cdot4}=37.9$ instead of $N=\frac{180}{5\cdot4}$.

[1] Dr. Royston-Pigott: *A further inquiry into the limits of microscopic vision and the delusive application of Fraunhöfer's optical law of vision.* Jour. Roy. Microsc. Soc., 1879, 9–20.

[2] Ernst Grundlach: *Working distance and its relations to focal length and aperture.* Amer. Mon. Microsc. Jour., II (1881), 32–33.

[3] Article 98, *supra.*

See also E. M. Nelson: *Virtual images and initial magnifying power.* Jour. Roy. Microsc. Soc., 1892, 180–185.

Malassez pointed out that there is no precise definition of the phrase magnifying power. He suggested that it be expressed in terms of unit distance from the posterior face of the lens. L. Malassez: *Etaluation du pouvoir grossissant des objectifs microscopiques.* Archives d' Anatomie Microsc., 1904, 274, 285. Comptes Rendus, CXLI (1905), 1004–1006.

Idem: *Sur le pouvoir grossissant des objectifs microscopiques.* Comptes Rendus, CXLI (1905), 880–881.

Idem: *Evaluation de la puissance des objectifs microscopiques.* Ibidem, CXLII (1906), 773–775.

150. Dry and Immersion Objectives.—Objectives may be classified as dry or immersion, depending upon the medium between them and the object. In the former, air only is used; in the latter, water, glycerine, oil, or some other fluid having about the same index of refraction as the glass, and the lenses are spoken of as water-, oil-, etc., immersion objectives. In petrographic work no great magnifying powers are required, and immersion lenses are not much used although they could be used to advantage in certain cases. Immersion lenses possess the advantage of transmitting more light,[1] since none is lost by partial reflection, but dry lenses gain nothing by being used with oil, since they were not constructed with that object in view.

The effect of the immersion oil upon the resolving power of the objective may be seen from Fig. 292, which shows a section through the front lens of the objective, the cover-glass, the Canada balsam, the object slide, and the condenser. The left half of the lens may be taken to represent the front lens of a Leitz 1/12-in. oil-immersion, and the right half, that of a Leitz No. 9 dry objective, the two being chosen since they have very similar front lenses and working distances. Corresponding to the immersion oil shown below the objective to the left, there is an oil film below the object slide and above the condenser, shown in the lower right half. This is necessary only for objectives having a numerical aperture greater than 1.0. If the refractive indices of the condenser, immersion fluid, object slide, mounting medium, and object lens are approximately equal, the ray H suffers no refraction in passing from H to L, and the angle at which it leaves the object is the same as the angular aperture, that is,

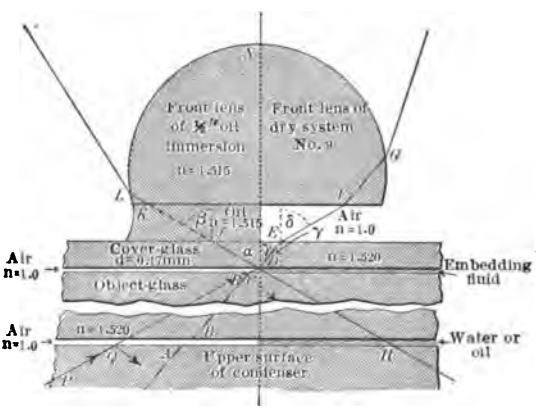

FIG. 292.—Passage of light through dry and oil-immersion objectives. (Leitz.)

$$\alpha = \beta. \qquad (1)$$

Such a system is spoken of as one of **homogeneous immersion.**

In the dry system, shown in the upper right and lower left halves of the figure, the path of the ray is entirely different. The extreme ray P, corresponding to the ray H of the oil-immersion lens, is totally reflected at Q, and

[1] W. H. Dallinger: *Dry v. immersion objectives.* Jour. Roy. Microsc. Soc., I (1878), 154.

only those rays whose angles of incidence are less than the critical angle can pass through. The ray *DEFG*, for example, passing from a denser to a rarer medium, follows the law

$$\frac{\sin\gamma}{\sin\delta}=\frac{1}{n}. \tag{2}$$

In the figure, for comparison, the angular apertures in the dry and immersion lenses have been taken to be the same, so that $\beta=\delta=60°$. But from equation (1) $\beta=\alpha$, therefore $\delta=\alpha$ and equation (2) becomes

$$\frac{\sin\gamma}{\sin\alpha}=\frac{1}{n}. \tag{3}$$

That is, the angular aperture being the same, the resolving power of the dry lens (which depends upon the obliquity and number of rays coming from the object (Art. 147)) is only $\frac{1}{n}$ that of the oil immersion.

The immersion fluid used is generally cedar oil (Juniperus virginiana). It should be kept in a well-corked bottle so that it may remain free from dust, and not thicken from evaporation. A bottle having a rod of glass or wire attached to the stopper (Figs. 293–294[1]) is very convenient, since it affords a means by which the oil may be applied to the objective, and also prevents its waste or accidental spilling.

FIG. 293.—Immersion oil bottle. (Bausch and Lomb.)

FIG. 294.—Immersion oil bottle. (Zeiss.)

Oil may be placed between the object and the objective in one of two ways. A drop may be placed on the surface of the cover-glass and the tube racked down until the objective dips in the oil, or it may be applied to the lens of the objective, which is then lowered until the oil touches the cover. The latter method is probably the better, since there is less danger of spreading the oil. Air bubbles should be avoided. If any occurs, the oil should be removed and another drop applied. After using an oil-immersion objective both the lens and the slide should be carefully cleaned at once. The greater part of the oil should be taken up with filter paper or soft linen, and the front lens and thin section wiped dry with very soft linen, moistened, if necessary, with a drop of xylene or benzene, never alcohol. Care must be taken not to use too much benzene for it may penetrate between the lenses and destroy the balsam film.

[1] W. Gebhardt: *Fläschen zur Aufbewahrung des Immersionsöls.* Zeitschr. f. wiss. Mikrosk., XIV (1897), 348–350.

151. Classification of Objectives According to Correction for Aberration.—Another method of classifying objectives is according to the amount of correction for aberration, and objectives may be achromatic, semi-apochromatic, or apochromatic. The former are corrected for primary spherical aberration and aberration for one color, the semi-apochromatics are corrected, in addition, for a second color, and the apochromats are chromatically corrected for three colors and spherically corrected for two (See Art. 93.)

Three types of achromatic objectives are made by Leitz and are shown in Figs. 295 to 297. They may be taken as representative of the objectives made by all opticians. The first (Fig. 295) consists of two members, each

FIG. 295.

FIG. 296.

FIG. 297.

FIGS. 295 TO 297.—Achromatic objectives. (Leitz.)

of which is made up of two or three cemented lenses, and is representative of low- and medium-power dry objectives. The second, a high-power dry objective, has a hemispherical front lens—the magnifying element—back of which there are two other members, each made up of two or three cemented lenses for correcting the spherical and chromatic aberration. Fig. 297 shows a 1/12-in. oil-immersion objective. The front hemispherical lens is succeeded by a meniscus, back of which the third and fourth members of two or three cemented lenses each act as correctors of aberration.

152. Effect of Cover-glasses of Different Thicknesses.—The effect of cover-glasses of different thicknesses upon the passage of light from Canada balsam to air is shown in Fig. 298. The rays of light, coming from the object *O*, are refracted away from the normal *N* when they emerge in air. Traced backward, it will be seen that the various rays, even with the same thickness of cover-glass, do not intersect in a point, but lie at various distances from the objective along the normal, and that with cover-glasses of different thicknesses, the vertical intercepts are different.[1] The intercepts represent

[1] See also Art. 208, *infra*.

M. D. Ewell made various experiments to test the effect of curved cover-glasses and found that in those purchased of good dealers it was practically *nil*. M. D. Ewell: *The effect of curvature of cover-glass upon microscopy*. Proc. Amer. Microsc. Soc., 13th meeting, Detroit. XII (1891), 79–93.

the positions of the image for the various rays, and, by comparison with Fig. 205, it may be seen that while objectives might be spherically undercorrected so as to compensate this effect for one thickness of cover-glass, it would not answer for some other. This effect of cover-glasses of different thicknesses is not noticeable in low-power objectives, but a variation of even 0.05 mm. may greatly impair the efficiency of the higher powers. The above remarks do not, of course, apply to homogeneous-immersion objectives, and only in a small degree to water-immersion objectives.

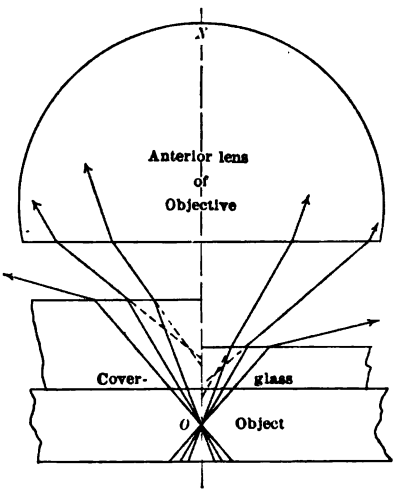

FIG. 298.—Effect of cover-glasses of different thicknesses.

Dry, non-adjustable objectives are generally corrected to compensate for cover-glasses 0.16 to 0.18 mm. in thickness.[1] If the covers are of a different thickness, this may be partially counterbalanced by slightly lengthening the body tube for those that are thin, and shortening it for those that are thick. Certain high-power objectives are made with a correction collar (Fig. 299) by means of which an adjustment may be made for cover-glasses of different thicknesses. By rotating the collar, the two posterior lenses separate, more or less, from the anterior. The divisions on the scale indicate, in hundredths of a millimeter, the cover-glass thickness corresponding to that adjustment.

To compensate an objective requires some practice. It is best accomplished by using a test plate of some kind, such as a slide of *pleurosigma angulatum* or an Abbe test plate. In the former a flat diatom is selected and examined for fine lines. If these are not seen, the correction collar should be turned sightly with one hand while the micrometer screw of the microscope is moved, above and below its focus, with the other. This process is carefully continued until the fine lines begin to appear, which they will do lying above or below the plane of the diatom. The fine lines are now kept in focus while the correction collar is turned until the lines reach the plane of the diatom. The process

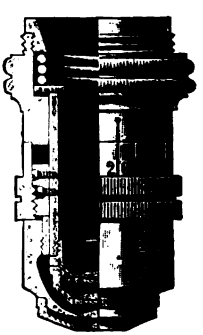

FIG. 299.—Objective fitted with correction-collar. (Zeiss).

[1] Bausch & Lomb 0.18 mm., Beck 0.006 in., Fuess 0.17 mm., Leitz 0.16–0.18 mm., Nachet 0.15 mm., Reichert 0.17 mm., Seibert 0.15–0.18 mm., Swift 0.18 mm., Zeiss 0.17 mm.

should be repeated several times, and the mean of the readings thus obtained should be noted on the slide as the proper correction for that slide with *that particular objective and ocular.*

The Abbe test plate[1] (Fig. 300), in its present form, consists of a long thin wedge of glass, silvered and engraved with a series of parallel lines on its under side, and cemented to an object slip. Owing to the wedge-shape of the cover-glass, there is a gradual increase in the thickness from one end to the other, which can be read directly to 0.01 mm., or less, from a scale engraved upon it. The edges of the engraving upon the silver, which is extremely thin, serve as the marks upon which to focus.

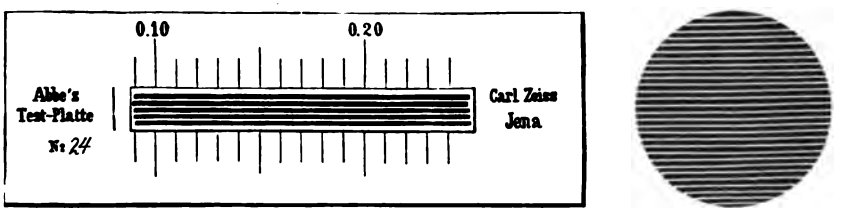

FIG. 300.—The Abbe test plate No. 24. (Zeiss.)

If one prepares his own thin sections, it is advisable to use cover-glasses of uniform thickness. Commercially they come assorted in sizes:

No. 1. Thickness, 0.13 to 0.16 mm.
2. Thickness, 0.16 to 0.25 mm.
3. Thickness, 0.25 to 0.50 mm.

The thicker glasses should be used for such objects as are to be examined only with low powers, the thin glasses for oil-immersion objectives. Both of these, however, are practically useless for petrographic work where a thickness of from 0.15 to 0.18 mm. is desirable. To determine the thickness of cover-glasses, a micrometer screw (Figs. 345–346) of some sort should be used,[2] and the thickness noted on the label of the slide, so that, if necessary, the objective may be properly compensated. For objects already mounted, the cover-glass thickness may be measured by the method of the Duc de Chaulnes, given in Article 208. Another method is given by Czapski.[3] Since the index of refraction of the cover-glass is generally unknown, it is not sufficient, in making this test, to focus upon the top and bottom and multiply

[1] E. Abbe: *Beiträge zur Theorie des Mikroskops und der mikroskopische Wahrnehmung.* Archiv. f. mikr. Anat., IX (1873), 434–437. Also in *Gesammelte Abhandlungen*, I, 1904, 66–68.

Anon: *Abbe's test-plate.* Jour. Roy. Microsc. Soc., III (1883), 281–283.

Anon: *Directions for using the Abbe test-plate.* Zeiss circular, Mikro 116.

[2] See also Edward Bausch: *The full utilization of the capacity of the microscope, and means for obtaining the same.* Microscope, X (1890), 289–296.

[3] S. Czapski: *Die Bestimmung von Deckglasdicken an fertigen Präparaten.* Zeitschr. f. wiss. Mikrosk., V (1888), 482–484.

the thickness thus found by 1.5 as an assumed refractive index of the glass. In Czapski's method it is necessary that cover-glasses of known thicknesses, or an Abbe test plate, be used as a gage. The objective of 0.6 to 0.9 N. A. is focussed on the top and bottom of the known glasses, with central illumination, and the amount of lift of tube for each thickness is noted. It makes no difference whether the true value of a division of the fine adjustment screw is known or not. The values thus determined are compared with the known thicknesses of the cover-glasses, and a mean reduction factor is obtained, a factor which is only to be used with the same combination of objective, ocular, diaphragm, and tube length. For example, if, with a cover-glass 0.220 mm. in thickness, the movement of the micrometer screw was 52 divisions, and with a thickness of 0.180 mm., 43 divisions, we have $\frac{22}{52} = 0.00423$ and $\frac{0.18}{43} = 0.00418$, a mean of 0.0042, which is the factor required. To make determinations, all that is necessary is to multiply the reading obtained through an unknown cover-glass by the factor, and the result is the thickness

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NOTES FOR TABLE 153

[1] Data obtained directly from the makers.

[2] Oculars used in obtaining the field of view: Bausch & Lomb 1 3/5 in. (with $\frac{250}{F} = 6.4$), Fuess No. 2 (5.6), Leitz No. 0 (4.0), Seibert No. 1 (5.0), Zeiss No. 2 (6.4), Beck No. 1 (5.0), Reichert No. II (6.0).

J. W. Stephenson: *On a table of numerical apertures, showing the equivalent angles of aperture of dry, water-immersion, and homogeneous-immersion objectives, with their respective resolving powers, taking the wave length of line E as the basis, $a = n \sin w$, n = refractive index, and $w = 1/2$ angle of aperture.* Jour. Roy. Microsc. Soc., II (1879), 839–841.

Anon: *Notes on aperture, microscopical vision, and the value of wide-angled immersion objectives.* Ibidem, N. S., I (1881), 303–360.

Anon: *Penetrating power of objectives.* Ibidem, I (1881), 831–832.

NOTES FOR TABLE 154

[1] Frank Crisp: *On the limits of resolution in the microscope.* Ibidem, V (1885), 968–973.

H. J. Detmers: *The numerical aperture of an objective in relation to its angle of aperture in air, water and balsam.* Proc. Amer. Microsc. Soc., 8th meeting, Cleveland, VII (1185), 199–202.

Edward M. Nelson: *On the limits of resolving power for the microscope and telescope.* Jour. Roy. Microsc. Soc., 1906, 521–531.

153. COMPARATIVE TABLE OF DRY ACHROMATIC OBJECTIVES OF DIFFERENT MAKERS[1]

No.	Maker	F. in mm.	N. A.	2 u	Free working distance	Field of view[2]	Magnification $\frac{\Delta}{F}$	Magnification $\frac{250}{F}$
00	Fuess	61.0	0.10	12°	70.00	10.00	.....	4.0
2	Nachet	50.0	0.09	10°	30.00	.....	.....	5.0
00	Reichert	50.0	0.06	7°	42.00	8.50	3.0	5.0
2"	Bausch & Lomb	48.0	0.08	9°	53.00	9.00	2.0	5.0
0	Fuess	31.0	.....	.....	39.00	4.75	.....	8.0
00	Seibert	45.0	0.07	8°	32.00	6.5	3.2	5.5
a_0	Zeiss	45.0	.....	.....	32.00	14.0	.....	5.5
1°	Leitz	42.0	0.08	9°	40.00	8.5	2.7	6.0
1	Leitz	40.0	0.11	12°	34.5	7.0	3.2	6.0
0	Reichert	40.0	0.06	7°	35.0	7.5	4.0	6.0
800	Beck	40.0	0.06	7°	34.0	6.5	.....	6.0
a_1	Zeiss	39.0	.....	.....	20.0	14.0	.....	6.4
75	Swift	38.0	0.20	23°	19.5	.....	7.5	6.5
a_2	Zeiss	37.0	.....	.....	30.0	8.0	3.7	6.5
0	Seibert	36.0	0.11	12°	30.0	5.2	5.2	6.5
1 1/3"	Bausch & Lomb	32.0	0.10	11° 29′	38.0	4.8	4.0	8.0
1	Fuess	32.0	0.10	11°	31.0	4.5	.....	8.0
1	Reichert	30.0	0.17	19°	25.0	5.0	5.0	8.0
a_2	Zeiss	28.0	.....	.....	33.0	4.5	6.5	9.0
aa	Zeiss	26.0	0.17	19.6°	14.0	4.0	8.0	9.5
1	Seibert	25.4	0.22	25°	14.0	3.6	8.6	10.0
3	Nachet	25.0	0.22	25°	8.0	.....	18.0	10.0
2	Leitz	24.0	0.19	24°	16.0	4.0	5.8	10.4
79	Swift	24.0	0.25	29°	14.5	.....	.....	10.5
801	Beck	22.0	0.13	15°	14.0	3.35	6.0	11.3
2	Fuess	22.0	0.21	24°	15.0	2.8	.....	11.3
3	Reichert	18.5	0.34	39°	5.5	2.2	8.5	13.5
3	Fuess	17.0	0.21	24°	11.0	1.9	.....	14.5
AA	Zeiss	17.0	0.30	35°	7.5	2.5	.....	14.5
3	Leitz	16.2	0.30	35°	5.5	2.1	10.3	15.4
2/3"	Bausch & Lomb	16.0	0.25	28.5°	7.0	1.85	10.0	15.6
802	Beck	16.0	0.15	17°	7.5	2.2	9.0	15.6
A	Zeiss	15.0	0.20	23.2°	9.0	2.0	14.0	16.7
4	Fuess	14.0	0.26	30°	8.0	1.6	.....	18.0
3a	Leitz	13.0	0.40	47°	3.2	1.6	14.1	19.0
2	Seibert	12.7	0.26	30°	4.0	2.2	14.2	20.0
B	Zeiss	12.0	0.35	41°	3.0	1.5	.....	20.8
802A	Beck	12.0	0.30	35°	7.0	1.6	16.0	20.8
4	Nachet	12.0	0.30	40°	3.5	.....	27.0	20.8
5	Fuess	10.0	0.32	38°	4.2	1.2	.....	25.0
4	Leitz	10.0	0.47	56°	2.0	1.1	18.2	25.0
3	Seibert	8.5	0.35	40°	2.5	1.5	21.4	30.0
4	Reichert	8.0	0.50	60°	2.0	1.45	20.0	31.0
1/3"	Bausch & Lomb	8.0	0.50	60°	1.6	0.9	20.0	31.0
C	Zeiss	7.0	0.40	47°	1.8	0.9	31.0	36.0
4	Seibert	6.4	0.60	74°	2.0	1.0	34.2	39.0
6	Fuess	6.3	0.80	106°	1.1	0.7	.....	39.6
803	Beck	6.0	0.68	85°	1.0	0.8	30.0	41.7
99	Swift	6.0	0.80	106°	1.0	.....	41.0	41.7
101	Swift	6.0	0.88	123°	0.7	.....	41.5	41.7
5	Nachet	6.0	0.60	80°	1.25	.....	50.0	41.7
5	Leitz	5.4	0.68	101°	0.76	0.70	33.3	46.0
5	Reichert	5.2	0.77	100°	0.5	0.6	30.0	48.0
7	Fuess	5.0	0.82	110°	0.85	0.55	.....	50.0
6	Reichert	4.3	0.85	116°	0.40	0.5	37.0	58.0
5	Seibert	4.2	0.85	128°	0.6	0.5	.....	60.0
D	Zeiss	4.2	0.65	81.2°	0.6	0.5	55.0	60.0
6	Leitz	4.0	0.82	110°	0.42	0.48	48.0	62.5
1/6L	Bausch & Lomb	4.0	0.65	81° 4′	0.6	0.43	43.0	62.5
1/6S	Bausch & Lomb	4.0	0.85	116° 26′	0.3	0.43	43.0	62.5
804	Beck	4.0	0.71	90°	0.64	0.50	44.0	62.5
6	Nachet	3.5	0.90	120°	0.60	.....	80.0	71.0
7a	Reichert	3.2	0.88	122°	0.35	0.40	50.0	78.0
5 1/2	Seibert	3.2	0.85	128°	0.30	0.43	80.0	78.0
8	Fuess	3.0	0.82	110°	0.75	0.40	.....	83.0
7	Leitz	3.0	0.85	116°	0.29	0.35	62.5	83.0
1/8"	Bausch & Lomb	3.0	0.85	116° 26′	0.2	0.32	57.0	83.0
805	Beck	3.0	0.82	110°	0.36	0.33	60.0	83.0
113	Swift	3.0	0.92	134°	0.4	.....	81.0	83.0
115	Swift	3.0	0.97	145°	0.35	.....	.....	83.0
7	Nachet	3.0	0.90	125°	0.45	.....	110.0	83.0
8a	Reichert	2.8	0.90	128°	0.30	0.32	57.0	89.0
9	Fuess	2.7	0.97	152°	0.60	0.30	.....	93.0
8	Nachet	2.3	0.90	130°	0.20	.....	130.0	109.0
6a	Seibert	2.1	0.90	128°	0.25	0.28	116.0	119.0
9	Reichert	2.0	0.95	142°	0.20	0.25	80.0	125.0

154. APERTURE TABLE[1]

Aperture ($n \sin u = N.A.$)	Corresponding angle ($2u$) for			Limit of resolving power, in lines to an inch			Illuminating power ($N.A.$)^2	Penetrating power $\frac{1}{N.A.}$
	Air ($n = 1.00$)	Water ($n = 1.33$)	Homogeneous immersion ($n = 1.52$)	White light $\lambda = 0.5269\,\mu$, line E)	Monochromatic (blue) light ($\lambda = 0.4861\,\mu$, line F)	Photography ($\lambda = 0.4000\,\mu$, near line h		
0.05	5° 44′	4° 18′	3° 46′	4,821	5,252	6,350	0.003	20.000
0.10	11° 29′	8° 38′	7° 34′	9,641	10,450	12,700	0.010	10.000
0.15	17° 14′	12° 58′	11° 19′	14,462	15,676	19,050	0.023	6.667
0.20	23° 4′	17° 18′	15° 7′	19,282	20,901	25,400	0.040	5.000
0.25	28° 58′	21° 40′	18° 56′	24,103	26,126	31,749	0.063	4.000
0.30	34° 56′	26° 4′	22° 46′	28,923	31,351	38,099	0.090	3.333
0.35	40° 58′	30° 30′	26° 38′	33,744	36,576	44,449	0.123	2.857
0.40	47° 9′	35° 0′	30° 31′	38,564	41,801	50,799	0.160	2.500
0.45	53° 30′	39° 33′	34° 27′	43,385	47,026	57,149	0.203	2.222
0.50	60° 0′	44° 10′	38° 24′	48,205	52,252	63,499	0.250	2.000
0.52	62° 40′	46° 2′	40° 0′	50,133	54,342	66,039	0.270	1.923
0.54	65° 22′	47° 54′	41° 37′	52,061	56,432	68,579	0.292	1.852
0.56	68° 6′	49° 48′	43° 14′	53,990	58,522	71,119	0.314	1.786
0.58	70° 54′	51° 42′	44° 51′	55,918	60,612	73,659	0.336	1.724
0.60	73° 44′	53° 38′	46° 30′	57,846	62,702	76,199	0.360	1.667
0.62	76° 38′	55° 34′	48° 9′	59,774	64,792	78,739	0.384	1.613
0.64	79° 36′	57° 31′	49° 48′	61,702	66,882	81,279	0.410	1.562
0.66	82° 36′	59° 30′	51° 28′	63,631	68,972	83,819	0.436	1.515
0.68	85° 41′	61° 30′	53° 9′	65,559	71,062	86,359	0.462	1.471
0.70	88° 51′	63° 31′	54° 50′	67,487	73,152	88,899	0.490	1.429
0.72	92° 06′	65° 32′	56° 32′	69,415	75,242	91,439	0.518	1.389
0.74	95° 28′	67° 37′	58° 16′	71,343	77,333	93,979	0.548	1.351
0.76	98° 56′	69° 42′	60° 0′	73,272	79,423	96,518	0.578	1.316
0.78	102° 31′	71° 49′	61° 45′	75,200	81,513	99,058	0.608	1.282
0.80	106° 16′	73° 58′	63° 31′	77,128	83,603	101,598	0.640	1.250
0.82	110° 10′	76° 8′	65° 18′	79,056	85,693	104,138	0.672	1.220
0.84	114° 17′	78° 20′	67° 6′	80,984	87,783	106,678	0.706	1.190
0.86	118° 38′	80° 34′	68° 54′	82,913	89,873	109,218	0.740	1.163
0.88	123° 17′	82° 51′	70° 44′	84,841	91,963	111,758	0.774	1.136
0.90	128° 19′	85° 10′	72° 36′	86,769	94,053	114,298	0.810	1.111
0.92	133° 51′	87° 32′	74° 30′	88,697	96,143	116,838	0.846	1.087
0.94	140° 6′	89° 56′	76° 24′	90,625	98,233	119,378	0.884	1.064
0.96	147° 29′	92° 24′	78° 20′	92,554	100,323	121,918	0.922	1.042
0.98	157° 2′	94° 56′	80° 17′	94,482	102,413	124,458	0.960	1.020
1.00	180° 0′	97° 31′	82° 17′	96,410	104,503	126,998	1.000	1.000
1.02	.....	100° 10′	84° 18′	98,338	106,593	129,538	1.040	0.980
1.04	.....	102° 53′	86° 21′	100,266	108,684	132,078	1.082	0.962
1.06	.....	105° 42′	88° 27′	102,195	110,774	134,618	1.124	0.943
1.08	.....	108° 36′	90° 34′	104,123	112,864	137,158	1.166	0.926
1.10	.....	111° 36′	92° 43′	106,051	114,954	139,698	1.210	0.909
1.12	.....	114° 44′	94° 55′	107,979	117,044	142,237	1.254	0.893
1.14	.....	118° 0′	97° 11′	109,907	119,134	144,777	1.300	0.877
1.16	.....	121° 26′	99° 29′	111,835	121,224	147,317	1.346	0.862
1.18	.....	125° 3′	101° 50′	113,764	123,314	149,857	1.392	0.847
1.20	.....	128° 55′	104° 15′	115,692	125,404	152,397	1.440	0.833
1.22	.....	133° 4′	106° 45′	117,620	127,494	154,937	1.488	0.820
1.24	.....	137° 36′	109° 20′	119,548	129,584	157,477	1.538	0.806
1.26	.....	142° 39′	111° 59′	121,477	131,674	160,017	1.588	0.794
1.28	.....	148° 42′	114° 44′	123,405	133,764	162,557	1.638	0.781
1.30	.....	155° 38′	117° 35′	125,333	135,854	165,097	1.690	0.769
1.32	.....	165° 56′	120° 33′	127,261	137,944	167,637	1.742	0.758
1.33	.....	180° 0′	122° 0′	128,225	138,989	168,907	1.769	0.752
1.34	.....	.....	123° 40′	129,189	140,035	170,177	1.796	0.746
1.35	.....	.....	125° 18′	130,154	141,080	171,447	1.823	0.741
1.36	.....	.....	126° 58′	131,118	142,125	172,717	1.850	0.735
1.37	.....	.....	128° 40′	132,082	143,170	173,987	1.877	0.729
1.38	.....	.....	130° 26′	133,046	144,215	175,257	1.904	0.725
1.39	.....	.....	132° 16′	134,010	145,260	176,527	1.932	0.719
1.40	.....	.....	134° 10′	134,974	146,305	177,797	1.960	0.714
1.41	.....	.....	136° 8′	135,938	147,350	179,067	1.988	0.709
1.42	.....	.....	138° 12′	136,902	148,395	180,337	2.016	0.704
1.43	.....	.....	140° 22′	137,866	149,440	181,607	2.045	0.699
1.44	.....	.....	142° 39′	138,830	150,485	182,877	2.074	0.694
1.45	.....	.....	145° 6′	139,795	151,530	184,147	2.103	0.690
1.46	.....	.....	147° 42′	140,759	152,575	185,417	2.132	0.685
1.47	.....	.....	150° 32′	141,723	153,620	186,687	2.161	0.680
1.48	.....	.....	153° 39′	142,687	154,665	187,957	2.190	0.676
1.49	.....	.....	157° 12′	143,651	155,710	189,227	2.220	0.671
1.50	.....	.....	161° 23′	144,615	156,755	190,497	2.250	0.667
1.51	.....	.....	166° 51′	145,579	157,800	191,767	2.280	0.662
1.52	.....	.....	180° 0′	146,543	158,845	193,037	2.310	0.658

If light between E and F ($=0.508\mu\mu$) is used, the N. A. will be a true measure of the resolving power, since it is exactly equal to the number of hundred thousands of lines to an inch. This will give 100,000 as the maximum for a dry objective, 133,000 for a water-immersion, and 153,000 for a homogeneous-immersion with crown-glass cover.[1]

155. Testing the Objective.—The value of an objective depends upon its definition and resolving power. In making a test one should have an objective of known value for comparison, and a series of test objects. The ocular employed should be the same in each case. The test plate most commonly used is made by J. D. Möller[2] and consists of a slide upon which are mounted a series of twenty diatoms[3] whose markings vary from 3 to 95 in a thousandth of an inch. They are as follows:

Diatom	Direction of striæ	Striæ in 1/1000 of an inch, after Morley
1. Triceratium favus Ehrbg		3.7
2. Pinnularia nobilis Ehrbg	transv.	13.0
3. Navicula lyra Ehrbg. var	transv.	16.0
4. Navicula lyra Ehrbg	transv.	24.5
5. Pinularia interrupta Sm. var	transv.	26.0
6. Stauroneis phoenicenteron Ehrbg		34.5
7. Grammatophora marina Sm	transv.	38.4
8. Pleurosigma Balticum Sm	transv.	33.1
9. Pleurosigma acuminatum (Kg.) Grun	transv.	46.4
10. Nitzschia amphioxys Sm		49.2
11. Pleurosigma angulatum, Sm	diagonal	47.0
12. Grammatophora oceanica Ehrbg = G. subtilissima	transv.	61.6
13. Surirella gemma Ehrbg	transv.	53.5
14. Nitzschia sigmoidea Sm	transv.	62.0
15. Pleurosigma fasciola Sm. var	transv.	58.0
16. Surirella gemma Ehrbg	longit.	67.0
17. Cymatopleura elliptica Breb		63.0
18. Navicula crassinervis Breb = Frustulia saxonica Rabh		86.0
19. Nitzschia curvula Sm		90.0
20. Amphipleura pellucida Kg	transv.	95.2

The process of testing an objective serves not only the purpose of determining its limit of capacity, but teaches a student, as nothing else will, how to bring out that capacity. While this is of much less importance in petrographic than in biologic work, it is, nevertheless, something that should be

[1] See J. W. Stephenson: *Op. cit.*

[2] Anon: *Moeller's test-plate (Probe-Platte).* Amer. Jour. Microsc., I (1875), 16–17. Made by J. D. Möller's Institut für Mikroskopie. Wedel i. Holstein, Germany.

[3] L. Dippel. Zeitschr. f. Mikrosk., II (1880), 4 plates.* Another test plate is described by Henri Van Heurck: *Nouvelle plaque d'épreuve (Test-Platte) pour la vérification des objectifs.* Zeitschr. f. angew. Mikrosk., IV (1898), 1–4.

understood by every user of a microscope. This was well expressed by Hirst[1] who said:

"The tyro, sitting down before his newly acquired instrument, places an object on the stage, turns on the full glare of light from his mirror and condenser, and fancies he sees everything to perfection. Let him try the same method of proceeding on some delicate diatom-valve; and where in the hand of the skilled manipulator a moment before, lines or beading were beautifully displayed, he sees a blank. He may spend long hours in trying every trick of illumination, moderating his light, varying its obliquity by altering the angle of his mirror, focussing and re-focussing the condenser, altering the adjustment of his objective; and at last, when his patience is well-nigh exhausted, the desired result is obtained, the delicate markings start suddenly into view, and he possesses the consciousness that, under his hands, mirror, condenser, and objective are now doing their best. Has this time been wasted? I think not."

The method of testing, briefly, is as follows. Place the objective to be tested in the microscope and examine all the diatoms of the test plate in order, beginning with No. 1. At the start use the greatest possible amount of light, placing the mirror in the axial line of the microscope and removing the polarizer. Examine the structure of the diatoms and note whether the outline and the markings appear to lie in a single plane. If they do not, adjust the correction collar (Art. 152). Proceed in the examination until a diatom is reached whose markings cannot be seen. Now swing the mirror-bar slightly to one side, thus giving more inclined illumination. If the markings do not yet appear, increase the inclination until they do. Proceed to the next diatom and so on until no striæ can be seen. It is quite probable that by tilting the mirror, changing the illumination, inserting a bull's-eye condenser, or moving the correction collar, they will appear. It is possible that the ocular is of too low a power. This may be determined by noting how close together the striæ were in the last diatom in which they could be seen. Successive trials will probably enable the student, with the same combination of objective and ocular, to see striæ where none appeared before.

One should be able to resolve the diatoms given below by means of objectives having the numerical apertures noted in the first column of the table.

N. A.	Diatom	Striæ in 0.001 in.	Remarks
0.45	Pleurosigma Balticum	33	Central illumination.
0.55	Pleurosigma acuminatum	45	Central illumination.
0.65	Pleurosigma angulatum	47	Central illumination.
0.75	Nitzschia sigmoidea	62	Central illumination.
0.85	Surirella gemma (longit.)	67	Central illumination.
0.95	Navicula crassinervis	86	Inclined illumination.
1.05	Nitzschia curvula	90	Inclined illumination.
1.20	Amphipleura pellucida	95	Inclined illumination.

[1] G. D. Hirst: *Notes on some local species of diatomaceæ.* Jour. and Proc. Roy. Soc. New South Wales, XI (1877), 272–277, in particular 276.

156. Cost of Objectives.—As a matter of comparison it may be said that objectives with a focal length of 25 mm. and over, cost approximately $4.00 each; between 25 and 10 mm., $5.50 to $10.00; 10 to 3 mm., $7.00 to $15.00; 3 to 2 mm., about $20.00. A 1/12-in. (1.9 mm.) oil-immersion objective costs about $27.00, and a 1/16-in., $40.00.

Apochromatic objectives are much more expensive. One of 16 mm. focal length will cost about $25.00, 8 mm., $32.00; 4 mm., $45.00; 3 mm., $50.00; 2 mm. oil-immersion of 1.30 N. A., $100.00, and the same with 1.40 N.A., $130.00.

While it will not be necessary to caution owners of microscopes in regard to the care of their objectives, the above prices may serve as a hint to students using University property. Instructions for the care of objectives are given in Article 198.

THE OCULAR OR EYEPIECE

157. Huygens Eyepiece.—The ocular of a microscope is not nearly so complicated as the objective. In most forms but two lenses are used. Three types are made, Huygens or negative, Ramsden or positive, and compensating eyepieces.

The Huygens eyepiece consists of two simple plano-convex lenses placed with their plane surfaces toward the eye. The upper lens (*e*, Fig. 301) is known as the eye-lens, the lower (*f*), as the collective or field lens. The focal length of the eye-lens is one-third that of the field-lens, and the two are separated a distance equal to the sum of their focal lengths. The Huygens eye-piece cannot be used to magnify an object directly, and it is, for this reason, called negative. As may be seen from the figure, the real image (O_3) is formed within the ocular by the field-lens. This collects the rays which come from the objective and which would normally have produced the real image at O_2. The image O_3 is smaller than the real image produced by the objective, consequently the field of view of the ocular is greater than it would be were the image O_2 viewed directly. When cross-hairs or micrometers are used, they must be placed in the plane of O_3 in order that they may be viewed simultaneously with the image.

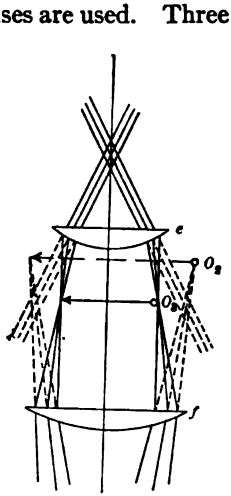


FIG. 301.—Huygens or negative eyepiece.

The rays of light emerging from the eye-lens are parallel, and thus cause the eye least fatigue. Under this condition the image appears to be that of an object infinitely distant, although it is customary, in computing magnifications, to consider the image as being formed at the distance of distinct vision (250 mm.).

The Huygens eyepiece is the one most commonly used in petrographic microscopes. It is achromatic in the sense that images of different colors appear of the same size. In most modern instruments the various oculars are so mounted that their lower focal points lie in the same plane when inserted in the tube. That is, the optical tube length, except so far as this is changed by the ocular itself, remains practically the same for the same objective, irrespective of the ocular used. This is a great convenience, as it makes re-focussing unnecessary when changing from one ocular to another of different power.

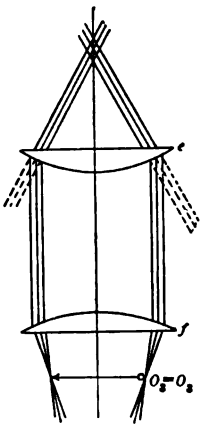


FIG. 302.—Ramsden or positive eyepiece.

158. Ramsden Ocular.—The Ramsden or positive ocular, like the Huygens, consists of two simple plano-convex lenses, but in this eyepiece they are placed with their convex sides toward each other (Fig. 302). Usually the focal lengths of the two are equal, and the distance between them is about one-third the sum of their focal lengths. The focal plane of the combination lies one-fourth the focal length of the collective lens *f* below it, consequently cross-hairs or micrometers, placed in the focal plane, are viewed directly through the eye-lens *e*.

This type of ocular is used principally for special work, such as making measurement with a micrometer.

In neither the Huygens nor the Ramsden ocular is any attempt made to correct spherical aberration, since they are used with small apertures and the distortion is slight. Chromatic aberration is corrected only so far as this is possible by varying the distance between the lenses.

159. Compensating Oculars.—Even in apochromatic objectives it has been found impossible to do away entirely with differences in the focal planes for different colors. To overcome this, Abbe invented compensating oculars. These are overcorrected just the proper amount to eliminate the error, whereby the field becomes entirely free from color up to the edge of the diaphragm, which itself shows an orange border. There is not a great deal of advantage in using compensating oculars in petrographic work since high-power objectives do not give perfectly flat fields up to the margin where chromatic aberration interferes.

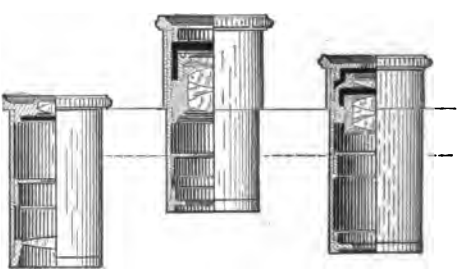


FIG. 303.—Compensating oculars. (Zeiss.)

Like the Huygens, the mounts of compensating oculars are made so that their lower focal points fall in the same plane (Fig. 303).

160. COMPARATIVE TABLE OF HUYGENS OCULARS OF DIFFERENT MAKERS

Number	Maker	Focal length in mm.	Magnification $\frac{250}{F}$	Magnification $\frac{\triangle}{F}$
0	Leitz	62.5	4.0	.....
1	Fuess	57.0	4.4	.....
1	Leitz	50.0	5.0	.....
1	Beck	50.0	5.0	.....
1	Reichert	50.0	5.0	3.0
0	Seibert	50.0	5.0	3.5
1	Zeiss	50.0	5.0	3.0
2″	Bausch & Lomb	50.0	5.0	.....
2	Fuess	45.0	5.6	.....
II	Leitz	41.65	6.0	.....
A	Swift	41.6	6.0	.....
1 3/5″	Bausch & Lomb	40.0	6.3	.....
1A	Beck	40.0	6.3	.....
II	Reichert	40.0	6.3	.....
2	Zeiss	40.0	6.3	4.0
1	Seibert	34.0	7.3	5.0
1 1/3″	Bausch & Lomb	33.0	7.9	.....
III	Leitz	31.25	8.0	.....
2	Beck	30.0	8.3	.....
3	Fuess	30.0	8.3	.....
III	Reichert	30.0	8.3	.....
3	Zeiss	30.0	8.3	5.5
B	Swift	27.7	9.0	.....
IV	Reichert	25.0	10.0	.....
2	Seibert	25.0	10.0	7.0
4	Zeiss	25.0	10.0	7.0
1″	Bausch & Lomb	25.0	10.0	.....
IV	Leitz	25.0	10.0	.....
4	Fuess	21.0	12.0	.....
V	Leitz	20.85	12.0	.....
C	Swift	20.8	12.0	.....
4/5″	Bausch & Lomb	20.0	12.5	.....
2A	Beck	20.0	12.5	.....
V	Reichert	20.0	12.5	.....
5	Zeiss	20.0	12.5	10.0
3	Seibert	17.0	14.7	10.0
D	Swift	16.7	15.0	.....
2/3″	Bausch & Lomb	16.7	15.0	.....
3	Beck	16.5	15.0	.....
E	Swift	13.8	18.0	.....
4	Seibert	12.5	20.0	14.0

Huygens oculars are worth from $1.50 to $2.00; compensating oculars, $6.00 to $7.00.

161. Oculars for Special Purposes.—Most oculars of special design are used for observations in polarized light. They will be described below (Chapters XXV–XXVI). The following eyepieces are used for observations in ordinary light.

162. Demonstration Oculars.—As long ago as 1848, Queckett[1] described an ocular fitted with a pointer for purposes of demonstration. In the Huygens ocular, designed by Professor Pfitzner[2] and shown in Fig. 304, the ex-

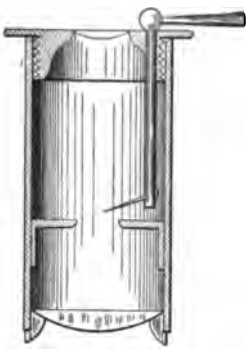


FIG. 304.—Demonstration ocular after Pfitzner. (Leitz.)

FIG. 305.—Demonstration ocular after Bourguet. (Reichert.)

tremity of a pointer, which is attached to a rod, lies in the plane of the image. By combining a rotation of the ocular within the tube with a rotation of the pointer, any part of the field may be shown to the student without centering

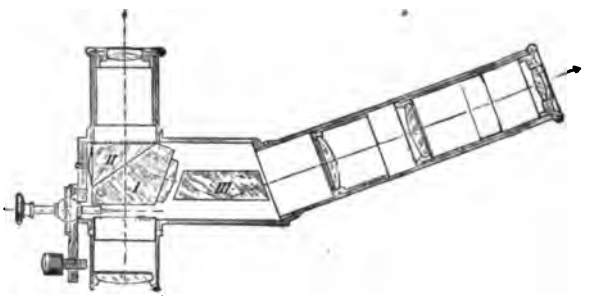


FIG. 306.—Double demonstration ocular after Edinger. (Leitz.)

it under the cross-hairs. In a similar ocular, after Bourguet[3] (Fig. 305), any mineral may be pointed out by inserting the rod, more or less, and rotating it.

A double demonstration ocular,[4] with the eye-lenses separated by 18 cm. (7 in.), is shown in Fig. 306. By means of reflecting prisms, it is possible

[1] Queckett: *Microscope*, 1st ed., 1848, 130, Fig. 91.*

[2] Martin Kuznitzky: *Facultative Demonstrations-Oculare*. Zeitschr. f. wiss. Mikrosk., XIII (1896), 145–146.

[3] Anon: *Neues Index-Okular nach Bourguet*. Zeitschr. f. angew. Mikrosk., VIII (1902), 33.

[4] L. Edinger: *Das Zeigerdoppelokular*. Zeitschr. f. wiss. Mikrosk., XXVII (1910), 336–338.

for student and instructor to view the same section at the same time. The pointer, shown at the left of the diagram, may be pushed in, more or less, and moved in azimuth in a sliding ring, thus covering every part of the field.

163. Focussing Cross-hairs in the Ocular.—The cross-hairs of the ocular are attached to a sliding sleeve within the tube, and are so placed that they lie in the plane O_3 (Figs. 301–302). While the focal plane occupies a different position for different eyes, it is ordinarily not necessary to move the sleeve, the adjustment being accomplished by sliding the eye-lens collar, which, in most oculars, is held by friction. It would be an improvement if the eye-lens ring were screwed in and held in place by a bearing screw. The easiest way to adjust the focus of the cross-hairs is to remove the eyepiece from the microscope and focus by looking through it against a light background. The cross-hairs should be seen in sharp focus at the first glance through the ocular, and before the eye has had time to accommodate itself. When in proper adjustment and with the ocular in the microscope, the cross-hairs will appear well defined and lie in the plane of the image of whatever object is viewed through the instrument.

Sometimes it is impossible to obtain a sharp focus by shifting the eye-lens. It is then necessary to move the sliding collar to which the cross-hairs are attached. It is a simple enough matter to slide it into proper position in the Ramsden ocular, but is more difficult in the Huygens, where it lies between the two lenses. The eye- or the field-lens should be removed, and the cross-hair collar shoved up by means of a pencil, care being taken not to touch the cobwebs. It may be necessary to make several trials before getting the proper position for the hairs, which should be in focus when the eye-lens slide is approximately in its intermediate position.

164. Replacing Cross-hairs.—The finest cross-hairs are made of spider web, the dark thread from the inside of a nest being the best. These nests, which may be found in the autumn hanging on bushes, should be torn open and the eggs removed, otherwise the newly hatched spiders will eat the web.

To replace cross-hairs, first remove the ring, which is to support them, from the ocular. It will be seen that there are two scratches, at right angles to each other, which indicate the proper positions for the cross-hairs. Take a single thread, an inch or two long, from a spider's nest, and attach, to each end, as heavy a weight as it will carry. Hold one weight in the fingers, and dip the thread in hot water or hold it in steam, to stretch it. Now move the ring against the center of the web and turn it into a horizontal position, leaving a weight to hang down on either side. If the hair is not quite in proper position, move it by means of a pin, then fasten it in place by a bit of wax or a touch of shellac. Replace the other hair in the same manner.

165. Magnification of the Compound Microscope.—According to Abbe,

the magnifying power of an objective is determined by the formula $\frac{250}{F_o}$ (Art. 149), and that of the eyepiece by $\frac{\Delta}{F_e}$ (Arts. 98 and 103). Many makers reverse the formulæ and give $\frac{250}{F_e}$ as the magnifying power of the eyepiece and $\frac{\Delta}{F_o}$ as that of the objective (Art. 149). The magnification of the compound microscope may be considered as the resultant of two successive magnifications, the first being the magnification produced by the objective (O_3, Fig. 229), the second that produced by the ocular, which magnifies the real image derived from the objective (O_2) and produces the final image O_4. Its value, consequently, regardless of whether Abbe's system or the reverse is used, will be:

$$N = \frac{250}{F_o} \times \frac{\Delta}{F_e} = \frac{250\Delta}{F_o F_e}. \tag{1}$$

But $\frac{F_o F_e}{\Delta} = F$ (Eq. 1, Art. 102),

therefore $$N = \frac{250}{F}, \tag{2}$$

which is the same equation as (2) Art. 102, as it should be.

In practice[1] the magnifying power of any combination of ocular and objective may be obtained by direct comparison as explained in Article 246, or it may be obtained by multiplying, according to equation 1, the known magnifying powers of the ocular and the objective obtained from the table in Articles 153 and 160.

If the tube length used is greater than that given in the table of computed magnifications (160 or 170 mm.), a correction must be made to the amount of magnification of the objective, as indicated in Article 149. It is not advisable, however, with an objective of focal length shorter than from 5 to 7 mm., to try to increase the magnification by changing the tube length from that for which it was designed. A difference of only 10 mm. with an oil-immersion lens will materially reduce its efficiency.

[1] Cf. Sir A. E. Wright: *On certain new methods of measuring the magnifying power of the microscope and of its separate elements.* Jour. Roy. Microsc. Soc., 1904, 279–288.

CHAPTER XI

VARIOUS MODERN MICROSCOPES

166. Introduction.—It is impracticable to describe all of the different kinds of petrographic microscopes made, and in the following pages only some of the more important instruments will be noted. While the stands described below are typical of those of the different makers, there are innumerable varieties, especially of more simplified form, and the catalogues of the different manufacturers[1] may be examined with profit by the student as a supplement to this chapter.

167. Leitz Stand AM.—One of the best petrographic microscopes manufactured is the Leitz's stand AM,[2] already described in part and represented in Figs. 230 and 231. The stand is of large dimensions, without being clumsy, and provides ample space for all of the accessories used in modern petrographic work, including v. Fedorow's universal stage. The body tube is unusually wide so that it may be used in photomicrographic work. The draw tube (*TA*, Fig. 231) is adjustable by means of a rack and pinion moved by the milled head *OcE*, and is graduated to show the mechanical tube length. The fine adjustment has been described above (Art. 115). The revolving stage may be read to minutes by means of a vernier, and may be moved slowly by means of a tangent screw (*TS*, Fig. 230), the latter movement being extremely valuable in reading small extinction angles and so on. The stage also has two lateral movements with a range of 20 mm., the motion being controlled by two screws, and its amount read by means of two scales set into the stage.

The polarizer (*P*, Fig. 231) and iris diaphragm, which are shown in detail in Figs. 258–260, may be raised or lowered by means of the screw *BT*. Both

[1] Some of the leading manufacturers of petrographic microscopes are:
America: Bausch & Lomb Optical Co., Rochester, N. Y.
Austria: C. Reichert, Bennogasse 24–26, Wien VIII.
England: R. & J. Beck, Ltd., 68 Cornhill, London. James Swift & Son, 81 Tottenham Court Road, London, W.
France: A. Nachet, 17 Rue St. Séverin, Paris.
Germany: R. Fuess, Steglitz b. Berlin. Düntherstrasse 8. E. Leitz, Wetzlar. (Branch, 30 East 18th St., New York.) W. & H. Seibert, Wetzlar. Carl Zeiss, Jena.
Switzerland: Société Genevoise pour la Construction d'Instruments de Physique et de Méchanique, 8 Rue des Vieux-Grenadiers, Genève.

[2] Gabriel Lincio: *Das neue Leitz'sche mineralogische Mikroskopmodell A.* Neues Jahrb., B. B., XXIII (1907), 163–186.

polarizer and analyzer are of the Glan-Thompson type with a large opening angle. They may be rotated and the amount of rotation read from a divided scale. Above the analyzer is a long focus lens to correct the displacement caused by the insertion of the prism. The Bertrand lens (*BL*, Fig. 230) slides in and out of the draw tube, and may be moved up or down to bring it into focus.

FIG. 307.—Berkey model microscope. (Leitz.)

The only improvements that might be suggested for this stand are the addition of a diaphragm, either sliding or iris, in the image plane of the Bertrand lens, and a detachable, rigid connection between the two nicols, permitting their simultaneous rotation, as in the instruments shown in Figs. 291, 312, and 313.

168. Leitz's Berkey Model.—A simpler microscope, and one most excellently adapted to the use of elementary students, is shown in Fig. 307. It was made after the specifications of Professor Berkey and embraces the most essential accessories. Being less elaborate than the preceding, it is less likely to get out of order, a considerable advantage in general class-room work. In this microscope the condensing lens is inserted or thrown aside by rotating the milled head beneath the stage. The polarizer may be placed in the 0°, 90°, 180° or 270° positions, or may be swung entirely aside. The revolving stage is graduated, the analyzer slides in and out of the tube, and the Bertrand lens may be slipped into a slot above it. The graduated draw-tube has an inside diameter of 24 mm.

168a. Leitz's New Stand.—A microscope, somewhat more complete than the above and embracing all the essentials of a modern instrument and at a reasonable price, is shown in its preliminary form in Fig. 308.

Instead of the usual tube, 24 mm. in diameter, this instrument has a tube of 30 mm., and thus has a field of view nearly twice as great. Both coarse and fine adjustments are provided. The latter is of the type shown in Fig. 245, and has divisions of 0.01 mm., permitting a reading to 0.0025 mm. with ease. There are two iris diaphragms, one above the lower nicol, and one above the Bertrand lens. The lower diaphragm and the whole condensing system may be raised or lowered by means of a rack and pinion, and the diaphragm may be displaced laterally so that inclined illumination may be used. The Bertrand lens is fastened in a slider in the inner tube whereby it may be raised or lowered to bring it into focus with any ocular. It has two adjusting screws at the side for accurate centering. The upper lenses of the condensing system may be thrown aside by rotating a milled head, and are so constructed that in changing from parallel to convergent light, it is not necessary to lower the polarizer. The angular aperture of the condenser is large, giving the first yellow ring around the interference figure of quartz. The Johannsen wedge, inserted in a slider in the accessory slot at 45°, makes unnecessary the picking up and laying down of mica plate or quartz wedge for each determination. Attached to each objective is a centering device which consists of two screws, working at right angles to each other, and adjusted by means of watch keys. When this is combined with the new objective tongs, in which a strong spring presses firmly against an inclined bar, permanent centering is obtained. An attachable mechanical stage may be used with the instrument if desired.

FIG. 308.—New petrographic microscope. (Leitz.)

In a later instrument made for the writer, the adjusting screws for the Bertrand lens, shown in the illustration, have been replaced by two square-end screws which may be turned by means of the same watch keys used in centering the cross-hairs. This prevents the accidental displacement of the screws by the finger. The Bertrand lens, also, is inserted from the other

side from that shown in the figure, to correspond in position to that of the analyzer. This arrangement places the levers for both iris diaphragms to the rear when they are open, instead of one to the front and one to the rear.

There is under consideration, furthermore, a new device for the simultaneous rotation of the nicols.

FIG. 309.—Petrographic microscope. 3/7 natural size. (Seibert.)

169. Seibert Microscope.—The Seibert microscope shown in Fig. 309 is another excellent instrument. It has coarse and fine adjustment screws, one division on the latter measuring 0.002 mm.; revolving stage, provided with degree divisions, vernier, and stage clamp; and centering screws working parallel to the cross-hairs, for adjusting the center. Both polarizer and analyzer are flat-end nicols. The former may be moved up or down by means of a screw, and may be swung aside by means of a hinge when observations are to be made by ordinary light. Between the polarizer and the condenser is an iris diaphragm. The Bertrand lens is inserted by raising the lever just above the analyzer. As in many microscopes, there is no upper diaphragm, although for modern petrographic work it is almost absolutely necessary.

170. Fuess Stand VIa.—One of the newest of the Fuess microscopes (VIa)[1] is shown in Fig. 310. It differs from those previously described in having an attachment by means of which the nicols may be simultaneously revolved. In older forms the rotating analyzer was a cap nicol which materially cut down the field of view. In this microscope the tube is double, the outer one being stationary while the inner one rotates, carrying with it the nicol prism and the slot above the objective clip (*k*). It thus permits any accessory placed in this slot to retain its orientation, with reference to the nicols, during the rotation. It is possible, also, to rotate (1) polarizer,

[1] J. Hirschwald: *Ueber ein neues Mikroskopmodell, etc.* Centralbl. f. Min., etc., 1904, 626–633.

Idem: *Die Prüfung der natürlichen Bausteine auf Ihre Wetterbeständigkeit.* Berlin.

Idem: *Handbuch der bautechnischen Gesteinsprüfung.* Berlin I, 1911, 142–147.

analyzer, and ocular with cross-hairs simultaneously, (2) analyzer and ocular, (3) polarizer and analyzer, or (4) analyzer alone. The polarizer is a modified nicol, the analyzer a Glan-Thompson. In the eyepiece there is a sliding diaphragm containing a circular and a square opening, the latter to facilitate the measuring of all of the constituents of a rock section. The stage is of the Hirschwald pattern, described above.[1] The graduations of the stage are in degrees, with a vernier reading to 5′. A novel arrangement is the electric light (*G*) for illuminating opaque minerals or rock slabs, a blue glass in front of the light reducing its yellow color to approximately the tone of daylight.

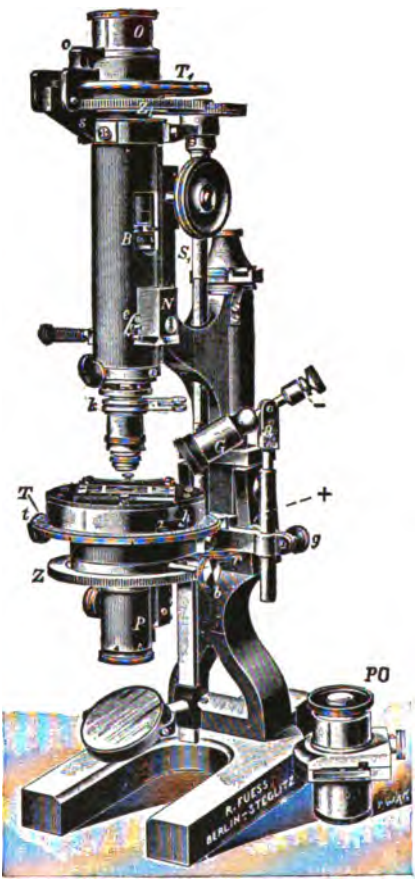


FIG. 310.—Microscope model VI *a*. (Fuess.)

171. Fuess Stand, IIIa.—One of the best moderate-priced instruments for students' use is the Fuess IIIa.[2] Fig. 311 represents the stand of the No. III which becomes IIIa by the substitution of the tube shown in Fig. 311a. The following description applies to the IIIa.

The rotating stage is divided into degrees, with verniers reading to 5′. The polarizer is a modified nicol prism with square cross-section and cemented with linseed oil. It is raised or lowered by means of a milled head on the side of the instrument not shown in the illustration. The upper lenses of the condensing system may be thrown in or out by means of the lever *b′*, also shown in Fig. 233. The objective clip is shown in Fig. 239. The fine adjustment is produced by means of a spring depressed by the milled head *n*, which is graduated and acts as a micrometer screw.

The particularly attractive feature of this microscope is its extremely wide field of view. The field, ordinarily, is limited by the size of the collec-

[1] Article 109, *supra*.

[2] C. Leiss: *Mikroskope mit sehr grossem Sehfeld für petrographische Studien*. Neues Jahrb., II (1897), 86–88.

tive lens of the ocular. In this microscope the tube is 30 mm. in diameter instead of 23.25 mm. as in most microscopes, and by this means it is possible to use larger oculars. The following table gives, in millimeters, the

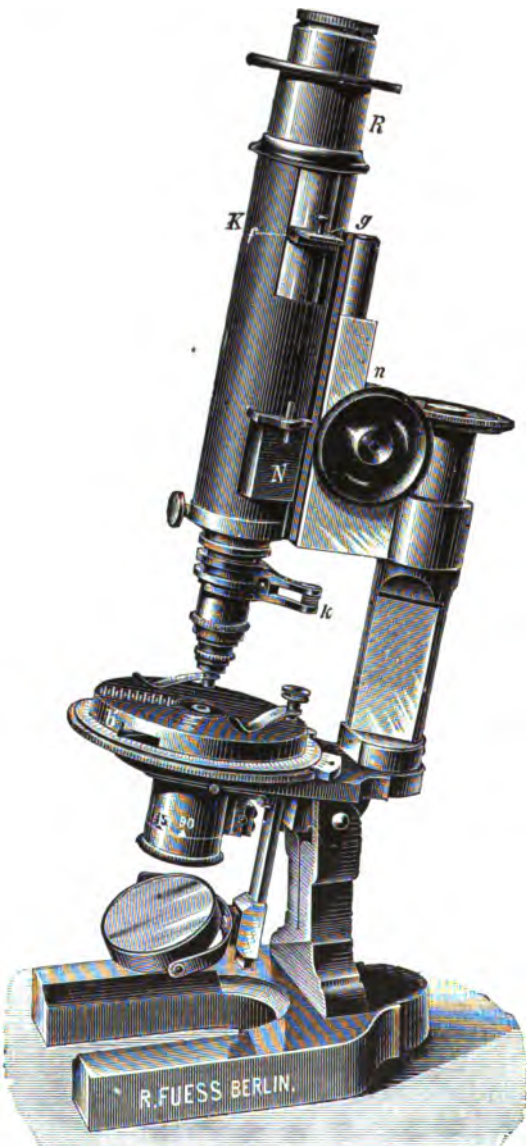


FIG. 311.—Microscope model III. 1/3 natural size. (Fuess.)

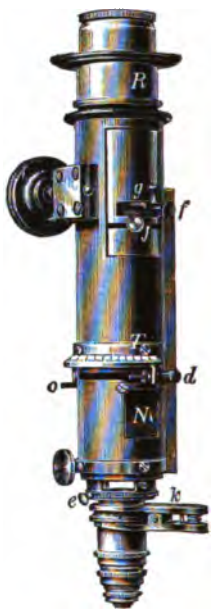


FIG. 311*a*.—Large tube for microscope III *a*. (Fuess.)

field of the new and of the older forms, and shows that it has been approximately doubled.

The analyzer is a Glan-Thompson prism which may be rotated by means of the lever *d*. The Bertrand lens, in the newer instruments, is permanently attached at *f*, and beneath it is an iris diaphragm *J*. A diaphragm above the polarizer (Fig. 255) should be specified in ordering.

Fuess' objective number	0	1	2	3	4	5	6	7	8	9
Fuess' ocular usual No. 2.	3.8	3.45	2.25	1.6	1.35	0.9	0.7	0.46	0.33	0.28
Fuess' large field ocular, No. 2.	6.0	5.5	3.31	2.5	2.0	1.5	0.15	0.7	0.55	0.4

It would be an improvement if the condensing lens and iris diaphragm were attached to a holder separate from the polarizer, so that the latter might be swung aside without the former. This might be so arranged that one could displace the center of the condenser and the diaphragm from the axis of the microscope for the production of inclined illumination. The iris diaphragm lever (*J*, Fig. 311a), shown in the illustration as straight, is usually made with a drop. It should be straight, for although not quite so easy of access, it does not continually twist and strain the threads as does the bent form.

172. Fuess Microscope, Model Ib.—On a previous page[1] there was illustrated a rigid bar connection, after designs by Sommerfeldt, by which the two nicols could be simultaneously rotated. A similar arrangement, on a more complicated instrument, is shown in Fig. 312. This microscope, after de Souza-Brandão,[2] consists of a large stand, similar to the Fuess No. Ia, and, like that instrument, has an Abbe[3] illuminating apparatus. The polarizer is an Ahrens prism, and over it are the condensers, which are centered by means of the screws *z*. The stage possesses, besides the usual rotation in azimuth, a second rotation in altitude, the amount being read, by means of verniers and the drum T_1, to 5 minutes. This movement is very convenient, especially for obtaining maximum extinction angles, which is usually possible since the opening in the stage is 6 cm. in diameter on the lower side, and the stage may be rotated as much as 60°. The mechanical stage mm_1 has micrometer divisions to 0.01 mm. and is detachable, a very good point since for most purposes it is more convenient to work without it. The tube, which cannot be lengthened, has an inside diameter of 24 mm., and takes the ordinary oculars. The analyzer is a Glan-Thompson prism which may be rotated independently through 135° when the bar connecting it with the polarizer is placed in the 90° position. The simultaneous rotation of the

[1] Article 139 and Fig. 291, *supra*.

[2] V. de Souza-Brandão: *O novo microscopio da commissão do serviço geologico* Communiçadoes da Commissão do Serviço Geologico de Portugal, V (1903–4), 118–250.

C. Leiss: *Ueber zwei neue Mikroskope für petrographische und krystalloptische Studien.* Zeitschr. f. Kryst., XLIX (1910–1911), 193–197.

[3] C. Leiss: *Neues Mikroskopmodell Ia für mineralogische und petrographische Studien.* Zeitschr. f. Kryst., XLIV (1908), 264–267.

Idem: *Neues Mikroskop Modell VIb für krystallographische und petrographische Studien.* Ibidem, XLVIII (1910), 240–242.

nicols is produced by means of the bar s, which is attached to the divided circle N, the connection with the nicols being made by means of the forked bars s_3 and s_4. The amount of rotation possible is 180°, the vernier reading to 5 minutes. Below the ocular is the iris diaphragm OJ. Accompanying the

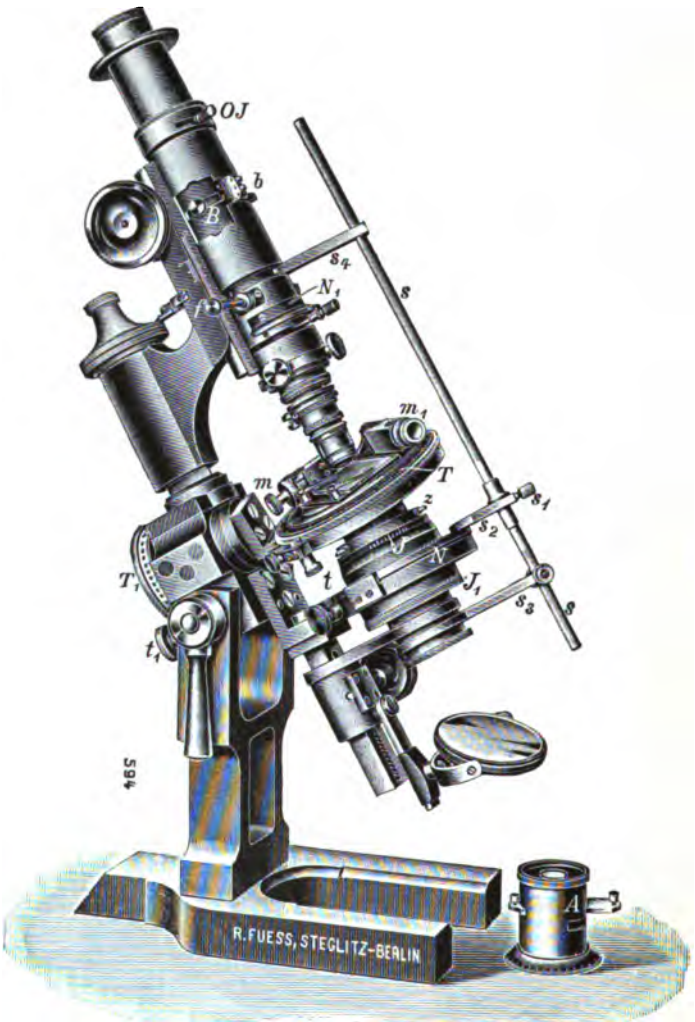


FIG. 312.—De Souza-Brandão microscope. (Model I *b*.) (Fuess.)

microscope are three Bertrand lenses adapted for different oculars, thus requiring their removal and insertion, a method less convenient than a permanently attached Bertrand lens set in a sliding sleeve, as in the Fuess IIIa microscope, and adapted for all oculars. Below the stage is an iris diaphragm J.

173. Fuess Microscope, Model IIa.—The latest microscope with simultaneously rotating nicols is the Fuess Model IIa[1] (Fig. 313). As may be seen from the illustration, a rigid bar connects hinged levers extending from polarizer to analyzer, the object of the hinges being to permit the end portions to be elevated and thus allow the nicols to be slipped in or out, or rotated independently. The amount of rotation of the nicols may be read from the graduated circle above the analyzer or from the graduations of the stage. The analyzer is a Glan-Thompson prism, the polarizer an Ahrens. If the rotating lever of the upper nicol were attached beneath the calcite prism, it would be advantageous since it would do away with the reflection of light from its upper surface. In this microscope the movable upper lenses

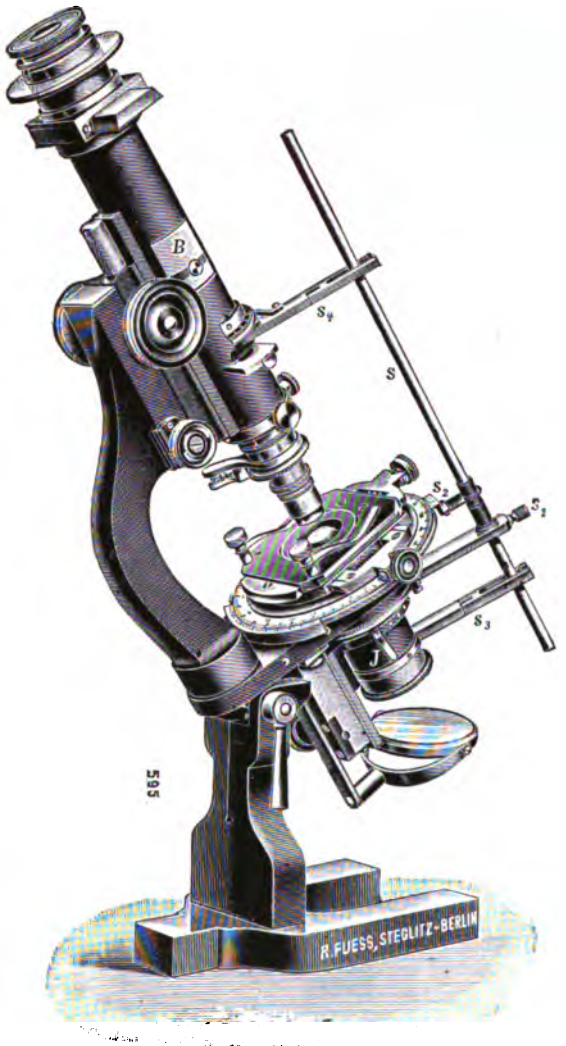


FIG. 313.—Microscope Model II *a*. (Fuess.)

[1] Fred Eugene Wright: *Neuere Verbesserungen am petrographischen Mikroskop.* Centralbl. f. Min. etc., 1911, 581–584.

C. Leiss: *Ueber zwei neue Mikroskope für petrographische und krystalloptische Studien.* Zeitschr. f. Kryst., XLIX (1911), 198.

See also:

Fred. Eugene Wright: *A new petrographic microscope.* Amer. Jour. Sci., XXIX (1910), 407–414.

Idem: *Methods of petrographic-microscopic research.* Carnegie Publication No. 158. Washington, 1911, 10–13.

of the condensing system, used in the other Fuess microscopes, are omitted, the Abbe illuminating apparatus making these unnecessary. At the upper end of the tube is a slide, similar to the Seidentopf compensator (Fig. 469,) for the insertion of accessories in the focal plane of the ocular.

174. Zeiss Crystallographic and Petrographic Microscope, III MD.—The Zeiss[1] crystallographic and petrographic microscope III MD (Fig. 314), has coarse and fine adjustments, the latter of the Berger[2] type, one division of the milled head corresponding to a variation of 0.002 mm. in the position of the tube. The inner tube is movable by means of a rack and pinion, and there are millimeter divisions for recording the tube length. The Bertrand lens is inserted in the lower end of the tube. There is a polarizer which swings out, an iris diaphragm below the stage, and a condensing system of 1.40 numerical aperture. Two analyzers are provided, one to swing out, and a cap. There is no upper diaphragm nor are there means for simultaneously rotating the nicols. The non-mechanical stage is of the revolving type, and is graduated. Above the objective there is a carrier in which the accessories may be inserted.

FIG. 314.—Microscope model III MD. (Zeiss.)

[1] S. Czapski: *Mikroskope von Carl Zeiss in Jena für krystallographische und petrographische Untersuchungen.* Zeitschr. f. Instrum., XI (1891), 94–99.

[2] Max Berger: *Ein neuer Mikroskop-Oberbau.* Zeitschr. f. Instrum., XVIII (1898), 129–133.

175. Zeiss Small Mineralogical Stand VM.—Smaller than the above is the microscope shown in Fig. 315. This instrument has a two-lens condensing apparatus with a numerical opening of 1.0, which, with the polarizer, may be entirely thrown out of the axis of the microscope. The lower lens of the condenser is attached to the polarizer, the upper is loosely placed above it and may be lifted out when the condenser is swung aside. This does not allow a very rapid change from parallel to convergent light or *vice versa*, since it is necessary to rack down the polarizer, swing it aside, insert the condensing lens with the fingers, swing it back, and rack it up. The polarizer is held in place by friction and may be rotated through 360°. In the lower part of the tube there are two slides, one for the analyzer, the other containing a circular opening into which is laid the gypsum or mica plate. There is neither draw-tube, Bertrand lens, nor upper diaphragm.

FIG. 315.—Small mineralogical stand VM. (Zeiss.)

176. Reichert Mineralogical Stand MI.—The Reichert microscope MI (Fig. 316) has a wide tube so that it may be used for photomicrographic as well as for ordinary petrographic work. It possesses a rotating upper nicol with degree divisions, a Bertrand lens with iris diaphragm, a slot at 45° for the insertion of the accessories, and an objective clutch. The revolving stage, 125 mm. in diameter, may be read to minutes, and may be slowly rotated, for the exact measurement of small angles, by means of a tangent screw which may be snapped into place. The mechanical stage is provided with verniers at right angles to each other and reading to 0.01 mm. The stand is large and has a heavy foot. The coarse adjustment is produced by means of a rack and pinion, the fine (Fig. 248), which is located above the arm so that the instrument may be carried by the latter, by means of a horizontal disk, thicker at one side than at the other, thus forcing upward a wheel carrying the tube. It may be read to 0.001 mm. Beneath the stage is a triple condensing system, the upper two lenses of which may be readily

thrown out of the line of collimation,[1] thus changing the light from convergent to parallel. The polarizer, with its iris diaphragm, may be raised

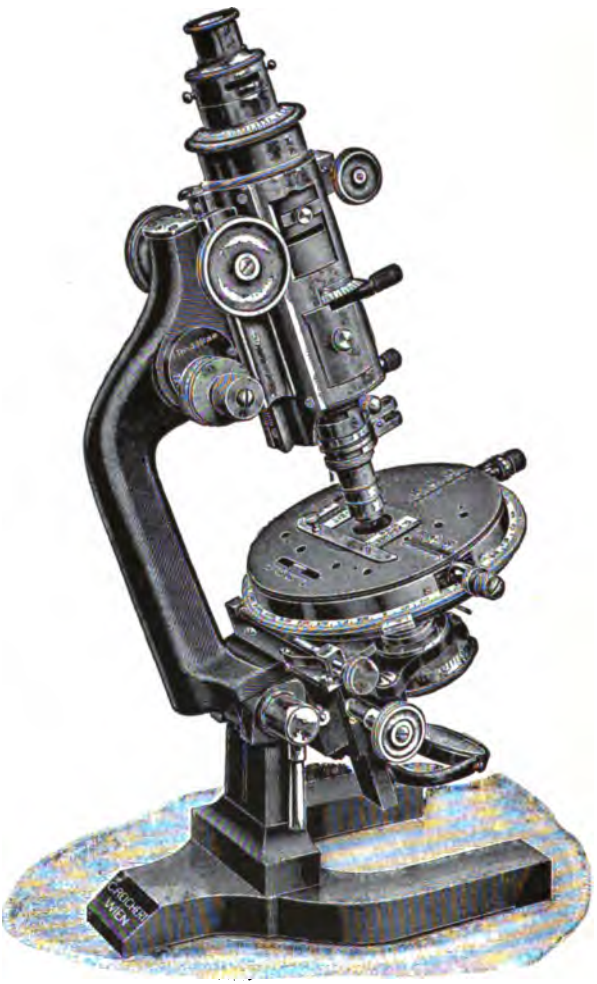


FIG. 316.—Mineralogical stand MI. (Reichert.)

or lowered by means of a milled head, and both may be removed and replaced by an Abbe illuminating apparatus.

[1] The **line of collimation** is the line joining the intersection of the cross-hairs and the optical center of the objective.

177. Reichert Mineralogical Microscope MVIII.—A smaller and cheaper microscope is shown in Fig. 317. The stand is intermediate in form between the German horseshoe and the English tripod, and is non-tilting. The illuminating apparatus, condenser, and iris diaphragm may be swung entirely aside by means of the vertical screw shown beneath the stage. The upper two lenses of the condensing lens may be moved aside independently to change from convergent to parallel light. Focussing is by means of a rack and pinion, there being no fine adjustment. In the tube, which is of fixed length, are inserted the analyzer and Bertrand lens on sliders, so that they may be readily inserted or removed. The centering screws for the objective work at 45°, and there is an objective clutch beneath them.

FIG. 317.—Mineralogical stand M VIII. (Reichert.)

178. Bausch & Lomb LCH Petrographic Microscope.—The Bausch & Lomb LCH stand (Fig. 318) has recently been improved and is now capable of doing most of the work ordinarily required of a petrographic microscope. The space above the stage is large, giving ample room for the use of stage accessories. The friction drawtube is graduated to millimeters, and has a slot for the Bertrand lens with an iris diaphragm above it. The oculars are of standard size (23 mm. in diameter). The upper nicol is capable of being rotated 90°, the lower nicol 360°. There are centering screws above the objective working parallel to the crosshairs, an objective clutch, and a fine adjustment screw of the lever type (Fig. 249) reading to 0.0005 mm. No strain comes on the adjustment screw when

the instrument is lifted, the mechanism being above the handle. The stage is 90 mm. inside, and 102 mm. outside the degree graduations, and the rotation is read by means of a vernier reading to 0.1°. The polarizer, a nicol prism with an angular field of 19°, may be swung entirely out of the

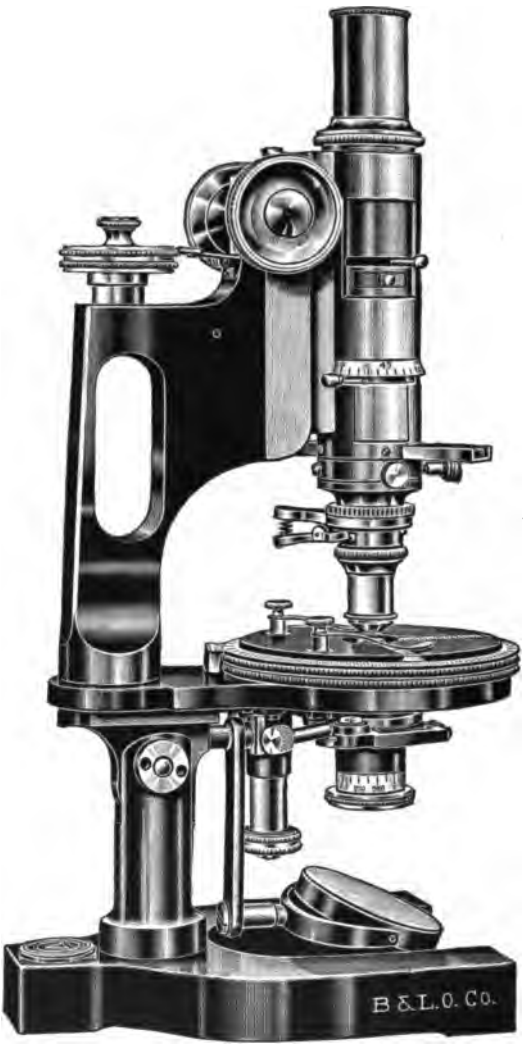


FIG. 318.—Petrographic microscope LCH. (Bausch and Lomb.)

optical axis when desired, which is a good point. The upper lenses of the condenser, which is of three lenses and N. A. 1.10, may be thrown out of the axis of the microscope without disturbing the polarizer or the iris diaphragm. A mechanical stage may be substituted for the one regularly used.

179. Nachet Microscope.—The Nachet[1] microscope (Fig. 319) is quite different, in some respects, from any of the instruments described above. The objective is connected, by means of a separate arm, with the rotating stage, making re-centering unnecessary, when using different objectives,

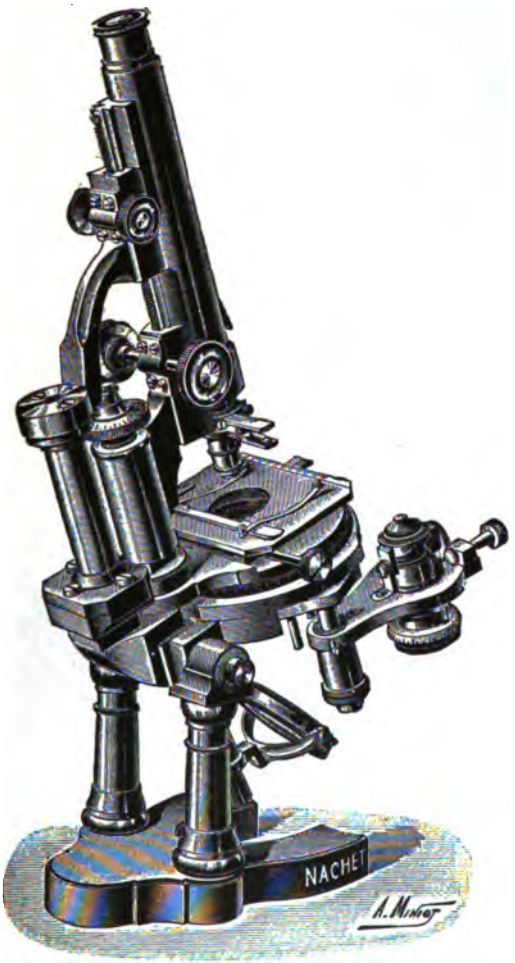


FIG. 319.—Petrographic microscope. (Nachet.)

since the whole arm rotates with the stage. This arrangement is convenient for centering small mineral fragments, but the arm attached to the stage is in the way when one wishes to use certain accessory apparatus, such as

[1] A. Nachet: *On a petrographical microscope.* Jour. Roy. Microsc. Soc., III (1880), 227–228.

Anon: *Nachet's petrographical microscope.* Ibidem, N. S., I (1881) 934–935.

that of von Fedorow, etc. Focussing is also inconvenient, since the milled head is not always in the same place. The stage is divided into degrees but may be read to 6 minutes by means of verniers. It may be rotated by hand

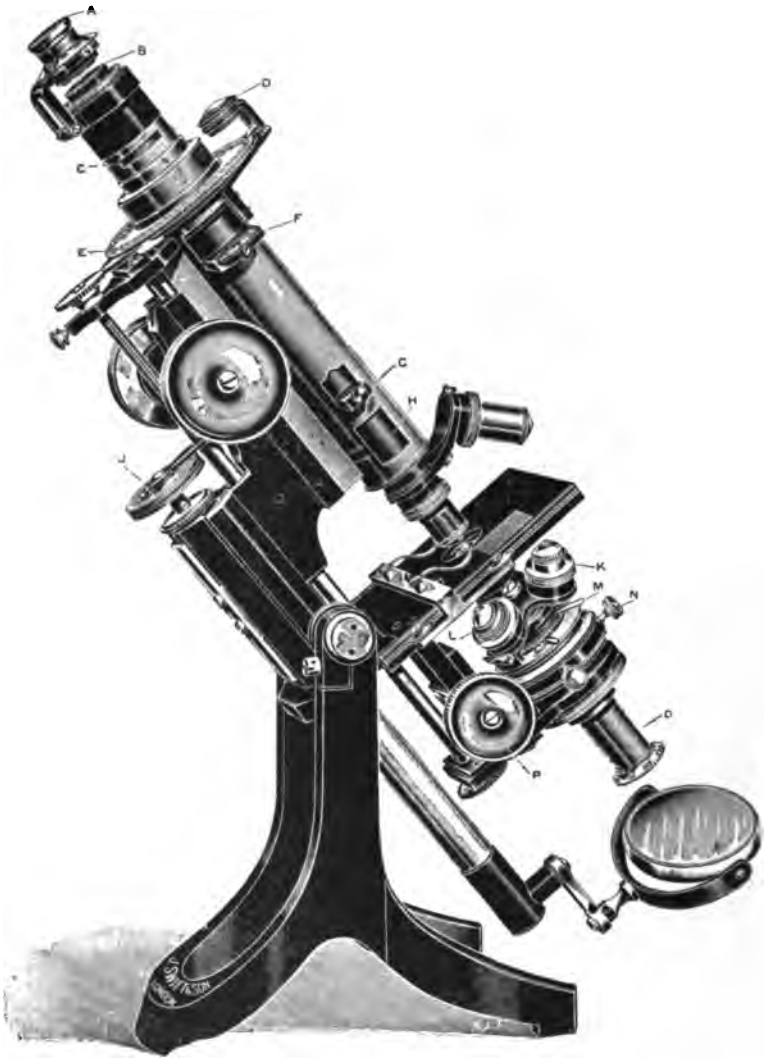


FIG. 320.—Improved Dick petrographic microscope. (Swift and Son.)

or by means of a tangent screw on the opposite side of the microscope from that shown in the figure. The mechanical stage is moved by two screws working at right angles to each other. The analyzer may be swung out on a hinge, the polarizer on a pivot, as shown in Figs. 256 and 319.

180. Swift's Improved Dick Petrographic Microscope.—As mentioned above,[1] the first petrographic microscope with simultaneously rotating nicols was designed by Allan B. Dick[2] and was made by James Swift & Son, London. In considerably improved form,[3] it is still made by the same firm (Fig. 320). The stage is fixed but, if desired, a rotating stage may be attached. The polarizer *O*, the analyzer *A* of the cap variety, and the ocular *B* with its cross-hairs, may be rotated together by means of the wheel *E*, which may be clamped in any position by means of a small screw at the back. The amount of rotation may be read to 5 minutes by means of a hinged magnifier *D*. Either nicol may be rotated independently or thrown out of the line of collimation. An alternative analyzer *H* is fitted in the tube, as are also two Bertrand lenses (*F*, *G*), the lower giving a large, the upper a small interference figure. The upper Bertrand *F* is fitted with a rotating diaphragm with six openings of different sizes. Beneath the stage is a triple revolver carrying three different condensers *K* and an iris diaphragm M. *N* are handles for rotating a disk into which may be set a variety of stops, and the whole condensing system may be raised or lowered by means of the screw *P*. The microscope has both coarse and fine adjustment (*J*), the latter reading to a thousandth of a millimeter of vertical movement. *C* is a slot for accessories and *D* a lens for reading the amount of rotation of the nicols.

181. Swift's Large Petrographic Microscope.—Another microscope manufactured by Swift & Son is shown in Fig. 321. It has a large tube and was designed especially for photomicroscopy but can be used equally well for all purposes. It differs from the other microscopes here described in its hinge, which is so constructed that the center of gravity remains low down, however the body may be inclined.[4] It may be clamped in any position by means of the screw *T*, and possesses the advantage that it is impossible to overturn it backward. The mechanical stage, whose rotation may be read to 5 minutes by means of verniers, may be clamped by the screw *N* in any position. The upper end of the large tube may be removed by unfastening the screw *F*, and a photographic lens inserted, the large tube preventing the cutting off of the outside rays. The fine adjustment screw

[1] Article 139, *supra*.

[2] Allan Dick: *A new form of microscope.* Mineralog. Mag., VIII (1888), 160–163.
Idem: *Notes on a new form of polarizing microscope.* London, 1890.*
Idem: *Additional notes on the polarizing microscope.* London, 1894.*
Anon: *Dick and Swift's patent petrological microscope.* Jour. Roy. Microsc. Soc., 1889, 432–436.
Anon: *Messrs. Swift and Son's improved Dick petrological microscope.* Ibidem, 1895, 97.

[3] G. W. Grabham: *An improved form of petrological microscope, etc.* Mineralog. Mag. XV (1910), 335–338, 347–348.

[4] (–Wenham): *A new microscope.* Northern Microsc., II (1882), 108–110.

reads to 0.002 mm. The polarizer *S* is large, has a graduated lower flange, and may be rotated. There are two analyzers, both of the Glan-Thompson type. The lower one swings in or out by means of the lever *K*. The upper one *A* may be revolved, the amount of rotation being indicated on a scale.

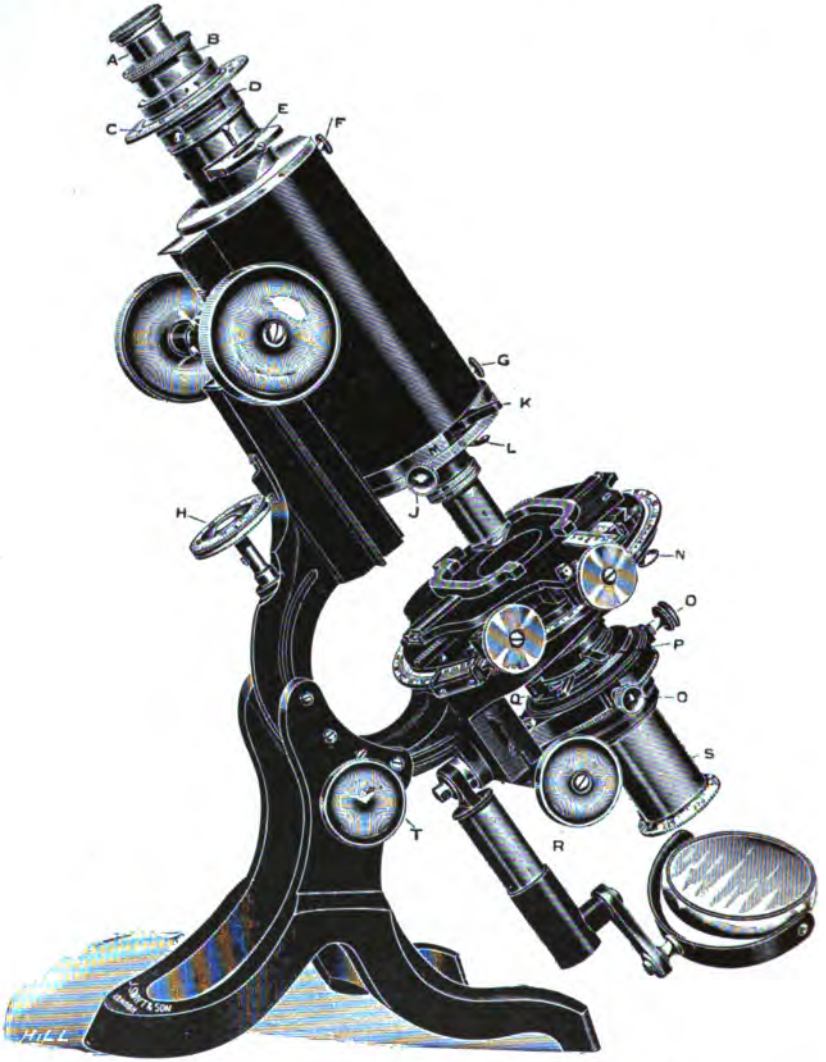


FIG. 321.—Large petrographic microscope. (Swift and Son.)

There are also two Bertrand lenses *L* and *E*. *P* is an iris diaphragm, *Q* the lever and *OO* two centering screws for the condensing system, and *R* a milled head by means of which this system may be raised or lowered.

182. Beck's London Petrographic Microscope.—Beck's "London" petrographical microscope, large model (Fig. 322), belongs to the class of instruments having nicols simultaneously rotating. The base is large, and the pillar *A* is so placed that when the instrument is inclined, it will not overbalance. The stage *B* is square and non-rotating. While a revolving stage is not a necessity in a microscope whose nicols rotate together, it is some-

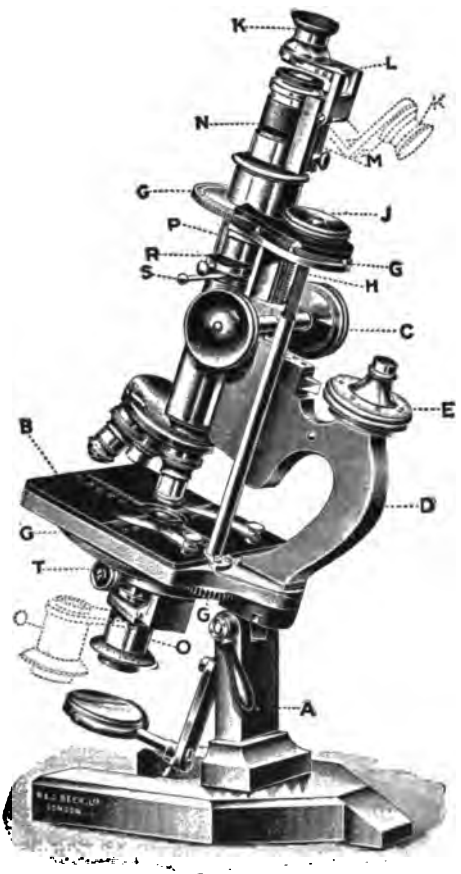


FIG. 322.—London petrographic microscope. (Beck.)

times a great convenience. The analyzer and polarizer rotate by means of the geared wheels *G*, and may be clamped in any position. The upper wheel *G* is graduated to degrees, and indicates the position of the nicols. The polarizer *O* may be revolved independently of the analyzer, if desired, and may be swung out, as shown by the dotted lines. The analyzer *K* is of the cap variety, and may be clapped back as shown. An objection to the nicol prism above the eyepiece is that it greatly cuts down the field of view, a

nicol within the body being preferable. Two forms of condensers may be obtained. The simpler form consists of a hemispherical lens, fitted in the top of the nicol sleeve, and a second hemispherical lens, pivoted on one side of the stage and capable of being swung out of the line of collimation by means of a lever. The larger condenser has a pivoted top lens which may be swung out of line. It also carries an iris diaphragm, and the whole condensing system may be raised or lowered by means of the milled head *T*. In the inner tube of the microscope is a slot *P* carrying a Bertrand lens *R* and an iris diaphragm *S*. The accessories may be inserted in the slot *L*, above the eyepiece, or in the one below it (*N*); both openings are at 45° with the cross-hairs.

183. Société Genevoise Universal Microscope.—The microscope shown in Figs. 323–325 is especially adapted for von Fedorow's methods, but may be transformed into an ordinary mineralogical microscope. In Fig. 323 it is shown with the objective clamp and fine adjustment screw (*C*) attached to the stage in the manner of the Nachet microscope, making centering independent of the objective. This clamp may be removed by the screw *d*, and the objective inserted in the clamp *p*, making the microscope similar to the German instruments (Fig. 324). In this case the objective is centered by the screws *V* (Fig. 325). As may be seen from the illustrations, the stage is of ample size, and to it may be clamped a von Fedorow stage (Fig. 325). In order to overcome the necessity of raising the tube unduly, and thus making the instrument top heavy, it is here possible to lower the entire stage by the screw *l*, Fig. 323. This makes it possible, likewise, to use the instrument as a metallographic microscope.

Besides these special features, the instrument possesses most of the attachments of the microscopes described above except simultaneously rotating nicols. When used with the stage fine adjustment, as shown in Fig. 323, the Bertrand lens *V* is inserted in the clutch *p;* when *p* is used for the objective, it is inserted in *A*. The nicol prisms are both capable of being rotated. At the upper end of the tube is a slot *q* at 45° to the cross-hairs for the insertion of the accessories, and corresponding to it there is one in the focal plane of one of the Huygens oculars. By means of the milled wheel *a* the tube may be extended. Rotation of the stage may be read from verniers, a tangent screw assisting in obtaining fine adjustments. Upper and lower diaphragms are provided.

184. Fuess Microscope for the Theodolite Method.—A microscope of an entirely different type[1] is shown in Fig. 326. It combines in itself a von Fedorow stage and a petrographic microscope with simultaneously rotating nicols. The universal stage in this instrument, however, is considerably

[1] C. Leiss: *Neue petrographisches Mikroskop für die Theodolit-Methode.* Centralbl. f. Min. etc., 1912, 733–736.

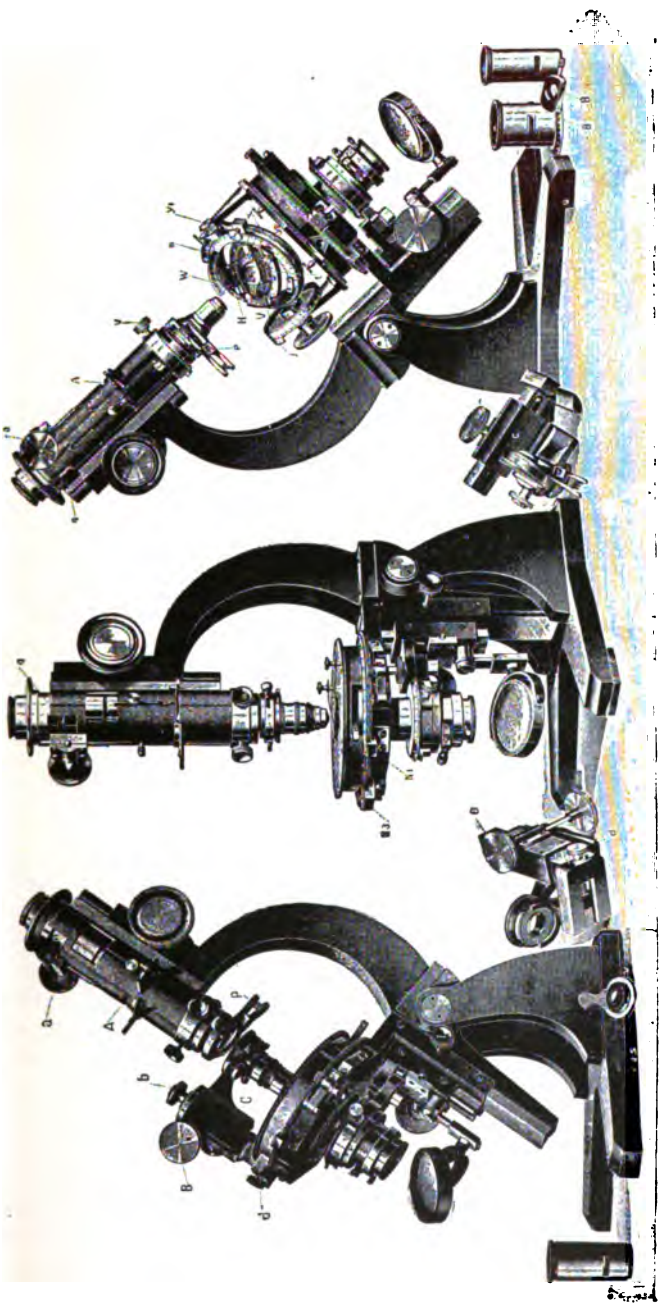


FIG. 323. FIG. 324. FIG. 325.

FIGS. 323 TO 325.—Universal microscope. (Société Genevoise.)

larger than in the detachable stage, being capable of taking sections 28×48 mm., thus doing away with the necessity of using circular sections. The construction of the stage is similar to that of the ordinary von Fedorow stage, and clearly appears from the illustration. The ordinary rotatory movement of the stage not being present, the nicols are made to rotate simultaneously

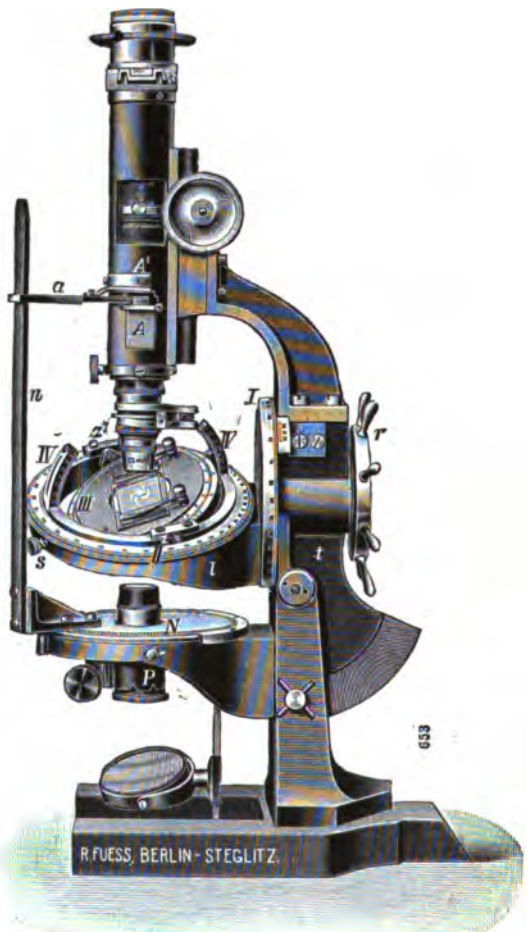


FIG. 326.—Microscope with universal stage. (Fuess).

by the rigid bar *n* in the same manner as in the microscopes shown in Figs. 312 and 313. Its motion may be read, by means of a vernier, to 5 minutes. The upper nicol, which is a Glan-Thompson prism, may be disconnected from the bar *n* by raising the tube until the bar *a* passes over its end. The instrument is made with or without a hinge for tilting, and without fine adjustment for focussing.

185. Beck's Rosenhain Metallurgical Microscope.—Still another type of microscope is necessary for metallurgical work because an artificial source of light is generally used, and it is inconvenient to change its position. For this reason the stage is made to move by means of a rack and pinion, thus focussing the instrument from below without disturbing the tube.

FIG. 327.—Rosenhain metallurgical microscope. (Beck.)

A microscope of this kind is the Rosenhain metallurgical microscope, shown in Fig. 327. At the side of the body tube is an opening, guarded by an iris diaphragm, to regulate the amount of light admitted. This instrument may be used for examining polished faces of rocks as well as of metals.

CHAPTER XII

SELECTING, USING, AND TAKING CARE OF A MICROSCOPE

186. Selecting a Microscope.—The general requirements of a good petrographic microscope are thus summarized by Wright:[1]

"(1) Firm, rigid stand for the support of the optical system.

"(2) Optical system centered; optic axis of the system to pass through the center of rotation of the stage.

"(3) Simple device for centering the objective; the centering screws to be parallel with, and not diagonal to, the cross-hairs of the ocular in order that the observer may have field coordinates as guides. To center the stage instead of the objective is wrong in principle as it displaces the one point to which the optical system is tied.

"(4) Easy passage from parallel to convergent polarized light.

"(5) Easy passage from low to high powers.

"(6) Bertrand lens centered and adjusted to proper focus.

"(7) Properly constructed coarse and fine adjustment screws for focussing the objective, the fine adjustment screws to record intervals of 0.001 mm. and to be free from lost motion.

"(8) Satisfactory arrangement for raising and lowering the sub-stage condenser.

"(9) Accurately constructed mechanical stage on which lateral movements of 0.01 mm. can be measured directly.

"(10) Degree circle of stage to be accurately divided and provided with vernier to read to 5′ at least.

"(11) The ocular, the upper nicol carriage, the Bertrand lens support—in short, all moving parts—to fit accurately, so that on insertion they invariably return to exactly the same point."

To these points may be added, accessible adjustment screws, plainly readable stage vernier, lower diaphragm, and diaphragm above the Bertrand lens. It is desirable also to have readily removable lower nicol, and nicols simultaneously rotating.

Which microscope to choose, from among the numerous instruments on the market, depends largely upon the use to be made of it, and upon the amount of money which is to be spent. The microscope which is chosen for individual use, and which can have the personal care of the owner, may not be the instrument one would put in the hands of a miscellaneous lot of undergraduates. Men doing advanced work require more elaborate microscopes, instruments capable of taking all of the attachments which may aid

[1] Fred Eugene Wright: *The methods of petrographic-microscopic research.* Carnegie Publication No. 158. Washington, D. C., 1911, 12–13.

in research. It may be that a single instrument will not answer the purpose, and several microscopes, adapted to specific uses, must be purchased. So far as the cost is concerned, if a certain selection of accessories will temporarily answer the purpose, a better grade of instrument may be purchased and additional equipment added as occasion demands. A study of catalogues, and a comparison of the instruments described in the previous chapter, may help in making a selection. The instrument to which one is accustomed is likely to appear the most satisfactory.

For students' use the following equipment is sufficient for most purposes.

A stand having coarse and fine adjustment, revolving stage, upper and lower nicols, upper and lower diaphragms, condensing lens, attached Bertrand lens, objective clutch, centering device, and slot for accessories.

Two Huygens oculars with magnifying powers of 5 and 10 times. A micrometer ocular with a magnification of 7.5 is often useful.

Three objectives of approximately the following focal lengths, 40 mm., 15 mm., and 5 mm. (See table, Art. 153.)

A quartz wedge and a selenite plate or a combination wedge such as is described in Article 298.

An objective of approximately 2.5 mm. focal length is often desirable for obtaining interference figures on small particles, but it is not at all necessary that each student's outfit should be equipped with one. If all of the microscopes used in a laboratory are of the same kind, it will be found that a single example of many accessories, such as special oculars, objectives, markers, etc., may be used in common by all the students.

USE AND CARE OF A MICROSCOPE

187. Light.—The best light for microscopic work is that coming from the north; next best, from the east. There should be no obstructing buildings or trees, and the mirror of the microscope should be able to reflect direct light from the sky. Direct sunlight should never be used.

On dark days, or where it is impossible to obtain proper daylight, an artificial light is a great convenience. The source of the light is immaterial provided that it is strong enough, and that it is properly corrected for color. If not corrected, the interference colors will appear abnormal and the light be unsatisfactory. The usual artificial lights are all too yellow and must, consequently, be corrected by a blue glass of proper intensity.

The lamp shown in Fig. 328, after Dr. O. Lassar,[1] is made for the use of oil or gas. With the latter, an Auer burner is used. It has a silvered reflector and a cobalt blue-glass front. The light is approximately of the tone of daylight but is hardly strong enough.

[1] Similar lamps are described: *Parkes's microscope lamp with cooling evaporator.* Jour. Roy. Microsc. Soc., III (1880), 528–529; and *Schieck's microscope lamps.* Ibidem. 1888. 490–491.

A light made by Baker[1] consists of a Nernst electric lamp mounted on a heavy tripod, and capable of being adjusted to any height or tilted to any angle. The globe covering the light is blackened except a small aperture in front through which the light passes. Colored screens are used to modify the light. A similar lamp, with a 60-watt incandescent bulb, and provided with blue, amber, and diffusing screens, is manufactured by Leitz (Fig. 329).

Wright[2] described an acetylene gas burner, fed by a J. B. Colt generator No. 102, and the writer has used both a Nernst and an 80-Watt 115-volt tantalum lamp, properly shaded, toned down by cobalt glass, and made

FIG. 328.—Microscope lamp after Dr. O. Lassar. 1/7 natural size. (Fuess.)

FIG. 329.—Artificial light. (Leitz.)

uniform by a finely ground glass screen. Either light is of sufficient strength, but the latter is too fragile if handled much. If attached to the wall where it is not likely to be jarred, it makes an ideal light.

Between any artificial source of light and the mirror, there should be placed a condensing lens of some sort, in order that the beams may be collected, although, as mentioned above, a ground-glass screen will do fairly well. This condenser may be nothing more than a Florence flask, 15 to 20 cm. in diameter, and filled with water or an ammonia copper sulphate solution, made by adding 50 c.c. of ammonia to 25 c.c. of a 10 per cent. copper sulphate solution, and then diluting it to fill a 6-in. flask.[3] If the solution is

[1] Anon: *C. Baker's electric lamp for the microscope.* Jour. Roy. Microsc. Soc., 1905, 252.

[2] Fred. Eugene Wright: *Artificial daylight for use with the microscope.* Amer. Jour. Sci., X (1909).

[3] Charles J. Chamberlain: *An artificial light for the microscope.* Jour. Appl. Microsc., VI (1903), 2663–5.

milky, add more ammonia. For class work, three or four globes may be used around one open light, such as a Welsbach burner. The globes should partly project through circular openings in blackened screens, which thus serve to keep out all direct light.[1]

More convenient than a glass globe, and not expensive, is a bull's-eye condenser (Fig. 330), 75 to 100 mm. in diameter. If mounted on a stand as shown in the illustration, it may be adjusted to any height or any angle.

The position of the artificial light is a matter of convenience, and it may be placed either to the front or at one side. With a light which requires attention, it is most convenient to have it at the side.

FIG. 330.—Bull's-eye condenser. (Central Scientific Co., Chicago.)

188. Table.—The table should be firm and of a height to suit the convenience of the individual. If one works with the microscope inclined, a height of 28 to 30 in. (71 to 76 mm.), and used with a chair of 17 to 17 1/2 in. (43 to 44 1/2 mm.), is generally satisfactory. If the instrument is used upright, the table must be lower. In the laboratory, a long table attached to and extending the length of the north wall will accommodate the greatest number of students. It should, however, be extremely rigid and firmly attached, so that no jar will be transmitted from one part to another. In height it may be 36 in. thereby permitting a student to stand or to regulate the height of his revolving stool as he finds most restful, and at the same time allowing the instructor to glance through the instrument with the least possible disturbance to a class. The working table should be fitted with drawers in which to keep accessories, and a cabinet or bell jar should be provided to protect the microscope from the dust. For laboratory classes, it is also desirable that at least one artificial light be provided for each two students.

METHOD OF WORKING

189. Position.—The least possible fatigue will be felt by the student if he sits perfectly upright, with the arms resting on the table, and so places the microscope that it will not be necessary to compress the chest or strain the neck in looking through it. The instrument should be placed squarely in front, so that both hands may be used to manipulate it.

190. Proper Eye to Use.—Use whichever eye is least fatigued by the work, and *keep the other eye open.* It may be difficult, at first, not to see

[1] A condenser of the kind here described is made by Bausch and Lomb.

with this eye, but after a short time no exertion will be necessary to let it remain passive. If both eyes can be used equally well, make a point of changing from one to the other. Keep the eye close to the eyepiece. The proper position is in the Ramsden disk (*EP*, Fig. 229), which is very close to the eye-lens in high powers, and slightly farther removed in low.

191. Eye Shade.—It not only adds materially to the comfort of working but makes a brighter image, by allowing the pupil of the eye to dilate, if much of the outside light is excluded by means of shades. If one works facing a window, a square of black cloth hung over a wire, and extending from 8 to 10 in. above the tube to about the level of the stage, is very convenient, and may be shoved aside when it is desired to work by incident light. Another good shade is made by cutting a 3/8-in. board into the form shown at *A*, Fig. 331. The hole should fit the tube snugly. A dark pasteboard hood (double-faced corrugated board does very well), with a curtain reaching to the stage, may be set on this board to exclude practically all of the light. It may be cut from one piece of paper as shown at *B*, the heavy lines indicating the cuts, the dotted lines, scorings along which to bend. The narrow strip *P* should be bent upward and fastened at *P'*. For observations by incident light the entire hood, but not the board, should be removed. If one wishes to work with the left eye, instead of the right, the board *A* may be reversed. If the light comes from the left side, instead of the right, the scorings should be made on the other side of the pasteboard, and the sides bent in the opposite direction. This will bring the curtain *P'* on the left side.

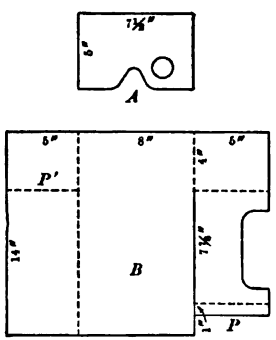


FIG. 331.—Eye shade.

Dr. J. Peiser[1] described a shade made as follows: A copper wire, 2 mm. in thickness and 25 cm. long, is fastened to a leather-covered ring which clamps to the tube below the eyepiece. This wire is curved backward and upward, and at its upper end, a hollow brass tube, 2 mm. in diameter and 66 cm. long and bent into the form of a parabola, is attached at its center. A black satin curtain, slit at the lower end to form two pendants, which may be held up with two snap fasteners, is attached to the cross-wire and forms the shade. A similar shade was described previously by Schiefferdecker.[2]

A very simple shade is shown in Fig. 332.[3] It is attached to the upper

[1] J. Peiser: *Ein Mikroskopierschirm.* Zeitschr. f. wiss. Mikrosk., XXI (1904), 467–469.

[2] P. Schiefferdecker: *Ueber einen Mikroskopirschirm.* Ibidem. IX (1892), 180–181.

[3] R. H. Ward: *An eye-shade for monocular microscopes.* Amer. Mon. Microsc. Jour., V (1884), 82–83.

A similar shade was described by E. Pennock: *Eye shade for monoculars.* Jour. Roy. Microsc. Soc., N. S., I (1881), 518.

part of the tube of the microscope and may be used for either eye. Oculars may be changed without removing it.

192. Amount of Light.—Use the lower diaphragm to cut off superfluous light, the amount depending upon the objective. Enough should be admitted so that structures may be seen without straining the eye, but not enough to produce a glare. If too much light is admitted, it conceals the finer detail. More light should be admitted when the nicols are inserted than when they are out.

193. Proper Magnifying Power to Use.—Begin work with low-power objectives, and increase the magnification as necessary. Do as much work as possible with the low powers and save your eyes. For the greater part of the work, no magnification greater than 50 to 60 diameters is necessary. For interference figures, 180 diameters is generally ample.

194. Objective Clutch.—When using an objective clutch of a pattern similar to that shown in Fig. 239, it is advisable to give the objective a slight rotation after insertion in order to insure its dropping into proper position. If the objective still appears markedly out of center, do not at once adjust the cross-hairs, but remove the objective and examine it and the clutch for foreign matter. It is a good plan to wipe each objective collar when beginning the day's work.

FIG. 332.—Dr. R. H. Ward's eye shade. (Bausch and Lomb.)

195. Focussing.—Become familiar with the free working distance of the objectives, so that they may automatically be set roughly in focus. For high powers, place the eye on a level with the stage, and look toward a window between the cover-glass and the lens, lowering the objective until but a narrow streak of light is seen. Now look through the ocular, and raise the tube very slowly until the section is in focus. *Always focus upward* and no thin sections will ever be broken. For colorless minerals, such as quartz, cut down the illumination, and look for bubbles or other inclusions. Use a low-power objective as a finder and place the mineral desired under the cross-hairs. In removing high-power objectives always raise the tube lest the cover-glass or the objective be injured.

Various devices for safe-guarding the slide against breakage by the ocular have been devised.[1] Most of them consist of a ring about the objective, to

[1] E. H. Griffith: *On several new microscopical accessories.* Proc. Amer. Microsc. Soc., 9th meeting, VIII (1886), 150–152.

S. Gelblum: *Discussion des conditions générales que doit remplir le dispositif d'arrêt du tube à tirage dans tout microscope, et déscription du moyen pratique pour arriver à ce résult.* Zeitschr. f. wiss. Mikrosk., XX (1903), 129–132.

S. E. Dowdy: *A focussing safeguard.* English Mechanic, LXXVIII (1903), 291.

which is attached a button or pin which, upon lowering the tube, comes in contact with the edge of the slide or a button on the stage.

196. Changing the Ocular.—When changing from one ocular to another, especially if they fit snugly, raise the tube, and take care not to press the objective through the thin section. Not only will the slide be broken but the objective may be ruined as well.

The method of centering the objective was described above (Art. 114).

HINTS ON THE CARE OF A MICROSCOPE

197. Care of the Stand.—Keep the stand of the microscope, especially the working parts, free from dust.

Do not carry the microscope by any part above the fine adjustment, unless you wish to ruin it. Do not, for example, carry microscopes with the prism type of fine adjustment by the arm (Figs. 307, 308, 310, 311, 322). They should be carried by the post. If the hinge only is below the arm, the latter is the most convenient part by which to carry it (Figs. 230, 309, 314, 316, 318, etc.).

Do not clean the stand with alcohol, for it will remove the yellow lacquer. Use benzene or xylene, and wipe with a soft cloth in the direction of the grain of the metal, never across. If the microscope has a vulcanite stage and the benzene stains it, clean it by rubbing with oil.

Lubricate the working parts with clock oil. If it becomes gummy, clean with benzene applied with a cloth. If the microscope has an inner tube, occasionally remove it, wet a cloth with a small amount of oil, and wipe the inside of the outer and the outside of the inner tube. Oil the slides, but not the teeth, of the rack and pinion. The latter should be kept free from dust and be cleaned with benzene.

If any part of the microscope is unscrewed, use great care, when replacing screws, to start the threads properly. If once cross-threaded, the screw is ruined.

Use a screw-driver which is neither too large nor too small, and see that it is of the same shape as the slot in the screw head.

198. Care of Nicols and Lenses.—Do not expose a microscope to sudden changes of temperature. If moved from a cold to a warm room, moisture is likely to gather on the lenses, or the balsam may crack.

Do not expose the lenses or the nicol prisms to direct sunlight, nor keep them near a steam radiator. The cement may soften.

Remember that the nicol prisms are made of calcite which is very soft and likely to become scratched. Dust them only with a soft camel-hair brush. Remember, too, that they are expensive.

Be sure that the lens surfaces are clean and free from dust. Remove

dust particles from oculars and objectives with a soft brush or by blowing upon them, then wipe, with a circular motion, with a soft cloth. Use soft linen, never silk or cotton, and keep, in a dust-proof box, separate cloths for lenses and stand.

If finger marks or dust cannot be removed with a dry cloth, breathe upon the lens and wipe, or wipe with a cloth moistened very slightly with benzene or xylene. Use great care to prevent any of the cleaning fluid from getting between the lenses. Never use alcohol.

Both sides of the field- and eye-lenses of the ocular may be cleaned if necessary, but remember that when the lenses are removed there is nothing remaining to protect the cobwebs.

Front and back surfaces of objectives may readily be cleaned. Dust is especially likely to settle on the back lens. Internal surfaces should be examined with a hand lens and, if any cloudiness exists, the objective may be unscrewed with great care. It is better, however, to return such lenses to the maker.[1] If separated by the owner they are likely to become decentered, or more dust may enter than is removed.

Objectives used with immersion oil should be cleaned immediately afterward.

Do not let the front lens of an objective come in violent contact with a cover-glass, and never let an objective fall. In order to permit the entrance of as much light as possible (Figs. 296–297), the amount of metal clasping the edge is very little, in some objectives none projects over the rim of the lower face, and it is held in place only by the pressure at the sides, and even here by only a very small piece.

TESTING AND ADJUSTING THE MICROSCOPE AND THE ACCESSORIES

199. Cross-hairs.—Some of the explanations given in this and the following sections may be in advance of students who have had no preliminary work in petrography. The methods of testing and adjusting, which do not more properly belong elsewhere, are inserted here, however, in order that all such methods may be brought together under one heading for easy reference.

Cross-hairs: focussing. See Art. 163.

Cross-hairs: replacing. See Art. 164.

Cross-hairs: centering. See Art. 114.

Cross-hairs. To set at right angles to each other and parallel to the directions of vibration of the nicols.—To determine whether the cross-hairs of the ocular are set at right angles to each other, a mineral with straight cleavage, an object micrometer, an object-slip with a straight scratch across it, or some such object is placed upon the stage, and it is rotated until one of the cross-

[1] William Wales: *The proper care and use of microscope lenses.* Jour. N. Y. Microsc. Soc., I (1885), 113–116.

hairs is parallel to it. The stage vernier is now read, and the stage rotated through 90°. In its new position the line should be parallel to the other cross-hair. The test should be repeated a number of times.

The cross-hairs should not only be at right angles to each other but parallel to the principal sections of the nicols as well. The nicols are first tested by the method given in Article 202, after which a slide consisting of a mineral having parallel extinction, such as anhydrite, anthophyllite, or needle-like quartz prisms,[1] is placed on the stage and rotated to the position of darkness. This position may be observed by the use of a gypsum test plate giving the sensitive violet. In this position the cross-hairs should be parallel to the cleavage of the mineral. Repeat the operation ten or a dozen times and, if the cross-hairs and nicols are not parallel, rotate the cross-hair support by means of a spanner.

To avoid the polarizing effect of the objective, it is better to remove it and use only the Bertrand lens in combination with the ocular, thus leaving no lens between the nicols. The test object should be rather large in this case, since the magnification of the ocular and Bertrand lens is not great. By pointing the microscope at the sun, the point of extinction may be seen much more clearly.

A Bertrand ocular may be used instead of a unit retardation plate to determine when the mineral is in the position of extinction.

200. Bertrand Ocular. *Testing the position of the division lines, which should be parallel to the vibration planes of the nicols.*—To set the separating lines of the Bertrand ocular parallel to the principal sections of the nicols, use is made of an anhydrite or anthophyllite section. The nicols are first tested for accurate position of crossing by some other means than by the Bertrand ocular, after which the mineral is placed on the stage in the position of extinction. Upon inserting the Bertrand ocular there should be uniform color in the four quadrants. If this is not found, the vibration planes of the nicols do not coincide with the divisions of the Bertrand ocular.

201. Bertrand Lens. *Centering.*—The center of the Bertrand lens should lie exactly on the axis of the microscope. If it does so, the center of the interference cross of a section of calcite, cut exactly at right angles to the *c* axis, will lie at the intersection of the cross-hairs of the ocular. If it does not do so it may be corrected by means of the centering screws inserted in the lens mounting. Be sure that the test plate of calcite is accurately cut at right angles to the axis.

202. Nicol Prisms.—*Determining the vibration directions of the nicol prisms.* See Art. 140.

[1] E. Weinschenk: *Eine Methode zur genaue Justirung der Nicol'sche Prismen.* Zeitschr. f. Kryst., XXIV (1904–5), 581–583.

To set the nicol prisms at right angles to each other.—The principal sections of the nicol prisms should be perpendicular to each other as well as parallel to the cross-hairs. To test this, use may be made of the Bertrand ocular. The polarizer is inserted with its knife edge engaged in the V notch of the casing. The analyzer is shoved out of the axis of the microscope and a cap nicol is placed over the Bertrand ocular and set at 0° (or 90°, depending upon the orientation of the polarizer and whether the eye is most sensitive to blue or orange tones). In this position the four quadrants of the ocular should appear exactly the same shade of color. If they do not do so, and the amount of rotation necessary to produce uniform color is greater than 1/2° to 1°, the nicol should be rotated in its casing by means of a spanner or by the set screws, if such are provided. The most convenient spanner for this purpose is a cylinder, at the upper end of which are two projecting points which engage in the notches in the nicol casing. The spanner may be placed in position and, since it is in the form of a tube, the nicol may be rotated with it while looking through the microscope. Great care must be observed not to scratch the lower surface of the nicol when the protecting glass is removed from below.

To correct the analyzer, the polarizer must be removed, the cap nicol turned to the 90° (or 0°) position, and the same process repeated as for the polarizer.

Another method of setting polarizer and analyzer at right angles, is to place upon the stage of the microscope a cleavage piece of anhydrite or anthophyllite, or a prismatic needle of quartz[1] mounted in balsam. The crystal is placed exactly parallel to one of the cross-hairs. It should appear perfectly dark between crossed nicols. Now, leaving polarizer and tube-analyzer in position, place a cap nicol above the eyepiece and rotate it. If in any position color appears in the crystal, it indicates that the nicols are not exactly crossed and should be corrected.

To test the two analyzers one proceeds in the reverse way, rotating the polarizer.

Another method is to remove from the microscope the ocular and objective, and unscrew from the top of the polarizer the condensing lens. If the microscope, with nicols crossed, is now pointed directly at the sun, the position of maximum darkness may be determined within a quarter of a degree. The sun will appear as a dull disk in the dark field.

203. Accessories.—*Determination of the direction of* c *in the one-fourth undulation mica plate.* Examine the interference figure produced by the mica plate, using it as a mineral section. The axis of least ease of vibration c is the line joining the loci of the hyperbolæ. b is at right angles to this line.

[1] E. Weinschenk: *Op. cit.*

Determination of the c *direction in the gypsum-plate* (unit retardation plate). Examine the interference figure, using the gypsum plate as a mineral section. The line joining the quadrants showing the lowest color (yellow) is the c direction.

Determination of the c *direction in a quartz or mica wedge.* Use the wedge as a mineral section and, with a mica plate whose c direction is known as an accessory, determine the elongation.

CHAPTER XIII

OBSERVATIONS BY ORDINARY LIGHT

204. Ordinary Light.—When we speak of ordinary light, we mean light which has not been polarized, consequently to obtain such, both nicol prisms should be removed from the microscope. As a matter of fact, in many instruments the lower nicol is removed with difficulty, and one makes his observations by plane polarized light. For most minerals this is of no great consequence since there is usually very little difference in their appearance by ordinary or by plane polarized light. There are certain minerals, however, as we shall see later, whose colors differ with the direction of light vibration, and their true colors must be determined by ordinary light. The intensity of the unpolarized light is nearly twice as great as the plane polarized. This occasionally may make it more advantageous to use the former.

Substances which are to be examined by ordinary light are of two classes, transparent and opaque.

Transparent minerals are examined by *transmitted* light for crystal form, cleavage, and color. By it, also, angles, refractive indices, lengths, and thicknesses are measured.

Opaque minerals are examined by *incident* light for crystal form, color, lustre, etc.

205. Determination of Crystal Form.—Crystal form, of both transparent and opaque minerals, is determined in the same way that it would be in cross-sections of large specimens, but while this determination is of great importance megascopically, it is of comparative unimportance in sections of rocks. In the latter, in the majority of cases, individual crystals have not had a chance for undisturbed development, but have had their growth hampered in all directions by the growth of other crystals. In certain classes of rocks, namely the porphyries, the development of certain individuals has been more or less perfect, and a study of their forms may sometimes be of assistance in their determination.

In hand specimens one has to deal with more or less perfect polyhedrons or, if cleavage flakes, polygons cut in a few definite directions from the solid forms. In rock sections one has only random cross-sections from more or less distorted solids from which to make determinations, cross-sections which depend not only upon the crystal form, but upon the direction in which they were cut, as well. Another difficulty is the fact that sections of the same shape may be cut from totally different crystals. In spite of these difficul-

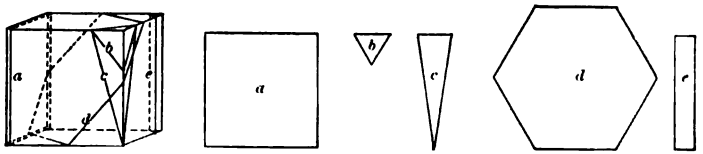


FIG. 333.—Isometric system. Cube and sections cut from it.

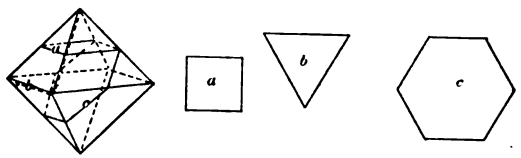


FIG. 334.—Isometric system. Octahedron and sections cut from it.

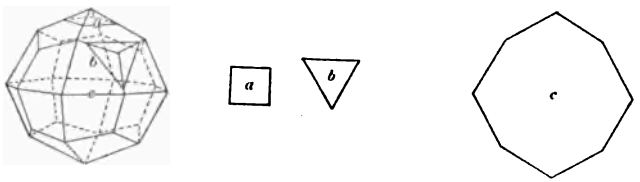


FIG. 335.—Isometric system. Icositetrahedron and sections cut from it.

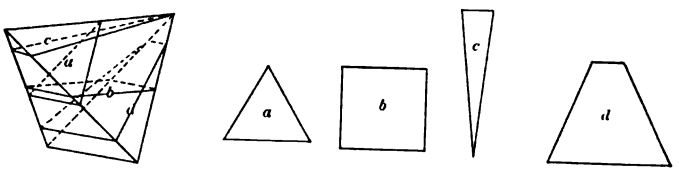


FIG. 336.—Isometric system. Tetrahedron and sections cut from it.

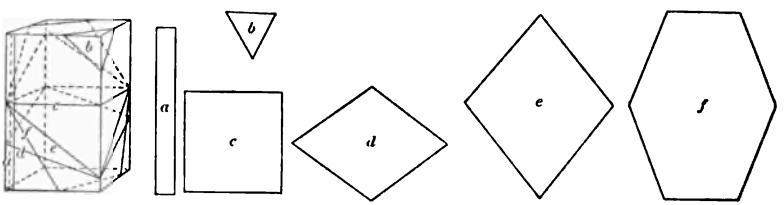


FIG. 337.—Tetragonal system. Prism and sections cut from it.

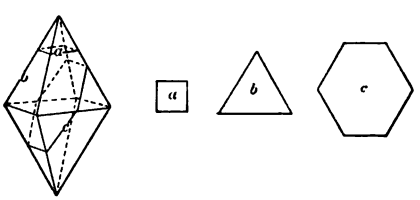


FIG. 338.—Tetragonal system. Bipyramid and sections cut from it.

ties, however, it is usually possible, by comparing a number of sections in the same rock slice, to determine the form of the crystal from which they were cut.

A comparison of Figs. 333–341 may be of assistance, especially to those who have not made a study of descriptive geometry. It is impossible to

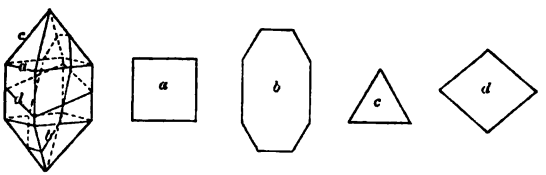


FIG. 339.—Tetragonal system. Bipyramid and prism, and sections cut from it.

give all sections which may be cut from crystals of the different systems, and only a few of the more common forms are here shown. The student may work out others for himself.

From these diagrams it may clearly be seen how it is possible to cut a hexagonal section from an isometric crystal, a square section from one that

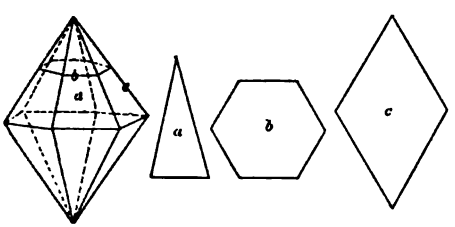


FIG. 340.—Hexagonal system. Bipyramid and sections cut from it.

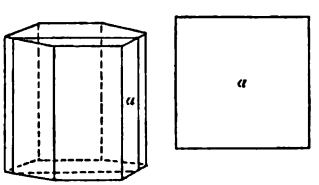


FIG. 341.—Hexagonal system. Prism and section cut from it.

is hexagonal, or a triangular section from one of any system. Too much dependence must not be placed on cross-sections, therefore, or it may lead to a wrong conclusion in regard to the crystal system to which the mineral belongs.

206. Cleavage and Parting.—Another property of minerals which is to be observed by ordinary light is cleavage, which is developed in characteristic directions in a thin section by the process of grinding. The direction and perfection of the cleavage cracks depend upon the crystal system and the substance itself. If the mineral possesses no cleavage, the cracks shown are irregular; if present, the cleavage lines are directions of least cohesion, and the cracks follow these directions and appear as parallel lines representing the traces of the cleavage planes.

Cleavage is described as **perfect** when the cracks are sharp and extend uninterruptedly for considerable distances. This cleavage is found in mica, fluorite, etc.

With **good,** or **distinct,** cleavage the cracks do not continue uninterruptedly for such great distances, but show off-sets, and then continue in the same direction as before. The off-sets may be irregular breaks but more likely pass along other cleavage planes, as in hornblende, augite, or orthoclase.

Indistinct, poor, or **imperfect** cleavage is very irregular. While the lines roughly follow certain directions, the cracks are more or less uneven. This cleavage is well shown in olivine.

Pinacoidal cleavage is generally well developed in one direction only. It is well shown in mica. Prismatic cleavage is usually parallel to two planes, as in hornblende or augite. In certain minerals of the isometric and hexagonal systems, three good cleavages are developed. In the former they are at right angles to each other, as in galena; in the latter they form rhombohedrons, as in calcite. With either of these cleavages, however, generally only two sets of lines are shown in the thin sections, although three may be. Certain isometric crystals have perfect octahedral cleavage, fluorite, for example.

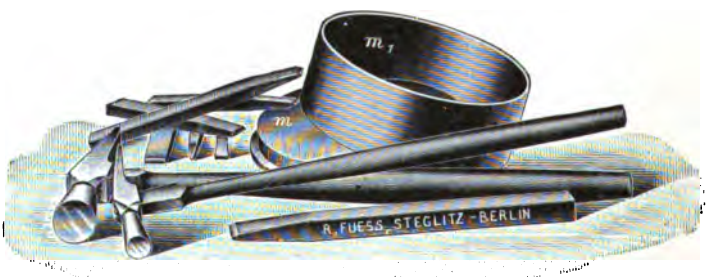


FIG. 342.—Apparatus for obtaining cleavage flakes of minerals, after Wülfing. 1/3 natural size. (Fuess.)

While cleavage angles are important in the determination of minerals, they must be used with caution under the microscope, since the angles depend upon the orientation of the random section shown in the rock slice. Where the sections are cut at right angles to the cleavage planes, the angles are characteristic. These sections may be recognized by noting, on raising or lowering the tube of the microscope, that there is no displacement of the cleavage cracks. As an example of two totally different cleavages appearing alike, amphibole and pyroxene may be cited. In the former the cleavage angle in a section at right angles to the prismatic faces is about 124°, in the latter about 93°, yet a section inclined about 56° to the normal will give, in pyroxene, an angle of 124°. The cleavage cracks, however, will not be perpendicular to the section, and will be laterally displaced upon changing the focus from the top to the bottom of the slide.

If, instead of using random sections in a rock slice, one employs cleavage

fragments, the determination is much simplified, since the flat faces will here bear definite relations to the crystallographic axes. In preparing such mineral fragments, one should crush, not pulverize, the mineral. A diamond mortar is convenient. Chisels and an iron plate with a guard ring, such as are described by Wülfing,[1] may be used for larger flakes (Fig. 342).

In some minerals there is occasionally developed a fracture parallel to a certain direction, but the mineral cannot everywhere be cleaved parallel to this plane. This **parting**, as it is called, occurs along lines of weakness, such as result from shearing, or develop along gliding planes. It is usually well shown in the small apatite crystals of granitic rocks.

DETERMINATION OF REFRACTIVE INDICES

207. Relief.—It has already been pointed out that there is a constant ratio between the angle of incidence and the angle of refraction of light passing from one transparent medium to another, and that this constant, expressed by the equation $n = \frac{\sin i}{\sin r}$, is called the index of refraction. Under the microscope, minerals of different indices, embedded in Canada balsam, appear more or less rough. These rough minerals, from their resemblance to shagreen, are said to have **shagreen surfaces**,[2] an effect which may be due, in part, to inequalities of the surface, each little elevation and depression reflecting and refracting the light at a different angle, with the result that certain spots are more, and others less, illuminated. It follows from the indices of refraction and critical angles of two media, that the greater the difference between them, the greater the contrast of the surface inequalities and the rougher it appears, whether the mineral be of a considerably higher or of a considerably lower index than the balsam. Another result of the difference in indices is the apparent elevation or depression of certain minerals from the plane of the section; that is, certain minerals stand out in relief. This is due to the fact that rays of light, from the lower surfaces of different minerals, appear to come from the points of intersection of the refracted rays (Fig. 343), consequently the minerals which have a higher refractive index appear to stand out above the others.

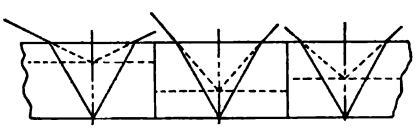


FIG. 343.—Relief in minerals.

If there be placed upon a thin section of a colorless mineral with a rough surface, and without a cover-glass, a drop of a liquid with an index of refraction exactly equal to that of the mineral, it will be found that the appearance

[1] Rosenbusch-Wülfing: *Mikroskopische Physiographie*, I_1, 4 Aufl., 1904, 29.

[2] J. Thoulet: *De l'apparence dite chagrinée présentée par un certain nombre de minéraux examinés en lames minces.* Bull. Soc. Min. France, III (1880), 62–68.

of roughness disappears, as is to be expected, since there will be neither reflection nor refraction at the contact, and the light will pass through without deflection. If a liquid with an index either greater or less be used, the relief reappears.

The index of refraction of a mineral is one of the most important properties for its identification, and many methods have been devised for its determination. Here only those methods which are applicable for use with the microscope will be discussed.

There are three microscopic methods open to the investigator. One may determine the index of refraction of the mineral directly, as by the method of the Duc de Chaulnes or one of its modifications, one may immerse fragments of the mineral in a fluid of known index, or one may determine the relation which the refractive index of the unknown mineral bears to that of one which is known and which is in contact with it.

208. The Method of the Duc de Chaulnes.—The method of the Duc de Chaulnes[1] is one which is applicable to the measurement of the mean indices of refraction of plane-parallel mineral plates. It depends upon the fact that if a medium or high-power objective is accurately focussed upon an object, and there is inserted between it and the objective a transparent plate with parallel sides, the image becomes blurred, and it is necessary to raise the tube of the microscope a certain amount in order that the image may again appear sharp. The amount of change necessary depends upon the index of refraction of the plate and upon its thickness.

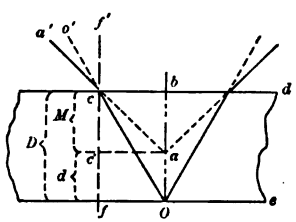


FIG. 344.—The Duc de Chaulnes' method for measuring refractive indices.

Let *c d e f*, Fig. 344, be a plate of an isotropic substance whose thickness has been accurately measured. A ray of light *Oc* will be refracted, upon reaching the air *c*, to the point *a'*, consequently a mark on the lower surface of the slide at *O* will appear to lie, not at *O*, but on the backward extension of the line *ca'*, at *a*. If, now, the tube of the microscope is raised and focussed upon a mark *b* on the upper surface, the amount of elevation is not *Ob*, the true thickness of the slide, but *ab*.

Let $M = ab$, the measured thickness of the mineral,

$D = Ob = fc$, the actual thickness,

$fco = i$, the angle of incidence,

$f'ca' = c'ca = r$, the angle of refraction.

[1] Le Duc de Chaulnes: *Sur quelques expériences relatives à la dioptique.* Histoire de l'Academie Royale des Sciences, 1767. Paris, 1770, 162–175.

Idem: *Mémoire sur quelques expériences relatives à la dioptique.* Mém. de l'Acad. France, Année 1767, Paris 1770, 423–470. In particular pages 430–435.

$$\frac{\tan i}{\tan r} = \frac{\frac{Of}{fc}}{\frac{ac'}{cc'}}.$$

But $Of = ac'$, and $cc' = ab = M$, wherefore

$$\frac{\tan i}{\tan r} = \frac{cc'}{fc} = \frac{M}{D}.$$

In the small angles here used, where i and r approach 0°, the tangent approaches the sine, and the latter may be substituted in the equation, whereby

$$\frac{\sin i}{\sin r} = \frac{M}{D}. \tag{1}$$

But $\frac{\sin i}{\sin r} = \frac{1}{n}$, when light passes from a denser to a rarer medium, therefore

$$\frac{M}{D} = \frac{1}{n} \text{ and } n = \frac{D}{M}. \tag{2}$$

That is, the index of refraction of the substance is equal to the value of the true thickness divided by the measured thickness. For example: By the micrometer screw on the microscope the apparent thickness of a basal section of quartz was found to be 0.5 mm., by actual measurement it was found to be 0.77. The index of refraction, therefore, was $\frac{0.77}{0.5} = n = 1.54$.

The weakness of the method lies in the uncertainty of the position of sharpest focus and inaccuracy in the micrometer reading, a difference of ±0.001 mm. in each would give, in the above example, $\frac{0.769}{0.501} = 1.535$, a result decidedly different even though a section half a millimeter in thickness was used. If the section were of the thickness of a normal rock slice, the error would be much greater. Another error is caused by lost motion in the micrometer screw, and a third by the fact that the section may not be of the same thickness throughout and the measurements may not be made at the exact spot where the indices of refraction are determined. Fairly accurate results may be obtained if the precaution is taken to avoid lost motion by screwing the fine adjustment in one direction only, in reading the top and bottom of the slice, and further that of taking a large number of readings for thickness at various places in the mineral, and averaging the results.

The measurement of the actual thickness of a plane parallel but unmounted mineral slice may be made by placing, upon the stage of the microscope, a glass plate having a reference mark upon its upper surface, and sharply

focussing upon it. The mineral to be measured is then placed above the mark by sliding it over to exclude the air, and its upper surface is brought into focus. The difference in the readings of the micrometer screw of the fine adjustment is the true thickness. Another method of measuring thickness is to use an ordinary micrometer screw (Fig. 345) or an interference sphaerometer[1] (Fig. 346). The latter has a scale *d* divided into 0.5 mm. spaces and a disk *c* with 250 divisions, permitting a reading to 0.002 mm. and an estimate to 0.001 mm. The instrument is not dependent upon the feeling of contact, as are ordinary micrometer screws, and it is, consequently, much more accurate. The substance to be measured is placed upon the glass plate *e*, which, in turn, rests upon a black glass plate *f*. A sodium light is placed beyond the instrument, and the instant the rounded end of the screw touches the substance to be measured, interference bands appear to move at the contact between the two glasses (*e* and *f*).

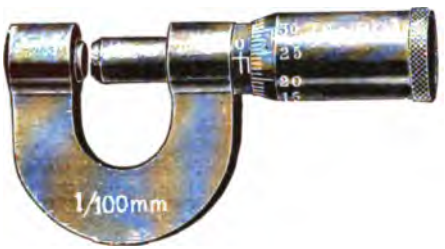


FIG. 345.—Micrometer calipers. (Central Scientific Co., Chicago.)

FIG. 346.—Interference sphaerometer. 3/5 natural size. (Fuess.)

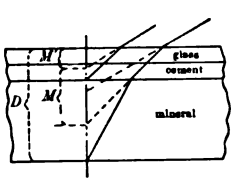


FIG. 347.—Diagram showing correction to be applied for cover-glass in measuring the index of refraction of a substance by the method of the Duc de Chaulnes.

The thickness of a doubly refracting mineral sometimes may be determined by means of its birefringence (Art. 301).

In determining the index of refraction of a thin section of a mineral, the cover-glass should be removed. If this is not done a correction must be applied for the combined cover-glass and balsam film. A value of 1.52 may be taken as a fair average of the indices of glass and Canada balsam,

[1] C. Leiss: *Mittheilungen aus der R. Fuess'schen Werkstätte. Interferenz-Sphärometer zur genauen Messung der Dicke von Kristallplatten.* Neues Jahrb., 1898 (II), 72–73.

and since $D' = 1.52\,M'$, instead of $D' = M'$ as it would in air, $0.52\,M'$ must be deducted from both D and M in formula (2), M' being the *measured* distance between the upper surface of the mineral and the upper surface of the cover-glass, D the *true* thickness of the mineral and the cover-glass, and M the *apparent* thickness of the crystal plate measured from its bottom to its top surface (Fig. 347). The formula becomes $n = \frac{D - 0.52\,M'}{M - 0.52\,M'}$.[1]

Various modifications of the method of the Duc de Chaulnes have been proposed in order to overcome the error produced by slight inaccuracies in measuring the true and the apparent thickness.

PROBLEM

Determine, by the method of the Duc de Chaulnes, the index of refraction of a cleavage plate of fluorite, about 0.5 mm. in thickness, first measuring the true thickness by means of the fine adjustment of the microscope.

209. Brewster's Method for Determining the Refractive Index of a Liquid (1813).—Sir David Brewster[2] determined, microscopically, the indices of refraction of fluids by placing, successively, two liquids in a glass trough with a perfectly flat bottom. Let n be the index of refraction of a known liquid and n' that of the one to be determined. If D, d, and d' are the distances, measured from the objective, to the upper surface of the glass bottom of the containing vessel through air only, through the known liquid, and through the unknown, then

$$\frac{n-1}{n'-1} = \frac{\frac{1}{D} - \frac{1}{d}}{\frac{1}{D} - \frac{1}{d'}} = \frac{1 - \frac{D}{d}}{1 - \frac{D}{d'}}.$$

210. Becquerel and Cahours' Method for Determining the Refractive Index of a Liquid (1840).—Becquerel and Cahours[3] used a similar method, but instead of measuring D, d, and d', they determined the number of divisions of a micrometer (P, p, and p',) which were included between two fixed lines in a micrometer ocular on examining different media. These values, as may easily be proved, are proportional to those given by Brewster, so that

[1] See Art. 152, *supra.*

[2] David Brewster: *A treatise on new philosophical instruments*, Chapter II, Book IV. *Description of an instrument for measuring the refractive powers of fluids, and of a method of determining the refractive powers of solids; with tables of the refractive powers of various substances.* Edinburgh, 1813, 240–288.

[3] Edmond Becquerel et Auguste Cahours: *Recherches sur les pouvoirs réfringents des liquides.* Comptes Rendus, XI (1840, Paris, 1841), 867–871.

Abstract: *Untersuchungen über das Brechvermögen einiger Flüssigkeiten.* Pogg. Ann., LI (XXI, 2nd series), 1840, 427–433.

$$\frac{n-1}{n'-1}=\frac{1-\frac{P}{p}}{1-\frac{P}{p'}}.$$

The standard used for comparison was distilled water whose mean index was taken as 1.333.

The index may be determined directly if a shallow tray of the liquid is inserted between the objective and a reference mark on a glass on the stage. If d represents the amount which the objective must be raised, and D the depth of the liquid screen, we have

$$d=D\left(\frac{n-1}{n}\right), \text{ or } n=\frac{D}{D-d}.$$

But $D-d=M$ (Fig. 344), and the equation becomes $n=\frac{D}{M}$, as before. Becquerel and Cahours say further, that "this very simple formula may also be used to determine directly the index of refraction of a solid," which is, then, of course, the method of the Duc de Chaulnes.

211. Bertin's Method (1849).—In the method of Bertin,[1] which is applicable to solids, no micrometer screw is necessary on the microscope, but the measurements are made with stationary objective and movable ocular. A finely divided glass scale is placed upon the upper surface of the mineral whose index of refraction is to be measured, the tube of the microscope is drawn out to its full extent, and the enlargement of the image of the scale is determined. Let G be its value. If the micrometer is now placed beneath the mineral, it will be found that the divisions are indistinct. Without changing the position of the objective, it will be found that by depressing the ocular (shortening the tube length) the micrometer may again be brought into focus, but the enlargement, in this case, differs from that first determined. Let γ be the new value. If the mineral is now entirely removed from the stage and the micrometer viewed through the microscope with air as the only intervening medium, a farther depression of the ocular is necessary, and a third enlargement g results.

From the general equation of lenses (Eq. 9, Art. 85) we have

$$\frac{1}{f}=-\frac{1}{f_1}+\frac{1}{f'_2}.$$

In a biconvex lens, f_1 will be negative and f'_2 positive, and our equation becomes

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f'_2}, \text{ or } 1+\frac{f_1}{f'_2}=\frac{f_1}{f}. \qquad (1)$$

[1] A. Bertin: *Sur la mesure des indices de réfraction des lames transparentes et des liquides a l'aide du microscope ordinaire.* Ann. Chim. et Phys., XXVI (1849), 288–296.

Review: *Messung der Brechungsindexe von durchsichtigen Platten mittelst des gewöhnlichen Mikroskops.* Pogg. Ann., LXXVI (1849) (XVI, 3d series), 611–612.

where f is the principal focus of the system, f_1 the distance of the object, and f'_2 the distance of the image from the lens. The magnification is expressed by the ratio of the size of the image to that of the object, which is equal to the ratio of f'_2 to f_1, whereby

$$G=\frac{f'_2}{f_1} \text{ or } \frac{1}{G}=\frac{f_1}{f'_2}.$$

Substitute in equation (1)

$$1+\frac{1}{G}=\frac{f_1}{f}. \tag{2}$$

When the microscope was placed below the mineral section, the apparent distance of the object from the lens (Fig. 348) was f_1+M, therefore

$$1+\frac{1}{\gamma}=\frac{f_1+M}{f}. \tag{3}$$

When the mineral section was removed from the stage, the distance was f_1+D, and

$$1+\frac{1}{g}=\frac{f_1+D}{f}. \tag{4}$$

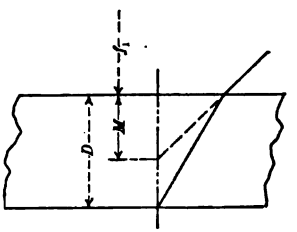


FIG. 348.

Subtracting (4) from (3),

$$\frac{1}{\gamma}-\frac{1}{G}=\frac{M}{f}. \tag{5}$$

Subtracting (2) from (4)

$$\frac{1}{g}-\frac{1}{G}=\frac{D}{f}. \tag{6}$$

Dividing (6) by (5) and combining with (2), Art. 208,

$$\frac{\frac{1}{g}-\frac{1}{G}}{\frac{1}{\gamma}-\frac{1}{G}}=\frac{\frac{D}{f}}{\frac{M}{f}}=\frac{D}{M}=n. \tag{7}$$

Simplifying, we have, as the index of refraction,

$$n=\frac{G\gamma-g\gamma}{Gg-g\gamma}. \tag{8}$$

In determining the index of refraction by this method, an object micrometer on very thin glass should be used. It should be placed with the engraved side down to determine G, and up to determine γ and g.

Bertin suggests that if a very thick plate is to be measured, it is better to compare it with a plate of known thickness and index by the formula

$$\frac{D\left(1-\frac{1}{n}\right)}{D'\left(1-\frac{1}{n'}\right)}=\frac{\frac{1}{g}-\frac{1}{\gamma}}{\frac{1}{g}-\frac{1}{\gamma'}}. \tag{9}$$

This equation is derived from those preceding as follows:
Subtracting (3) from (4) we have

$$\frac{1}{g} - \frac{1}{\gamma} = \frac{D-M}{f}. \tag{10}$$

But $M = \frac{D}{n}$ (Eq. 2, Art. 208), whereby (10) becomes

$$\frac{1}{g} - \frac{1}{\gamma} = \frac{D\left(1 - \frac{1}{n}\right)}{f}. \tag{11}$$

For another substance with measurements D' and γ', and index n', we obtain

$$\frac{1}{g} - \frac{1}{\gamma'} = \frac{D'\left(1 - \frac{1}{n'}\right)}{f}. \tag{12}$$

Dividing (11) by (12), we obtain equation (9).

PROBLEM

Check, by Bertin's method, the index of refraction of the fluorite plate used in the previous problem.

212. Sorby's Method.—By a modification of de Chaulnes' method, Sorby[1] was enabled to measure not only the single refractive index of isotropic substances, but the two different indices of those that are anisotropic, as well. To make his determinations he equipped his microscope with a scale and vernier whereby he was able to read the vertical movement of the tube to 0.001 in. (0.025 mm.). In modern microscopes such measurements may be made by means of the fine focussing adjustment which, in some instruments, give readings to 0.0005 mm. Underneath the stage, and as far below the lenses of an achromatic condenser as possible, was placed a glass plate (Fig. 349) upon which were engraved two sets of fine lines. These were ruled in two directions at right angles to each other and 0.01 in. (0.254 mm.) apart. The lines of this grating could be brought to a focus by means of the condenser, either upon the lower or upper surface of the specimen or anywhere within it, and appeared there as a much reduced image. Close to the glass grating was an iris diaphragm whereby a circular image of any diameter could be brought into focus in the same plane as the ruled lines. Below the diaphragm was a nicol

[1] H. C. Sorby: *On a simple method for determining the index of refraction of small portions of transparent minerals. Preliminary notice.* Mineralog. Mag., I (1877), 97–98.

Idem: *President's Address, Mineralogical Society.* Ibidem, 193–208.

Idem: *On some hitherto undescribed optical properties of doubly refracting crystals. Preliminary notice.* Proc. Roy. Soc. London, XXVI (1877), 384–386.

Idem: *On the determination of the minerals in thin sections of rocks by means of their indices of refraction.* Mineralog. Mag., II (1878), 1–4.

Idem: *Further improvements in studying the optical characters of minerals.* Ibidem, II (1878), 103–105.

Idem: *On a new method for studying the optical properties of crystals.* Ibidem, XV (1909), 189–215.

prism and another was above the eyepiece, and either or both could be rotated or thrown out of position. A 2-in. (50 mm.) eyepiece and a 2/3-in. (16.9 mm.) objective were used, the latter stopped down to a 13° aperture by means of a cap with a small opening. Another cap, with a semi-circular opening cutting off exactly one-half of the front lens in any desired direction, was used to determine the plane of polarization of any beam that had passed through the mineral under examination.

Determinations were made both on mineral sections cut with plane-parallel faces and on natural crystals, the latter possessing the advantage of having opposite faces truly parallel. If the surfaces were rough, a drop of oil, of approximately the same index as the mineral, was placed above and below it, and protected by a cover-glass. This gave rise to a small error, but with a specimen from 1/10 to 1/2 in. in thickness, it was of no great moment.

FIG. 349. FIG. 350. FIG. 351. FIG. 352. FIG. 353. FIG. 354.

FIGS. 349 TO 354.—Images seen through mineral plates by the method of Sorby.

With the microscope so arranged, the phenomena observed are as follows:

Isotropic Substances.—On looking at the image of the grating without any intervening object, both sets of lines are seen at the same focus, as shown in Fig. 349.[1] If an isotropic mineral or a transparent amorphous body with plane-parallel faces is placed on the stage, the two sets of lines can still be seen in one plane although at a different focus than before. No matter how much the stage is rotated, the lines remain in view and the circle is not distorted. Isotropic substances, consequently, have no special focal axis. They are also **unifocal** because all parts of the only image lie at the same focal distance.

The index of refraction of an isotropic substance, consequently, is determined, as explained above, by the formula

$$n = \frac{D}{M}$$

Anisotropic Crystals.—The phenomena observed in minerals having double refraction are totally different, and in order to examine separately the two rays, which are polarized in opposite planes, it is necessary to use a rotating analyzer, either within the tube or above the eyepiece, and so turned that it permits either one or the other of the rays to pass through. In every case it will be found that the ordinary ray is unifocal and acts as does the light in isotropic substances.

UNIAXIAL CRYSTALS

Section cut Perpendicular to the Optic Axis.—If a section of calcite, 0.25 in. in thickness and cut at right angles to the optic axis, is examined, two images ap-

[1] For photographic reproductions of these figures, see the beautiful illustrations given by Dr. Hans Hauswaldt: *Interferenzerscheinungen im polarisirten Licht*, 3te Reihe, Magdeburg, 1908, plates 35–36–37.

pear, each showing both sets of ruled lines. They are directly superimposed but lie in different focal planes, as though there were two sets of lines ruled on opposite sides of a glass plate. On bringing one image into focus, the circle appears sharp and undistorted, but the other image, which is seen out of focus, appears as a large blurred circle surrounding the first (Fig. 352). On changing the focus, the second image becomes sharp and the first forms the blurred halo. Looking straight down, the ordinary cannot be distinguished from the extraordinary image, but if the section be somewhat inclined the images separate and the two rays may be differentiated.

Placing the semi-circular stop over the objective, with the straight cut of the opening parallel to one of the sets of lines in the grating, produces the effect of slightly inclining the section by causing the light to pass through obliquely. It thus shows the ordinary image to be unifocal and the extraordinary image to be slightly bifocal, as explained below.

The index of refraction of the ordinary ray, in the section of calcite examined by Sorby, was found to be equal to 1.659; the apparent, but not the true value for the extraordinary, 1.335. The value of the apparent extraordinary ray in various directions should be, according to Stokes,[1] equal to the square of the true index of the extraordinary ray divided by the true index of the ordinary. In this case

$$\frac{(1.487)^2}{1.659} = 1.333.$$

Section Parallel to the Cleavage.—The images of the circular opening, seen through a section of calcite cut parallel to the cleavage, appear widely separated in the plane of the principal axis (Fig. 353), and lie at different focal distances. The image due to the ordinary ray is in no way distorted and lies in the center of the field, that due to the extraordinary ray is elongated and appears to lie at a lower level and to one side. It will be found that there is no single adjustment in which this image is completely in focus. In one position of the objective it appears as an elongated band with two sides parallel to each other and parallel to the axis of the crystal and with illy defined ends. On raising the tube of the microscope, the band changes into a poorly defined circle several times larger than the real one, and then into a band elongated in a direction perpendicular to the former.

With the analyzer arranged so that only the ordinary image appears, it will be found to be unifocal, and both sets of lines of the grating will appear, no matter what the azimuth of the crystal. The index of refraction was found by Sorby to be 1.657.

When the crystal is so turned that the lines of the grating are parallel and perpendicular to the axis of the crystal, and the analyzer so arranged that only the extraordinary image appears, there will be two widely separated focal points at each of which *only one system* of lines can be seen. That at which the lines parallel to the axis appear, give an index of 1.412, while that at which those perpendicular appear give approximately 1.578, but the latter are poorly defined unless light passed through red glass is used. The extraordinary image is therefore truly **bifocal.**

[1] G. G. Stokes: *On the foci of lines seen through a crystalline plate.* Proc. Roy. Soc., London, XXVI (1877), 386–401.

In sections cut at greater inclinations with the axis, the bifocal image becomes more and more nearly unifocal until in sections perpendicular to the axis, it is entirely so, as explained above.

Section Parallel to the Principal Axis.—On examining a section of calcite, 0.2 in. in thickness and parallel to the optic axis, it will be found, when the analyzer permits only the extraordinary ray to pass through, that there are two different foci at which the lines of the grating are visible. The circular hole is elongated first in one direction (Fig. 350) and then in the other (Fig. 351), and in each case only one system of rulings can be seen, and then only when the grating is so arranged that the lines are parallel and perpendicular to the axis of the crystal. The image of the extraordinary ray is bifocal, and since the rulings disappear when the stage is rotated, it has a definite focal axis. The ordinary ray gives an image not distorted and at a single focus.

The index of refraction of the ordinary image is its true index. That for the lines parallel to the principal axis of the crystal is the true index of the extraordinary, while that of the lines perpendicular to this axis is the apparent index and is equal, according to Stokes, to the square of the index of the ordinary ray divided by that of the extraordinary, in this case producing a result of 1.868, which is *greater than that of the ordinary ray.*

Sorby, in regard to the apparent index, says: "The phenomenon seen with the microscope depends entirely on the power of the object glass to collect divergent rays. In the case of substances having no double refraction, this divergence merely obeys the laws of ordinary refraction, and enables us to measure the index in the manner already explained; but in the case of the extraordinary ray, the light is bent from the normal line unequally and in opposite directions, and may thus enter the object glass at an angle of divergence greater or less than that depending on the index of refraction.[1]

BIAXIAL CRYSTALS

Section Perpendicular to the Principal Axis.—Crystals of aragonite and orpiment were used. The circular hole of the diaphragm appears as two crosses (Fig. 354) lying in widely different focal planes, each cross being itself bifocal and polarized in opposite planes. There may thus be four different apparent indices, but in sections cut in particular directions one or two pairs may become equal and have the appearance of a unifocal image, differing, however, from unifocal images due to an ordinary ray, in becoming bifocal when one-half the front lens of the objective is covered with the semi-circular stop. There is no ordinary ray.

If the section is inclined away from the axis, the image becomes much less symmetrical.

Sections Parallel to the Principal Axis.—Sections parallel to the principal axis give different figures, depending upon their orientation with respect to the other axes. When parallel to the principal and to one of the secondary axes, a cross with unequal arms, at four different foci, is obtained; when cut parallel to the principal and along the diagonal of the secondary axis, one image is decidedly bifocal and one unifocal. The latter, however, is caused by an extraordinary ray, as may be shown by passing an inclined ray through it.

[1] H. C. Sorby: *Op. cit.*, Mineralog. Mag., I (1877), 199.

Determination of Indices of Refraction.—In determining the real value of the indices of refraction, the following facts must be remembered.

1. A crystal having no double refraction has no bifocal image, and its index of refraction is the true index.

2. The ordinary ray of a uniaxial crystal gives a unifocal image, and its index of refraction is its true index, no matter what may be the orientation of the section.

3. Biaxial crystals have two bifocal images whose focal axes are always perpendicular to the plane of polarization of the images. In any bifocal image one apparent index is true when the corresponding principal focal axis is parallel to the plane of the section. If, therefore, a biaxial crystal is cut parallel to two principal axes, each image will give one true index; the third may be calculated. If the crystal is cut parallel to only one axis, only one true index can be determined, and if parallel to no axis, none of the true indices can be obtained.

CHAPTER XIV

OBSERVATIONS BY ORDINARY LIGHT (Continued)

Determination of the Refractive Indices of a Mineral by the Immersion or Embedding Method

213. Maschke (1872–1880).—If a crystal is immersed in a liquid of a different refractive index, and is examined under the microscope, it will be seen that its borders are either dark or colored, due to the reflection of the light at the edges. If the index of the immersion liquid is exactly the same as that of the mineral, the borders are lost and, if the mineral is colorless, the latter disappears from view. This fact had long been known but its applicability to the separation of microscopic mineral fragments appears first to have been recognized by Maschke[1] in 1872, while engaged in a study of quartz and tridymite. He determined the fact that as the index of the immersion fluid approaches that of the mineral, the dark borders give way to colors, which he ascribed to interference. He stated that when the index of the liquid is lower than that of the mineral, the latter appears bluish or bluish-green with a reddish rim, and that when the index of the liquid is greater than that of the mineral, the latter appears reddish with a bluish or bluish-green rim. He also suggested that just as we now have a scale of hardness, so might also a series of immersion liquids be prepared for the comparison of refractive indices. He proposed, as such, cassia oil, turpentine, and poppy oil, or mixtures of these, alcohol, and a solution of mercuric nitrate of various degrees of dilution.

In a later paper, Maschke[2] correctly recognized the colors as microprismatic, and indicated how they might be brought out by inclined illumination. To produce this he displaced the lower diaphragm laterally or, more simply, fastened across the front lens of the objective, by means of a touch of wax on either side of the casing, a thin, dull-black strip of paper, 1.5 to 2 mm. in width and with sharp edges. The paper was pressed into close contact with the lens and, since a low power was used, the opening was sufficiently large. The diaphragm was now closed until the paper appeared as a narrow, black bar across the middle of the field.

For the measurement of the indices of doubly refracting minerals, Maschke made use of a polarizer, and determined the values in different direc-

[1] O. Maschke: *Ueber Abscheidung krystallisirter Kieselsäure aus wässrigen Lösungen.* Pogg. Ann., CXLV (5 ser. XXV, 1872), 549–578, in particular 568–569.

[2] O. Maschke: *Ueber eine mikroprismatische Methode zur Unterscheidung fester Substanzen.* Wiedem. Ann., N. F. XI (1880), 722–734.

tions. Among the fluids used in his later work were water, amyl alcohol, glycerine, almond oil, and cassia oil, the latter two mixed in varying proportions. He thus had a series of indicators with values from 1.333 to 1.606.

214. Sorby (1877).—No further use was made of the immersion method for determining the relative refractive indices of a fluid and a solid until the method was rediscovered in 1900 by Schroeder van der Kolk.[1] The method of reducing the dark borders by immersion had been employed, however, and Sorby[2] made use of a diaphragm and of inclined illumination. In his Presidential address to the Royal Microscopical Society, in 1877, he called attention to the fact that when a mineral is immersed in a fluid having a refractive index but slightly different, no outline is seen if the angle of convergence of the light is considerable, but by cutting down the cone of light, the outlines become more and more distinct and the shading greater and greater. He spoke of the importance of having the means of varying the angle of deviation from a direct line by means of a diaphragm below the condenser.

215. Thoulet (1870).—In 1879, Thoulet[3] described a heavy solution of potassium mercuric iodide, previously used by Sonstadt but now generally known as Thoulet's solution,[4] for determining specific gravities. It has a very high index of refraction, the maximum being 1.7333 for sodium light. Being miscible with water in all proportions, a range of indices from 1.333 to 1.733 may be obtained. Goldschmidt[5] computed the values given below, for sodium light and at 18° C.

TABLE SHOWING THE RELATIONS BETWEEN SPECIFIC GRAVITY AND REFRACTIVE INDEX OF THOULET'S SOLUTION IN SODIUM LIGHT AND AT 18° C.

Specific gravity	n_D	Specific gravity	n_D	Specific gravity	n_D	Specific gravity	n_D
3.2	1.7333	2.7	1.6395	2.2	1.5457	1.7	1.4551
3.1	1.7145	2.6	1.6207	2.1	1.5270	1.6	1.4371
3.0	1.6956	2.5	1.6020	2.0	1.5090	1.5	1.4186
2.9	1.6768	2.4	1.5832	1.9	1.4910	.....	.....
2.8	1.6582	2.3	1.5645	1.8	1.4731	1.0	1.3333

In 1880, Thoulet[6] used the immersion method, to a certain extent, and

[1] See footnote 24, Art. 228.

[2] H. C. Sorby: *Anniversary Address of the President of the Royal Microscopical Society.* Mon. Microsc. Jour., XVII (1877), 117–118.

[3] J. Thoulet: *Séparation méchanique des divers éléments minéralogiques des roches.* Bull. Soc. Min. France, II (1879), 17–24.

[4] For the method of preparation see Art. 454. The solution is decomposed by metallic iron. It is also extremely poisonous and should be used with great caution.

[5] V. Goldschmidt: *Ueber Verwendbarkeit einer Kaliumquecksilberjodlösung bei mineralogischen und petrographischen Untersuchungen.* Neues Jahrb., B. B., I (1881), 179–238.

[6] J. Thoulet: *De l'apparence dite chagrinée présentée par un nombre de minéraux examinés en lames minces.* Bull. Soc. Min. France, III (1880), 62–68.

mentioned that the shagreen surface of minerals disappears when the refractive indices of fluid and solid are the same. He used water, alcohol, glycerine, olive oil, beech-nut oil, clove oil, cinnamon oil, bitter-almond oil, and bisulphide of carbon.

216. Stephenson (1880).—In order to obtain relief in biologic specimens, Stephenson,[1] in 1880, immersed them in phosphorus, bisulphide of carbon, or solutions of sulphur He gave a table of the refractive indices of various immersion substances but made no attempt to determine the index of the embedded material.

217. Rohrbach (1883).—Rohrbach,[2] in 1883, proposed a solution of barium mercuric iodide, now generally known as Rohrbach's solution, for the determination of the specific gravity of minerals, and as one having a high refractive index. He gives 1.7932 to 1.7928 at 23° C. and in sodium light. The relation between specific gravity and refractive index is shown in Fig 722.

218. Brauns (1886).—Methylene iodide, introduced by Brauns[3] in 1886 as a heavy solution and as one having a high index of refraction, is a light yellow fluid, unaltered by contact with air and miscible in all proportions with benzol. Undiluted, its indices of refraction at different temperatures and by sodium light are as follows:

REFRACTIVE INDICES OF METHYLENE IODIDE AT DIFFERENT TEMPERATURES BY SODIUM LIGHT

Temp.	n_D	Temp.	n_D	Temp.	n_D	Temp.	n_D
5°	1.74873	11°	1.74447	17°	1.74021	23°	1.73595
6°	1.74802	12°	1.74376	18°	1.73950	24°	1.73524
7°	1.74731	13°	1.74305	19°	1.73879	25°	1.73453
8°	1.74660	14°	1.74234	20°	1.73808	.....	.....
9°	1.74589	15°	1.74163	21°	1.73737	.....	.....
10°	1.74518	16°	1.74092	22°	1.73666	31°	1.7300

219. Bertrand (1888).—Bertrand[4] increased the index of refraction of methylene iodide by dissolving in it a large quantity of sulphur by means

[1] J. W. Stephenson: *On the visibility of minute objects mounted in phosphorus, solutions of sulphur, bisulphide of carbon and other media.* Jour. Roy. Microsc. Soc., III (1880), 564–567.

[2] Carl Rohrbach: *Ueber eine neue Flüssigkeit von hohem specifischen Gewicht, hohem Brechungsexponenten und grosser Dispersion.* Wiedem. Ann., N. F., XX (1883), 169–174. For the method of preparation see Art. 456.

[3] R. Brauns: *Ueber die Verwendbarkeit des Methylenjodids bei petrographischen und optischen Untersuchungen.* Neues Jahrb., 1886 (II), 72–78. For the method of use, see Art. 457.

[4] Emile Bertrand: *Liquides d'indices supérieurs à* 1.8. Bull. Soc. Min. France, XI (1888), 31.

of heat. On cooling, large crystals of sulphur were formed, leaving a liquid having a refractive index above 1.8. On dissolving iodine and sulphur in methylene iodide, a liquid having an index greater than 1.85 was obtained. The proportions of iodine and sulphur, in the latter liquid, are not given by Bertrand, and the writer has been unable to obtain a higher refractive index than 1.82 after the fluid becomes cold.

220. Klein (1890).—Klein,[1] in 1890, 1891, and later, used the immersion method to get rid of the boundaries of crystals in making various examinations under the microscope, such as extinction, optic angles, and so on, but he did not specifically apply it to the determination of refractive indices.

221. Schroeder van der Kolk (1892).—Schroeder van der Kolk,[2] in 1892, used inclined illumination to bring out certain properties of minerals, but as yet had not applied it to the determination of their indices.

222. Zirkel (1893).—Zirkel,[3] in 1893, gave a list of twenty-six immersion fluids, but suggested no way by which to determine whether fluid or solid has the greater index. The accuracy of some of the higher indices is questioned. No references are given to the authority for the data.

223. Retgers (1893).—Retgers,[4] in 1893, proposed phosphorus in a molten condition or as a concentrated solution in carbon bisulphide as a medium of high refractive index. A grain of colorless to yellow phosphorus, the size of a pin head, is rapidly dried with a piece of linen or filter paper, and is placed on the object-slide and quickly covered with a cover-glass. Upon heating, high up over a small naked flame, the phosphorus melts, and, if the precaution is taken to press down firmly upon the cover-glass, it will spread out into a flat drop, 1 or 2 cm. in diameter. There is no danger of ignition since no air is admitted. Even if a small quantity is squeezed out beyond the cover-glass and ignites, it burns out without igniting the part covered from the air. The phosphorus should not be heated above the melting-point (44° C.) or it will turn dark yellow or red. After the phosphorus is fluid, it will remain so for a considerable time and have an index of

[1] Carl Klein: *Ueber eine Methode, ganze Krystalle oder Bruchstücke ders·lben zu Untersuchungen im parallelen und im convergenten polarisirten Lichte zu verwenden.* Sitzb. Akad. Wiss. Berlin, 1890 (I), 347–352.

Idem: *Ueber die Methode der Einhüllung der Krystalle zum Zweck ihrer optischen Erforschung in Medien gleicher Brechbarkeit.* Reprinted, with additions by the author, from Stizb. Akad. Wiss. Berlin, 1890, 703, in Neues Jahrb., 1891 (I), 70–76.

[2] J. L. C. Schroeder van der Kolk: *Ueber die Vortheile schiefer Beleuchtung bei der Untersuchung von Dünnschliffen im parallelen polarisirten Lichte.* Zeitschr. f. wiss. Mikrosk. VIII (1891–2), 456–8.

[3] F. Zirkel: *Lehrbuch der Petrographie,* I. 2te. Aufl., Leipzig, 1893, 40.

[4] J. W. Retgers: *Der Phosphor als stark lichtbrechendes Medium zu petrographischen Zwecken.* Neues Jahrb., 1893 (II), 130–134, and correction, Ibidem, 1894 (I), 424.

refraction in sodium light of 2.075. On cooling, the phosphorus remains perfectly clear and will not form a crystalline aggregate although it is isometric. Its index of refraction is 2.144 by sodium light. After having made a determination, object and cover-glass may be freed from phosphorus by dipping them into nitric acid, which will reduce the phosphorus to phosphoric acid.

Dissolved in carbon bisulphide, phosphorus is in no danger of ignition if properly used, nor does it oxidize into the red or opaque form. It should not, however, be kept in stock, but one should proceed as follows: A grain of the mineral to be examined is placed on the object-slide, and with it a piece of phosphorus about 1 mm. in diameter. It is then quickly covered with a cover-glass, and one or two drops of carbon bisulphide are permitted to flow beneath the edge, pressure being applied at the same time to the cover-glass. The phosphorus soon dissolves and is much more transparent than in the molten state, although its index of refraction is only about 1.95 (P, $n=2.14$, CS_2, $n=1.63$) at room temperature. Object and cover-glass may be cleaned by dipping in carbon bisulphide.

224. Ambronn (1893).—By the methods used before 1893, the process of finding immersion fluids of the proper indices was extremely tedious, especially when working with anisotropic minerals for which it is necessary to make observations above a nicol prism, placing the plane of polarization parallel first to one and then to another vibration direction, and selecting refractive fluids corresponding to each. Ambronn[1] said that it is much easier to find a fluid with an index of refraction intermediate between the indices in two directions at right angles to each other in the mineral, than to find two that exactly coincide. In such a fluid the boundaries of the mineral do not disappear unless the stage is rotated to a particular position with reference to the direction of vibration of the polarizer. If, then, one determines the amount of rotation in azimuth necessary in each of two such intermediate fluids with different indices, he can determine the indices of refraction of the mineral from the equations:

$$\omega_1^2 = \frac{n_2^2 \cos^2 \varphi_1 - n_1^2 \cos^2 \varphi_2}{\cos^2 \varphi_1 - \cos^2 \varphi_2}$$

$$\epsilon_1^2 = \frac{n_2^2 \cos^2 \varphi_1 - n_1^2 \sin^2 \varphi_2}{\sin^2 \varphi_1 - \sin^2 \varphi_2}$$

in which ω_1 and ϵ_1 are the indices of refraction to be determined in two directions at right angles to each other, n_1 the index of refraction of the first immersion liquid, n_2 that of the second liquid, φ_1 the angle between c and the

[1] H. Ambronn: *Ueber eine neue Methode zur Bestimmung der Brechungsexponenten anisotroper mikroskopischer Objecte.* Ber. Gesell. Wiss., Leipzig., Math.-phys. Kl., XLV (1893), 316–318.

position where the boundaries of the mineral disappear in the first liquid, and φ_2 the angle between c and the position of disappearance in the second. This method is applicable only to very thin sections of minerals, an appreciable error arising if they are thick.[1] The accuracy of the method does not appear to be very great, Ambronn's results varying in the second decimal place.

225. Ambronn (1896).—In a later paper, Ambronn[2] called attention to the colored borders seen at the contact between a mineral and an immersion fluid when the indices of refraction differ but slightly, say in the third decimal place. As an example he gives the contact between glass and a mixture of monobromnaphthylene and xylol. The indices of the two for different rays are

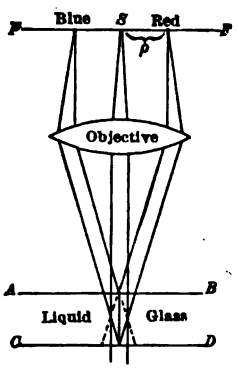


FIG. 355.—The cause of colored borders around minerals.

	Glass	Fluid	α
n_B	1.5134	1.5097	4° 0′
n_C	1.5144	1.5116	3° 30′
n_D	1.5170	1.5170	0° 0′
n_E	1.5204	1.5236	3° 40′
n_F	1.5234	1.5296	5° 10′
n_G	1.5290	1.5406	7° 0′
n_H	1.5335	1.5500	8° 20′

Consequently, when white light is used (Fig. 355), since the dispersion for fluids, in general, is greater than for solids, the yellow rays (n_D) will pass through both media without change of direction, the red will be bent toward the glass, and the blue toward the liquid.[3] Ambronn suggests, for the determination of refractive indices, that a series of refractive fluids, differing by 0.005, be prepared. When the color effects are well marked, the refractive index for the yellow cannot differ greatly in the two media. The observation may now be conducted by sodium light, and the fluids changed until the border totally disappears. For anisotropic crystals the polarizer is used to transmit the light in a single plane.

226. Marpmann (1896).—For embedding diatoms and other biological

[1] Compare Sorby's work, Art. 212 and Pauly's, Art. 232.

[2] H. Ambronn: *Farbenerscheinungen an den Grenzen farbloser Objecte im Mikroskop.* Ber. Gesell. Wiss. Leipzig, Math.-phys. Kl., XLVIII (1896), 134–140.

[3] Compare the explanation given by Schroeder van der Kolk, Art. 228.

specimens, so that their structures would stand out in relief, Marpmann,[1] in 1896, used cinnamon, cassia, and other oils.

227. Schroeder van der Kolk (1898).—Schroeder van der Kolk,[2] in 1898, made use of the immersion method for determining refractive indices, but he had not yet discovered its quantitative possibilities. He depended upon the width and strength of the dark border to determine the difference in the indices—the wider the border, the greater the difference—and called it a rapid, even though not very exact, method. He suggested the use of a series of fluids of known indices as indicators and, since certain fluids act as solvents for certain salts, he gave two lists of immersion liquids, one for inorganic substances, and one for organic. Each series is composed of liquids which, in most cases, can be mixed with each other in any quantity, thus giving the possibility of preparing fluids of any desired index.

For inorganic salts	n	Sp. gr.	Boiling point	For organic salts	n	Sp. gr.	Boiling point
Hexane	1.37	0.66	68°	Methyl alcohol	1.32	0.81	66°
Heptane	1.39	0.71	98°	Water	1.34	1.00	100°
Cajeput oil	1.46	0.92	174°	Ethyl ether	1.36	0.72	35°
Olive oil	1.47	0.92	.....	Ethyl alcohol	1.37	0.81	78°
Castor oil	1.49	0.96	265°	Amyl alcohol	1.40	0.83	132°
Benzol	1.50	0.89	80°	Chloroform	1.45	1.50	61°
Xylol	1.50	0.86	136°	Glycerine	1.47	1.26	290°
Beech nut oil	1.50	0.92	.....	Creosote	1.54	1.06	200°+
Cedar oil	1.51	0.98	237°	Aniline	1.60	1.04	183°
Clove oil	1.53	1.05	253°	Cadmium borotungstate	1.70	3.60	.....
Anise oil	1.56	0.99	220°	Potassium mercuric iodide	1.72	3.20	.....
Bitter almond oil	1.60	1.04	180°	Barium mercuric iodide	1.79	3.59	.....
Carbon bisulphide	1.63	1.29	47°	Mercuric iodide in aniline and quiniline	2.20	.....	.....
α Monobromnaphthalene	1.66	1.50	277°				
Methylene iodide	1.76	3.34	180°				
Phenyl sulphide	1.95	1.12	272°				

Intermediate fluids may be prepared by the mixture of two according to the formula

$$v_1 n_1 + v_2 n_2 = (v_1 + v_2)\, n,$$

where v_1 and v_2 are their respective volumes. Thus 9 volumes of heptane and 2 of benzol give a fluid having a refractive index close to 1.41. Only fluids having approximately the same boiling-points should be combined, otherwise, on account of the evaporation of one component, the index of the mixture may change rapidly. He suggests mixtures of olive and castor,

[1] G. Marpmann: *Ueber die Anwendung von Zimmtöl, Cassiaöl, und anderen Einschlussmitteln in der Mikroskopie.* Zeitschr. f. angew. Mikrosk., II (1896–7), 335–338.

[2] J. L. C. Schroeder van der Kolk: *Kurze Anleitung zur mikroskopischen Krystallbestimmung.* Wiesbaden, 1898, 11–14.

clove and cedar, clove and bitter almond, and anise and bitter-almond oils, and α-monobromnaphthalene and bitter almond oil. Mixtures of clove and anise oil become cloudy and should, consequently, not be used. Carbon bisulphide, being highly volatile, should not be mixed with other components. He further states that phenyl sulphide appears not always to have the same index and that mercuric iodide in aniline and quiniline were not personally tried by him.

To determine the index of refraction of a mineral he worked, in the beginning, with the condenser inserted. The process is, under this condition, less sensitive, and the borders disappear with greater differences between the indices of the solid and the liquid. When this had taken place, the condenser was removed and the limits determined with greater accuracy. For still greater accuracy, a small diaphragm was inserted, and finally monochromatic light was used.

Van der Kolk[1] speaks of colored borders, due to dispersion, appearing when the black border disappears, but makes no further use of them.

228. Schroeder van der Kolk (1900).—No great use was made of the immersion method until it received its great impetus by the publication of Schroeder can der Kolk's *Tabellen.*[2] After the issue of his *Anleitung*, which was intended primarily for chemists, he greatly developed the method, and it was here made use of for the rapid determination of minerals, some 300 being given in the order of their indices.

According to former methods, the dark borders enabled one to determine that solid and immersion fluid were of different refractive indices, yet one might be uncertain whether the index of the fluid was too low or too high. The method here described is based on the principle of the dispersion of light by prisms, since the grains of crushed minerals have, in general, more or less wedge-shaped edges.

If the condenser and polarizer are removed from the microscope and a beam of monochromatic light is directed, by means of the plane mirror, squarely upon a more or less lens-shaped mineral fragment embedded in a liquid of a different refractive index, one of two things will take place. If the immersion fluid has a refractive index which is lower than that of the mineral, the latter will act as a double convex lens and the rays will first converge, then cross, and finally diverge (Fig. 356). If, on the other hand, the refractive index of the mineral is less, it will act as a double concave lens, and the rays, after passing through, will diverge (Fig. 357) If the objective is one of rather low power and has a considerable focal length, the appearance is the same in either case since the border rays will be deflected too much to enter the lens, consequently a dark border will appear around the

[1] *Op. cit.*, page 45.

[2] J. L. C. Schroeder van der Kolk: *Tabellen zur mikroskopischen Bestimmung der Mineralien nach ihrem Brechungsindex.* 2 Aufl. Wiesbaden, 1906.

mineral.[1] So far as the border is concerned, one can determine only that the refractive indices of the two media are different. If, now, the mirror is swung to one side so that the illumination is inclined (in Figs. 358 and 359 from the right), the light will converge or diverge as before, but the appearances as seen under the microscope are different. In the case where the index of the mineral is greater than that of the liquid (Fig. 358), the rays will cross and diverge, it is true, but the one on the opposite side from that from which the light proceeded will more or less directly enter the lens, and that side of the mineral, or the opposite side of the image seen in the microscope, will appear bright. When the refractive index of the mineral is lower than that of the liquid, the reverse phenomenon will take place (Fig. 359).

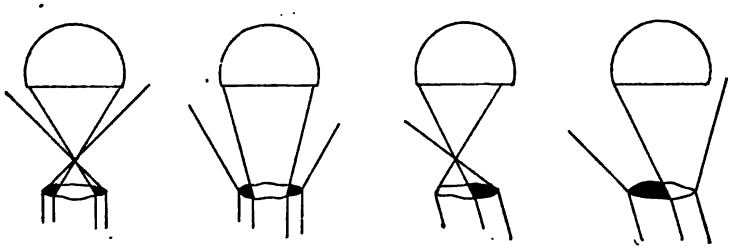


FIG. 356. FIG. 357. FIG. 358. FIG. 359.

FIGS. 356 TO 359.—The cause of dark or light borders.

Instead of displacing the mirror, inclined illumination may be produced much more readily by inserting the condenser and placing above it, but below the section, an opaque screen of thin metal or of cardboard, extending to a greater or less distance beyond the middle.

From these phenomena is derived the following rule for observations made without the condenser, the *apparent* position of the shadow due to the inversion of the image by the microscope being taken into account. *When the dark shadow appears, in the image, on the same side as that from which the screen was actually inserted, the index of refraction of the mineral is greater than that of the immersion fluid; when it appears on the opposite side, the refractive index of the mineral is less.* When the condenser is inserted and somewhat lowered, the phenomenon is reversed.[2]

Instead of using a screen between condenser and object, inclined illumination[3] may be produced by simply shutting off part of the light by holding the finger between the mirror and the condenser, by a sliding diaphragm such

[1] Compare, here, the Becke method, Art. 236.

[2] This method of illumination and the phenomena observed had already been described by Becke. See Art. 239, *infra*.

[3] J. L. C. Schroeder van der Kolk: *Ueber die Vortheile schiefer Beleuchtung bei der Untersuchung von Dünnschliffen im parallelen polarisirten Lichte.* Zeitschr. f. wiss. Mikrosk., VIII (1891), 458.

as are shown in Figs. 251, 252, and 253, or by a sliding diaphragm in the Ramsden disk above the ocular.[1]

From the above method it is easy to determine whether the refractive index of the mineral is higher or lower than that of the immersion fluid. To determine how much they differ, use is made of the color effect produced by the difference in the dispersion of white light in the two substances. In general this is greater in liquids than in solids, a phenomenon to which attention had already been called by Ambronn.[2] If the refractive indices of the two substances are nearly the same for yellow light, the solid will have a higher index for reds and a lower for blue, consequently the edge of the mineral, in the image, will appear blue on the same side as that from which the screen was inserted, and orange on the opposite edge.

Isotropic Substances.—Since isotropic substances have but a single refractive index, the method described above may be used by simply immersing a fragment of the mineral successively in fluids of different indices. The mineral should be crushed, not powdered; the size of the grains being such that they can be totally submerged in a drop of the liquid placed upon an object-glass. The minimum size that may be used is such that the two boundaries of the grain may still be distinguished with an objective whose magnification $\left(\frac{\Delta}{F}\right)$ is about 15. Begin with a fluid of intermediate index, and, after having determined whether the index of the mineral is above or below that of the fluid, notice whether the borders are heavy and black, or whether they are colored. If the former is the case, the indices of the two substances are far apart, if colored, close together. Choose, now, another immersion fluid of a lower index if the one first tried was too high, or of a higher index if it was too low. If the index of the second fluid is still on the same side, choose a third, and so on. If it falls on the opposite side, work between the values of the last two trials. When the color phenomenon is produced, use monochromatic light and change the immersion fluid until the dark boundaries of the mineral totally disappear. (Compare the Becke method, Chapter XV.)

Anisotropic Minerals. Uniaxial Crystals.—For some purposes, the mean value of the refractive indices of an anisotropic mineral $\left(\frac{2\omega+\epsilon}{3} \text{ or } \frac{\alpha+\beta+\gamma}{3}\right)$ may be sufficient. If it is required to obtain the exact values, the determination is more difficult. Both analyzer and polarizer should first

[1] For a further discussion of inclined illumination see

H. Schneiderhöhn: *Die Beobachtung der Interferenzfarben schiefer Strahlenbündel als diagnostisches Hilfsmittel bei mikroskopischen Mineraluntersuchungen.* Zeitschr. f. Kryst., L (1912), 231–241.

F. E. Wright: *Oblique illumination in petrographic microscope work.* Amer. Jour. Sci., XXXV (1913), 63–82.

[2] Art. 226, *supra.*

be inserted and the stage rotated until the mineral is at extinction. In this position its vibration directions correspond with those of the nicols. The analyzer should now be removed and the refractive indices be determined in these two directions.

Of the two values determined, one will be that of the ordinary ray. It may be recognized by being the same in every grain, whatever may be the orientation, and its value is the true value of ω. In grains in which there is obtained an interference figure[1] showing the emergence of the optic axis at the center, the values will be the same regardless of the azimuth to which the stage is rotated.

The value of the other refractive index may, or may not, be the one desired, since a section through the index surface of the extraordinary ray is an ellipse, and the real value of the extraordinary index is along its maximum or minimum vibration axis, depending upon whether the mineral is negative or positive. To obtain the accurate value of ϵ, one should determine the maximum or minimum value of a large number of grains. If it is known that certain fragments are elongated in the direction of crystallographic c, the value of ϵ may be determined at once.

Biaxial Crystals.—The index surface of a biaxial crystal is an ellipsoid of three axes, consequently there are three indices to be determined. The process is very similar to that just given. Determinations are made on a large number of grains, and the highest and lowest values, assumedly the maximum and minimum, are taken for γ and α. The value of β may be computed by the formula

$$\tan^2 V = \frac{\alpha^2(\gamma^2-\beta^2)}{\gamma^2(\beta^2-\alpha^2)},$$

when the size of the optic angle can be measured. Under the microscope β may be determined, in a mineral fragment which shows the emergence of a bisectrix at the center of the field, by making the measurement in a direction at right angles to the plane of the optic axes.

There is no ordinary ray in biaxial crystals.

229. Immersion Fluids.—A great many different substances have been proposed for immersion fluids. Not only simple substances may be used, but mixtures of several as well, and it is thus possible to prepare a series, differing by any desired amount. It is usually necessary that the liquids chosen should retain constant indices during the process of measurement, and only in special cases is a changing index permissible. The boiling-points of two liquids which are to be mixed should be approximately the same, otherwise one will evaporate more rapidly than the other, and the refractive index will vary. The liquids, also, should be unaffected by air, so that the indices of the stock material will remain constant and will not require testing every

[1] Chapter XXIX, *infra*.

time that they are used. Neither should the liquids act upon the minerals under examination, the lens, or the lens casing. Fulfilling these requirements, the various oils, in particular, are well adapted for immersion fluids.

In the following table the values given are, in general, those at 20° C. (68° F.), or room temperature. In some of the fluids there is a decided variation with temperature, and if the work is performed in a very cold or a very warm room, it may be necessary to check the values of the stock material. A greater variation, however, than that produced by heat, is to be found in different lots of the same material, even of the same make, and it is necessary, consequently, to test the values of each new purchase. With unquestionably accurate measurements, different determinations have given, with some fluids, results varying as much as 0.04, and while the values in the list below were probably accurate for the material tested, it is not a safe procedure to accept them for the proper indices in preparing a set of immersion fluids.

In some of the older text-books, materials of very high indices are apparently incorrectly given. Thus phenyl sulphide has, according to Zirkel[1] and Behrens,[2] a value of 1.95,[3] while Himmelbauer[4] found it to be 1.638 at 18.5° C. by sodium light. Mercury iodide in aniline and quiniline is given by Zirkel as 2.2, but Schroeder van der Kolk,[5] in spite of repeated attempts to attain a refractive index so high, was unable to succeed. Wright[6] reached a value of only 1.8.

TABLE OF REFRACTIVE INDICES OF VARIOUS IMMERSION LIQUIDS

(Arranged in the order of increasing values)

Substance	Index	Temp.	Light	Sp. gr.	Boil.-pt.	Formula	Authority
Air	1.00000						
Water	1.33358	18.75	D	1.00	100°	H_2O	Fraunhofer.
Water	1.33392	15.25	D				Baille.
Ethyl ether	1.35210	21.3°	D	0.71	35°	$C_4H_{10}O$	Lorenz.
Acetone	1.35932	20.0°	D	0.82	57°	C_3H_6O	Korten.
Ethyl alcohol	1.36164	20.0°	D	0.79	78°	C_2H_6O	Ketteler.
Ethyl alcohol	1.36138	20.0°	D				Korten.
Hexane	1.37536	20.0°	D	0.66	55° ±	C_6H_{14}	Brühl.
Heptane	1.3867	23.0°	D	0.68	98°	C_7H_{16}	Gladstone.
Chloroform	1.44366	20.0°	D	1.49	61°	$CHCl_3$	Lorenz.

[1] F. Zirkel: *Lehrbuch der Petrographie.* I, 2te Aufl., Leipzig, 1893, 40.

[2] Wm. Behrens: *Tabellen zum Gebrauch bei mikroskopischen Arbeiten.* Braunschweig, 3te Aufl., 1898, 5c.

[3] Perhaps the 9 in the original reference was, by a typographical error, an inverted 6.

[4] Alfred Himmelbauer: *Bemerkungen über das Phenylsulfid.* Centralbl. f. Min., etc., 1909, 396.

[5] Schroeder van der Kolk: *Tabellen*, etc., p. 11.

[6] Fred Eugene Wright: *The methods of petrographic-microscopic research.* Carnegie Publication No. 158, Washington, 1911, 98, footnote.

TABLE OF REFRACTIVE INDICES OF VARIOUS IMMERSION LIQUIDS.—*Continued*

Substance	Index	Temp.	Light	Sp. gr.	Boil.-pt.	Formula	Authority
Ethylene chloride	1.44439	20.0°	D	1.25	84°	$C_2H_4Cl_2$	Weegmann.
Petroleum	1.45						Wright.
Lavender oil[1]	1.461			0.88	188°		S. v. d. Kolk.
Carbon tetrachloride	1.4656	12.3°	D	1.61	77°	CCl_4	Gladstone.
Turpentine[1]	1.47212	20.7°	D	0.89	162°	$C_{10}H_{16}$	v. d. Willigen.
Glycerine	1.47293	20.0°	D	1.26		$C_3H_8O_3$	Landolt
Olive oil	1.4763	0.0°	D	0.92			Olds.
Beechnut oil	1.477			0.92			S. v. d. Kolk.
Almond oil	1.4782	0.0°	D				Olds.
Castor oil	1.478			0.96	265°+		S. v. d. Kolk.
Castor oil	1.481	16.0°					Behrens.
Toluol	1.49552	20.0°	D	0.87	110°	C_7H_8	Brühl.
Xylol, ortho-	1.4966	18.0°	D	0.86	136°	C_8H_{10}	Gladstone.
Xylol, meta-	1.5020	15.5°	D	0.87		C_8H_{10}	Gladstone.
Xylol, para-	1.4846	16.0°	D	0.85		C_8H_{10}	Gladstone.
Benzol[2]	1.4979	21.5°	D	0.88	80°	C_6H_6	Gladstone.
Pseudocumol	1.4801	12.0°	D	0.84	170°	C_9H_{12}	Gladstone.
Sandal wood oil	1.507	16.0°					Behrens.
Ethyl iodide	1.51307	20.0°	D	1.93	73°	C_2H_5I	Haagen.
Cedar wood oil[1]	1.516			0.98	237°		S. v. d. Kolk.
Cedar wood oil	1.510	16.0°					Behrens.
Monochlor benzol	1.527			1.13	132°		S. v. d. Kolk.
Ethylene bromide[3]	1.53789	20.0°	D	2.18	130°	$C_2H_4Br_2$	Weegmann.
Fennel oil	1.538			0.96	188°		S. v. d. Kolk.
Canada balsam	1.54±						See Art. 243.
Clove oil[1]	1.544			1.05	253°		S. v. d. Kolk.
Clove oil	1.533						Behrens
Bitter almond oil	1.54638	20.0°	D	1.05		C_7H_6O	Landolt.
Anise oil	1.54754	21.4°	D	0.98	220°		v. d. Willigen.
Nitro benzol[4]	1.55291	20.0°	D	1.20	209°	$C_6H_5NO_2$	Brühl.
Dimethyl aniline	1.55873	20.0°	D	0.96	192°	$C_8H_{11}N$	Brühl.
Monobrombenzol	1.561			1.52	155°		S. v. d. Kolk.
Orthotoluidine[4]	1.572			0.99	198°		S. v. d. Kolk.
Aniline[1, 4, 6]	1.58629	20.0°	D	1.02	138°	C_6H_7N	Brühl.
Bromoform[4, 5]	1.5890	20.0°	D	2.82	151°	$CHBr_3$	Jahn.
Monochloranaline	1.592			1.24	207°		S. v. d. Kolk.
Cassia oil	1.6026	22.5°	D	1.04			Baden-Powell.
Cassia oil	1.58624	20.0°	D				Wiedemann.
Quiniline[1, 7]	1.6171	20.0°	D	1.09	237°	C_9H_7N	Berliner.
Quiniline[1, 7]	1.6262	10.0°	D	1.10		C_9H_7N	Gladstone.
Cinnamon oil[1, 6]	1.61879	23.5°	D	1.06	225°		v. d. Willigen.
Monoiodbenzol	1.621			1.83	188°		S. v. d. Kolk.
Carbon bisulphide[8]	1.62761	20.0°	D	1.26	46°	CS_2	Ketteler.
Phenyl sulphide	1.635	18.5°	D				Himmelbauer.
α-Monochlornaphthalene	1.639			1.20	260°		
α-Monobromnaphthalene.	1.64948	20.0°	D	1.50	277°		Walter.
α-Monobromnaphthalene.	1.65114	16.5°					Nasini.
α-Monobromnaphthalene.	1.66102	23.5°					Dufet.
Phosphorus tribromide.	1.6866	25.0°					Zirkel.
Cadmium borotungstate solution.[9]	1.70						Zirkel.
Potassium mercuric iodide solution.[10]	1.7167	18.0°	D	3.11			Goldschmidt.

See end of table for notes.

TABLE OF REFRACTIVE INDICES OF VARIOUS IMMERSION LIQUIDS.—*Continued*

Substance	Index	Temp.	Light	Sp. gr.	Boil.-pt.	Formula	Authority
Methylene iodide[11]	1.7421	19.0°	D	3.32	1.81°	CH_2I_2	Gladstone.
Methylene iodide[11]	1.7559	10.5°	D	3.34	.....	CH_2I_2	Gladstone.
Barium mercuric iodide solution.[12]	1.7931	23.0°	D	3.564	.....	.....	Rohrbach.
Sulphur in methylene iodide	1.8	.....	.....	.....	.....	.....	Bertrand.
	1.83	.....	.....	.....	.....	.....	S. v. d. Kolk.
Sulphur in methylene iodide	1.79	.....	.....	.....	.....	.....	Wright.
Molten sulphur	1.89±	130°	.....	.....	.....	.....	S. v. d. Kolk.
Molten sulphur	1.93±	110°	.....	.....	.....	.....	S. v. d. Kolk.
Mercury methyl	1.93	.....	.....	.....	.....	.....	Zirkel.
Phosphorus in CS_2	1.95	20.0°	.....	.....	.....	.....	Retgers.
Molten phosphorus	2.075	44.0°	D	1.75	.....	P	Retgers.
Molten phosphorus	2.11311	44.0°	H_β	.....	.....	.....	Damien.
Selenium	2.92	.....	D	.....	.....	Se	Merwin & Larsen.

[1] Oxidizable. Should be kept from air.

[2] Very useful for cleaning oil from minerals.

[3] Extremely poisonous.

[4] Sensitive to light.

[5] If the grains float, use a cover-glass.

[6] Strong dispersion.

[7] Hygroscopic. Add a piece of KOH to the liquid.

[8] Very volatile and can be used only with a cover-glass. It should be allowed to flow under the edge after the cover-glass has been placed over the mineral.

[9] Klein's solution.

[10] Thoulet's solution. Very poisonous.

[11] Sensitive to light. The iodine which separates may be removed with copper.

[12] Rohrbach's solution.

In the above table the names printed in italics are those recommended by Schroeder van der Kolk, and most of them are miscible.

For practical use in petrographic work, the difference between the indices of each fluid and the one next succeeding it need not be less than 0.005. They should be kept in well-stoppered bottles, systematically arranged in a wooden rack or, better, in a covered box. The bottles, doubly closed by stopper and cap, and provided with a convenient glass dropper, should be small enough so that no great amount of fluid is necessary to fill them, the half-ounce (15 c.c.) size being ample. Kept in such bottles, the amount of change in values is not great. A set of oils, prepared by the writer and tested after two years, showed a maximum change of 0.003. De Lorenzo and Riva[1] determined the indices of a set of oils after three to six months. No statement is made in regard to the care taken of the liquids in the meantime. The following values were found, tests being made with an Abbe-Pulfrich refractometer.

[1] De Lorenzo and Riva: Review in Zeitschr. f. Kryst., XXXV (1902), 501–502. *Die Krater von Vivara auf den Phlegreïschen Inseln*, Mem. Roy. Acc. Sci., Napoli, X (1901), 1–60.

	I(18°)	II (18°)
Lavender oil	1.4650	1.4644
Cedar oil	1.4738	.....
Juniper oil	1.4850	1.4855
Fennel oil	1.5090	1.5095
Mixture of lavender, fennel, and cinnamon oils	1.5170	1.5178
Mixture of lavender, clove, and cinnamon oils	1.5208	1.5193
Clove oil	1.5270	1.5280
Mixture of clove and cinnamon oils	1.5347	1.5336
Wintergreen oil	1.5363	1.5365
Almond oil	1.5396	1.5412
Anise oil	1.5562	1.5563
Mixture of clove and cinnamon oils	1.5590	.....
Mixture of clove and cinnamon oils	1.5750	1.5751
Cinnamon oil (Goa)	1.5840	1.5830
Cinnamon oil (Ceylon)	1.6033	1.5980

Among the various combinations of liquids which may be used, those proposed by Wright[1] are very good. He prepared a set of immersion fluids as follows and found a change of not over 0.002 in a year. For temperature, there is a decrease of about 0.001 for every 3° C.

Mixtures of petroleum and turpentine	1.450–1.475
Mixtures of turpentine and ethylene bromide or clove oil	1.480–1.535
Mixtures of clove oil and α-monobromnaphthalene	1.540–1.635
Mixtures of α-monobromnaphthalene and α-monochlornaphthalene	1.640–1.655
Mixtures of α-monochlornaphthalene and methylene iodide	1.660–1.740
Sulphur dissolved in methylene iodide	1.740–1.790
Methylene iodide, antimony iodide, arsenic sulphide, antimony sulphide, and sulphur	1.790–1.960

For minerals having very high refractive indices, Merwin and Larsen[2] used molten sulphur, molten selenium, and mixtures of the two, these substances being miscible in all proportions when in a molten condition. The mixtures are prepared by placing the required weight of powdered selenium in a 3-in. test-tube, heating it until the mineral is thoroughly fused, and allowing it to cool. The proper amount of pure flowers of sulphur is now added, and the mixture heated just enough to allow thorough mixing with a glass rod. As the material cools it is gathered on the rod, and finally cut into small fragments. These may now be returned to the tube, which should be corked, and preserved for use. One or two grams are sufficient to examine a hundred minerals.

To determine refractive indices with this preparation, a small piece of it and a little of the mineral, finely pulverized, are heated together on an object-

[1] Fred. Eugene Wright: *Op. cit.*, 96.

[2] H. E. Merwin and E. S. Larsen: *Mixtures of amorphous sulphur and selenium as immersion media for the determination of high refractive indices with the microscope.* Amer. Jour. Sci., XXXIV (1912), 42–47.

Both sulphur and selenium had long previously been used as immersion fluids with high refractive indices. Mixtures of selenium and sulphur or arsenic were used by Marpmann. [G. M(arpmann): *Das Selen als Einschlussmittel für Diatomaceen.* Zeitschr. f. angew. Mikrosk., IV (1898), 6–8.] Marpmann also used selenium dissolved in selenium-ethyl $Se(C_2H_5)_2$.

and under a cover-glass, over a small flame, until the preparation is liquid, when the two are mixed and pressed into a thin film. The film is again heated for half a minute until bubbles begin to appear when it is again pressed thin and cooled, after which the determination is made in the usual manner. With care no appreciable amount of sulphur will be vaporized. The cooled mixtures, rich in selenium, have a deep red color and remain amorphous for months, those very rich in sulphur are yellow to orange, and may crystallize immediately on cooling. With less than 15 per cent. Se this crystallization takes place so readily that they are not well adapted to accurate work.

Owing to the high dispersion of the selenium, it is desirable to use monochromatic light for accurate work, a simple method being to make a screen by pressing a bit of heated selenium between an object- and cover-glass, and placing it on the eyepiece. The transmitted light gives a wave length approximately equivalent to that of lithium. With white light and colorless minerals, the light and shade effect may not be seen in large grains owing to the excess of illumination. In such cases the smaller grains, which are more deeply covered by the mixture, may be used.

The following table gives the refractive indices of different mixtures for lithium and sodium flames.

Per cent. Se	n_{Li}	n_{Na}	Equivalent wave length in $\mu\mu$
0.0	1.978	1.998	.....
9.0	2.000	2.022	.....
17.6	2.025	2.050	.....
25.0	2.050	2.078	.....
31.8	2.075	2.107	.....
37.5	2.100	2.134	.....
43.2	2.125	2.163	580
48.2	2.150	2.193	605
53.0	2.175	2.220	615
57.0	2.200	2.248	620
64.0	2.250	2.307	630
70.0	2.300	2.365	633
75.0	2.350	2.423	636
80.0	2.400	2.490	640
87.7	2.500	2.624	645
93.8	2.600	2.755	652
99.2	2.700	2.90	662
100.0	2.716	2.92	665

To fill the gap between fluids having refractive indices from 1.33 to 1.80 and from 2.1 to 2.4, Merwin[1] proposed solutions of iodoform, tri-iodide of arsenic, tri-iodide of antimony, tetra-iodide of tin, and sulphur in methylene iodide. With various proportions dissolved in 100 parts of methylene iodide,

[1] H. E. Merwin: *Media of high refraction for refractive index determinations with the microscope; also a set of permanent standard media of lower refraction.* Jour. Washington Acad. Sci., III (1913), 35–40.

fluids of refractive indices between 1.764 and 1.868 were obtained. Fluids from 1.74 to 2.28 were obtained by dissolving arsenic trisulphide in methylene iodide near its boiling-point. Merwin also prepared resin-like substances with indices between 1.68 and 2.10 by dissolving tri-iodides of arsenic and antimony in piperine. For media between 2.1 and 2.6 he used mixtures of amorphous sulphur and arsenic trisulphide. Other media were mixtures of piperine and rosin for indices between 1.546 and 1.682, and mixtures of rosin and camphor for 1.510 to 1.546.

DETERMINATION OF THE REFRACTIVE INDICES OF FLUIDS

230. Introductory.—In the previous method for the determination of the indices of refraction of minerals, it is required to have liquids of known indices. The determination of the indices of these liquids may be made most accurately with an Abbe-Pulfrich refractometer, but such an instrument is not always available, and a method for determining them by means of the microscope itself is a great convenience, especially for checking the values after the liquids have been kept on hand for a number of years.

231. Smith's Method (1885).—As long ago as 1813 Brewster[1] determined the refractive indices of liquids by an application of the Duc de Chaulnes' method, and a similar method was given by Becquerel and Cahors[2] in 1840. Both methods require the use of a considerable amount of the fluid whose refractive index is to be determined, and the result must be computed mathematically. A method, based on the same principle, but requiring only a small amount of material and no calculations, was devised by Smith[3] in 1885.

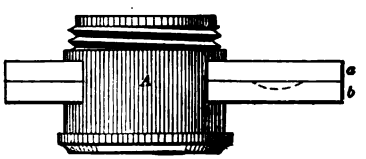


FIG. 360.—Smith's apparatus for determining the refractive index of a fluid.

The instrument, by means of which the refractive indices of fluids are measured, consists of a short cylinder (*A*, Fig. 360) which is inserted at the lower end of the tube of the microscope, just above the objective clutch. Sliding in this cylinder are two slips of crown glass *a* and *b*, 2 in. long, 1/2 in. wide, and 1/10 in. thick, and having a refractive index as nearly as possible the same as that of the cover-glass. One of these slips *b* has a polished concave depression, one-third or more of the thickness of the glass, ground in it near one end.

[1] Art. 209, *supra.*

[2] Art. 210, *supra.*

[3] H. L. Smith: *Device for testing refractive index.* Amer. Mon. Microsc. Jour., VI (1885), 181–182.

Idem: *Device for testing refractive index of immersion fluids.* Proc. Amer. Soc. Microsc., 8th annual meeting, Cleveland, VII (1885), 83–85.

The method of determining the refractive index of a fluid is to place a drop of it in the depression of the lower glass slip, place above it the other, thus squeezing a thin film of the medium between the two, and insert it in the slot of the adaptor. With the slips in the position shown in the figure and with a 1 in. objective inserted in proper position beneath it, it will be found that there has been no appreciable change in the focus of the instrument. The microscope is now focussed sharply upon some clearly defined object, then the slips are pushed in until the liquid lens lies directly back of the objective. If the medium is homogeneous with the glass of the slips, there will be no change in focus or definition, and no chromatic aberration. Since no immersion oil known is strictly homogeneous in this sense, although it may have the same refractive index as the glass, the focus may be unchanged although a colored rim will appear. If the fluid being tested is of a different refractive index it will be necessary to change the focus of the microscope, the amount depending upon the value of the index. Working with a few fluids of known refractive indices, one may mark, upon the side of the tube, the positions of the focus for different values, the rack and pinion adjustment being used, the fine adjustment remaining continually the same. Thus if the cavity is filled with cinnamon oil we get a certain mark for a value of 1.6; using the same object, objective, and eyepiece, we get another of 1.33 for water, and still others for cedar-oil, glycerine, clove-oil, etc.[1] With these points scratched on the side of the tube, by interpolation the intermediate values may be easily determined, the distance between 1.3 and 1.6 being about half an inch.

232. Pauly's Method (1905).—Pauly's[2] method for determining refractive indices under the microscope is extremely simple, and he claims that it is correct to 2 or 3 in the fourth decimal place. It is based on a modification of Ambronn's[3] method.

The indicatrix of uniaxial crystals is an ellipsoid of rotation, and

$$\epsilon_1 = \frac{\omega\epsilon}{\sqrt{\omega^2\sin^2\varphi + \epsilon^2\cos^2\varphi}} \qquad \text{(Eq. 9, Art. 53.)}$$

is the equation of the index of refraction of a wave whose normal makes an angle of φ with the c axis. Every section of the indicatrix is an ellipse and, in a plate cut parallel to the principal axes, all values of indices, intermediate between the values of the principal indices, will lie on the ellipse.

If a plate of calcite, cut parallel to c, is placed on the stage of the micro-

[1] See table Art. 229.

[2] Anton Pauly: *Ueber eine einfache Methode zur Bestimmung der Brechungsexponenten von Flüssigkeiten.* Zeitschr. f. wiss. Mikrosk., XXII (1905), 344–348.

[3] H. Ambronn: *Ueber eine neue Methode zur Bestimmung der Brechungsexponenten anisotroper mikroskopischer Objecte.* Ber. Akad. Wiss. Leipzig, 1893, 316–318. The method is described in Art. 224, *supra.*

scope, and upon it is placed a drop of the fluid whose index (between $\omega = 1.6585$ and $\epsilon = 1.4864$ of the calcite) is to be determined, and it is then covered with a cover-glass, it will be found, when the polarizer only is inserted and the diaphragm beneath the stage is partially closed, that upon rotating the stage a certain amount, practically all of the inequalities on the surface of the calcite, as well as the border of the drop, will disappear.[1] The cause of this is that the polarizer permits only rays vibrating parallel to one direction to pass through the calcite plate, and the rays which reach the eye have an index of refraction equal to that radius of the calcite index-ellipse which is parallel to the vibration direction of the nicol. When the index of refraction of the calcite plate exactly equals that of the fluid, which is isotropic, all inequalities between the two disappear and the light passes through as though there were but a single medium. The angle A at disappearance is read from the stage graduations, the stage is turned until the lines again disappear, and the angle B is read. $1/2\ (A-B) = \varphi$ or $90° - \varphi$, as the case may be (Fig. 361). The value of the index may be computed from the formula, or it may be determined graphically from the diagram (Fig. 362). One must note whether the angle φ or $90° - \varphi$ was read, which is easily done by making a mark on the calcite in the direction of the c axis (the direction of ϵ). The diagram is computed for angles from c to ω.

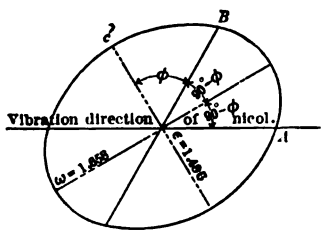


FIG. 361.—Diagram illustrating Pauly's method for determining refractive indices.

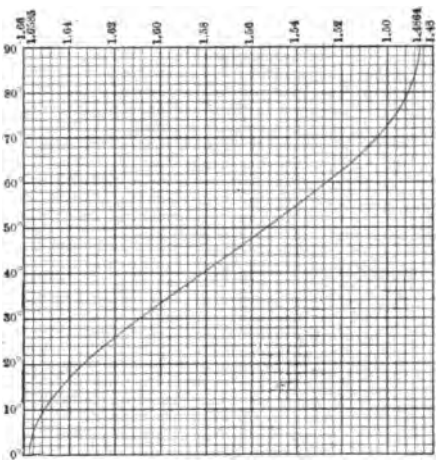


FIG. 362.—Diagram for determining the indices of refraction of liquids by the amount of rotation of the stage necessary to produce a disappearance of the boundaries when placed on a calcite plate ($90° - \varphi$).

If no orientated section is at hand, a cleavage plate of calcite may be taken, but here the range in indices is only from 1.567 to 1.658. For fluids having a higher index than ω of calcite, siderite, in which $\epsilon = 1.643$ and $\omega = 1.872$ may be substituted. Pure calcite or siderite must be used, otherwise the refractive indices will be different from those here given.

The writer has been unable, by Pauly's method, to obtain results closer than 3 in the second decimal place.

[1] Cf. the method of Schroeder van der Kolk, Arts. 227–228.

233. Michel-Lévy's Indicators.—Since the refractive index of a mineral may be determined by immersing it in a fluid of known index, inversely that of the fluid may be found by immersing in it a mineral of known index.

A series of such indicators was proposed by Michel-Lévy,[1] in 1894. Each fragment was oriented in a definite direction and all were mounted on a number of glass plates in the order of increasing indices. The scale was composed of the following minerals:

Fluorite	1.433
Hauynite	1.496
Leucite	1.508
Orthoclase	1.526 and 1.519
Microcline	1.529 and 1.523
Albite	1.540 and 1.532
Cordierite	1.589 and 1.532
Oligoclase	1.542 and 1.543
Nephelite	1.547 and 1.543
Quartz	1.553 and 1.544
Andesine	1.556 and 1.549
Labradorite	1.562 and 1.554
Anorthite	1.588 and 1.575
Melilite	1.641 and 1.621
Apatite	1.638 and 1.634
Andalusite	1.643 and 1.632

The objection to the indicators of Michel-Lévy is that not only may the refractive indices of the same minerals be different in specimens from different localities, but even in those from the same quarry, consequently each mineral used must be carefully tested before being mounted. Another objection is that it is difficult to find minerals differing by uniform amounts, and a third, that anisotropic crystals with different indices in different directions must be used for some of the indicators.

234. De Souza-Brandão's Indicators.—The indicators proposed by de Souza-Brandão[2] and prepared by Fuess are made of small squares of glass, 2 mm. on a side and 1 mm. thick, and of different refractive indices. Being isotropic, the values are the same in all directions, and, since glass is amorphous and homogeneous, many squares can be cut from one specimen whose index of refraction is accurately determined, a great advantage in the commercial preparation of such scales. The scale consists of 35 different indicators mounted, 2.5 mm. apart, in Canada balsam, on seven object slips, 47 mm. × 27 mm., and with the index values engraved on the glass opposite each. The seven slides are as follows:

[1] A. Michel-Lévy: *Étude sur la détermination des feldspaths.* Première fascicule, Paris, 1894, 62–63.

[2] V. de Souza-Brandão: *Ueber eine Skala von Lichtbrechungs-Indicatoren.* Centralb. f. Min. etc., 1904, 14–18.

I	II	III	IV	V	VI	VII
1.434	1.494	1.523	1.552	1.590	1.631	1.680
1.450	1.501	1.531	1.558	1.604	1.648	1.693
1.465	1.509	1.536	1.564	1.614	1.657	1.702
1.478	1.512	1.539	1.573	1.620	1.666	1.717
1.486	1.516	1.548	1.580	1.625	1.673	1.735

To determine the index of an unknown liquid, a few drops are placed on the indicator and covered with a cover-glass, and the relative indices noted, either by inclined illumination or by the Becke method described below. Another method is to fill a small glass tray to a depth of 1/2 mm. with the liquid whose index is to be determined and invert in it one of the test plates. Such a tray, 11 mm. deep and 2 mm. longer and broader than the indicator slips, is furnished with each scale. The determinations of the relative indices are made through the object slip and are accurate to about 0.003.

De Souza-Brandão also suggested that instead of using so many different refractive oils, one would better use a single, dilutable fluid. For this purpose Sonstadt's (Thoulet's) solution[1] is excellent. It is miscible with water in all proportions, and does not act upon Canada balsam, the cement of the scale. The action upon Canada balsam, after a time, of many of the oils and of α-monobromnaphthalene and methylene iodide, is a great objection to their use with these indicators. Methylene iodide is not well adapted for general use in the determination of refractive indices, since the fluids with which it is miscible are very volatile, and the refractive index of the mixture changes rapidly. Sonstadt's solution may most conveniently be made up in ten different strengths, having specific gravities of 1.5, 1.7, 1.9, 2.1, 2.3, 2.7, 2.9, 3.0, and 3.1, and corresponding to indices ranging from 1.42 to 1.72.[2] The mixtures should be kept in not too small pipette flasks. In each bottle two specific gravity indicators[3] may be placed, such that one just floats and one sinks when the liquid is of the proper specific gravity, consequently of proper index.

The advantage of using but one kind of fluid is that after approximately determining the refractive index of the mineral by immersion tests, a considerable quantity of the stock solution, nearest this value, may be slightly diluted until it reaches the exact index of the mineral. Its exact value may now be determined by means of a refractometer, Pauly's method, or the above mentioned scale. The used material may then be evaporated a trifle on the water bath[4] until it reaches a specific gravity slightly greater than that of the stock material from which it was taken. After pouring back, a

[1] Art. 454.
[2] Art. 215.
[3] Art. 481.
[4] Art. 454.

few drops of water will bring it to the proper specific gravity. The objection to Sonstadt's solution is that its first cost is considerable and it is extremely poisonous.

235. Clerici's Method (1907).—A simple method of directly determining the refractive index of a fluid, under the microscope, was proposed by Clerici.[1] It has the advantage of requiring no change of indicators, and may be used to determine the refractive index of a volatile fluid with changing index the instant it corresponds with that of the solid immersed.

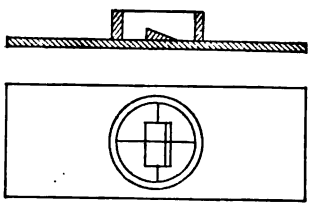


FIG. 363.—Clerici's apparatus.

The apparatus (Fig. 363) consists, simply, of an object slip, in the center of which two lines are engraved crossing at right angles. Above this cross, and with its refracting edge parallel to one of the lines, is cemented a small glass prism, and around this a short section of a glass tube, making, thus, a shallow vessel with a prism cemented in the bottom. The microscope is focussed sharply upon the engraved lines, which are rotated until they are parallel to the cross-hairs. The fine adjustment screw is read, after which the ring is filled with the unknown fluid. It will be found that the engraved lines are now displaced a certain amount, the distance depending upon the refractive index of the fluid, and in order to bring them back into position, it is necessary to move the adjustment screw. By making determinations upon a series of fluids of known indices, a curve may be constructed by the aid of which the index of any unknown liquid may be found, using, of course, the same combination of ocular, objective, and tube length. Clerici claims the method to be accurate to the third decimal place.

[1] Enrico Clerici: *Sulla determinazione dell'indice di rifrazione al microscopio.* Rendiconti della Reale Accad. dei Lincei, Roma, XVI (1907), 336–343.

CHAPTER XV

OBSERVATIONS BY ORDINARY LIGHT (Continued)

Determination of the Refractive Indices of a Mineral by the Becke Method

236. Becke (1893).—In 1893, Becke[1] called attention to the fact that at the contact between two transparent minerals of different refractive indices in thin sections, under certain conditions of illumination, the total reflection of some of the rays of light produces a characteristic phenomenon.

If one focusses accurately, with a medium-power objective, on the contact between two minerals having different indices of refraction, condenser and analyzer being removed, it will appear as a sharp line when it lies at right angles to the section. If, now, some of the light entering from below be cut off by means of a diaphragm, and the tube of the microscope be very slightly raised so as to throw the image somewhat out of focus, there will appear along the contact, but within the mineral having the higher index, a bright line which broadens upon raising the tube still farther and then disappears. On depressing the tube, the bright line appears at the edge of the mineral having the lower index.

The phenomenon observed depends upon the total reflection of the rays incident at more than the critical angle when passing from the denser to the rarer medium, and is explained, by Becke, by means of the illustration reproduced as Fig. 364.

Let AB and BC be two minerals in contact at B, and let the refractive index of A be less than that of C. Let 0 to 12 be convergent rays of light entering from below, and let the refraction of the rays above and below the mineral be disregarded since this is of no importance in the explanation.

The ray 0, entering the minerals perpendicular to their surfaces, suffers no refraction but passes straight through. The rays 1, 3, 5, 7, 9, and 11, travelling from the rarer medium A at the left, to the denser medium C, are bent toward the normal, and pass out to the right. The rays entering from the right, however, pass from a denser to a rarer medium. In such cases all rays impinging on the second at more than the critical angle, such

[1] F. Becke: *Ueber die Bestimmbarkeit der Gesteinsgemengtheile, besonders der Plagioklase. auf Grund ihres Lichtbrechungsvermögens.* Sitzb. Akad. Wiss., Wien, CII (1893), Abth. I, 358–378.

Idem: *Petrographische Studien am Tonalit der Rieserferner. Untersuchungsmethoden.* T. M. P. M., XIII (1892–3), 385–389.

as 2, 4, and 6, are totally reflected,[1] and emerge upon the same side, while those reaching the second surface at a smaller angle of incidence, such as the rays 8, 10, and 12, are refracted. In consequence, therefore, of the total reflection of certain rays, the light is unevenly disturbed, and the elevation of the objective shows the concentration of the light on the side of the mineral having the higher refractive index. The smaller the cone of entering light, down to the limit of the critical angle, the clearer will be the phenomenon observed, for if only the rays from 1 to 6, in the figure, enter from below, there will be no ray passing to the left as against six to the right. The less the difference between the indices of the two minerals, the greater will be the critical angle, consequently the smaller must be the size of the diaphragm

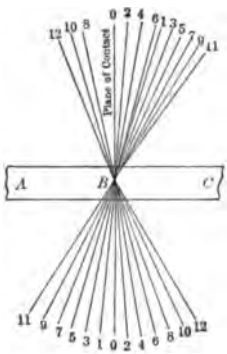


FIG. 364.

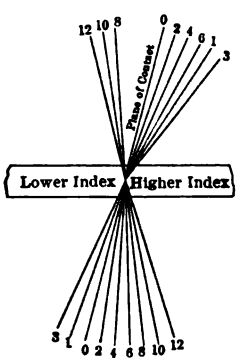


FIG. 365.

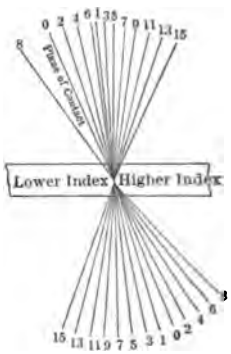


FIG. 366.

FIG. 364.—Becke's explanation of the bright line effect.

FIG. 365.—Becke line. The contact between the two minerals is inclined, the mineral with lower index lying above the other.

FIG. 366.—The Becke line. The contact between the two minerals is inclined, the mineral with higher index lying above the other.

used. Based upon this, there was proposed by Viola[2] a quantitative measure of the refractive indices.

Becke further calls attention to the fact that there is no difference in the phenomenon observed even if the contact between the two minerals is not quite vertical, provided the medium having the lower index lies above (Fig. 365). If, however, it lies below (Fig. 366), and the inclination is great enough, the bright line may appear to move the wrong way. In practice this is of no consequence since such contacts may be clearly recognized, under the microscope, by the shifting of the line when the focus is changed from the bottom to the top.

237. Hotchkiss' Explanation.—Hotchkiss[3] gives a somewhat different

[1] Art. 41 *supra*.

[2] Art. 241 *infra*.

[3] W. O. Hotchkiss: *An explanation of the phenomena seen in the Becke method of determining index of refraction.* Amer. Geol., XXXVI (1905), 305–308.

explanation as follows: Let AB, Fig. 367, be a cross-section of two minerals with indices of 1.50 and 1.70 respectively, and let the plane of contact be perpendicular to the page and represented by the line CD. Let the convergent light come from below and pass through the section. Ray 1 is refracted so as to meet the plane between the two media at the point n, at a distance above the point x of 1.87 times the length x–y. Ray 2 meets it at m, a distance equal to 1.56 times x–y. Ray 3 and 4, since B has the higher index, are refracted to meet the plane at points m' and n', higher than the similar rays in A, or at distances above y of 1.76 and 2.18 times x–y, respectively.

At the surface of contact between A and B the critical angle is 62° 10′, whereby all rays incident on y–z from B at an angle greater than 62° 10′, are totally reflected back into B. On the other hand, a portion of the light from A, incident upon y–z, is refracted into B. The ratio between the amount reflected and the amount refracted depends upon several factors. In proportion as the contact surfaces of A and B are highly polished, more light is reflected and less refracted; as the angle of incidence increases, more light is reflected and less refracted; and as the difference in the indices increases the amount of light reflected becomes greater. Since the contact surface of minerals in rocks is seldom smooth, the tendency is for a large part of the light from A to be refracted into B, and the condition obtains as shown in the figure—that for a certain vertical distance along the contact, approximately equal to mn', nearly all the light will be on the side of the mineral having the higher index. If the microscope is focussed within this vertical distance a band of light will be seen. If the tube is now raised, the band will be seen to broaden, as is evident from the directions of the refracted and totally reflected rays. If, on the other hand, the objective is lowered, the band becomes narrower and, finally, is brighter on the side of the mineral having the lower index, which is explained by the fact that the light in A, which is approximately the same in amount as that in B at this distance above the base of the section, is concentrated in a band of width mr, which is shorter than ms, and will, therefore, show greater intensity.

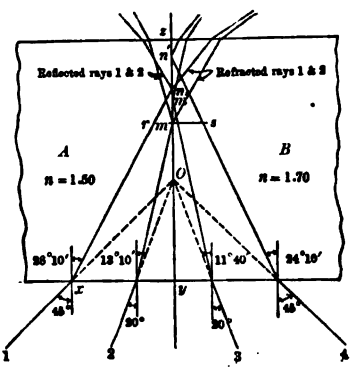


FIG. 367.—Hotchkiss' explanation of the Becke line.

If rays from B are incident upon y–z at an angle less than the criitcal angle (62° 10′ in the case illustrated), they will not be totally reflected, but will partly pass through into A. If there is sufficient light thus refracted, a bright band will be seen in A as well as in B when the objective is raised. It is important, therefore, to diaphragm the light entering the condensing

system to such an extent that all the light from B is totally reflected at the contact surface. This increases the relative brightness of the band seen in B.

Hotchkiss, further, computed the different values of the distances from y to m and n for other indices, and from these showed how, theoretically, the indices of minerals might be determined from the differences in value.[1] Practically the magnitude of the elevation of the tube, perhaps 0.0005 mm. for the change from 1.54 to 1.56, is too small to be measured accurately with the microscope.

238. Grabham's Explanation.—In the previous explanations of the Becke line, convergent light and more or less vertical contact was necessary. An explanation based on parallel rays, such as are ordinarily used, and inclined junction planes, was first suggested by Anderson to Grabham.[2]

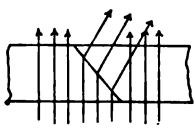


FIG. 368.

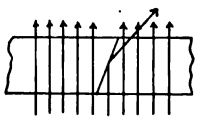


FIG. 369.

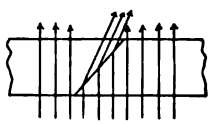


FIG. 370.

FIG. 368 TO 370.—Grabham's explanation of the Becke line.

When the plane of contact between the two minerals is inclined to that of the section, two cases may occur, depending upon whether the mineral of greater or less refractive index overlaps the other. In the former case (Fig. 368) the rays, coming from below, pass from a rarer to a denser medium and are, therefore, bent toward the normal at the point of contact. Under the second condition there are two cases. The rays, passing from a denser to a rarer medium, may fall upon the contact at more than the critical angle (Fig. 369) and be totally reflected, or they may reach it at a less angle and pass through but will be bent away from the normal. In any case the light is increased on the side of the mineral having the higher index of refraction.

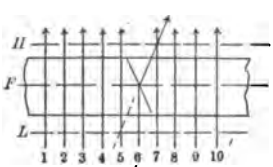


FIG. 371.

If, now, the objective is focussed upon the point where the light meets the contact (F, Fig. 371), the section will appear in focus and no bright line will be seen. If the tube is raised so that the focal plane is at H, the combination of ray 7 and the refracted ray 6 produces an increase of light at that point. If the tube is raised still farther, the point of light moves, progressively, farther and farther toward the mineral having the higher index, according as each vertical ray of light is crossed by the refracted ray. If,

[1] Cf. C. Viola: *Ueber eine neue Methode zur Bestimmung des Brechungsvermögens der Minerale in den Dünnschliffen.* T. M. P. M., XIV (1894–5), 554–562. See Art. 240 *infra.*

[2] G. W. Grabham: *An improved form of petrological microscope with some general notes on the illumination of microscopic objects.* Mineralog. Mag., XV (1910), 341–347.

on the other hand, the tube of the microscope is lowered so that the focal plane lies at L, the bright line will appear at the junction of ray 5 and the backward projection of the refracted ray 6. Since, under the microscope, ray 6 will appear to come from a point on its dotted backward extension, and not from the point 6, as the tube is lowered more and more, the bright line will pass progressively through its intersections with rays 5, 4, 3, etc.

239. Inclined Illumination.—Becke[1] called attention to the fact that the differences between the indices of refraction of two minerals could be brought out by the use of inclined illumination. For this purpose he displaced the lower diaphragm laterally and found that the edge of the image, opposite to the direction in which the diaphragm was displaced, became dark when the refractive index of the mineral was greater than that of the adjacent one, a law which was later similarly stated by Schroeder van der Kolk.[2] Becke produced inclined illumination, likewise, by an adaptation of the Exner[3]

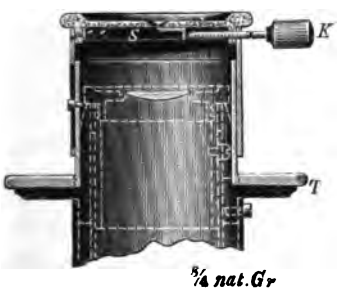


FIG. 372.—The Becke-Exner mikrorefractometer. 3/4 natural size. (Fuess.)

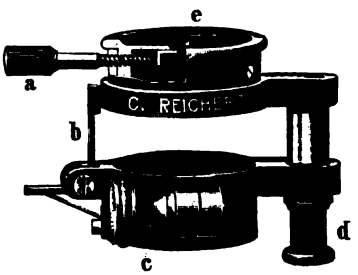


FIG. 373.—The Becke-Exner mikrorefractometer. (Reichert.)

microrefractometer. This instrument is shown, in the simplified form suggested by Becke,[4] in Fig. 372. It is placed on the end of the tube (T) of the microscope, and its upper part is extended until the opening of the diaphragm lies in the Ramsden disk; a position which may be recognized by the disappearance of the blue halo around the field. The disk S, which cuts off the rays from one side, is now moved laterally across the opening, by means of the screw K, until the wished-for effect appears. By the use of this instrument a difference of 0.001 in the refractive indices of two adjacent minerals may be recognized.

Another form of the Becke-Exner microrefractometer is shown in Fig. 370. This instrument is attached to a pivot d, so that it may be swung in or out of the field, the clip b holding it in position on the axis of the microscope.

[1] F. Becke: *Op. cit.*, Sitzb. Akad. Wiss. Wien, CII (1893), and T. M P. M., XIII (1892–3), 387.

[2] Art. 228.

[3] Sigm. Exner: *Ein Mikro-Refractometer.* Arch. Mikrosk. Anatomie, XXV (1885), 97–112.

[4] C. Leiss: *Die optischen Instrumente der Firma R. Fuess.* Leipzig, 1899, 246.

240. Viola-de Chaulnes-Becke Method.—Viola,[1] in 1895, worked out a combination of the de Chaulnes and Becke methods by means of which, theoretically, the refractive indices of a mineral can be determined.[2]

Let M_2 and M_1 (Fig. 374) be two minerals in contact whose indices of refraction are n_2 and n_1, respectively, and let n_2 be less than n_1. A high-power objective is focussed on the upper surface of one mineral, and the micrometer adjustment of the microscope is read. The objective is then lowered, and the Becke line, on the side of the mineral with the higher refractive index (M_1), will be seen to become gradually narrower until it crosses the line of contact, which will be in sharp focus at a_1. The micrometer is again read, the difference between the two readings giving the value of e_1. The distance e is now measured, and the refractive index of M_1 determined by the equation $n_1 = \frac{e}{e_1}$. The index of the mineral (M_2) having the lower index is determined by measuring the distance e_2 from the surface to the point where the dark shadow disappears and the bright line begins to show on the side of M_2. Its value is found from the equation $n_2 = \frac{e}{e_2}$. The writer has found it impossible, in practice, to locate the positions of a_1 and a_2 closely enough for accurate determinations.

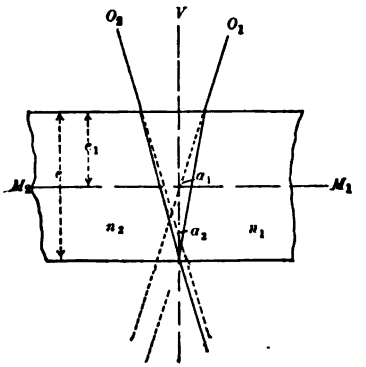


FIG. 374.—Viola-de Chaulnes-Becke method.

241. Viola-Becke Method.—The difference between the refractive indices of two abutting minerals in a thin section, according to Viola,[3] are proportional to the square of the opening of the iris diaphragm necessary to see the Becke line, and may be determined by the formula

$$n_2 - n_1 = kD^2,$$

where n_2 and n_1 are the indices of refraction of the two minerals, D the diameter of the diaphragm opening, and k a constant.

If, then, the refractive index of one mineral is known, that of any mineral in contact with it may be determined by measuring the greatest diameter of the iris diaphragm at which the Becke line is visible. The constant k may be determined by means of two known substances. For example, it

[1] C. Viola: *Ueber eine neue Methode zur Bestimmung des Brechungsvermögens der Minerale in den Dünnschliffen.* T. M. P. M., XIV (1894–5), 554–562.

[2] Cf. Hotchkiss, Art. 237 *supra.*

[3] C. Viola: *Methode zur Bestimmung des Lichtbrechungsvermögens eines Minerales in den Dünnschliffen.* T. M. P. M., XVI (1896–7), 150–154.

may be measured in a section of apatite cut at right angles to the c axis ($\omega = 1.638$) and embedded in a fluid having an index of 1.549. The objective is first sharply focussed, then very slightly raised, and the diaphragm slowly closed until a bright line appears in the apatite at the contact with the fluid. D, in this case, let us assume, was 15 mm.; the formula becomes

$$k = \frac{1.638 - 1.549}{(15)^2} = 0.00039.$$

As an example of measurement, a crystal of pyroxene in contact with Canada balsam was taken. The crystal was turned until its γ direction was parallel with the vibration direction of the lower nicol. The iris diaphragm was now closed until the Becke line appeared, when D was found to be 20.5 mm., from which the value of γ was found to be 1.713. The section was now turned 90° so that α was parallel with the nicol. Here $D = 19$ mm., whereby $\alpha = 1.690$.

Viola claims that as soon as the eye is sufficiently trained, this method is accurate to the third decimal place, and that it is preferable to any previously discovered method on account of its simplicity, and because it is not necessary to determine the thickness of the section. By this method he determined that the small inclusions in certain leucites—crystals so small that no interference colors were shown and no other property than their refractive indices were determinable—were pyroxene and apatite.

242. Practical Applications of the Becke Method.—The Becke method is capable of wider application than simply to determine which of two adjacent minerals has the higher refractive index. If that of one mineral is known, the relation of the other to it is known. If two adjacent minerals are known and one has a refractive index higher, and one a lower than the unknown, there are established definite limits for the unknown. The Becke line[1] may be used also with the immersion method. It is more sensitive and easier to see than the light and dark borders produced by inclined illumination, and, at the same time, it may be seen over the whole field of the microscope at once. It makes, for example, the process of determining the relative amounts of orthoclase and quartz in a fine granular groundmass very simple, for if the objective is very slightly thrown out of focus, either up or down, the two minerals stand out clearly, one from the other. No special preparation of material is necessary, no attachments not ordinarily provided with a microscope are needed, and no elaborate computations are required to obtain accurate results. A series of tests, made by de Lorenzo and Riva,[2] show it to be accurate to ± 0.001.

[1] The term **Becke line** was proposed by W. Salomon: *Ueber die Berechnung des variablen Werthes der Lichtbrechung in beliebig orientirten Schnitten optisch einaxiger Mineralien von bekannter Licht- und Doppelbrechung.* Zeitschr. f. Kryst., XXVI (1896), 182.

[2] G. de Lorenzo and C. Riva: Mem. Acc. Sci. Napoli, X (1901), 1–60.* Review in Zeitschr. f. Kryst., XXXV (1902), 501–2.

The measurement of the refractive indices in two directions in anisotropic crystals, and their comparison with the two refractive indices of some known substance, was applied by Becke[1] to the determination of the feldspars, and was found to give good results. If adjacent sections of quartz and feldspar, with extinction directions parallel, are chosen, the vibration direc-

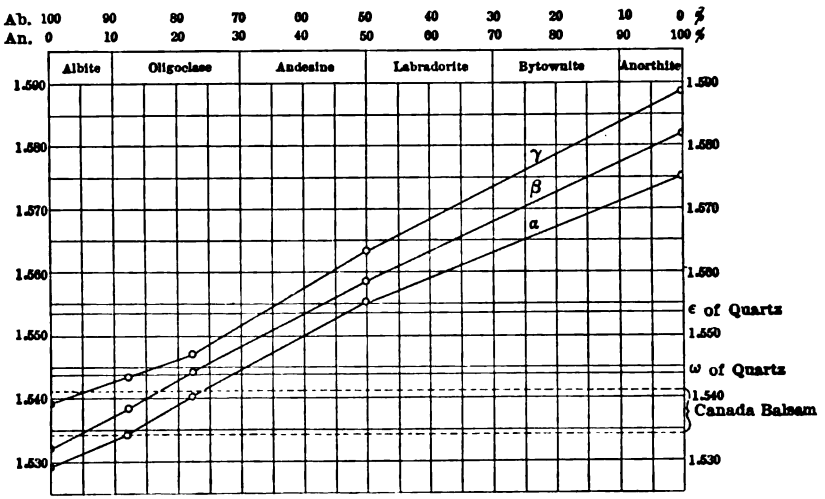


FIG. 375.—Curve showing the refractive indices of the lime-soda feldspars. (Modified from Rosenbusch-Wülfing.)

tions, necessarily, must be parallel also, consequently if the sections happen to be cut along the maximum and minimum directions (known by their maximum interference colors), ω and ϵ will be parallel or at right angles to α and γ. Taking the values of α, β, and γ of the plagioclases and ω and ϵ of quartz as given by Rosenbusch, the relationships are shown by Fig. 375, from which the following grouping is derived:

Group	Parallel position	At right angles	Feldspars
I.	$\omega > \alpha$ $\epsilon > \gamma$	$\omega > \gamma$ $\epsilon > \alpha$	Albite, $Ab - An_8An_1$
II.	$\omega > \alpha$ $\epsilon > \gamma$	$\omega \leqslant \gamma$ $\epsilon > \alpha$	Oligoclase, $Ab_8An_1 + Ab_3An_1$
III.	$\omega \leqslant \alpha$ $\epsilon > \gamma$	$\omega < \gamma$ $\epsilon > \alpha$	Oligoclase, $Ab_3An_1 - Ab_2An_1$
IV.	$\omega < \alpha$ $\epsilon \leqslant \gamma$	$\omega < \gamma$ $\epsilon > \alpha$	Andesine, $Ab_2An_1 - Ab_1An_2$
V.	$\omega < \alpha$ $\epsilon < \gamma$	$\omega < \gamma$ $\epsilon \leqslant \alpha$	Andesine, $Ab_1An_2 - Ab_1An_1$
VI.	$\omega < \alpha$ $\epsilon < \gamma$	$\omega < \gamma$ $\epsilon < \alpha$	Labradorite, Bytownite, Anorthite. $Ab_1An_1 - An$

From this it appears that the calcic plagioclases always have higher refract-

[1] F. Becke: *Op. cit.*, Akad. Wiss. Wien, CII (1893), and T. M. P. M., XIII (1892–3), 387–396.

ive indices than quartz, and albite and oligoclase-albite always lower. The other plagioclases may be separated from each other as shown.

But not only may quartz be used, but any other known mineral as well, for example, nephelite. Potassium feldspars and anorthoclase have all their refractive indices lower than ϵ of nephelite; the plagioclases bear to it the following relations.[1]

(Nephelite: $\omega = 1.542$, $\epsilon = 1.537$.)	In parallel position	At right angles
Albite,	$\omega > \gamma$ $\epsilon > \alpha$	$\omega > \alpha$ $\epsilon \lesseqgtr \gamma$
Oligoclase-albite,	$\omega > \gamma$ $\epsilon \lesseqgtr \alpha$	
Oligoclase,	$\omega \lesseqgtr \gamma$	$\omega \lesseqgtr \alpha$
Andesine, Labradorite, Bytownite, Anorthite,	$\omega < \gamma$ $\epsilon < \alpha$	$\omega < \alpha$ $\epsilon > \gamma$

A drawback to this method of Becke is that the sections of quartz used for comparison must be cut approximately parallel to crystallographic c, or the values of ω and ϵ will not be at their maxima. If those sections are chosen which give high interference colors, as was suggested by Becke, they may still not be parallel to c but may show that the section is thicker than normal. Salomon[2] overcame this difficulty by calculating the maximum value of the extraordinary ray for any section of quartz (Fig. 376) by means of the equation

$$\epsilon_1 = \frac{\epsilon\omega}{\sqrt{\epsilon^2 \sin^2\varphi + \omega^2 \cos^2\varphi}}$$

(Eq. 9a, Art 53),

where φ is the angle between the section and the optic axis. To determine the angle φ, Salomon made use of interference figures and measured the amount of inclination of the optic axis from the axis of the microscope. While the description of the method is in advance of the description of inter-

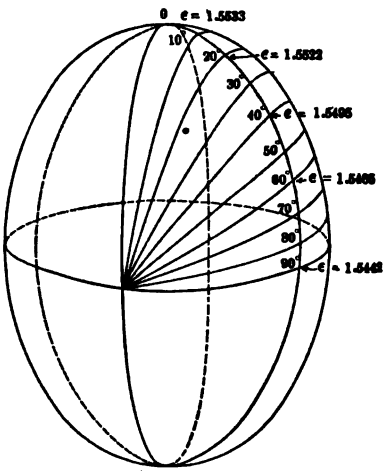


FIG. 376.

[1] Albert Johannsen: *Determination of rock-forming minerals.* New York, 1908, 76.

[2] W. Salomon: *Ueber die Berechnung des variablen Wertes der Lichtbrechung in beliebig orientirten Schnitten optisch einaxiger mineralien von bekannter Licht- und Doppelbrechung.* Zeitschr. f. Kryst., XXVI (1896), 178–187.

ference figures, it is, nevertheless, inserted here as the most convenient place for reference.

Four cases occur.

1. The center of the interference figure lies within the field of view.

2. The center of the interference figure lies beyond the field of view but the bars are sharp enough to permit, with sufficient accuracy, the measurement of their distances from the center of the field.

3. Similar to case 2, but the bars are too indistinct to permit of accurate measurements.

4. The section shows the characteristic figure of a section cut approximately parallel to crystallographic *c*.

In the first case, with the aid of a Bertrand lens and a micrometer ocular, the distance between the emergence of the optic axis and the center of the field is measured. The displacement is reduced to the true value of φ by means of Mallard's formula.[1]

$$\sin \varphi = \frac{kd}{\omega}, \tag{1}$$

in which k is a constant for the particular combination of lenses, tube length, etc., used, and d the measured distance. The constant should be determined previously by making a number of measurements on substances whose values of ω and φ are known.

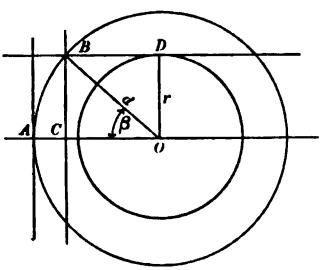


FIG. 377.

In the second case, the distance between centers may be determined in the following manner. The stage is first carefully centered and is then rotated until one of the black bars (*OC*, Fig. 377) passes exactly through the center *O*. The vernier is read at this position, after which the stage is rotated until the same bar passes through *D*, the limit of the field of view. Obviously the point of emergence of the optic axis has moved from *A* to *B*, about *O* as a center and with a radius of $OA = OB = d$, and through an angle β. If $BC = OD = r$, the radius of the field of view, then $\sin \beta = \frac{r}{d}$, and $d = \frac{r}{\sin \beta}$. Substituting in equation (1), we have

$$\sin \varphi = \frac{kr}{\omega \sin \beta}. \tag{2}$$

The accuracy of the measurements, in this case, depends upon the sharpness of the axial bars, which themselves increase in distinctness with increasing strength of double refraction, thickness of section, and convergence of rays from the condenser.

[1] Art. 411 *infra*.

In the third case, it is not possible to determine the value of φ accurately.

In the fourth case, the dark bars are generally too indefinite for the determination of how nearly the section is parallel to the optic axis. If exactly parallel, the value of φ, of course, is 0°.

All measurements, in the above determinations, should be repeated a number of times to insure accuracy.

In Becke's method for the determination of the plagioclase feldspars they are separated into six groups; by Salomon's method they may easily be separated by means of their birefringence, into nine groups, each differing from the next by 0.001. This would correspond to sections of quartz differing by 10° in their inclination to crystallographic c, whereby errors of less than 5° would make but slight differences in the results obtained.

The computed values of ϵ_1 for various angles of inclination are given in the table below and, graphically, in Fig. 378.

φ	ϵ_1	φ	ϵ_1
90°	1.5442	40°	1.5495
80°	1.5445	30°	1.5510
70°	1.5453	20°	1.5522
60°	1.5465	10°	1.5530
50°	1.5480	0°	1.5533

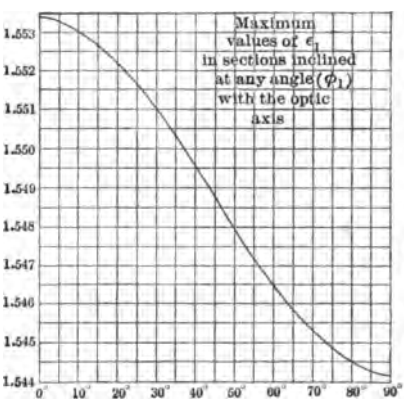


FIG. 378.—Maximum values of ϵ_1 in sections inclined at any angle (φ) with the optic axis.

From these determinations the value of the birefringence ($e-\omega$) for any particular section may be obtained, and, from that, the thickness of the section may be accurately determined.

A second drawback to the method of plagioclase determination, as proposed by Becke, is that in a rock section in which there is but little quartz and feldspar, few or no cases may be found in which adjacent grains of the two minerals extinguish simultaneously. The value for the refractive index of quartz in a direction parallel to the vibration directions of the unknown mineral may, however, be computed, and thus permit the use of any section.

When a plane polarized ray of light reaches a section of quartz cut in any direction except at right angles to the optic axis, it is broken up into two rays. These two rays, the extraordinary and the ordinary, on leaving the quartz, emerge at different points, it is true, but, from some other rays of light impinging at a slightly different point, there will likewise arise ordinary and extraordinary rays, so that, with a beam of light entering from below, from every point of the upper surface of the quartz, two rays having different indices will emerge. If, then, the vibration direction of the polarizer is not

parallel to one of the principal vibration directions of the quartz, the refractive index seen by the eye is not the true but only an apparent index, and consists of a combination of the two.

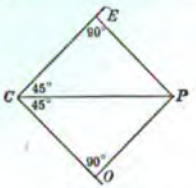


FIG. 379.

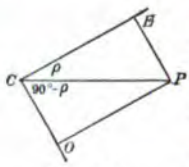


FIG. 380.

For example, let *CP*, Fig. 379, represent the amount and direction of the vibration of the light entering the quartz from the polarizer, and let the vibration directions of the latter lie exactly at 45° to the former. Then, by the resolution of forces, *CE* and *CO* will represent the extraordinary and ordinary components into which this ray is broken by the quartz. Since the amplitudes and intensities of the two rays are here equal, the resulting apparent refractive index will be just half that of the ordinary ray alone plus half that of the extraordinary alone, or $\frac{\omega+\epsilon_1}{2}$. If, however, the angle *ECP* has a different value, represented by ρ, we have (Fig. 380),

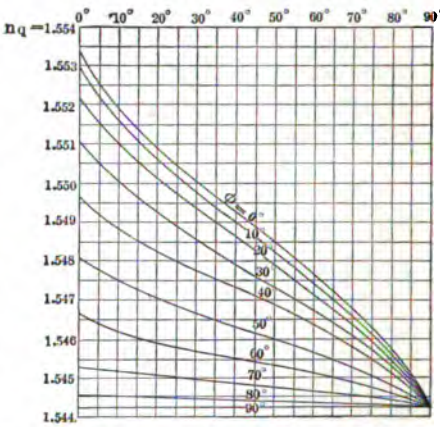
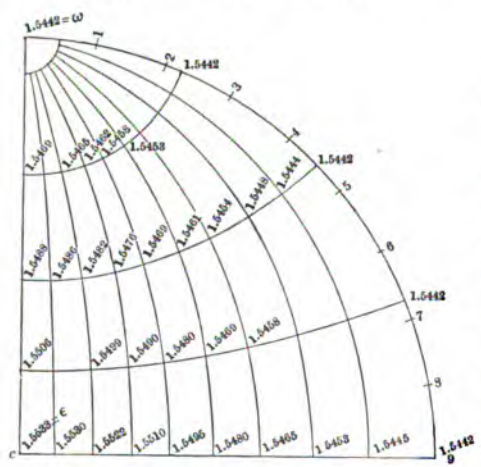


FIG. 381.—Diagram giving values of n_q for all values of ρ and ten values of φ.


1.5442=ω
1.5442
1.5442
1.5442

$$n_q = \frac{\omega + \epsilon_1 \operatorname{cotan} \rho}{1 + \operatorname{cotan} \rho}.$$

From this equation the apparent index of refraction in any section of quartz may be computed, consequently it is not necessary, in order to determine the refractive index of an unknown mineral, that the sections of quartz should have extinctions parallel with it. The proportion of usable sections in a rock-slice is thus greatly increased.

Computed by this formula, the following values of n_q were obtained.

Values for n_q for various values of ρ.

$\rho = 0°$	1.5442	1.5445	1.5453	1.5465	1.5480	1.5495	1.5510	1.5522	1.5530	1.5533
$\rho = 10°$	1.5442	.....	.....	.....	.....	.....	.....	.....	.....	1.5519
$\rho = 20°$	1.5442	.....	.....	.....	.....	.....	.....	.....	.....	1.5508
$\rho = 22\ 1/2°$	1.5442	.....	.....	1.5458	1.5469	1.5480	1.5490	1.5499	.....	1.5506
$\rho = 30°$	1.5442	.....	.....	.....	.....	.....	.....	.....	.....	1.5500
$\rho = 40°$	1.5442	.....	.....	.....	.....	.....	.....	.....	.....	1.5492
$\rho = 45°$	1.5442	1.5444	1.5448	1.5454	1.5461	1.5469	1.5476	1.5482	1.5486	1.5488
$\rho = 50°$	1.5442	.....	.....	.....	.....	.....	.....	.....	.....	1.5484
$\rho = 60°$	1.5442	.....	.....	.....	.....	.....	.....	.....	.....	1.5475
$\rho = 67\ 1/2°$	1.5442	.....	.....	.....	1.5453	1.5458	1.5462	1.5465	.....	1.5469
$\rho = 70°$	1.5442	.....	.....	.....	.....	.....	.....	.....	.....	1.5467
$\rho = 80°$	1.5442	.....	.....	.....	.....	.....	.....	.....	.....	1.5456
$\rho = 90°$	1.5442	1.5442	1.5442	1.5442	1.5442	1.5442	1.5442	1.5442	1.5442	1.5442
$\phi =$	90°	80°	70°	60°	50°	40°	30°	20°	10°	0°

They are shown graphically in Fig. 381, and stereographically in Fig. 382. The former figure clearly brings out the fact that at 45° the value for n_q is the mean between $\rho = 90°$ and $\rho = 0°$.

The above method may be used, not only with quartz, but with any uniaxial mineral if the proper values are computed. It makes possible the determination of the refractive index at every contact, no matter how the crystal may be oriented, between an unknown mineral and a uniaxial crystal of known refractive index; the directions chosen for determination in the unknown being naturally along the principal vibration directions.

243. Refractive Index of Canada Balsam.—One of the most convenient standards with which to compare the refractive index of an unknown mineral in a thin section is the medium in which it is mounted, usually Canada balsam. While this substance is amorphous, unfortunately its index is not absolutely constant but varies with the method of preparation, the amount of heating during the process of mounting the rock-slice, and the age of the preparation, especially if air has had access to it. The variations, however, are too small to be taken into consideration for most minerals, and it is only for minerals whose indices of refraction fall within the limits of variation of the balsam, that accurate determinations are necessary.

Canada balsam has been used as a means of comparison ever since the

Becke method came into use in 1893, but it was only recently that extensive determinations of its refractive index and its variation have been made. The values given in the older works differ decidedly, in most cases being given much too high. Brewster[1] gave 1.549, Behrens[2] 1.528–1.540, Klein[3] 1.536, Zirkel[4] 1.549, Becker[5] 1.5393, Rosenbusch-Wülfing $\pm$1.54[6] and 1.542 to 1.550.[7] In 1909 Calkins[8] compared the index of Canada balsam with various minerals in 300 thin sections from one to eight years old, and found that in only one case out of a hundred did the index of balsam exceed 1.544. The lowest value obtained was between 1.535 ± 0.002. He gives 1.54 as a fair mean value, and says the refractive index is rarely less than 1.535 nor more than 1.545. Schaller[9] made a number of determinations with a refractometer, on blank slides prepared for the purpose, and found that uncooked balsam in sodium light has an index of 1.524, soft-cooked an average of 1.5387, as usually cooked in the mounting of thin sections 1.5377, and overcooked an average of 1.5412 with a maximum value of 1.543. Wülfing[10] made determinations by comparison with minerals in thin sections prepared thirty to forty years previously and also with an Abbe-Pulfrich total refractometer on fresh balsam obtained from six different firms. He found, in a collection thirty years old, that the central portions had a value of 1.538 ± 0.002, while the borders, which had become yellow with age, averaged 1.5416. Other sections gave values between 1.5330 and 1.5382, the mean being 1.537 ± 0.004. He concluded that the index in the majority of the slides of the Heidelberg collection lies between 1.533 and 1.541, and only in rare cases does it reach 1.544 or fall below 1.533, both cases being due to fault of manufacture. Balsam which has turned yellow does not always have a high index, but all balsam when exposed to air discolors, becomes brittle, and increases in index. Balsam protected by a cover-glass or by a crust of balsam may retain its sticky consistency and low index even for forty years; it therefore

[1] Sir David Brewster: *A treatise on new philosophical instruments.* Edinburgh, 1813. Book IV, Chapter II tables.

[2] Wm. Behrens: *Tabellen zum Gebrauch bei mikroskopischen Arbeiten.* Braunschweig, 1 Aufl., 1887, Tabelle XXVII.

[3] Carl Klein: *Ueber die Methode der Einhüllung der Krystalle zum Zweck ihrer optischen Erforschung in Medien gleicher Brechbarkeit.* Neues Jahrb., 1891 (I), 70–76.

[4] F. Zirkel: *Lehrbuch der Petrographie.* I, 2te Aufl., Leipzig, 1893, 40.

[5] G. F. Becker: *Reconnaissance of the gold fields of southern Alaska.* 18 Ann. Rep. U. S. Geol. Survey, pt. III, Washington, 1898, 30. Determination of the refractive index of balsam by Prof. J. E. Wolff.

[6] Rosenbusch-Wülfing: *Mikroskopische Physiographie,* I_1, 4te Aufl. 1904, 150.

[7] Idem: *Ibidem,* I_2, 345.

[8] F. C. Calkins: *Refractive Index of Canada balsam.* Science, N. S. XXX (1909), 973.

[9] Waldemar T. Schaller: *The refractive index of Canada balsam.* Amer. Jour. Sci., XXIX (1910), 324.

[10] E. A. Wülfing: *Ueber die Lichtbrechung des Kanadabalsams.* Sitzb. Akad. Wiss. Heidelberg, Math.-naturw. Kl., 1911, 20 Abhandl., 1–26.

is altered only on the surface or at the borders. Commercial balsams are so uniform that in the preparation of thin sections the limiting values of the index need not fall outside the limits 1.533 and 1.541, and, with practice, should be between 1.534 and 1.540.

244. Relation between Refractive Index and Density.—Various formulæ have been empirically determined to express the relation between the refractive indices of substances and their densities, the simplest one being that of Gladstone and Dale[1] which is,

$$\frac{n-1}{d}=K,$$

where K is a constant.

A formula, determined independently by Lorentz[2] and Lorenz,[3]

$$\frac{n^2-1}{n^2+2}\cdot\frac{1}{d}=K_2$$

is more complex than that of Gladstone and Dale and, according to Larsen,[4] no more accurate, one formula holding as well as the other.

245. The Examination of Opaque Minerals.—While opaque minerals are of comparatively slight importance in ordinary petrographic work, they are of great importance in the study of ore deposits. Until within a comparatively recent period, no serious attempts were made to study them microscopically. With the development of microscopical metallographic methods, however, the possibility of studying opaque minerals by the same means was opened up, and while the methods are even now not fully developed, what has been done is sufficient to indicate the possibilities. The

[1] J. H. Gladstone and T. P. Dale: *Researches on the refraction, dispersion, and sensitiveness of liquids.* Phil. Trans. Roy. Soc., London, CLIII (1863), 317–343, especially 320.

[2] H. A. Lorentz: *Ueber die Beziehung zwischen der Fortpflanzungsgeschwindigkeit des Lichtes und der Körperdichte.* Wiedem. Ann., IX (1880), 641–665.

[3] L. Lorenz: *Ueber die Refractionsconstante.* Ibidem, XI (1880), 70–103.

[4] Esper S. Larsen: *The relation between the refractive index and the density of some crystallized silicates and their glasses.* Amer. Jour. Sci., XXVIII (1909), 263–274.

See also

H. L. Barvír: *Ueber die Verhältnisse zwischen dem Lichtbrechungsexponenten und der Dichte bei einigen Mineralien.* Sitzb. Gesell. Wiss. Prag, 1904, No. 3.*

F. Slavik: Review of above in Zeitschr. f. Kryst., XLII (1905–6), 410–411.

M. Sprockhoff: *Beiträge zu den Beziehungen zwischen dem Krystall und seinem chemischen Bestand.* Neues Jahrb., B. B., XVIII (1903–4), 117–154.

Michael Stark: *Zusammenhang des Brechungsexponenten natürlicher Gläser mit ihrem Chemismus.* T. M. P. M., XXIII (1904), 536–550.

scope of this work is too limited to insert these methods here, and the student is referred to the papers mentioned below.[1]

Another method, which promises to be of value for the study of opaque minerals, is that of staining employed by Leo,[2] who gives the characteristic colors produced on a limited number of minerals.

[1] Joh. Koenigsberger: *Zur optischen Bestimmung der Erze.* Centralbl. f. Min., etc., 1901, 195–197.

William Campbell: *The microscopic examination of opaque minerals.* Econ. Geol., I (1905–6), 751–766.

W. Campbell and C. W. Knight: *A microscopic examination of the Cobalt nickel arsenides and silver deposits of Temiskaming.* Ibidem, 767–779.

Wm. Campbell and C. W. Knight: *On the microstructure of nickeliferous pyrrhotites.* Ibidem, II (1907), 350–366.

Joh. Königsberger: *Ueber einen Apparat zur Erkennung und Messung optischer Anisotropie undurchsichtiger Substanzen und dessen Verwendung.* Centralbl. f. Min., etc., 1908, 565–573, 597–605. Translation in Winchells' *Elements of optical mineralogy.* New York, 1909, 465–475.

Francis Church Lincoln: *Certain natural associations of gold.* Econ. Geol., VI (1911), 247–302.

L. C. Graton and Joseph Murdoch: *The sulphide ores of copper.* Trans. Amer. Inst. Mining Eng., New York meeting, Feb., 1913, 741–809.

[2] Max Leo: *Die Anlauffarben. Eine neue Methode zur Untersuchung opaker Erze und Erzgemenge.* Dresden, 1911, 68 pp.

CHAPTER XVI

MEASUREMENTS UNDER THE MICROSCOPE

246. Measurement of Enlargement.—It has already been shown[1] that the magnifying power of a microscope is represented by the equation

$$N = \frac{250}{f},$$

consequently it may be computed from this equation or from the known magnifying powers of ocular and objective.[2] If these are unknown, it may be determined by direct comparison, with a scale, of the magnified image of an object of known size.

Upon the stage of the microscope is placed a so-called object-micrometer, which consists of a thin glass slide upon which there has been engraved or photographed a millimeter divided into ten or a hundred parts. After careful focussing, the image seen through the microscope is compared with an ordinary millimeter scale which is placed alongside the microscope at right angles to the axis of the instrument and at a distance of 250 mm. (the distance of distinct vision) from the exit pupil. If one now observes the microscopic image with one eye and the scale with the other, by shifting the scale, the two may be made to appear to lie together, and the enlargement determined. For example, if 25 divisions (0.25 mm.) of the object micrometer correspond to 70 divisions (70 mm.) of the scale, the enlargement, in diameters, for that particular combination of ocular, objective, and tube length, will be

$$\frac{70}{.25} = 280.$$

Instead of using both eyes, a camera lucida may be employed, and the length of a certain part of the object-micrometer may be drawn on a sheet of paper placed at a distance of 250 mm. from the eye. Care must be taken to tilt the microscope to the proper angle to give an undistorted image, the amount depending upon the kind of camera lucida used. The distance, 250 mm., must be measured from the eye point, consequently it must be the actual length of the path of the rays, traced through all the angles of its reflection in the camera lucida. The line traced upon the paper is measured, finally, and the computation made as before.

247. Measurement of the Field of View.—By an application of the measure of enlargement, the comparative values of the fields of view of different oculars may be made. The apparent diameter of the field at 250 mm. is

[1] Arts. 98 and 165.

[2] W. Le Conte Stevens: *Microscope magnification.* Amer. Jour. Sci., XL (1890), 50–62.

determined by comparison with a scale, or a circle is drawn, with the aid of a camera lucida, around the periphery.

248. Measurement of Lengths.—The actual size of a microscopical object may be determined in several ways. With a microscope fitted with a micrometer stage, one may determine dimensions directly by making first one side of an object coincide with the cross-hairs, and then the other. The difference between the two readings of the vernier is the required length. This is the quickest method of measurement and, with some stages, readings to 0.0005 mm. are possible.

More accurate measurements may be made by means of micrometer oculars. They are of two kinds, scale-micrometer oculars and screw-micrometer oculars. Scale-micrometers may be attached to Huygens, Ramsden, or compensating oculars. In the Seibert Huygens scale-micrometer ocular, shown in Fig. 383, the casing is made to unscrew in the middle, so that a micrometer scale *M*, shown alone at the right, may be placed within it. The position of the rabbet is such that when the scale is inserted with the engraved side uppermost, the latter coincides with the image formed by the microscope, and is magnified with it by the eye-lens. By drawing out the eye-lens, more or less, the micrometer divisions may be brought sharply into focus.

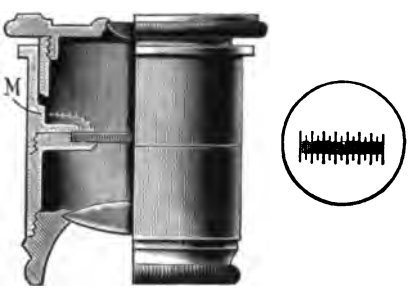


FIG. 383.—Scale micrometer ocular. (Seibert.)

Since the scale, so inserted, is magnified by the ocular, it must be calibrated for each different combination of ocular, objective, and tube length. This may be done by placing an object-micrometer upon the stage and noting the number of divisions of the former corresponding to a certain number of the latter. For example, if five divisions of the ocular-scale correspond with one of an object-scale which is divided into 0.01 mm., then one division of the former corresponds to 0.002 mm. (2.0μ).[1] Instead of determining the number of divisions corresponding with a single division of the object-micrometer, it is better to choose a larger number, since it reduces the error of the determination.

In the Zeiss compensating oculars, which are used with apochromatic objectives, the divisions of the scale are so calculated that with a tube length of 160 mm. each division is almost exactly equal to as many microns as there are millimeters in the focal length of the objective.[2]

[1] A *micron*, represented by μ, is a thousandth of a millimeter. $\mu\mu$ is a millionth of a millimeter.

[2] S. Czapski: *Compensationsocular 6 mit 1/1 Mikron-Theilung zum Gebrauch mit den apochromatischen Objectiven von Carl Zeiss in Jena.* Zeitschr. f. wiss. Mikrosk., V (1888), 150–155.

A variety of the scale-micrometer ocular is the net-, coordinate-, or cross-grating-micrometer ocular. This differs only from the preceding in having the glass plate engraved with cross-section lines (Fig. 384) instead of with a simple scale.[1] In the ocular shown in Fig. 383 the glass scales are interchangeable.

For still more accurate results, a screw-micrometer ocular[2] may be used (Fig. 385). Between, or beneath, the lens-combination, as the case may be, depending upon the type of eyepiece used, is fitted a scale, usually marked with 0.5 mm. divisions. Immediately above or below this plate is another, marked with a single line, and capable of being moved along the former by means of a micrometer screw. A complete revolution of the drum in most screw-micrometer oculars moves the scale 0.5 mm.; with 100 divisions upon it, each one indicates a movement of 0.005 mm. (5μ). Like the scale-micrometer ocular, this also must be calibrated by means of a stage-micrometer, and the number of divisions of the drum corresponding to one division of the scale determined.

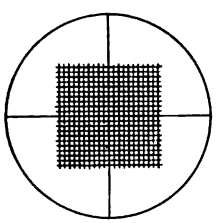


FIG. 384.—Net grating for micrometer ocular. (Fuess.)

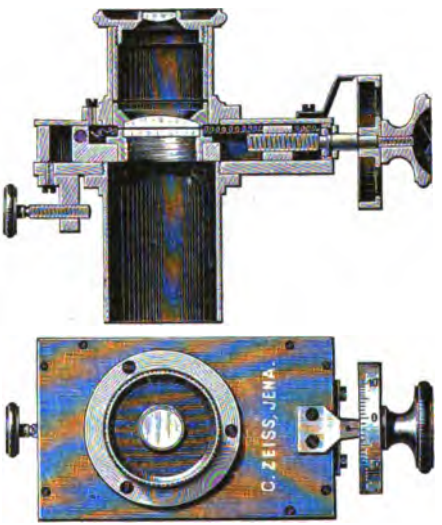


FIG. 385.—Screw-micrometer ocular. (Zeiss.)

[1] C. Leiss: *Mittheilungen aus der R. Fuess'schen Werkstätte. Ocular zur Messung der Mengenverhältnisse verschiedener Minerale in einem Dünnschliff.* Neues Jahrb., 1898 (II), 70.

[2] Hugo von Mohl: *Ueber eine neue Einrichtung des Schraubenmikrometers.* Arch. f. mikrosk. Anat., I (1865), 79–100.

Alfred Koch: *Eine Combination von Schraubenmikrometer und Glasmikrometerocular.* Zeitschr. f. wiss. Mikrosk., VI (1889), 33–35.

W. A. E. Drescher: *New accessories of the Bausch & Lomb Optical Company. Filar micrometer.* Proc. Amer. Microsc. Soc., 12th Ann. meeting, Buffalo, XI (1889), 132–133. (Describes a screw micrometer ocular after the designs of M. D. Ewell.)

Anon: Bulloch's improved filar micrometer. Jour. Roy. Microsc. Soc., 1891, 106–107.

For lost motion see V. Knorre: *Untersuchungen über Schraubenmikrometer.* Zeitschr. f. Instrum., XI (1891), 41–50, 83–93.

To determine the size of an object, one side is made to coincide with some mark on the scale, the full divisions between this and the other side are counted, the fractional part remaining is measured by the micrometer screw, and the true value computed. If the object is small, the movable index mark may be made to coincide, successively, with its two sides, and the number of revolutions of the micrometer screw noted. Like in an ordinary eyepiece, the eye-lens is movable to permit of accurate focussing upon the scale. The instrument is inserted in the draw tube of the microscope and is rigidly clamped by means of the screw at the side.

FIG. 386.—Wright's double screw-micrometer Ramsden ocular. 1/2 natural size. (Fuess.)

The ocular shown in Fig. 386 was designed by Wright[1] for the special purpose of measuring axial angles, though it may be used for all the purposes to which the preceding can be put. In the place of a movement in one direction only, this ocular has two movements at right angles to each other. Its use will be discussed more fully below.

249. Measurement of Areas.—The principal purpose for which areas are measured in petrographic work is the determination of the volume percentage of the constituents of rocks. One of the earliest methods proposed was that of Delesse,[2] which is based on the assumption that the sum of the areas of each of the constituents in a section of a uniformly homogeneous rock is proportional to the actual volume of that constituent. His method was to make, first, a drawing of each constituent in the rock by tracing carefully, on thin oiled paper, the outlines of each mineral as shown in a polished slab. Each kind of mineral was then differently colored in the drawing, and the whole was pasted on a piece of tin foil, after which it was carefully cut apart on the lines. The different colors were now carefully sorted, the tissue paper and gum were soaked off, and the tin foil was weighed for each constituent.

[1] Fred. Eugene Wright: *The measurement of the optic axial angle of minerals in the thin section.* Amer. Jour. Sci., XXIV (1907), 336.

Idem: *Das Doppel-Schrauben-Mikrometer-Okular und seine Anwendung zur Messung des Winkels der optischen Achsen von Krystalldurchschnitten unter dem Mikroskop.* T. M. P. M., XXVII (1908), 299.

Idem: *The methods of petrographic-microscopic research.* Carnegie Publication No. 158, Washington, 1911, 155.

[2] A. Delesse: *Procédé méchanique pour déterminer la composition des roches.* Comptes Rendus, XXV (1847), 544–545. Brief of following.

Idem: Same title. Ann. d. Mines, XIII (1848), 379–388.

Sollas[1] improved this method by making his drawing by means of a camera lucida.

A somewhat similar method was used by Joly[2] for determining the proportions of hard and soft constituents in paving material. Instead of using a camera-lucida, he made use of a photographic apparatus, and traced with ink the outlines of any particular constituent upon the back of a photographic plate upon whose front side a positive of coordinate paper was printed. Upon holding the transparent plate to the light, the number of square millimeters or square centimeters contained within the ink outlines could be estimated. The whole circular area being equal to $\frac{\pi D^2}{4}$, the area occupied by the mineral could be estimated as a percentage of that of the field, several drawings being made and an average taken of all.

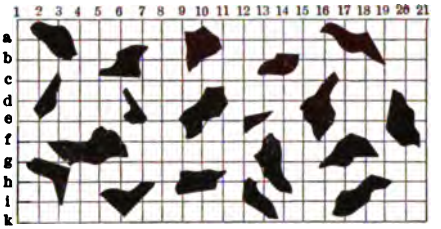


FIG. 387.—Comparison of linear measurements with total areas.

FIG. 388.—Hirschwald's planimeter ocular. (Fuess.)

Rosiwal[3] still further improved the method by reducing his measurements to linear series in two directions. His method is based on the principle that the total length of all measured lines, as *a* to *k* and 1 to 21 (Fig. 387), bears the same relation to the portions intercepted on these lines by each constituent, as the volume of the whole rock does to that of each constituent. His actual method of procedure was to draw rectangular coordinates upon the cover-glass, to add together all the intercepts, and finally to compare them with the total length of lines measured.

Hirschwald[4] simplified the measurement

[1] W. J. Sollas: *Contributions to a knowledge of the granites of Leinster.* Read Nov. 30, 1889. Trans. Roy. Irish Acad., Dublin, XXIX (1887–1892), 427–512, in particular 471–473.

[2] J. Joly: *The petrological examination of paving-sets.* Proc. Roy. Dublin Soc., X (1903–5), 62–92.

[3] August Rosiwal: *Ueber geometrische Gesteinsanalysen. Ein einfacher Weg zur ziffernmässigen Feststellung des Quantitätsverhältnisses der Mineralbestandtheile gemengter Gesteine.* Verh. d. k. k. geol. Reichsanst., Wien, 1898, 143.*

[4] J. Hirschwald: *Ueber ein neuer Mikroskopmodell und ein "Planimeter-Ocular" zur geometrischen Gesteinsanalyse.* Centralbl. f. Min., etc., 1904, 626–633.

Idem: *Handbuch der bautechnischen Gesteinsprüfung.* I, Berlin, 1911, 146–147, 163–172.

by the use of his planimeter ocular (Fig. 388). This consists of a Huygens ocular, in the focal plane of which there are two glass micrometer scales, 10 mm. long, and perpendicular to each other. Of the two scales, one is movable, in a direction at right angles to its length, by means of a milled head, the other is stationary. This ocular is much more convenient to use than a net micrometer ocular (Fig. 384) since one is much less likely to lose sight of the place under count. It is, however, less convenient, though more rapid, than an ordinary screw-micrometer ocular.

To test the accuracy of this method of measuring the area of any constituent in a rock, Hirschwald, in a manner similar to that already used by Rosiwal, cut a piece of paper, 10 sq. cm. in area, into small irregular pieces, and pasted them, haphazard, upon a quadratic ruled sheet of 50 sq. cm. area (Fig. 387). The exact proportion of the irregular portion, as compared with the larger sheet, was 20 per cent.; as determined by the linear measurements, it was 411 to 2000, or 20.6 per cent.

This method of determining areas by linear measurements, generally spoken of as the "Rosiwal method," is very convenient for determining the composition of a granular rock. If the individual components are of known composition, the complete analysis of the rock can be computed. The linear measurements are first reduced to 100, the values thus representing the relative volume of each component. The volumes are then multiplied by the specific gravity of the corresponding mineral, and the total again reduced to 100, to give the percentage weights, or masses, of each.

The following mechanical analysis of the "Butte granite," given by Cross, *et al.*,[1] may be taken as an example.

MECHANICAL ANALYSIS OF THE "BUTTE GRANITE"

	Total diameters	Relative volumes	Sp. gr.	Weights
Quartz	2,954	23.17	2.65	22.55
Orthoclase	1,373	18.62	2.57	21.07
Plagioclase	5,492	43.10	2.68	42.47
Biotite	1,130	8.87	3.00	9.77
Hornblende	482	3.78	3.20	4.44
Pyroxene	252	1.97	3.30	2.37
Magnetite	51	0.40	5.17	0.76
Pyrite	6	0.04	5.00	0.07
	12,740	99.95		99.98

In this analysis the 12,740 units of the micrometer scale represent the total distance measured and are the sum of 604 grains traversed. An idea may hereby be obtained of the number of readings necessary.

[1] Cross, Iddings, Pirsson, Washington: *Quantitative classification of igneous rocks.* Chicago, 1903, 226.

250. Measurement of Thicknesses.—Thicknesses may be measured by means of the fine adjustment screw of the microscope. The instrument should first be carefully focussed on a scratch on an object slip, after which the mineral to be measured should be placed above the mark, by sliding it over to exclude the air, and its upper surface brought into focus. The difference between the micrometer readings gives the measure of the thickness. In order to correct any lost motion which may be present in the screw, the microscope should be brought into focus, in both cases, by turning the screw in one direction only.

If the measurement is to be made by focussing through the mineral, the method of the Duc de Chaulnes[1] may be used, whereby $D=nM$ (Eq. 2, Art. 208), D being the true thickness, M the measured thickness, and n the index of refraction of the mineral. In using this method the measurements should be made near the center of the field, otherwise the curvature of the image may produce a considerable error.

A much more delicate measure of thickness is by means of the birefringence of a mineral.[2]

251. Measurement of Plane Angles.—In measuring plane angles under the microscope, it must be remembered that the apparent angle of cleavage[3] may not be the true angle. Wherever possible, the section for measurement should be so chosen that the planes, whose intersection is to be measured, lie parallel to the axis of the microscope. When this occurs, the junction line between the two will not be displaced upon focussing successively upon the bottom and top of the slide.

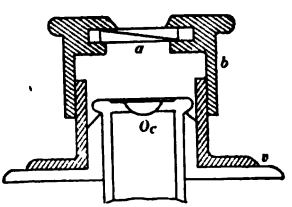


FIG. 389.—Leeson prism.

In making a measurement, the apex of the angle is set on the intersection of the cross-hairs, and one leg is made to coincide with one of them. The stage vernier is now read and the stage rotated until the other leg of the angle coincides with *the same cross-hair*. The difference between this reading and the former gives the angle. Instead of making the edges of the mineral and the apex of the angle coincide exactly with the cross-hairs, whereby a slight angle may be concealed by the thickness of the hairs, it will be found better to place them just a trifle to one side, the parallel position being determined by the uniform width of the hair-line of light between the two.

A method which is especially useful for small crystals is measurement with the aid of a Leeson prism.[4] This instrument, called a double refracting

[1] See also Art. 208, *supra*.

[2] Art. 301, *infra*.

[3] Art. 206, *supra*.

[4] H. B. Leeson: *On crystallography, with a description of a new goniometer and crystallonome*. Mem. and Proc. Chem. Soc., London, III (1848), 486–560, in particular 550–552.

goniometer by the inventor, depends on the action of a doubly refracting prism, either of Iceland spar or of quartz, of such thickness that it will only partially separate the two images of the angle which is to be measured. It may be used as a separate instrument, or attached to a microscope, or even attached to a telescope to measure the dip of strata. Used on a microscope, it is slipped over the ocular, after the manner of a cap nicol, and the amount of rotation is read from a vernier. It is shown in section in Fig. 389 in which *a*

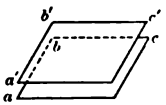


FIG. 390.

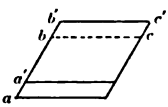


FIG. 391.

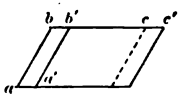


FIG. 392.

is an achromatic prism of Iceland spar or a Rochon quartz prism, *b* a sliding collar for adjustment, *v* the vernier, and *Oc* the ocular of the microscope. When a crystal or a cleavage angle is viewed through the prism, polarizer and analyzer of the microscope being removed, two images, somewhat separated, will appear. Upon rotating the cap, the extraordinary image will revolve about the ordinary, and will occupy various positions as shown in Figs. 390 to 392.

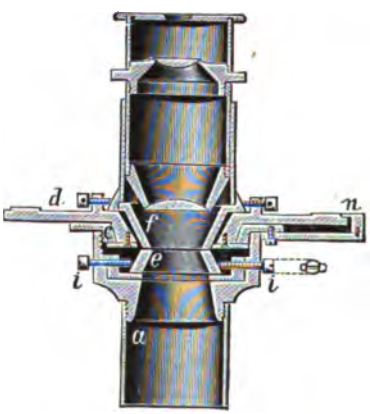


FIG. 393.—Ocular goniometer. 3/4 natural size. (Fuess.)

FIG. 394.—Ocular goniometer. (Reichert.)

Let *abc*, Fig. 390, be the angle to be measured. The vernier is set at 0° and clamped, and the tube containing the prism is revolved until the lines forming one side of the angle to be measured coincide in both images (*ab* and *a'b'*, Fig. 391). The vernier is now released and the whole instrument is revolved on the graduated scale until the two lines forming the other side of the angle coincide (*bc* and *b'c'*, Fig. 392). The amount of rotation is the measure of the angle or of its complement according to the direction in which

the prism was revolved. It is, of course, not at all necessary to set the vernier at 0° for the first reading; the difference between the two is sufficient.

Another method of measuring plane angles is by means of an ocular goniometer (Fig. 393). A diaphragm *e*, adjusted by means of four centering screws *i*, carries on its upper edge a single cross-hair passing exactly through the axis of the microscope. Above this the diaphragm *f*, likewise adjustable by centering screws, carries, on its lower edge, another cross-hair, also passing accurately through the axis. By means of the screw *c*, the two are brought as nearly as possible together, in which position the two hairs lie in the focal plane of the Ramsden ocular above. To read an angle, one leg is first placed parallel to the lower, and stationary, cross-hair, after which the upper cross-hair is rotated until it coincides with the other leg. The angle may be read to minutes by means of the vernier *n*. Another form of ocular goniometer is shown in Fig. 394.

If one uses a microscope with a cap nicol simultaneously rotating with the polarizer, consequently with an ocular likewise rotating at the same time, one may read the angle to within 5 minutes by the vernier there provided.

252. Measurement of Optic Axial Angles.—The measurement of optic axial angles is discussed below.[1]

[1] Chapters XXXIV–XXXV.

CHAPTER XVII

DRAWING APPARATUS

253. Drawing Apparatus.—All drawing instruments for use with the microscope are based on the principle of the camera lucida. The drawing paper and the image seen through the microscope appear to lie superimposed, in the same plane, due to reflection through a prism; consequently it is an easy matter to make a drawing of a rock section by tracing the outlines.

The simplest kind of drawing apparatus is in the form of a single prism which may be permanently attached to an ocular (Figs 395–396) or removable (Fig. 397). Such prisms are of two types. In one[1] the edge of the prism just reaches the axis of the microscope (Fig. 395) and but half the light is used. The rays of light, coming from the drawing, meet the lower surface of the prism at right angles and, after being twice reflected, emerge at right angles to the upper surface. With an instrument as shown in Fig. 395 the image is projected close to the stand of the microscope. In an improved form, shown in Fig. 396, the prism is so modified that when the microscope is inclined 45°, the image appears, without distortion, on the horizontal surface back of the stand. The light from the image and from the paper may be equalized by means of two tinted glasses which swing on a pivot below the prism.

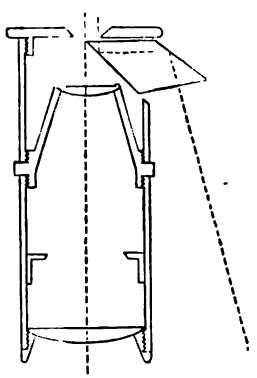


FIG. 395.—Section through drawing apparatus. (Leitz.)

The Nachet[2] camera lucida (Figs. 397–398) is simple and satisfactory. It is so constructed that the light from the whole field passes through the prism. This is accomplished by cementing a small triangular prism to one of the faces of a rhombic prism so that the light, coming from the image, strikes the lower face at right angles, passes through without refraction, and emerges at right angles to the upper surface. The rays from the drawing likewise enter and emerge at right angles to the faces, but suffer two reflections in their course. The image seen with this camera lucida is of the entire field of view

[1] P. Schiemenz: *Die neue Zeichenoculare von Leitz.* Zeitsch. f. wiss. Mikrosk., XII (1895), 289–292.

[2] Anon: *Nachet's improved camera lucida.* Jour. Roy. Microsc. Soc., II (1882), 260–261. The same instrument is described as Swift's by Frank Crisp: *On some recent forms of camera lucida.* Jour. Roy. Microsc. Soc., II (1879), 21–24.

and it is undistorted when the microscope is vertical and the drawing table at right angles to the line of projection. A blue glass neutralizes part of the

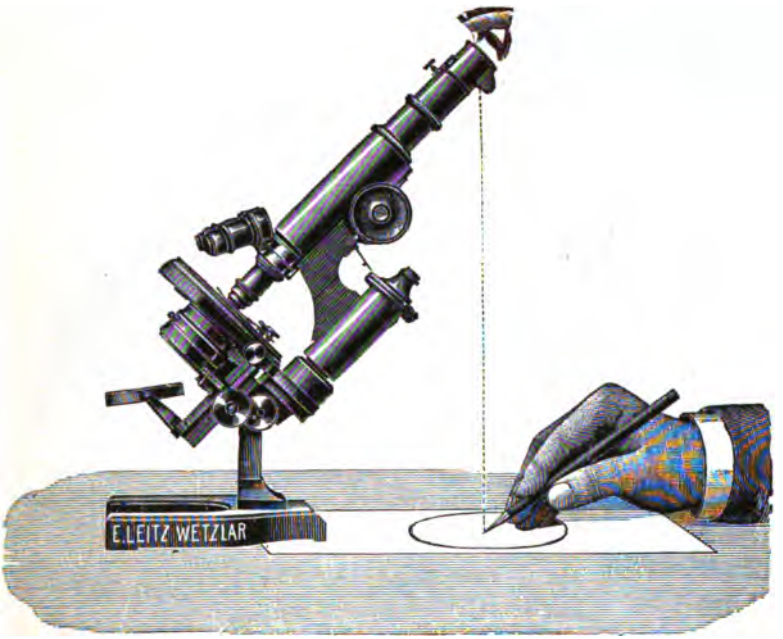


FIG. 396.—Improved drawing apparatus. (Leitz.)

light coming from the drawing and permits the image to be clearly seen. In another form[1] the prism is so cut that the microscope may be inclined.

FIG. 397.—Camera lucida. (Nachet.)

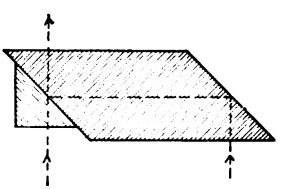


FIG. 398.—Passage of light through the Nachet camera lucida.

The Abbe type of drawing apparatus[2] (Fig. 399) consists of two triangular prisms, silvered on the plane of contact between them with the exception

[1] Anon: *Nachet's camera lucida.* Jour. Roy. Microsc. Soc., VI (1886), 1057.

[2] S. Czapski: *Ueber einen neuen Zeichenapparat und die Construction von Zeichenapparaten im allgemeinen.* Zeitschr. f. wiss. Mikrosk., XI (1894), 289–298.

Anon: *Directions for using the Abbe drawing apparatus.* Zeiss' circular Mikro 118, pp. 8, Jena, 1911.

of a small opening on the axis of the microscope. The light from the object passes through this opening while the images of the drawing paper and pencil are reflected by a mirror and by the silvered surface of the prism. The light from the paper may be moderated by one or more tinted glasses, and the

FIG. 399.—The Abbe drawing apparatus. 2/3 natural size. (Zeiss.)

whole instrument may be tilted out of the way on a hinge. When the mirror is set at 45°, a position indicated by a stop in the instruments of some makers, and the microscope is placed in an upright position, the image appears undistorted upon the paper. The part of the drawing nearest the microscope,

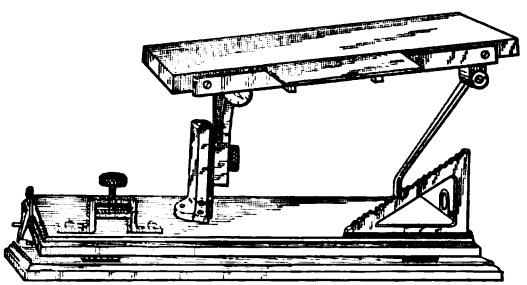


FIG. 400.—Tilting drawing-board. (Bausch and Lomb.)

however, is cut off by the foot and, if the entire field is to be drawn, it is necessary to change the inclination of the mirror. This introduces an error, however, for when the inclination of the mirror to the drawing-board is not exactly 45°, a distortion is produced in the image, and it is necessary to correct this by means of a tilting drawing-board such as is shown in Fig.

400.[1] This may be inclined in various directions and securely clamped in such positions. It may be raised or lowered, also, in order to modify the size of the image produced. When the drawing-board lies at a distance of 250 mm. from the exit pupil of the microscope, the distance being measured through all the changes of path of the ray in passing through the drawing apparatus, the image will be drawn with the so-called magnification of the microscope.

The drawing-board shown in the figure is 24 by 37 1/2 cm., and has an adjustable arm rest on the front edge. The base is 28 by 51 cm.

To test the accuracy of the setting of a camera lucida[2] use may be made of test objects, such as circles, squares, or parallel lines drawn on a glass slip. The drawings made from such objects may be measured: a compass set in the center of the circle should exactly follow the lines drawn, the square should have equal sides, and the parallel lines be truly parallel.

[1] For other forms see Wilhelm Bernhard: *Ein Zeichentisch für mikroskopische Zwecke.* Zeitschr. f. wiss. Mikrosk., IX (1892), 439–445.
Idem: *Zusatz zu meinem Aufsatz "Ein Zeichentisch, etc.,"* Ibidem, 298–301.
Dr. Giesenhagen: *Ein Zeichenpult für den Gebrauch am Mikroskop.* Zeitschr. f. wiss. Mikrosk., VII (1890), 169–172.

[2] J. Anthony: *On drawing prisms.* Jour. Roy. Microsc. Soc., IV (1884), 697–703.

CHAPTER XVIII

ROTATION APPARATUS

254. Rotation Apparatus.—Under the heading of rotation apparatus are included here all those appliances, accessory to a microscope, by which crystals or thin sections may be tilted from the horizontal so that the axis of the microscope passes through them at different angles than before. While the principal use of rotation apparatus is for the examination of minerals by polarized light, yet for certain purposes, such as the measurement of interfacial angles of small crystals, or of cleavage angles, they are used in ordinary light.

Probably the first rotation apparatus used with a microscope was that invented by Leeson[1] in 1848, and used by him to tilt crystals into proper positions for measuring angles. It had three movements, two of them horizontal and one vertical, whereby a crystal could be turned to any position in two planes.

The next instrument, eight years later, by Highley,[2] was not a detachable stage, but a built-in part of an inverted chemical microscope. Instead of the ordinary revolving stage, this instrument carried two concentric graduated rings, one of which had the usual movement in azimuth, while the other was pivoted and had a movement in altitude.

Valentin,[3] in 1861, used an apparatus which could be clamped to the stage of the microscope. It consisted of a rotating disk attached to an arm by which it had a movement in altitude. No graduated circle was provided, consequently there was no means of measuring the amount of rotation.

The rotating stage described by Nägeli and Schwendener,[4] while an improvement on the two preceding, was not as complete as Leeson's. It consisted of a horizontal plate and a vertical graduated circle. The movement was about a horizontal axis, the amount being indicated by a pointer without a vernier. A suggestion was made that for certain purposes it would be advisable so to arrange the apparatus that it could be rotated in a trough under water or some other liquid.

[1] H. B. Leeson: *On crystallography, with a description of a new goniometer and crystallonome.* Mem. and Proc. Chem. Soc. London, III (1848), 486–560. In particular 550–552.

[2] Samuel Highley: *Contributions to micro-mineralogy.* Quart. Jour. Microsc. Soc., IV (1856), 277–286.

[3] G. G. Valentin: *Die Untersuchung der Pflanzen und Thiergewebe in polarisirtem Lichte.* Leipzig, 1861, 166.*

[4] Carl Nägeli und S. Schwendener: *Das Mikroskop.* Leipzig, 1 Aufl., 1867, 2. Aufl., 1877. English translation, *The Microscope.* New York, 2nd ed., 1892, 315–319.

An appliance resembling the preceding was constructed by von Ebner[1] in 1874. The whole instrument was attached to the glass bottom of a trough, 75 mm. by 35 mm. and 18 mm. deep, which could be filled with an immersion fluid. The amount of rotation, about 50° each way, was indicated by a pointer, without vernier, and was necessarily only approximate.

A simple rotating stage, without graduated scales, was made by West.[2] Bertrand,[3] in 1880, described an instrument very similar to that of Nägeli and Schwendener, except that the graduated half circle was outside the immersion trough and that there was a vernier attached to the end of the pointer. A modern instrument, constructed by Fuess[4] on this principle, is shown in Fig. 401. The specimen is held in the pincette *P* or on an object carrier *O*, the latter consisting of a glass slip and a fork-shaped spring. *S* is a screw and *Sch* a sliding bar by means of which the holder *P* is brought into the axis of rotation. To Bertrand's instrument there was the objection that the horizontal axis penetrated the side of the vessel containing the immersion fluid, which made it almost impossible to prevent the escape of some of the latter upon the stage of the microscope. This is overcome, in the Fuess apparatus, by the horseshoe-shaped piece *i* which, however, only permits a rotation through 125°. The circle is graduated to degrees.

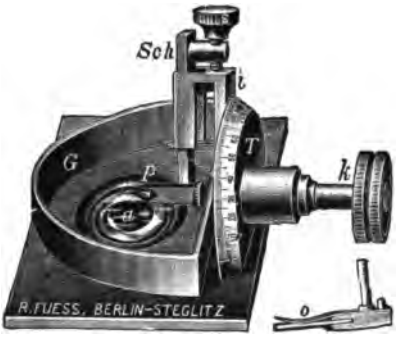


FIG. 401.—Small rotation apparatus. 2/3 natural size. (Fuess.)

In 1884 Brögger[5] described an instrument which he inserted in the central opening of the microscope and used in orienting small crystals for goniometric measurements. It is shown in section, twice the size of the original, in Fig. 402. A plate *d*, which rests upon the stage of the microscope, carries a disk *a* into which fits the hemisphere *c*. The latter is drilled out in the center and in it is placed the table *b*, upon which the crystal to be examined is placed, and which may be raised or lowered. A modified form of this instrument, which may be used for other purposes as well, is constructed by Fuess,[6] and is shown in Fig. 403. On a round disk, which is attached to the stage of the

[1] V. von Ebner: *Untersuchungen über das Verhalten des Knochengewebes im polarisirten Lichte.* Sitzb. Akad. Wiss. Wien, Math.-naturwiss. Kl., LXX (1874), iii Abth. 111–115.

[2] Anon: *West's universal-motion stage and object holder.* Jour. Roy. Microsc. Soc., III (1880), 331–332.

[3] Emile Bertrand: *Nouveau minéral des environs de Nantes.* Bull. Soc. Min. France, III (1880), 96–100.

[4] C. Leiss: *Die optischen Instrumente der Firma R. Fuess.* Leipzig, 1899, 230–231.

[5] W. C. Brögger und Gust. Flink: *Ueber Krystalle von Beryllium und Vanadium.* Zeitscher. f. Kryst., IX (1884), 225–237, in particular 227–228.

[6] C. Leiss: *Op. cit.*, 228–229.

microscope by the spring clips, is an upright *i* through which passes a horizontal axis. At one end is a milled head *k*, and at the other a disk *v*. The amount of rotation may be read in degrees from the graduated circle *T*. The disk *v* has a milled edge *w*, and may be rotated, carrying with it a central hemisphere *h*. The latter is loosely placed in the opening of the disk and may

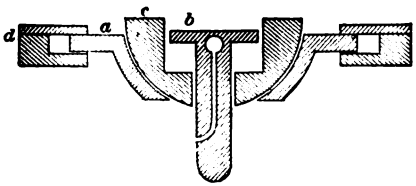


FIG. 402.—Section through Brögger's micro-goniometer.

FIG. 403.—Brögger's micro-goniometer. 2/3 natural size. (Fuess.)

be tilted at any angle, being retained in its position by a coating of some heavy oil, such as vaseline. Through the center of the hemisphere is bored a cone-shaped hole, larger on the under side than on the upper. Projecting into its center, and lying in the plane of the flat surface, is a blunt needle *N* which may be rotated by means of the screw *S*. The instrument, being intended primarily for adjusting the position of crystals to read their interfacial angles either by the stage vernier or by an ocular goniometer (Figs. 393–394), is provided with no graduations except the single circle *T*.

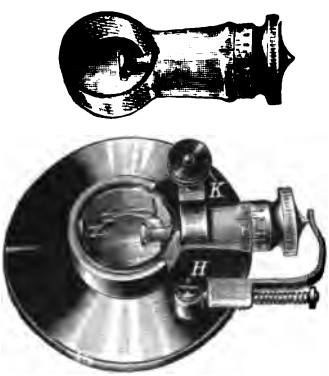


FIG. 404.—Klein's small rotation apparatus. 1/2 natural size. (Fuess.)

Nachet's[1] tub goniometer resembles that of Bertrand, but instead of inserting the horizontal axis through the side of the vessel, he attached it to a geared wheel within it. Another wheel, above the rim, transmitted the rotation to the graduated circle.

In 1891, Klein[2] described the first of his many rotation apparatus, and it is chiefly to him and to von Fedorow that our modern instruments are due. Klein's earliest apparatus, as constructed by Fuess,[3] is shown in Fig. 404. It consists of a circular metal plate, in the center of which is a hole, partially surrounded by a metal collar. Into this is inserted, and held by the clamp *K*, the immersion vessel, of which two sizes are

[1] A. Nachet: *Cuve goniomètre.* Bull. Soc. Min. France, X (1887), 186–187.

[2] C. Klein: *Krystallographisch-optische Untersuchungen. Ueber Construction und Verwendung von Drehapparaten zur optischen Untersuchung von Krystallen in Medien ähnlicher Brechbarkeit.* Sitzb. Akad. Wiss. Berlin, 1891 (I), 435–444, in particular 435–437.

[3] C. Leiss: *Op. cit.*, 231.

provided. The crystal to be examined is attached to the blunt end of a horizontal glass rod, at whose other end is a circle graduated to degrees. *H* is a spring to keep the rotating axis in close contact with the shoulder and thus prevents the escape of the immersion fluid, which should have a refractive index as nearly as possible the same as that of the substance under examination.

In the same year appeared von Fedorow's[1] first description of his "Universaltisch." It was described in greater detail in 1893, in the second of a long series of articles[2] on "Universal methods in mineralogy and petrography," and two types were illustrated. Both of these were considerably

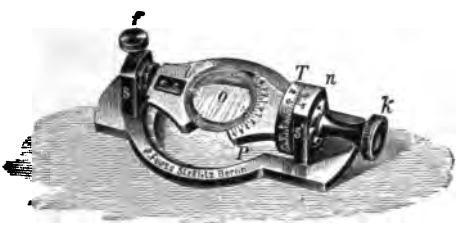


FIG. 405.—The von Fedorow small model universal stage. 1/2 natural size. (Fuess.)

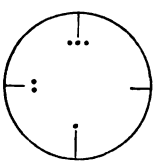


FIG. 406.—Object-glasses for von Fedorow's small universal stage.

changed at various times until, at present, the two types appear as shown in Figs. 405 and 407.

The first, or small model[3] (Fig. 405), may be attached to the stage of even the smallest microscopes, since the height of the object slip above the base is only 11 mm. Two vertical supports *ss'* carry the horizontal axis, which is rotated by the milled head *k*, the amount being read to 5 degrees from the

[1] E. von Fedorow: *Eine neue Methode der optischen Untersuchung von Krystallplatten in parallelem Lichte.* T. M. P. M., XII (1891), 505–509.

[2] (a) E. von Fedorow: *Universal- (Theodolith-) Method in der Mineralogie und Petrographie. I Theil.* Zeitschr. f. Kryst., XXI (1892–1893), 574–714.
(b) Idem: Same title, *II Theil.* Ibidem, XXII (1893–1894), 229–268.
(c) Idem: *Die einfachste Form des Universalischchens.* Ibidem, XXIV, 602–603.
(d) Idem: *Optische Mittheilungen. Noch ein Schritt in der Anwendung der Universalmethode zu optischen Studien.* Ibidem, XXV (1895–1896), 351–356.
(e) Idem: *Universalmethode und Feldspathstudien. I. Methodische Verfahren.* Ibidem, XXVI (1896–1897), 225–261. In particular 226–230, 241–242.
(f) Idem: Same general title, *II. Feldspathbestimmungen.* Ibidem, XXVII (1897–1898), 337–398.
(g) Idem: Same general title. *III. Die Feldspäthe des Bogoslowsk'schen Bergreviers.* Ibidem, XXIX (1897–1898), 604–658.
See also the following:
(h) C. Leiss: *Vervollständigte neue Form des E. v. Fedorow'schen Universaltisches.* Neues Jahrb., 1897 (II), 93–94.
(i) Idem: *Universaltische einfachster Form nach E. v. Fedorow.* Neues Jahrb., B. B., X (1895–1896), 420–423.
(j) Idem: *Die optischen Instrumente, etc.*, Leipzig, 1899, 233–236.
(k) Fred. Eugene Wright: *The measurement of the optic axial angle of minerals in the thin section.* Amer. Jour. Sci., XXIV (1907), 317–369, in particular 343.

[3] See references c, e, i, j, above.

graduated circle T, and to single degrees from the vernier n. A screw f clamps the axis in any desired position. As in all of von Fedorow's stages, circular object slips, 20 mm. in diameter and 1 mm. in thickness, are used. In this stage they are made to do double duty. A rabbet in the plate lP receives them and permits a rotation in azimuth. The round object-glasses O are marked, near the periphery, as shown in Fig. 406, with four scratches, separated by 90°, and distinguished by one, two, three, or no dots. They are pressed against the graduated side of the plate P by means of a spring, thus permitting more accurate reading of the scale, which is divided into 5° spaces. The rotation may be estimated to one degree. If the table is considerably tilted, two lenses, less than hemispheres by the thickness of the object- and cover-glasses, and with an index of refraction of 1.7 to 1.8, may be attached below and above by means of a drop of glycerine, to increase the angle of vision.[1]

FIG. 407.—The improved von Fedorow large universal stage. 1/2 natural size. (Fuess.) A form without the arcs V V_1 is shown in Fig. 695.

The large model (Fig. 407)[2] has four movements. The base may be clamped to the stage of the microscope by means of the thumb screws tt. Two uprights l support, and a screw f clamps the horizontal axis, the amount of whose rotation may be read to five minutes on the circle T and the vernier n. The disk T_1 may be rotated by means of a tangent screw,[3] read to five minutes at n, and clamped in position by the screw g. The tilting stage K may be clamped by the screw d, and the amount of inclination read to degrees on two hinged graduated segments V and V_1, suggested by von Fedorow[4] but first constructed by Wright.[5] The inner disk S is of glass and rotates within the graduated circle K.[6] It may be read to degrees. If a Hirschwald stage is used, the instrument may be permanently clamped to a blank sliding plate, which itself may be held securely in position on the microscope by a single thumb screw.

As in the small model, two rather less than hemispheres of glass, with or without holders, may be placed above and below the preparation and so

[1] See page 353 of reference d, and page 229 of c.
[2] See references e, h, and j, above.
[3] An addition suggested by Albert Johannsen in an instrument purchased from Fuess for the U. S. Geological Survey.
[4] See reference e.
[5] See reference k.
[6] See reference e.

arranged that the thin section forms the center of the sphere. This is on the same principle as those first used by Adams[1] in his polariscope.

In 1893, Klein[2] described three new rotation apparatus. The first (Fig. 408) is so arranged that a crystal attached to the rod F may be tilted and rotated in any desired direction. When used with an immersion fluid, the microscope is inclined backward to a horizontal position, the rotation

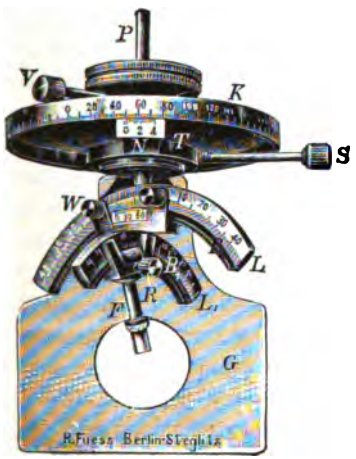


FIG. 408.—Klein's Universaldrehapparat. 1/2 natural size. (Fuess.)

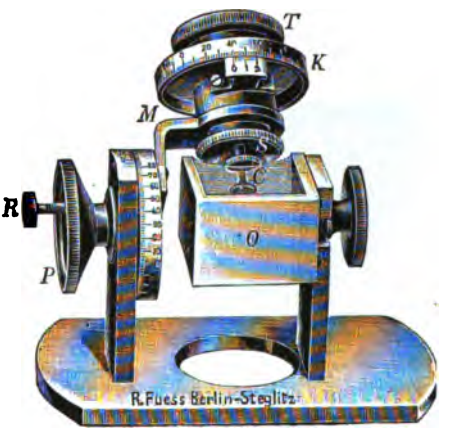


FIG. 409.—Klein's apparatus for the examination of gems. 1/2 natural size. (Fuess.)

apparatus being held firmly to the stage by two strong clamps, and the liquid is placed in a vessel which is held in position on a separate support. The instrument has three rotation motions, one around a complete circle, measured by graduations on K, and two 90° rotations, L and L_1. All readings may be made by means of verniers to 5 minutes. P is a rod by means of which the mineral attached to F may be lowered, and V and S are clamping screws.

The second instrument (Fig. 409), with two motions at right angles to each other, was designed especially for the examination of gems. Like the preceding it must be used with the microscope in a horizontal position. The containing vessel for the immersion fluid may be readily clamped on or removed. The motion P is of 90°, that of K, 360°. Verniers on each make possible readings to 10 minutes. In addition, by means of the screws S and T, one may adjust the crystal exactly in the axis of the microscope.

[1] W. G. Adams: *A new polariscope.* Phil. Mag., L (1875), 13–17.
Abstract of same article: *Ueber ein neues Polariskop*, Pogg. Ann. CLVII (1876), 297–302.

[2] C. Klein: *Der Universaldrehapparat, ein Instrument zur Erleichterung und Vereinfachung krystallographisch-optischer Untersuchungen.* Sitzb. Akad. Wiss. Berlin, 1895 (I), 91–107.
C. Leiss: *Ueber Neuconstructionen von Instrumenten für krystallographische und petrographische Untersuchungen.* Neues Jahrb. B. B. X (1895–6), 187–189.
Idem: *Die optischen Instrumente, etc.*, 232–233.

The third instrument[1] (Fig. 410) differs somewhat from the preceding in being designed principally for the examination, in parallel light, of thin sections immersed in a highly refracting liquid. The tub for the immersion fluid is large enough to permit the examination of every part of a rock section, 15 mm. square, when mounted on an object slide 28 by 48 mm. The microscope is placed in a vertical position, and the apparatus is attached by means of two strong clamps. In the bottom of a hemispherical tub B is a glass plate a for the transmission of the light from below. A horizontal disk T_1, in the center of which there is a glass plate S, may be rotated in azimuth by means of the button k, which transmits, through the horizontal axis z, a motion to a geared wheel. The latter engages in a circular rack T_1 around the periphery of the disk, and the amount of rotation may be read, to five minutes, by the vernier n_1. Two spring clips, e and e_1, hold the section in place. A movement in altitude is produced by the wheel T, and may be read to five minutes by the vernier n, c being the axis by means of which this motion is produced. In a later paper, Klein[2] described two further motions which were added to the inner disk by cutting it into concentric circles, to one of which was imparted a tilting motion in altitude, and to the other a horizontal rotary motion, the latter of which was still later[3] graduated.

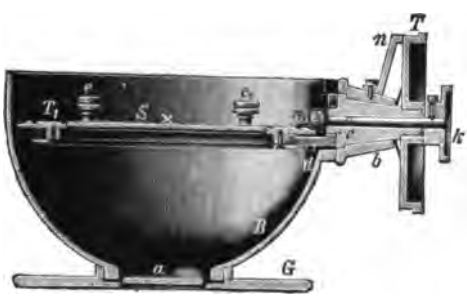


FIG. 410.—Klein's tub goniometer. 1/2 natural size. (Fuess.)

In 1895 Schroeder van der Kolk[4] described a very simple device by means of which a crystal or thin section might be tilted for examination in any direction. It consisted simply of a hemisphere of glass, 30 mm. in diameter, the convex side of which rested in the opening of the stage of the microscope. The slide was placed upon the flat surface and was held in place by a drop of glycerine or oil. If it was desired to place the hemisphere so that the upper

[1] C. Klein: *Ein Universaldrehapparat zur Untersuchung von Dünnschliffen in Flüssigkeiten.* Sitzb. Akad. Wiss. Berlin, 1895 (II), 1151–1159.
C. Leiss: *Ueber neuere Instrumente und Vorrichtungen für petrographische und krystallographische Untersuchungen.* Neues Jahrb. B. B., X (1895–6), 423–425.
Idem: *Die optischen Instrumente, etc.*, 237–238.
[2] C. Klein: *Ueber Leucit und Analcim und ihre gegenseitigen Beziehungen.* Sitzb. Akad. Wiss. Berlin, 1897 (I), 290–354, in particular 328–330.
Idem: Same title. Neues Jahrb. B.B., XI (1897), 475–553, in particular 522–525.
[3] Idem: *Die Anwendung der Methode der Totalreflexion in der Petrographie.* Sitzb. Akad. Wiss. Berlin, 1898 (1), 317–331, in particular footnote 2, page 321.
[4] J. L. C. Schroeder van der Kolk: *Zur Systembestimmung mikroskopischer Krystalle.* Zeitschr. f. wiss. Mikrosk., XII (1895), 188–92.
Idem: *Kurze Anleitung zur Mikroskopischen Krystallbestimmung.* Wiesbaden, 1898, 37–39.

surface was exactly horizontal, all that was necessary was to press down upon it, very gently, with the front lens of a medium power objective. No means of measuring the amount of tilting was possible.

A variation of the above, made by ten Siethoff,[1] permits examinations to be made by convergent as well as by parallel light. This device is especially convenient for the examination of small crystals, which may be fastened to the upper surface of the hemisphere by means of balsam or oil. This hemisphere forms the upper member of a triple-lens condenser (Fig. 411) and is entirely detached from the casing so that it may be rotated in any direction. In order that high power objectives, consequently those of short focal lengths, may be used, the upper edges are beveled, and the whole instrument is of such size that it may be slipped into the central opening of the stage of the microscope, being held in place by the pressure of the spring object clips on the collar *t*.

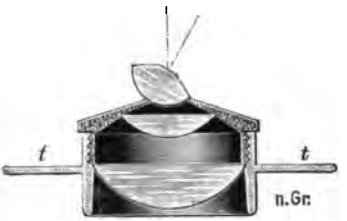


FIG. 411.—Ten Siethoff's rotation apparatus. Full size. (Fuess.)

Another variation of the Schroeder van der Kolk instrument is that of Arschinow,[2] who surrounded the upper edge of a glass hemisphere, 50 to 60 mm. in diameter, by a metal band to which were pivoted, at right angles to each other, two graduated arcs 5 mm. wide. Upon the face of the hemisphere two lines were engraved, joining the pivot points of the arcs. An ebonite ring, cut out to receive the glass hemisphere and lined with chamois skin, is clamped upon the stage of the microscope and holds the device in place. The thin section to be examined is fastened, cover-glass down, to the surface of the instrument with glycerine or cedar oil, and it is tilted to the desired position. If it is necessary to incline the section greatly, a small plano-convex lens, differing from a hemisphere by the thickness of the object glass, is placed above the object[3] to enlarge the field of view. After the crystal or thin section has been examined, the inclination of the hemisphere is determined by raising the two graduated arcs until they intersect directly under the cross-hairs of the microscope, and reading the values there indicated with the same objective as that with which the examination of the mineral was made. The graduations of the arcs are in degrees with 0° at the center and 90° at the hinges.

Wright[4] was able to determine, within a degree, the amount of rotation

[1] E. G. A. ten Siethoff: *Beitrag zur Krystalluntersuchung im convergenten polarisirten Lichte.* Centralbl. f. Min., etc., 1903, 657–658.

[2] Wladimir Arschinow: *Ueber die Verwendung einer Glashalbkugel zu quantitativen optischen Untersuchungen am Polarisationsmikroskope.* Zeitschr. f. Kryst., XLVIII (1910–11), 225–229.

[3] Cf. footnote 23, *supra*.

[4] Fred. Eugene Wright: *The methods of petrographic-microscopic research.* Carnegie Publication No. 158, Washington, 1911, 175.

of a hemisphere of glass, 63 mm. in diameter, by having it engraved with parallels and meridians, 5° apart (Fig. 412). The opening of the stage in which the hemisphere rested (Fig. 413) coincided exactly with the 20° parallel, and two small notches, cut in the edge of this opening, indicated the zero meridian. Across the flat surface of the hemisphere two lines were engraved, crossing at right angles at the center, to assist in centering.

A very simple rotating instrument was suggested by Jaggar[1] in 1897. It contains no graduated circles and was designed simply to tilt sections

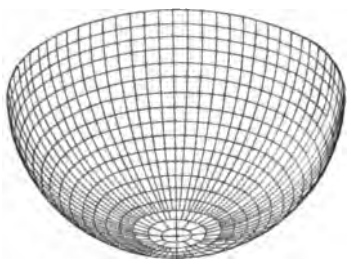


FIG. 412.

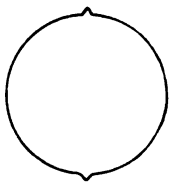


FIG. 413.

FIGS. 412 and 413.—Wright's hemispherical rotation apparatus.

into position to obtain maximum extinction angles, maximum interference colors, or to change slightly the orientation of interference figures in such cases where a measure of the amount of the rotation is not required. The instrument is attached to the stage by means of pins in the object-clip holes. It consists of a pair of spring clips, supported 15 mm. above the stage by a ball-and-socket joint which may be moved in any direction by means of a long removable key. A thumb-screw controls the tension on the ball-and-socket joint by means of pressure against a brass plate faced with cork which fits the ball. The amount of rotation possible is about 45°.

Recently a rotation microscope[2] has been placed upon the market, an instrument especially convenient since thin sections of the usual size may be used. It is described and figured in Article 184.

[1] T. A. Jaggar, Jr.: *A simple instrument for inclining a preparation in the microscope.* Amer. Jour. Sci., III (1897), 129–131.

[2] Cf. also the microscope described in Art. 183. In this, however, the universal stage is of extraordinary size.

CHAPTER XIX

THE COLOR OF MINERALS

255. Idiochromatic and Allochromatic Minerals.—Color[1] is another property of minerals which may be determined by ordinary light, and all colored minerals may be divided into two classes, those that are *idiochromatic* and those that are *allochromatic.* In the first the color is due to a property of the mineral itself, namely its power to absorb light of certain wave lengths. This property of absorption, however, may not be the same in every direction, or different wave lengths of light may be absorbed and, in consequence, the mineral may show what is known as dichroism or pleochroism.[2] In the second, the color is due to minute inclusions. The latter may be of such size that they can be distinguished under the microscope, or they may be so small and so sparsely distributed that they cannot be seen even with the highest powers. They are then spoken of as "dilute"[3] colors.

As to the nature of the coloring material, there exists great diversity of opinion. Being so dilute, attempts to analyze it years ago resulted in producing the opinion that they were volatile organic substances. Thus Schneider[4] thought the color of gems due to hydrocarbons.

Among the advocates of the organic nature of the coloring material were Wyrouboff, who experimented on fluorite, and Kraatz-Koschau and Wöhler, who determined the presence of carbon, nitrogen, and hydrogen in zircon, smoky quartz, amethyst, fluorite, apatite, calcite, microcline, baryte, rock-salt, and topaz. Among those who insisted on the inorganic nature of the pigment were Becquerel and Moissan, and Loew, who found free fluorine in fluorite; Weinschenk, Lehmann, Rosenbusch, and Spezia,[5] who found traces of iron in brown zircon.

There seems to be no doubt that the color of minerals of the second class is always produced by some foreign substance although its nature may or may not be known. Without a doubt, in some minerals, it is inorganic; less clearly proven, in others, is its organic nature. Whatever the pigment may be, it is distributed, in some places evenly, in others irregularly, through the mineral, but often so sparingly that a thin section appears absolutely

[1] See General Bibliography at end of Chapter.
[2] See Chap. XXI.
[3] H. Fischer: *Op. cit.*, II Abth. See General Bibliography at end of Chapter.
[4] J. Schneider: *Ueber Phosphorescenz durch mechanische Mittel.* Pogg. Ann., XCVI (1855), 282–287.
[5] Giorg. Spezia: *Sul colore del zircon.* Atti della Reale Accad. delle Scienze di Torino, XII (1876).* Review in Neues Jahrb., 1877, 303–305.
Idem: Same title. Atti della Accad. etc. Torino, XXXV (1889).* Review in Neues Jahrb., 1900 (II), 344.

colorless. Not only may the coloring matter be irregularly distributed, but two colors may appear in the same crystal. That radium has some effect in bleaching or coloring minerals may be seen by the pleochroic halos about certain included radioactive minerals, such as zircon in biotite or cordierite, but that radium can produce a color in a naturally colorless mineral is not proven.

Partly owing to the fact that the color of minerals is not a constant property, and partly because no simple color tables have been available, colors are named, at the present time, just as they were by Werner, over a hundred years ago.[1]

256. Determination of Color.—Long ago Fischer[2] attempted to classify definitely the color of minerals, and found in Radde's color scale[3] a means of comparison. This work, now long out of print, consisted of a series of colors arranged in the order of the spectrum. The main divisions, vermilion, orange, yellow, yellowish green, grass-green, bluish green, blue, violet, purple, and carmine, graded into each other and formed 30 transition members. In addition to these, there were twelve other colors, neutral gray, vermilion gray, brown, orange-gray, yellowish gray, yellowish greenish gray, greenish gray, bluish greenish gray, bluish gray, violet-gray, purplish gray, carmine-gray. At right angles to these forty-two colors were arranged twenty-one tones of each, ranging from black to nearly colorless, and lettered from *a* to *v*. To designate any particular color it was simply necessary to use a number and a letter, as 30*c*, 21*f*, etc.

Owing to the fact that Radde's color scale is now almost unobtainable, Möller[4] proposed Klincksieck et Valette's *Code des Couleurs* as a standard. There has recently appeared a work by Ridgway[5] in which 1115 color tints are shown arranged by tints in a manner similar to Radde's. An objection to any color scale of this kind is the impossibility of matching the colors seen under the microscope by transmitted light with the opaque colors of the scale seen by incident light.

Numerous devices have been suggested for producing the colors of the spectrum for comparison with the transparent colors seen under the microscope. While such instruments are of considerable value in the comparison of interference colors, and as such will be described below, they do not greatly

[1] The list of colors with their subdivisions may be found in many mineralogies, for example in:
Gustav Tschermak: *Lehrbuch der Mineralogie.* Wien, 3 Aufl., 1888, 156–157.
Wilhelm Haidinger: *Handbuch der bestimmenden Mineralogie.* Wien, 1 Aufl., 1845, 332–343.

[2] H. Fischer: *Ueber die Beziechnung von Farbenabstufungen bei Mineralien.* Neues Jahrb., 1879, 854–857.

[3] Internationalen Farbenskale von Radde in Hamburg. Société stenochromique, Paris.*

[4] Hans Jakob Möller: *International Farbenbestimmungen.* Ber. deutsch. pharmazeut. Gesell., 1910, 358–368. Review in Neues Jahrb., 1911 (II), 162.

[5] Robert Ridgeway: *Color standards and nomenclature.* Pp. iii+44, pl. 53. Washington, D. C., 1913.* Review by W. J. Spillman, Science, XXXVII (1913), 985–989.

help in the determination of the ordinary colors of minerals, the latter not being the pure colors of the spectrum.

This method of comparison was suggested, in 1849, by Brücke,[1] who proposed a gypsum wedge, mounted between glass plates, as a means of producing the colors. Later[2] he invented a simple apparatus which he called a "Schistoskop," in which interference colors, produced by gypsum plates, were used for comparison. Arons,[3] in 1910, described a "chromoscope" in which the colors are produced by the passage of light, between crossed nicols, through quartz plates of different known thicknesses and cut at right angles to the axis, the variation being produced by rotating the nicol through some angle less than 180°. A quartz wedge may be used instead of quartz plates of different thicknesses. Wright[4] suggested that the Ives colorimeter might be used with the microscope. This consists of red, green, and blue ray filters so arranged that when the three are simultaneously viewed, the light is white. The screens are mounted on a disk driven rapidly by an electric motor, and the amount of each light is regulated by shutters so made that the percentage of each, used in producing the proper color, can be determined.

Nutting[5] described and illustrated an apparatus in which the spectral colors are used for comparison; the various shades being produced by the admission of more or less white light.

257. Determination of the Color of Opaque Minerals.—For the observation of the colors of opaque minerals, an apparatus was invented by Inostranzeff.[6] As originally made, the color of the mineral, viewed directly through the microscope, was compared with a standard mineral by means of reflection through two prisms from a known mineral in another microscope. The device was not satisfactory, however, since the comparison was made between a color seen directly and one seen only by reflection. Inostranzeff, therefore, improved the double ocular by placing the eyepiece intermediate between the two microscopes (Fig. 414), from each of which appears one-half the field, one with the unknown and one with the known mineral for comparison. If two opaque minerals of the same kind are brought to the center of the field, no separating line will be seen between them. If there is even a very slight difference, the line will appear. The scale for comparison, instead of being made up of the natural minerals, which would be very expensive, is prepared from the powder of the minerals, and reproduces both color and luster very well.

[1] Ernst Brücke: *Ueber die Aufeinanderfolge der Farben in den Newton'schen Ringen.* Pogg. Ann., LXXIV (1849), 582–586.

[2] Idem: *Die Physiologie der Farben für die Zwecke der Kunstgewerbe.* Leipzig, 1887.

[3] Leo Arons: *Ein Chromoskop.* Ann. d. Phys., 4 ser., XXXIII (1910), 799–832.

[4] Fred. Eugene Wright: *The methods of petrographic-microscopic research.* Carnegie Publication No. 158, Washington, 1911, 69.

[5] P. G. Nutting: *Outline of applied optics.* Philadelphia, 1912, Chapter VI.

[6] A. v. Inostranzeff: *Ueber eine Vergleichungskammer zur mikroskopischen Untersuchung undurchsichtiger Mineralien.* Neues Jahrb., 1885 (II), 94–96.

A modification of this apparatus, giving a field divided horizontally into halves (Fig. 415), was made by Van Heurck[1] for comparing diatoms. This suggests the use of such a device for comparing thin sections of rocks from any region, or sections of similar rocks from different regions.

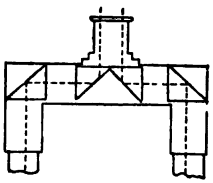


FIG. 414.—Inostranzeff's comparateur.

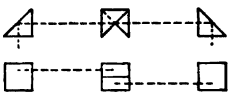


FIG. 415.—Van Heurck's comparateur.

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[1] Van Heurck: Bull. Soc. Belg. Microsc., XIII (1886), 76–78.* Review *Van Heurck's comparator.* Jour. Roy. Microsc. Soc., 1887, 463–464.

CHAPTER XX

MONOCHROMATIC LIGHT

258. The Production of Monochromatic Light.—For most petrographical determinations, ordinary white light answers the purpose, but for very exact measurements it is necessary to use monochromatic light. As an example, the case of refractive indices may be cited. From the well-known phenomenon of the spectrum, it may be seen that light, in passing through a prism, is broken up into rays having greater or less angles of refraction (Fig. 78). But with an increased angle of refraction there is also an increased value for the sine, and with this a decreased value in the refractive index. The consequence is that white light, passing from a rarer to a denser medium, emerges with rays having different refractive indices, that of the violet, which is bent most from its original course and has the least angle of refraction, is the highest, and that of the red, which is bent least, is the lowest. Thus in crown glass the index for the *A* line (red) is 1.5089, for *D* (yellow) 1.5146, and for *H* (violet) 1.5314.

TABLE OF WAVE LENGTHS[1]

Color	Fraunhofer line	Wave length λ	Produced by
Red	.....	769.93$\mu\mu$	.....
	.....	766.56	K
	A	759.40	.....
	B	686.74	.....
	.....	670.82	Li
Orange	*C*	656.30	H
	.....	610.38	Li
Yellow	D_1	589.62	Na
	D_2	589.02	Na
Green	.....	535.06	Tl
	E_1	527.05	.....
	E_2	526.97	.....
	b_1	518.38	.....
Blue	*F*	486.15	H
	f	434.00	.....
Indigo	*G*	430.79	.....
Violet	*h*	410.19	H
	H	396.81	.....

[1] Louis Bell: *The absolute wave length of light.* Phil. Mag., XXV (1888), 245–263, 350–372.

Henry A. Rowland: *A new table of standard wave lengths.* Astron. and Astrophys., XII (1893), 321–347.

The usual methods of producing monochromatic light are as follows:

1. By means of ray filters.
2. By the vaporization of certain solids.
3. By means of incandescent gases.
4. By means of certain rays of the spectrum, separated by a monochromator.

259. Ray Filters.—Truly monochromatic light cannot be produced by the absorption of the other colors by means of ray filters, although such devices suffice for many purposes. Thus a glass coated with a thin film of copper oxide will transmit light between the Fraunhofer lines *a* and *D;* blue cobalt glass will permit blue and extreme red to pass. More satisfactory are ray filters made of colored solutions enclosed in parallel-walled glass vessels, 15 to 20 mm. between walls, and used in combinations to give any desired color. Landolt[1] gives the following:

Color	Thickness of filter mm.	Aqueous solution of	Grams per 100 c.c. water	λ in $\mu\mu$	Mean λ
Red	20	Crystallized violet, 5 BO	0.005	639–718	665.9
	20	Potassium chromate	10.0		
Yellow	20	Sulphate of nickel ($NiSO_4$ +7 aq)	30.0		
	15	Potassium chromate	10.0	574–614	591.9
	15	Potassium permanganate	00.025		
Green	20	Copper chloride ($CuCl_2$+2 aq)	60.0		
	20	Potassium chromate	10.0	505–540	533.0
Blue (light)	20	Double green SF	0.02	green 494–526	
	20	Copper sulphate ($CuSO_4$+5 aq)	15.0	blue 458–494	488.5
Blue (dark)	20	Crystallized violet 5BO	0.005		
	20	Copper sulphate ($CuSO_4$+5 aq)	15.0	410–478	448.2

Crystallized violet 5BO is the trade name for the chlorhydrate of hexmethyl pararosaniline. Double green SF is chlormethyl hexmethyl pararosaniline chlorhydrate with zinc chloride. The pale blue light is not satisfactory since the band, from 458 to 526$\mu\mu$, is too broad and includes green and blue. All of the solutions are in water alone except the crystallized violet 5BO, the crystals of which should be dissolved in a small quantity of alcohol and then diluted with water to one liter. The stock of the two aniline solutions should be kept in the dark, the others do not alter except the potassium permanganate which must be made up fresh frequently.

For some purposes, one or more 20-mm. cells cannot be used, either on account of their thickness or on account of the reduction of light by reflection

[1] H. Landolt: *Methode zur Bestimmung der Rotationsdispersion mit Hülfe von Strahlenfiltern.* Ber. d. d. chem. Ges., XXVII (1894), 2872–2887, in particular 2884.

Idem: Sitzb. Akad. Wiss. Berlin, 1894, 923.

Idem: *Das optische Drehungsvermögen.* Braunschweig, 2 Aufl., 1898, 387–390.

from the many glass cell-walls. Nagel[1] gives a list of fluids which can be used in single cells, and which need not be more than 1 cm. in thickness. The materials are common, the solutions are easily made, and will keep for weeks without precipitation in closed cells. No proportions are given, the spectroscope being used to determine the proper light transmitted. The solutions are as follows:

Red: Lithium-carmine such as is used for microscopic coloring material. A thickness of 1 mm. gives a pure red, 1/2 mm. red with a tinge of orange.

Orange: No single fluid known which transmits only orange. Aniline orange lets the red rays pass through; a solution of potassium bichromate 1 cm. thick passes the red, orange, yellow, and yellowish green. A monochromatic orange filter may be made by preparing a not quite saturated solution of copper acetate acidified with a few drops of acetic acid. Add slowly, drop by drop, enough strong saffranin solution to extinguish the pure yellow, as shown by the spectroscope. With a thickness of 1 cm. the visible line will begin near the *C* line and end with the *D*, the pure orange, with a wave length of 640–600$\mu\mu$, being the only bright color transmitted.

Yellow: Pure yellow is a difficult color to obtain since the band is so narrow. A single cell 1 cm. thick which permits rays having a wave length between 620–570$\mu\mu$ to pass, that is orange-yellow, yellow, and greenish yellow, may be made by adding a saturated aqueous solution of orange G to an acidified copper acetate solution. The solution is of a brown color and does not keep well.

Greenish yellow and yellowish green: A very transparent filter may be made by boiling an excess of crystals of copper acetate in a saturated solution of potassium bichromate which has been acidified with acetic acid. The solution should be filtered after cooling. The 580–530$\mu\mu$ waves will pass through a cell 1 cm. thick.

Green: If one dissolves as much copper acetate as possible in a non-saturated solution of potassium bichromate or picric acid, one may obtain green filters. Increasing the amounts of the potassium bichromate or picric acid used cuts off more and more from the blue-green end.

Pure green or yellow-green: To a saturated solution of copper ammonium sulphate with an excess of ammonia, add, drop by drop, a saturated solution of potassium chromate, until the entire red, orange, yellow, and yellow-green rays are extinguished. The rays transmitted through a filter 0.7 mm. in thickness have a wave length of 535 to 495$\mu\mu$. The blue-green rays are removed (535–510$\mu\mu$ transmitted) by adding to the above a few drops of a weak alkaline aqueous solution of fluorescine.

Blue-green and cyan-blue: In an acidified copper acetate solution drop strong methyl green solution. 500 to 460$\mu\mu$ transmitted.

Cyan blue: A few drops of gentian violet solution added to the preceding makes a pure and strong blue. Transmitted rays 460–430$\mu\mu$.

Blue and violet: 470–410$\mu\mu$ may be cut out by copper ammonium sulphate solution, and blue and violet will be transmitted. By passing the rays through another cell containing a dilute solution of potassium permanganate, pure violet results.

[1] Wilibald A. Nagel: *Ueber flüssige Strahlenfilter.* Biol. Centralbl., XVIII (1898), 649–655.

Colored gelatine plates have been used as ray filters by Kirschmann[1] and others,[2] but they are not so satisfactory as liquid films, since they are not completely transparent, can stand neither heat nor moisture, and are not permanent, the aniline color fading. They are, however, usually more convenient to use than liquid filled cells. The simplest way to prepare such a filter is to fix an unexposed dry plate in hyposulphite of soda, as if it were a negative, then place the gelatine-coated plate in the desired color and dry in a dust-proof room.

For yellow use 1 grm. Mars yellow in 200 c.c. of 70 per cent. alcohol, or a saturated solution of aurantia in alcohol. For red dissolve (a) 2 grm. aurantia in 40 c.c. absolute alcohol, (b) 5 grm. rose Bengal in 20 c.c. methyl alcohol. Mix 20 c.c. of (a) with 10 c.c. of (b) and 90 c.c. of 4 per cent. collodion. For green use copper nitrate 160 grm., chromic acid 14 grm., distilled water 250 grm. Another green is eosine or malachite green. For blue use methylene blue.

If the color does not stain the gelatine well, it should first be mixed with 4 per cent. collodion.

260. Incandescent Vapors of Solids.—A limited number of colors may be produced by the vaporization of certain solids. The method is very simple and the colors are essentially monochromatic. The salts ordinarily used are lithium sulphate for red ($\lambda = 670\mu\mu$), sodium sulphate, sodium chloride, or sodium carbonate for yellow ($\lambda = 589\mu\mu$), and thallium sulphate for green ($\lambda = 535\mu\mu$). Yellow light is the one most commonly used. The sodium chloride gives the most intense light but the carbonate lasts longer.

For any of these colors the salt may be enclosed in a coil of platinum wire, or a piece of pumice may be saturated with a solution, and placed over a Bunsen or alcohol burner, or any one of the many more or less handy devices for the vaporization of the salt may be used.[3] Fig. 416 shows a very handy

[1] A. Kirschmann: *Ueber die Herstellung monochromatischen Lichtes.* Philos. Studien von W. Wundt. Bd. VI (1891), 543–552.*

[2] J. William Gifford: *An inexpensive screen for monochromatic light.* Jour. Roy. Microsc. Soc., 1894, 164–167.

K. Diederichs: *Die Herstellung von gegossenen Gelatineplatten als Strahlenfilter.* Zeitschr. f. angew. Mikrosk., IX (1903), 197–198.

Ernst Pringsheim, jun.: *Ueber die Herstellung von Gelbfiltern und ihre Verwendung zu Versuchen mit lichtreizbaren Organismen.* Ber. deutsch. Bot. Gesell., XXVI A (1908), 556–565.

J. Jullien: Bull. Soc. Zool. de Genève, 1908, 104.* Review *Economical monochromatic filters.* Jour. Roy. Microsc. Soc., 1909, 522.

[3] See H. Landolt: *Das optische Drehungsvermögen.* Braunschweig, 2 Aufl, 1898, 353–359.

Dr. Pribram: *Ueber einen neuen Brenner für Natriumlicht.* Zeitschr. f. analytische Chemie, XXXIV (1895), 166.

H. Landolt: *Natriumlampe für Polarisationapparate.* Zeitschr. f. Instrum., IV (1884), 390.

H. E. J. G. du Bois: *Ein Intensivnatronbrenner.* Zeitschr. f. Instrum., XII (1892), 165–167.

burner. It consists of a telescopic Bunsen burner, which may be raised or lowered, and a metal chimney to preserve a steady flame. The salt is placed in a platinum cup so arranged on a rod that it may be instantly thrown in or out of the flame by means of the pivot *c*.

Another burner, arranged for three different salts, each in a platinum or asbestos cup, is shown in Fig. 417. The change may be made quickly from one colored flame to another.

A number of very elaborate devices on the principle of an atomizer, are given by Beckmann.[1]

It is very desirable that a hood to carry off fumes be arranged above any burner producing monochromatic light; thallium fumes because they are poisonous, and sodium because the minute particles will long remain suspended in the air and will overpower, for hours afterward, any other flame that may be used.

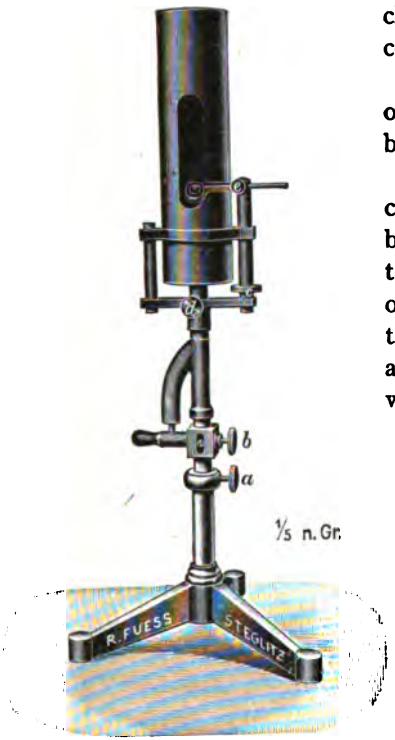

FIG. 416.—Burner for producing monochromatic light. 1/5 natural size. (Fuess.)

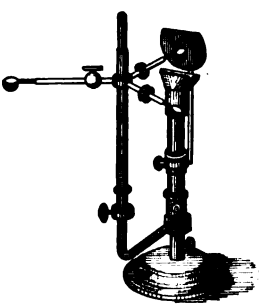

FIG. 417.—Burner for producing three different monochromatic lights. (Steeg und Reuter.)

261. Incandescent Gases.—Electricity from an induction coil, passed through a Geissler tube filled with hydrogen, will give a spectrum of four lines only, namely $\lambda = 656.3\mu\mu$, $486.1\mu\mu$, $434.0\mu\mu$, and $410.2\mu\mu$, corresponding to the Fraunhofer lines *C*, *F*, *f*, and *h*. By the use of suitable ray filters, any one of these lines may be separated from the others, but the light obtained is not very intense.

262. Dispersed White Light Produced by a Monochromator.—The purest monochromatic light that can be produced is that derived from the dispersion of white light. It is, however, not much used in ordinary petro-

[1] Ernst Beckmann: *Ueber Spektrallampen.* Zeitschr. f. phys. Chemie, XXXIV (1900), 593–611; XXXV (1900), 443–458, 652–660.

graphic research because it is necessary to use an elaborate and expensive piece of apparatus. Where such an instrument is at hand for mineralogical work, it may well be used, also, with the microscope. This instrument breaks

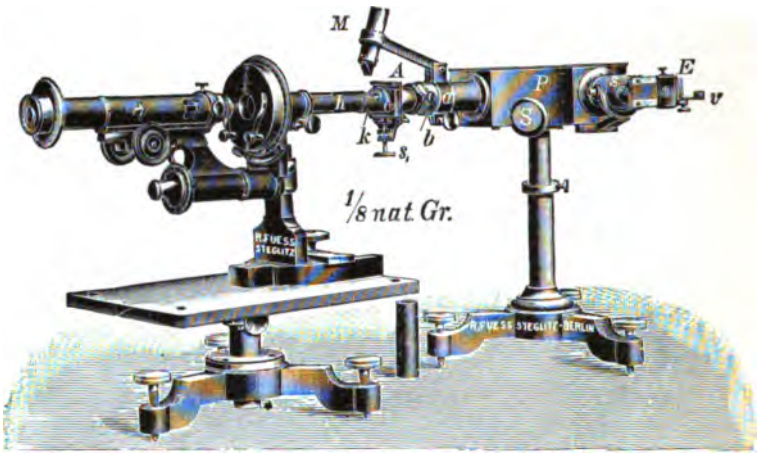

FIG. 418.—Wülfing's monochromator. 1/8 natural size. (Fuess.)

up white light into a continuous spectrum but permits only the desired rays to pass out through a narrow slit, perhaps a tenth of a millimeter in width.

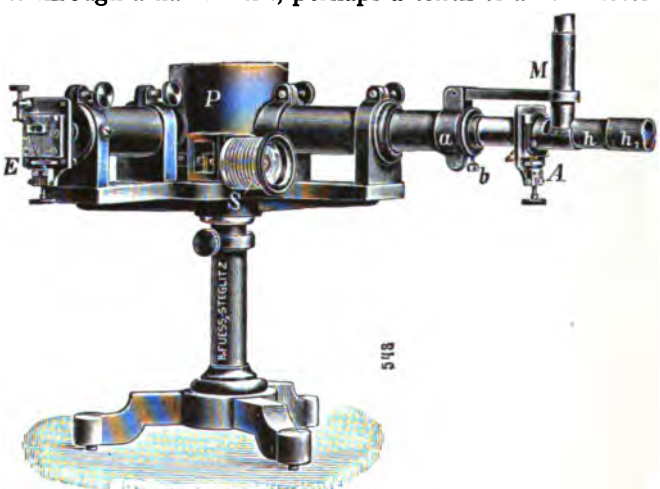

FIG. 419.—Monochromator. 1/6 natural size. (Fuess.)

An instrument of this kind, really a spectroscope with the addition of an adjustable slit, was described by Tutton.[1] The source of light is an electric

[1] A. E. Tutton: *An instrument of precision for producing monochromatic light of any desired wave-length, and its use in the investigation of the optical properties of crystals.* Phil. Trans. Roy. Soc. London, (A), CLXXXV (1894), 913–941.

Idem: Same title in German. Zeitschr. f. Kryst., XXIV (1894–5), 455–474.

arc, the prism is one of 60°, perfectly colorless, with refracting faces 4 1/2 by 2 1/2 in. The diffusion of the light is produced by ground-glass screens of two degrees of fineness. Wülfing[1] described a similar instrument (Fig. 418) in which, however, two prisms are used to produce the spectrum and a lens to diffuse the light. This instrument possesses the advantage that neither light source nor examining instrument need be moved to change from one color of light to another, the change being produced by means of a rotation of the prisms. Daylight or electric light may be used. In the Fuess[2] monochromator (Fig. 419), the prism used is a Pellin and Broca constant deviation prism with a dispersion of 3°. The wave length of the light emitted may be read directly from the large drum S, graduated from 390.0$\mu\mu$ to 775.0$\mu\mu$, which controls the rotation of the prism.

[1] E. A. Wülfing: *Ueber einen Spectralapparat zur Herstellung von intensivem monochromatischem Licht.* Neues Jahrb. B. B., XII (1898–9) 343–404.

[2] C. Leiss. *Zwei Spektralapparate (Monochromatoren) zur Beleuchtung mit homogenem Licht.* Zeitschr. f. Instrum., XXIX (1909), 68–72.

CHAPTER XXI

EXAMINATION BY PLANE POLARIZED LIGHT

Absorption, Dichroism, Pleochroism

263. Absorption of Light in Crystals.—Upon its emergence from any substance, the intensity of light is more or less reduced from that with which it entered. That is to say, a certain amount of light, in the course of its transmission, is absorbed by the body through which it travels. If this absorption is very slight and the amount is the same for rays of every wave length, the body is said to be transparent and colorless. If the absorption of certain rays is greater than others, the body is colored. If the absorption is so great that even in very thin sections no light passes through, the body is opaque.

If we construct geometrical solids to represent the amount of light absorbed after passing through crystals in every direction, we will obtain figures resembling the indicatrices. These figures are called **absorption surfaces** and differ for different crystal systems.

264. Isotropic Substances.—Since light travels with equal ease in every direction in isotropic substances, the absorption, for any color, must necessarily likewise be the same in every direction, whereby the absorption surface will be a sphere, and all sections of the same thickness cut from a mineral will appear of the same shade and color. For any other wave length of light, the absorption surface is a sphere whose diameter differs from the first.

265. Anisotropic Substances.—In anisotropic minerals, absorption may differ in different directions, whereby sections of a crystal, cut in different directions but of the same thickness, may appear of different colors, as, for example, cordierite or tourmaline. This property of crystals is called **pleochroism** (or **dichroism**[1]), and is possessed, to a certain extent, by a great many minerals. It is a valuable means of diagnosis, and may be determined very simply, under the microscope, by inserting the nicol below the thin section so as to produce plane polarized light, that is, light which vibrates parallel to one plane only. If the analyzer were inserted instead of the polarizer, the phenomenon would be obscured by the partial polarization produced by reflection from the mirror below the mineral. The reason that one does not see this difference in color without a polarizer is that the eye observes the resultant of the rays vibrating in both directions, and only when one set of

[1] See General Bibliography at end of chapter.

rays is cut out can it be perceived. In certain minerals the absorption is so complete in one direction that the phenomenon is visible without the nicol.

266. Uniaxial Crystals.—The colors apparent in viewing a section of a colored uniaxial mineral, cut at right angles to the optic axis, are those due to the ordinary ray. Since these travel with the same ease in every direction, they have the same absorption coefficient, consequently, no matter how the stage of the microscope is rotated, the color remains the same. If the section is cut at an angle with the optic axis, a difference in color may appear on rotating the stage, and this difference is at its maximum when the section is cut parallel to the optic axis. If the latter section is placed on a rotating apparatus, and it is turned in altitude about the optic axis, no change in color appears. In other words, the form of the absorption surface is that of an ellipsoid of rotation with the optic axis as its axis. The optic axis is also the direction of least or greatest absorption, consequently we have oblate or prolate absorption ellipsoids, depending upon whether this axis is that of minimum or

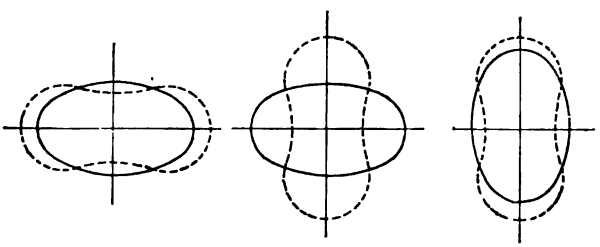

FIG. 420. FIG. 421. FIG. 422.

FIGS. 420 to 422.—Indicatrix and absorption surface compared in tourmaline, apatite, and melilite. Solid line = indicatrix, broken line = absorption surface. FIG. 420, Negative, absorption $\omega > \epsilon$; Fig. 421, negative, absorption, $\omega < \epsilon$; FIG. 422. positive, absorption $\omega < \epsilon$.

maximum absorption. The two cases are ordinarily written, *Absorption* $O > E$ (or *Absorption*, $\omega > \epsilon$), and *Absorption*, $E > O$ (or *Absorption* $\epsilon > \omega$), where O and E represent the directions of vibration of the ordinary and extraordinary rays, and ω and ϵ the directions of their respective indices. Thus tourmaline is negative (indices $\epsilon < \omega$) and it is darkest when the vibrations take place at right angles to c ($c = \epsilon$). The absorption, therefore, is greatest parallel to the direction of vibration of the ordinary ray and we have Absorption $\omega > \epsilon$ or Absorption $O > E$ (Fig. 420). In apatite, likewise negative ($\epsilon < \omega$), the mineral is darkest when the c axis lies parallel to the vibration direction of the lower nicol, therefore we have Absorption $\epsilon > \omega$ (Fig. 421). In melilite, which, in some cases, is positive ($\epsilon > \omega$), the extraordinary ray is most absorbed (Fig. 422), therefore Absorption $\epsilon > \omega$. Care must be taken, in writing descriptions, not to confuse the values of absorption and of refractive indices; the word absorption should always be written before the former.

Ordinarily, in uniaxial crystals, the absorption is greatest in the direction of the greatest refractive index (Figs. 420 and 422), whether the crystal be

positive or negative, a rule first given by Babinet[1] who recognized, however, that there were many exceptions to it.

267. Biaxial Crystals.—If a section of a colored biaxial crystal, cut at right angles to an optic axis, be examined, it will be found that the absorption is the same in every direction. This was to have been expected, since in such sections the ease of vibration is likewise the same in every direction. If, however, any other section be examined, it will be found, in many cases, that there is a difference in color in two directions, and that the colors in different sections likewise differ one from another. If a surface of absorption be constructed, it may, in general, be represented by a triaxial ovaloid, the values of whose axes (**absorption axes,** *Laspeyres*[2]) bear no relation to the values of the axes of the indicatrix. It was formerly held, and to a certain extent is still so taught, that the absorption axes coincide with the vibration axes. They do, certainly, in uniaxial crystals; in biaxial crystals in which the vibration directions coincide with the crystallographic axes, namely in the orthorhombic system; and along the *b* axis of the monoclinic system. Laspeyres[3] made determinations which seemed to prove that the absorption axes are, like the vibration axes, always at right angles to each other. In monoclinic crystals, one axis coincides with crystallographic *b*, the other two may or may not coincide with the directions of vibration. Thus, in piedmontite he found that the absorption axes made an angle of 20° with the axes of vibration. In triclinic crystals none may coincide. Voigt,[4] Becquerel[5] and Ramsay[6] came to the same conclusion, but Ehlers[7] who made examinations of certain uniaxial and monoclinic crystals, the latter being salts of cobalt, found that in those examined the absorption axes coincided with the vibration axes.

As the pleochroism of uniaxial crystals is divided into two classes, so may also that of biaxial crystals be divided, according to whether the maximum absorption lies in the plane of the optic axes or at right angles to it.

[1] M. Babinet: *Sur l'absorption dans les milieux colorés biréfringents.* Comptes Rendus, VII (1838), 832–833.

Idem: Abstract of preceding. *Ueber die Absorption in farbigen doppeltbrechenden Mitteln.* Pogg. Ann., XLVI (1839), 478–480.

[2] H. Laspeyres: *Mineralogische Bemerkungen.* Zeitschr. f. Kryst. IV (1879–80), 433–467, in particular 454.

[3] H. Laspeyres: *Op. cit.*, particularly 444–460, and especially 454–460.

[4] W. Voigt: *Erklärung der Farbenerscheinungen pleochroitischer Krystalle.* Neues Jahrb., 1885 (I), 119–141.

[5] Henri Becquerel: *Sur les lois de l'absorption de la lumière dans les cristaux et sur une méthode nouvelle permettant de distinguer dans un cristal certaines bandes d'absorption appartenant à des corps différents.* Comptes Rendus, CIV (1887), 165–169.

[6] W. Ramsay: *Ueber die Absorption des Lichtes im Epidot vom Sulzbachthal.* Zeitschr. f. Kryst., XIII (1887–8), 97–134.

[7] Johannes Ehlers: *Die Absorption des Lichtes in einigen pleochroitischen Krystallen.* Neues Jahrb., B. B., XI (1897–8) 259–317.

268. Pleochroic Halos.—In certain minerals, surrounding small inclusions of other minerals, there appear rounded spots or halos which are more strongly pleochroic than their host, although the maximum absorption directions of the two are parallel. As a matter of fact the "halos" are not circular but spherical, for they show the same outlines, no matter what the direction in which the section cuts the mineral. If the included grain is decidedly elongated, these spots are ellipsoidal instead of spherical, though such occurrences are quite rare. The borders around irregular grains, which are approximately equidimensional, are also spherical.

The minerals in which these pleochroic halos have been observed are andalusite,[1] augite, biotite,[1, 5] chlorite,[1] cordierite,[1] diopside,[2] glaucophane,[3] hornblende,[4, 5] ottrelite,[1] and tourmaline,[6] and the inclusions around which they occur are allanite,[5] apatite,[12] biotite,[9] cassiterite,[7] dumortierite,[8] pleonaste,[9] rutile,[6, 7] titanite,[8] topaz,[7] and zircon.[8]

As to the cause of these halos, there has, until recently, been great diversity of opinion. They have been thought to be organic, [1, 4, 6] or due to a local increase in the amount of the iron molecule,[8, 10] but within the last few years the belief has become general that they are due to radioactive emanations.[11] That they are not due to diffusion or aggregation is clearly evident from the fact that the sphere extends across, as well as in the direction of the cleavage in such minerals as biotite, or even from one mineral to another. The probability that the halos are due to the radioactive property of the included mineral was first pointed out by Joly,[12]

[1] H. Rosenbusch: *Die Steiger Schiefer und ihre Contactzone an den Graniten von Barr-Andlau und Hohwald.* Strassburg, 1877, 221, 281.*

[2] Idem: *Mikroskopische Physiographie.* 2te Aufl., Stuttgart, 1885, 191.

[3] Konstantin Anton Ktenas: *Die Einlagerungen im krystallinen Gebirge der Kykladen auf Syra und Sifnos.* T. M. P. M., XXVI (1907), 277.

[4] A. Michel-Lévy: *Propriété optiques des auréoles polychroïques.* Comptes Rendus, CIX (1889), 973–976.

[5] E. Cohen: *Ueber pleochroitische Höfe im Biotit.* Neues Jahrb., 1888 (II), 166–169.

[6] H. Traube: *Ueber pleochroitische Höfe im Turmalin.* Neues Jahrb., 1890 (I), 186–188.

[7] H. Rosenbusch: *Mikroskopische Physiographie,* 3te Aufl., 1892, 210.

Johannsen found, in a topaz granite, a pleochroic band at the contact between crystals of biotite and topaz.

[8] A. Michel-Lévy: *Sur les noyaux d polychroïsme intense du mica noir.* Comptes Rendus, XCIV (1882), 1196–1198.

Idem: *Propriétés optiques des auréoles polychroïques.* Comptes Rendus, CIX (1889), 973–976.

[9] O. Mügge: *Radioaktivität und pleochroitische Höfe.* Centralbl. f. Min., etc., 1909, 66.

[10] Hj. Gylling: *Några ord om Rutil och Zirkon med särskild hänsyn till deras sammanväxning med Glimmer.* Geol. Fören. i Stockholm Förh., VI (1882–3), 162–168.

[11] J. Joly: *Radioactivity and Geology.* New York, 1909, 64–69.

[12] Idem: *Pleochroic halos.* Phil. Mag., XIII (1907), 381–383.

and it has been shown, experimentally, that when biotite[1] or cordierite[2,3] are exposed to the rays of a small particle of radium, similar colored and pleochroic patches are produced. Another evidence for this theory is the fact that in size they never exceed 0.05 mm., and average 0.04 mm., which is almost exactly the distance that radium can affect a photographic plate through a medium having the density of biotite.[4] The actual change produced by the radium in the mineral, causing this intense pleochroism, is not known. Whatever it is, it causes a difference in the double refraction, which may be greater or less than that of its host,[5] and perhaps, also, a change in the dispersion.

269. Pseudo-pleochroism, Pseudo-dichroism, or Pseudo-absorption.—Certain minerals appear colorless in one direction and dirty brown in another, giving an appearance of absorption, although the phenomenon is not due to absorption at all. According to v. Fedorow,[6] it is shown to some extent by all minerals which have strong double refraction, especially by those which have very good cleavage and fine lamellation, as calcite, dolomite, and magnesite, and is due to the great difference in the refractive indices in two directions, thus permitting the rays whose vibrations are parallel to the lamellation to be totally reflected and those which enter at right angles to it to pass through, giving, in consequence, an appearance of partial absorption. According to Schroeder van der Kolk,[7] pseudo-pleochroism is due to the fact that innumerable sub-microscopic inclusions are arranged in parallel position within the mineral, so that when light enters in one direction it passes through without change, but when it enters in another, it is refracted and produces a brown tone.

270. Interference Phenomena, without the Analyzer, Produced by an Overlying Pleochroic Mineral.[8]—When a doubly refracting mineral occurs

[1] J. Joly: *Pleochroic holos.* Nature, LXXVI (1907), 589.

[2] O. Mügge: *Radioaktivität als Ursache der pleochroitischen Höfe des Cordierit.* Centralbl. f. Min., etc., 1907, 397–399.

[3] Idem: *Radioaktivität und pleochroitische Höfe.* Centralbl. f. Min., etc., 1909, 65–71, 113–120, 142–148.

[4] Georg Hövermann: *Ueber pleochroitische Höfe in Biotit, Hornblende und Cordierit, und ihre Beziehungen zu den α Strahlen radioaktiver Elemente.* Neues Jahrb., B.B., XXXIV (1912), 321–400.

See also R. J. Strutt: *A study of the radio-activity of certain minerals and mineral waters.* Nature, LXIX (1904), 473–475.

Idem: Same title as preceding Proc. Roy. Soc., London, LXXIII (1904), 191–197.

[5] E. A. Wülfing: *Rosenbusch-Wülfing: Mikroskopische Physiographie*, 4te Aufl., 1904, 347.

[6] E. v. Fedorow: *Pseudoabsorption.* Zeitschr. f. Kryst., XXXII (1900), 128–130.

Idem: *Ueber Pseudochroïsmus und Pseudodichroïsmus.* T. M. P. M., XIV (1895), 569–571.

[7] J. L. C. Schroeder van der Kolk: *Samml. Geol. Reichsmuseum, Leiden*, VI (1900), 89.

[8] J. L. C. Schroeder van der Kolk: *Eine eigenthümliche Folge des Pleochroismus in Gesteinsschliffen.* Zeitschr. f. wiss. Mikrosk., VII (1890), 30–32.

underlying a thin layer of a strongly pleochroic mineral, the latter acts as an analyzer by absorbing the rays vibrating in one direction, and, as a consequence, interference colors appear in the thin section. If the analyzer is inserted, the combined minerals, on account of the strong absorption in two directions, will show extinction but twice, instead of four times, on a rotation through 360°.

271. Determination of Pleochroism.—Pleochroism can be seen, with the unaided eye, in but very few minerals. It can easily be seen, under the microscope, by permitting only rays vibrating in one direction to pass through, as by inserting the polarizer alone.[1] In this way, first one color and then the other can be observed by rotating the stage. The objection to this method, which is the one usually followed, is that when the pleochroism is very slight, the eye is unable to perceive it, especially when the stage, and

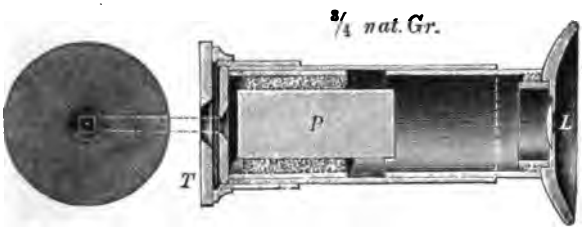

FIG. 423.—Dichroscope. 3/4 natural size. (Fuess.)

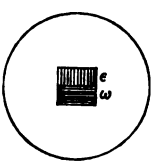

FIG. 424.

not the polarizer, is rotated. A much more delicate way of determining pleochroism is by means of a dichroscope ocular.

The ordinary dichroscope (Fig. 423) is an instrument which was invented by Haidinger.[2] It consists essentially of a calcite prism *P* in a metal casing, at one end of which is a rectangular opening and at the other a lens. The length of the calcite is so chosen that the two images of the rectangular opening in *T* are just in contact with each other (Fig. 424). Since one image of the opening is produced by the ordinary ray, and the other by the extraordinary, the vibration directions will be at right angles to each other, consequently, if a mineral is attached by a bit of wax over the opening in *T*, and it is viewed through the lens *L*, the two absorption colors produced by

[1] Gustav Tschermak: *Mikroskopische Unterscheidung der Mineralien aus der Augit-, Amphibol- und Biotitgruppe.* Sitzb. Akad. Wiss., Wien, LX (1869), 5–16.

[2] W. Haidinger: *Ueber den Pleochroismus der Krystalle.* Pogg. Ann., LXV (1845), 1–30.

See also V. von Lang: *Optische Notizen: Verbesserte dichroscopische Lupe.* Sitzb. Akad. Wiss. Wien, LXXXII (2), 1880, 174.

Gustav Halle: *Neues vervollständigtes Dichroskop.* Neues Jahrb., 1895 (II), 247–248.

Idem: *Eine neue Form des Dichroskopes.* Zeitschr. f. Instrum., XV (1895), 28.

A. Cathrein: *Vervollkommung des Dichroskopes.* Ibidem, XVI (1896), 225–226.

C. Leiss: *Mittheilungen aus der R. Fuess'schen Werkstätte. Verbindung eines Dichroskops mit einem Spectroskop.* Neues Jahrb., 1898 (II), 68–69.

the vibrations at right angles to each other will be seen at the same time. By rotating the end of the tube T, it is an easy matter to find the positions of maximum difference in absorption. At 45° from this position the two colors will be the same. By seeing the two colors thus, side by side, even very slight differences in absorption can be observed.

FIG. 425.—Ocular dichroscope. 1/2 natural size. (Fuess.)

The dichroscope ocular[1] (Fig. 425) is an attempt to combine the advantages of the dichroscope with the magnifying power of the microscope for the determination of the pleochroism of small mineral fragments in thin sections. It consists of a Huygens ocular in which there is inserted a calcite prism K, beneath which is a diaphragm with a rectangular opening. As in the ordinary dichroscope, the length of the calcite is so chosen that the two images produced by double refraction appear side by side. To use the instrument, both analyzer and polarizer must be removed. Sometimes the partial polarization of the light by the mirror affects the results. It is then advisable, if possible, to tilt the microscope backward and use the light directly reflected from the sky or clouds.

272. Determination of the Absorption Coefficient.—It is possible to determine the values of the coefficients of absorption in different directions in a crystal, but this belongs rather to the province of mineralogy than to petrology, and it will not be discussed here.[2] It may simply be mentioned that quantitative values of the intensity of the two transmitted rays are obtained by a combined spectroscope and photometer, either a Glan[3] spectrophotometer or a Königsberger[4] microphotometer being used.

[1] C. Leiss: *Miltheilungen aus der R. Fuess'schen Werkstätte. Ocular-Dichroscop für Mikroskope.* Neues Jahrb., 1897 (II), 92.

[2] See Johann Ehlers: *Die Absorption des Lichtes in einigen pleochroitischen Krystallen.* Neues Jahrb. B. B., XI (1897–98), 259–317.

[3] P. Glan: *Ueber ein neues Photometer.* Wiedem. Ann., I (1877), 351–360.

Louis Duparc et Francis Pearce: *Traité de technique minéralogique et pétrographique*, I, Leipzig, 1907, 423–425.

A. E. H. Tutton: *Crystallography and practical crystal measurement.* London, 1911 823–824.

[4] J. Koenigsberger: *Ueber ein Mikrophotometer zur Messung der Absorption des Lichtes.* Zeitschr. f. Instrum., XXI (1901), 129–133.

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Idem: *Pleochroismus an einigen zweiaxigen Krystallen in neuerer Zeit beobachtet.* Ibidem, 306–331.

H. de Sénarmont: *Versuche über die künstliche Erzeugung von Polychroïsmus in krystallisirten Substanzen.* Pogg. Ann., XCI (1854), 491.

Gustav Tschermak: *Mikroskopische Unterscheidung der Mineralien aus der Augit-, Amphibol- und Biotitgruppe.* Sitzb. Akad. Wiss. Wien, LIX (1869), 5–16.

Viktor v. Lang: *Optische Notizen. Verbesserte dichroskopische Lupe.* Ibidem, LXXXII (1880), 174.

H. Laspeyres: *Mineralogische Bemerkungen.* Zeitschr. f. Kryst., IV (1880), 444.

Carl Pulfrich: *Photometrische Untersuchungen über Absorption des Lichtes in anisotropen Medien.* Ibidem, VI (1882), 142–159.

W. Ramsay: *Ueber die Absorption des Lichtes im Epidot vom Sulzbachtahl.* Ibidem, XIII (1888), 97–134.

W. Voigt: *Erklärung der Farbenerscheinungen pleochroitischer Krystalle.* Neues Jahrb., 1885 (I), 119–141.

Er. Mallard: *Sur le polychroïsme des cristaux.* Bull. Soc. Min. France VI (1883), 45–52.

Henri Becquerel: *Sur l'absorption de la lumière au travers des cristaux.* Ibidem, X (1887), 120–124.

CHAPTER XXII

INTERFERENCE COLORS

273. Interference.—As we have already seen,[1] when two light waves of the same wave lengths and in the same plane differ by half a wave length, the resultant is zero and the light is extinguished. But white light is composed of rays of many different wave lengths (Figs. 440–441), and the conditions which would cause a difference of half a wave length for one color would not cause it for another. The consequence will be, naturally, that under such conditions the light seen is the complementary color of that extinguished. We have also seen that if two light waves differ by any other amount than half a wave length, or a multiple thereof, the resultant wave is of a different amplitude from the original wave. Whether the resultant of the combination of several waves is an increase or à decrease in the amount of light, the waves are said to interfere, and the phenomenon observed is spoken of as **interference.**

274. Color of Thin Plates.—If two plates of glass which are not perfectly true planes, such as panes of ordinary window glass, are pressed together, it will be found that there occur certain dark spots surrounded by concentric curves, rather far apart at the center but closer and closer together toward the outer rings. The colors, from the center outward, gradually diminish in brightness, and the outer rings approach what is known as "white of the higher orders." By pressing the glass plates closer together the inner rings broaden and the whole colored series becomes larger. The same phenomenon may be observed if a piece of thin glass, such as an object-slip or a cover-glass, be pressed against a glass sphere of large radius, such as a bell-jar or a reading glass. In this case, owing to the regularity in the increase in thickness of the air film between the two glasses, the curves are perfect circles. The colored rings observed in this experiment are known as **Newton's rings,**[2] and the series of colors, from the center outward, as **Newton's scale of colors.** Less symmetrically distributed, on account of the irregular variation in the thickness of the film, are the colors observed in a soap bubble, or in a film

[1] Art. 28, *supra.*

[2] Sir Isaac Newton: *Opticks.* Reprinted in Klassiker der exakten Wissenschaften, Nos. 96–97, edited by W. Ostwald. Leipzig, Book. II, Pt. I.

of oil spread upon the surface of water. The colors vary, as we shall see, with the thickness of the film, and thus is produced the gradual change in color of a soap bubble, which reaches a neutral tint just before breaking.

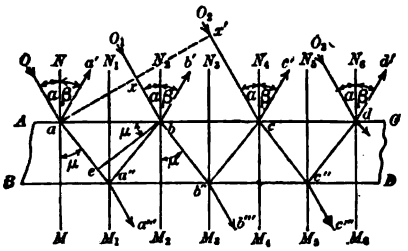


FIG. 426.—Passage of light through a thin film of air between two plates of glass.

Let $ABCD$ (Fig. 426) represent a thin film of some substance (*e.g.*, air) lying between two films of a medium having a higher index of refraction (*e.g.*, glass). Any ray of light, such as O, upon reaching the surface of different density at a, will be partially reflected and partially refracted, and if α represents the angle of incidence OaN, β the angle of reflection Naa', and μ the angle of refraction Maa'', we will have, since the light is passing from a denser to a rarer medium,

$$\frac{\sin \alpha}{\sin \mu} = \frac{1}{n}, \text{ and } \sin\alpha = \frac{\sin \mu}{n}. \tag{1}$$

We also have, since the angle of incidence is equal to the angle of reflection,

$$\alpha = \beta. \tag{2}$$

Consider first the refracted portion of any ray O. Upon reaching the second surface of the film BD, it will again be partially reflected and partially refracted toward b and a'''. Since the two surfaces AC and BD are parallel, $Maa'' = aa''N_1 = \mu$, and since the angle of incidence is equal to the angle of reflection, $aa''N_1 = N_1a''b = \mu$. Upon reaching, from below, the surface AC at b, the reflected portion of the ray $a''b$ will again be partially reflected and partially refracted to b' and b''. Here the angle $a''bM_2 = M_2bb'' = \mu$. At b the portion of the ray $a''b$ refracted upward into the air will make an angle such that

$$\frac{\sin \mu}{\text{sine angle of refraction at } b} = n,$$

for the light is now passing from a rarer to a denser medium.

Transposing, we have

$$\text{sine angle of refraction at } b = \frac{\sin \mu}{n}. \tag{3}$$

Uniting equations (1) and (3), we have,

$$\text{sine of refraction at } b = \sin \alpha,$$

therefore the angle of refraction at b $(N_2bb') = \alpha$, or

$$N_2bb' = \alpha = \beta. \tag{4}$$

Consider now a ray O_1, parallel to the ray O. When it reaches the film AC at b it will likewise be partially reflected and partially refracted. It makes an angle of α with the normal N_2, and its reflected ray b makes an angle $N_2bb' = \beta = \alpha$. But from equation (4) the angle of refraction of the

first ray at b is also $\alpha=\beta$, therefore the two rays coincide and travel along the same path from b to b'. The two rays, however, have traveled different distances. When the ray O is at a, the ray O_1 is at x, since the two rays are parallel and the ray front ax is at right angles to them. The ray O_1 continues on at the same rate to b, but the ray O, passing to a rarer medium, travels a greater distance ae, the amount depending upon the density of the medium. The ray O, therefore, which must travel from e to a'' to b, before it can start on the path traveled by O_1, is just the distance $ea''+a''b$ behind the other, or, as we say, is that much retarded. The consequence is that the two rays traveling along the same path bb' are in different phase, the lag depending upon the thickness of the film, the angle of incidence of the light, and the refractive index of the substances.

Suppose, for the moment, that monochromatic light having a wave length of λ is used. If the film is of such a thickness that $ea''+a''b$ will cause a difference of phase just equal to $\frac{\lambda}{2}$, the rays will so interfere that complete darkness is produced.[1] If the thickness of the film is somewhat greater, so that the retardation is $\frac{3\lambda}{2}$, $\frac{5\lambda}{2}$, $\frac{7\lambda}{2}$, etc., the effect is, of course, the same.

If a wedge-shaped film is used, instead of one which is plane parallel, there will be successive dark bands where the phase difference is a multiple of $\frac{\lambda}{2}$. This is well seen in the experiment of Newton's rings of which mention was previously made. Here the film is of no thickness at the center, where the contact is good, and darkness occurs. Surrounding this there is a succession of bands of light and darkness, the latter occurring wherever the phasal difference is a multiple of $\frac{\lambda}{2}$.

When white light is employed, colors or colored bands will be seen instead of darkness. This is due to the fact that white light is made up of rays of different wave lengths (Figs. 440–441) which interfere at different distances, and, as a result, the color seen at any point is that due to the subtraction of one color from the original white light. This is beautifully seen in soap bubbles, in which, as the film becomes thinner and thinner, successive wave lengths interfere.

275. Newton's Color Scale.—As we have seen, white light is made up of many rays of different wave lengths, which travel with different velocities and are differently refracted. As laid down by Newton,[2] the colors are divided into the following orders:

1. Black, blue, white, yellow, red.
2. Violet, blue, green, yellow, red.
3. Purple, blue, green, yellow, red.

[1] Art. 28, *supra*.

[2] Sir Isaac Newton: *Opticks*, Bk. II, Pt. I, obs. 4.

4. Green, dirty red.
5. Greenish blue, red.
6. Greenish blue, pale red.
7. Greenish blue, reddish white.

276. Color Scale according to Quincke.—Since Newton's time, the color scale has been worked out in great detail, and the numerical values for the retardations, and the thicknesses of the air films necessary to produce the colors, have been determined.[1] The values obtained by different observers are not all alike, owing to the fact that the positions of the different bands in the scale vary somewhat for different modes of illumination. Thus the values for the retardation of the sensitive violet is given by Wertheim and by Quincke as 575$\mu\mu$, by Rollet as 556$\mu\mu$, and by Kraft as 535.6 to 557.6$\mu\mu$ for clear sky. In most of the petrographic test-books, Quincke's values have been given.[2] They are as follows:

NEWTON'S COLOR SCALE
(Modified from Quincke)

No.	Retardation $\lambda = 589$	Order	Interference colors between crossed nicols	Interference colors between parallel nicols
1	0$\mu\mu$	0	Black	Bright white
2	40	.....	Iron-gray	White
3	97	.....	Lavender-gray	Yellowish white
4	158	1/4	Grayish blue	Brownish white
5	218	.....	Clearer gray	Brownish yellow
6	234	.....	Greenish white	Brown
7	259	.....	Almost pure white	Light red
8	267	.....	Yellowish white	Carmine
9	275	.....	Pale straw-yellow	Dark reddish brown
10	281	.....	Straw-yellow	Deep violet
11	306	1/2	Light yellow	Indigo
12	332	.....	Bright yellow	Blue
13	430	.....	Brownish yellow	Grayish blue
14	505	3/4	Reddish orange	Bluish green
15	536	.....	Red	Pale green
16	551	.....	Deep red	Yellowish green

[1] G. Wertheim: *Mémoire sur la double réfraction temporairement produite dans les corps isotropes, et sur la relation entre l'élasticité mécanique et entre l'élasticité optique.* Ann. Chim. et Phys., XL (1854), 156–221, in particular 180.

G. Quincke: *Experimental-Untersuchungen. Ueber Newton'sche Farbenringe u. s. w.* Pogg. Ann., CXXIX (1866), 177–218, in particular 180.

Alexander Rollet: *Ueber die Farben welche in der Newton'schen Ringsystemen aufeinanderfolgen.* Sitzb. Akad. Wiss. Wien, LXXVII (1878), 3 Abth., 177–261.

Camille Kraft: *Badania doświadczalne nad skala barw interferencyjnych* (*Études expérimentales sur l'échelle des couleurs d'interference*). Bull. internat. de l'Académie des Sci. de Cracovie. Cl. sci. math. et nat. (Anzeiger d. Akad. d. Wiss. Krakau. Math-naturw. Cl.) Cracovie, 1902, 310–353.

[2] A. Michel-Lévy: *Mineraux des roches*, 64. Groth: *Physikalische Krystallographie*, 4te Aufl., 47. Rosenbusch-Wülfing: *Mikroskopische Physiographie*, 4te Aufl., 228. Duparc et Pearce: *Traité d? tech. Min. et pétr.*, 182. Johannsen: *Rock-forming Minerals*, 10. Winchell: *Elements*, etc., 62–63.

NEWTON'S COLOR SCALE—(*Continued*)
(Modified from Quincke)

No.	Retardation $\lambda = 589$	Order	Interference colors between crossed nicols	Interference colors between parallel nicols
17	565	.....	Purple.....	Lighter green
18	575	.....	Violet.....	Greenish yellow
19	589	1	Indigo.....	Golden yellow
20	664	.....	Sky-blue.....	Orange
21	728	.....	Greenish blue.....	Brownish orange
22	747	.....	Green.....	Light carmine
23	826	.....	Lighter green.....	Purplish red
24	843	.....	Yellowish green.....	Violet-purple
25	866	.....	Greenish yellow.....	Violet
26	910	3/2	Pure yellow.....	Indigo
27	948	.....	Orange.....	Dark blue
28	998	.....	Bright orange-red.....	Greenish blue
29	1101	.....	Dark violet-red.....	Green
30	1128	.....	Light bluish violet.....	Yellowish green
31	1151	2	Indigo.....	Impure yellow
32	1258	.....	Greenish blue.....	Flesh colored
33	1334	.....	Sea-green.....	Brownish red
34	1376	.....	Brilliant green.....	Violet
35	1426	5/2	Greenish yellow.....	Grayish blue
36	1495	.....	Flesh-color.....	Sea-green
37	1534	.....	Carmine.....	Green
38	1621	.....	Dull purple.....	Dull sea-green
39	1652	.....	Violet-gray.....	Yellowish green
40	1682	.....	Grayish blue.....	Greenish yellow
41	1711	.....	Dull sea-green.....	Yellowish gray
42	1744	3	Bluish green.....	Lilac
43	1811	.....	Light green.....	Carmine
44	1927	.....	Light greenish gray....	Grayish red
45	2007	.....	Whitish gray.....	Bluish gray
46	2048	.....	Flesh-red.....	Green

277. Color Scale according to Kraft.—The determinations by Kraft were made in great detail for illumination by Argand lamp or incandescent electric light, Auer burner, electric arc, sunlight reflected from snow, gray sky, and clear sky. He found that not only does the same retardation produce different colors with different methods of illumination, but the widths of the bands of color, as plotted for different retardations, differ. Thus the sensitive violet was found to have the following retardations:

Argand or incandescent electric	576.4	to 590.0
Auer burner	567.0	582.6
Electric arc	554.1	571.3
Snow illuminated by the sun	551.2	571.0
Gray, cloudy sky	541.8	563.0
Clear sky	535.6	557.6

For comparison there are given, in the following tables, Kraft's values for clear and for cloudy sky, the conditions under which microscopic illumination is most commonly obtained. The numbers which have been added after the colors correspond to the same numbers, as nearly as it is possible to determine, in the table in Article 276.

[1] C. Kraft: *Op. cit.*

TABLE OF INTERFERENCE COLORS, LIGHT FROM A CLEAR SKY
(According to Kraft) $\lambda = 550\mu\mu$

Order	Interference color, nicols crossed	Retardation	Interference color, nicols parallel	Order
I	Black (1), passing through iron-gray (2) to	000.00	The color of the source of the light, passing through white (7) to	I
	Lavender-gray (3)	40.00		
	Grayish blue (4)	130.00	Yellowish white (11)	
		150.00	Brown (13)	
	White, tinted with greenish blue (6)	230.2	Reddish orange (14)	
		237.3	Red (15)	
	White, with traces of greenish blue	243.8	Dark carmine (16)	
		246.2	Deep purple (17)	
	Greenish white (7)	256.5		
	White with tint of yellowish green	263.4	Deep violet (18)	II
		277.0		
	Light greenish yellow	281.9	Indigo (19)	
	Light yellow (12)	347.3		
	Brown (13)	360.5		
	Orange	378.7	Blue (20)	
	Reddish orange (14)	457.1		
	Light red (15)	473.1	Greenish blue (21)	
	Carmine (16)	488.8		
	Purple (17)	496.3		
II	Violet (18)	501.0	Bluish green	
	Indigo (19)	520.0	Green (22)	
	Blue (20)	535.6		
	Greenish blue (21)	557.6	Yellowish green (24)	
		559.1		
	Bluish green	617.5	Greenish yellow (25)	
	Green (22)	630.0	Yellow (26)	
	Yellowish green (24)	692.7	Orange (27)	
	Greenish yellow (25)	724.0	Reddish orange (28)	
	Yellow (26)	735.5	Light red (29)	
	Orange (27)	755.3	Carmine (29)	
	Reddish orange (28)	761.4		
	Light red (29)	780.7	Purple	
	Carmine (29)	811.7	Violet (30)	III
	Purple (30)	836.8	Indigo (31)	
III	Violet (30)	846.2		
	Indigo (31)	885.3		
		890.0	Blue	
		943.1		
		947.8	Greenish blue (32)	
		996.1		
		1016.0		
		1027.0	Bluish green (33)	
		1052.3		
		1088.0	Green (34)	
		1114.2	Yellowish green	
		1123.7		
		1161.4	Greenish yellow (35)	
		1166.1		

TABLE OF INTERFERENCE COLORS, LIGHT FROM A CLEAR SKY—(*Continued*)

(According to Kraft) $\lambda = 550\mu\mu$

Order	Interference color, nicols crossed	Retardation	Interference color, nicols parallel	Order
		1161.4		
	Blue	1166.1		
		1192.1	Very pale, impure yellow	
	Greenish blue (32)	1203.0		
		1223.5	Flesh color (36)	
		1232.1		
	Bluish green (32)		Very light red	
		1279.3		
			Light carmine (37)	
		1291.4		
		1311.6		
	Green (34)		Light purple (38)	
		1366.5		
			Pale violet (39)	
	Yellowish green	1400.4		
		1405.1	Pale indigo	
	Greenish yellow (35)			
		1433.8		
	Very pale, impure yellow			IV
		1444.5	Pale blue (40)	
	Flesh color (36)	1451.0		
		1468.7	Greenish blue (41)	
	Light red	1503.6		
		1535.0	Bluish green (42)	
	Light carmine (37)	1550.3		
		1570.0	Green (43)	
	Light purple (38)	1628.6		
		1659.5		
	Pale grayish violet (39)		Yellowish green (44)	
		1691.2		
	Impure grayish indigo		Greenish yellow	

TABLE OF INTERFERENCE COLORS, LIGHT FROM A GRAY CLOUDY SKY

(According to Kraft) $\lambda = 550\mu\mu$

Order	Interference color, nicols crossed	Retardation	Interference colors, nicols parallel	Order
	Black (1), passing through iron-gray (2) to	0.0	The color of the source of the light, passing through white (7) to	I
		50.0		
	Lavender-gray (3)			
		110.0	Yellowish white (11)	
	Grayish blue (4)	160.0		
		222.2	Brown (13)	
	White, tinted with greenish blue (7)	226.4		
		236.8	Reddish orange (14)	
	White with traces of greenish blue (9)	245.0	Red (15)	
		250.2	Dark carmine (16)	
		252.8		
	Greenish white (10)		Deep purple (17)	
		263.7		
	White with a tint of yellowish green (11)	268.6		II
		277.3	**Deep violet** (18)	
	Light greenish yellow	282.1		
		326.7	Indigo (19)	
	Pale yellow (12)	347.5		
		366.0	Blue (20)	
	Brown (13)	406.0		
		435.7	Greenish blue (21)	
	Orange			
		472.9		
	Reddish orange (14)			
		488.5		

TABLE OF INTERFERENCE COLORS, LIGHT FROM A GRAY CLOUDY SKY—(*Continued*)
(According to Kraft) $\lambda = 550\mu\mu$

Order	Interference color, nicols crossed	Retardation	Interference colors, nicols parallel	Order
	Pale red (15)	500.0		
	Carmine (16)	508.3	Bluish green	
	Purple (17)	515.2	Green (22)	
		531.0		
II	Violet (18)	541.8	Yellowish green (24)	
	Indigo (19)	554.9		
		563.0	Greenish yellow (25)	
	Blue (20)	598.3		
		617.0	Yellow (26)	
	Greenish blue (21)	677.1	Orange (27)	
		720.0	Reddish orange (28)	
	Bluish green	735.3	Light red (29)	
		758.0	Carmine (29)	
		767.9		
	Green (22)	777.5	Purple (30)	
	Yellowish green (24)	808.6		
		817.8		
	Greenish yellow (25)	836.6	Violet (30)	III
		846.0	Indigo (31)	
	Yellow (26)	880.4		
		883.7	Blue	
	Orange (27)	922.8		
		936.3	Greenish blue (32)	
	Reddish orange (28)	942.8		
	Light red (29)	992.4		
		1020.0		
	Carmine (29)	1031.8	Bluish green (33)	
	Purple (30)	1036.7		
		1077.0	Green (43)	
III	Violet (30)	1096.0	Yellowish green	
	Indigo (31)	1114.6		
		1126.0	Greenish yellow (35)	
	Blue	1158.0		
	Greenish blue (32)	1186.1	Very pale, impure yellow	
		1202.8		
	Bluish green (32)	1210.6	Flesh color (36)	
		1231.9	Very light red	
		1285.5	Light carmine (37)	
	Green (34)	1299.7	Light purple (38)	
		1363.4		
	Yellowish green	1373.9		
	Greenish yellow (35)	1400.7	Pale violet (39)	
		1405.7	Pale indigo	
	Very pale impure yellow	1430.3	Pale blue (40)	IV
	Flesh color (36)	1444.4		
	Light red	1468.4	Greenish-blue (41)	
	Light carmine (37)	1488.4		
		1539.8	Bluish-green (42)	
	Light purple (38)	1581.5	Green (43)	
	Pale grayish violet (39)	1610.0		
	Impure grayish indigo	1662.6	Yellowish-green (44)	
		1690.9		
		1695.9	Greenish-yellow.	

CHAPTER XXIII

EXAMINATION BETWEEN CROSSED NICOLS

278. Isotropic Substances.—The light which emerges from a nicol prism vibrates parallel to one plane only, that is, it is plane polarized. If another nicol is placed with its vibration directions at right angles to that of the first, and in the path of the rays coming through it, all light will be cut off and the field will appear dark. If a thin section of a colorless, isotropic substance be placed on the stage of the microscope, there will be no change in the appearance of the field, since all such substances permit the rays to vibrate with equal ease in every direction, consequently it will have no effect upon the vibrations of the light.

279. Anisotropic Substances.—If one of the nicol prisms is slightly rotated so that it is not at right angles to the other, a small amount of light will be found to pass through. Let AA' (Fig. 427) represent the vibration direction of the analyzer, PP' that of the rotated polarizer, and OP the amount of light passing through the latter. Obviously, the light cannot pass through the analyzer so long as it vibrates in the direction PP', but it may be resolved into two rays vibrating at right angles to each other, as Oy and Ox. Of these, the component Ox is totally reflected by the balsam film of the upper nicol and is lost, but the component Oy will pass through, the amount being represented by the distance from O to y.

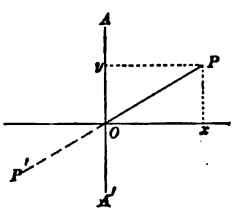


FIG. 427.—Passage of light through two nicol prisms.

PROBLEMS

1. On the stage of the microscope place a thin section of a colorless or pink garnet. Note that the mineral permits the light to pass through. Insert the analyzer. The field now remains dark during a complete rotation of the stage. Try to obtain an interference figure. If a uniaxial figure is obtained remove the mineral and see if the blank slide will not also give the same figure. This is caused by the polarizing effect of the lenses (Art. 356).

2. Place a basal section of a uniaxial crystal (*e.g.*, calcite or quartz) on the stage. Is the section isotropic? Is the mineral isotropic? Determine the latter by obtaining an interference figure (Chapter XXIX). On comparison with an isotropic mineral we see that the basal section of a uniaxial crystal is an isotropic section of an anisotropic mineral.

If the polarizer and analyzer are set at right angles, no light will pass through, as we shall see, unless there is placed upon the stage of the microscope a mineral which is anisotropic. Since such minerals transmit vibrations in two planes at right angles to each other, it follows, if the mineral is so placed upon the stage of the microscope that its vibration directions do not coincide with those of the nicols, that the effect is the same as though the polarizer were placed at an angle, and a certain amount of light will be transmitted, the amount depending upon the angle which the principal sections of the nicols make with those of the mineral. Between crossed nicols, therefore, the transmission of light is a means of separating anisotropic crystals from those that are isotropic.

280. Retardation in Anisotropic Media.—When light passes through an anisotropic medium, the two rays into which it was broken up do not emerge at the same time, but one lags behind the other. This retardation depends upon the wave velocities and the thickness of the section. The wave velocities themselves are dependent upon their respective refractive indices (n_1 and n_2), and vary as $\frac{1}{n_1}$ and $\frac{1}{n_2}$. If V_1 is the greater velocity, and V_2 the lesser, then $\frac{1}{n_1} > \frac{1}{n_2}$ and $n_1 < n_2$, n_1 and n_2 being, respectively, the refractive indices of the rays having velocities of V_1 and V_2.

Let t_1 be the time for the ray A_1, and t_2 the time for A_2 to pass through the crystal. If M is the thickness,

$$M = t_1 \frac{1}{n_1}, \quad t_1 = n_1 M, \tag{1}$$

and

$$M = t_2 \frac{1}{n_2}, \quad t_2 = n_2 M. \tag{2}$$

Whereby the retardation R is given by the equation

$$R = t_2 - t_1 = M(n_2 - n_1), \tag{3}$$

a result given in millimeters if M is so given.

281. Phasal Difference.—If λ is the wave length of the light considered, the phasal difference P between the two emerging rays may be expressed by

$$P = \frac{R}{\lambda} = \frac{t_2 - t_1}{\lambda} = \frac{M(n_2 - n_1)}{\lambda}. \tag{4}$$

282. Interference of Polarized Light.—The interference of polarized light was studied by Arago and Fresnel[1] who found (a) that two rays of light, polarized in planes at right angles to each other and afterward brought into the same plane, will interfere if they originally belonged to the same beam of polarized light, but (b) two rays of light, from different beams, polarized at

[1] Arago et Fresnel: *Mémoire sur l'action que les rayons de lumière polarisée exercent les uns sur les autres.* Ann. Chim. et Phys., X (1819), 288–305.
A. Fresnel: *Oeuvres complètes*, Paris, 1866, I, 521.

right angles to each other, may be brought into the same plane of polarization without interfering.

Let PP' (Fig. 428) be the vibration direction of the polarizer of a microscope, and let OP and OP' each represent one-half the amplitude of the vibrations of the light emerging from it. Let UU' and VV' be the vibration directions in an anisotropic crystal lying in the path of the polarized ray. The plane polarized light, upon entering the crystal, is broken into two rays whose rate of transmission is not the same, one of them being transmitted with greater difficulty than the other. As a result, when the two rays emerge, they will differ by some part of a wave length. Let us first consider the case of a retardation of a single wave length (λ) or a multiple thereof.

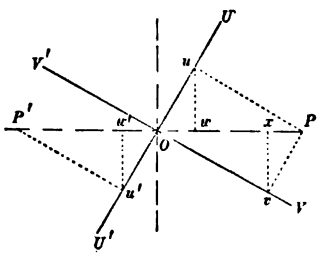


FIG. 428.

Case I. The vibrations differ by λ or a multiple of λ ($N\lambda$), and the nicols are parallel. Since the retardation is just one wave length, the two components emerge at O at the same time. One component will start toward v, the other toward u, and, owing to their different amplitudes, will reach these points at the same time, the locations being determined by the resolution of forces. If, now, an analyzer, with its vibration directions parallel to that of the polarizer, be inserted above the mineral, the two components will be brought back into the plane in which they were previously vibrating. The amplitudes are represented by Ox and Ow. The rays, differing by $N\lambda$, have their vibrations in the same direction, hence they have the same sign and, by their interference, the resultant is equal to their sum ($Ox+Ow$).

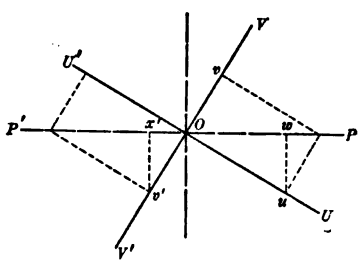


FIG. 429.

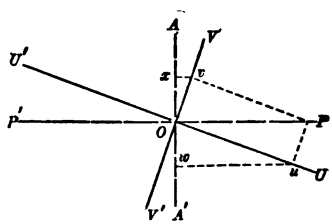


FIG. 430.

Case II. The vibrations differ by $\frac{N\lambda}{2}$, and the nicols are parallel. If the two component rays emerge with a difference in phase of $\frac{N\lambda}{2}$, where N represents any whole number, the first component to pass through has already traveled to v and back at O (Fig. 429) when the second component arrives there. When the first has reached v', the second has just reached u. Upon

inserting the analyzer parallel to the polarizer, the two components are brought back to the same plane and are represented by Ox' and Ow. But here the two components are in opposite directions and, in consequence, have different signs. The resultant, produced by their interference, is $-Ox'+Ow$.

Case III. The vibrations differ by $N\lambda$ and the nicols are crossed. As in Case I, the emerging rays reach u and v (Fig. 430) at the same time. Upon inserting the analyzer with its vibration direction (AA') at right angles to that of the polarizer, the resulting components are Ox and Ow, and, since they lie in opposite directions, the resultant is $Ox-Ow=0$. That is, with monochromatic light and crossed nicols, the interference of two components differing by $N\lambda$ will produce darkness.

Case IV. The vibrations differ by $\frac{N\lambda}{2}$ and the nicols are crossed. As in Case II, one emerging ray reaches u' (Fig. 431) when the other reaches v. Upon inserting the analyzer AA', the two new components vibrate in the same direction and the resultant is $Ow+Ox$.

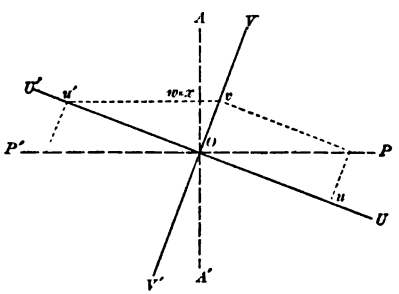


FIG. 431.

The above four cases prove Fresnel's first proposition.

Case V. Observations with an analyzer only inserted. When ordinary light, which vibrates with equal ease in every direction, passes through an anisotropic crystal, it is separated into two rays vibrating in planes at right angles to each other. If an analyzer is placed above such a crystal, the rays which reach it from below are made up of those which are parallel to the analyzer and those which are at right angles to it, and if, at one instant, the interference is with the same path difference, the next it is with the opposite. Since the changes take place extremely rapidly, we can perceive no interference phenomena.

This case proves the second statement of Fresnel.

283. Extinction Angles.—Place a thin section of a doubly refracting mineral on the stage of the microscope between crossed nicols and in such a position that its vibration directions coincide with those of the nicols (Fig. 432). Let Oa represent half the amplitude of the light entering through the polarizer $P'P$. Upon reaching the anisotropic crystal the tendency is for the light to be resolved into two planes at right angles to each other, Oa and Ob. Since these directions are parallel to those of the nicols, the Ob component of OP will be zero while all of the Oa component will pass through without change of direction. Upon reaching the analyzer, whose vibration direction AA' is at right angles to PP', the tendency is again to resolve the light into

two rays, one of which will pass through, and the other will be reflected out. The components of the ray Oa, along the vibration direction of the analyzer, will be zero, and Oa in the direction at right angles to it. But the OA component is the only one which is permitted to pass the analyzer, consequently the field of the microscope will be dark just as though there were no mineral on the stage, or as though the mineral were isotropic.

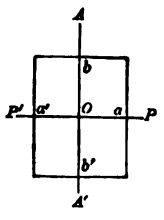


FIG. 432.

If the mineral section be slightly rotated (Fig. 433) so that its vibration directions form angles with those of the two nicols, then any ray of light from the lower nicol, having a half amplitude OP, will pass through the mineral by means of vibrations Ou and Ov, or Ou and Ov', depending upon the phasal difference between them. Let the phasal difference, for example, be $\frac{N\lambda}{2}$ (Figs. 433, 434, 435, 436), then the first component will have traveled from O to v and back to O when the second component emerges at O. When the first is at v' the second is at u. Upon reaching the analyzer AA', each of the two rays is again resolved into two others, and the components passing through are represented by Ow and Ox. Moving in the same direction, the resultant which reaches the eye is represented by $Ow+Ox$. The amplitude of this resultant governs the brightness of the light and, as may be seen by Figs. 433 to 436, it is at its maximum when the inclination of the vibration axes of the mineral to those of the nicols is 45°. Beyond that angle

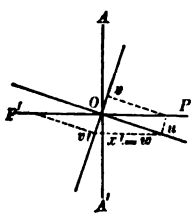


FIG. 433.

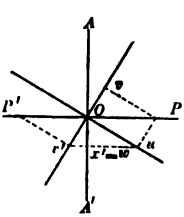


FIG. 434.

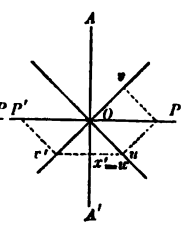


FIG. 435.

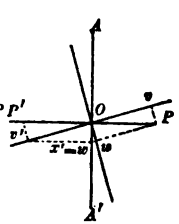


FIG. 436.

the intensity decreases, since the major part of the light falls to the component Ov, which is reflected out. If the thickness of the crystal is greater, so that the phasal difference of the two original components is $N\lambda$, (Cf. Case IV, *supra*), then the vibrations lie in opposite directions and the resultant is zero or darkness.

It follows from this, that when a section of an anisotropic mineral is placed upon the stage of the microscope and it is rotated through 360°, at the four positions in which its vibration directions coincide with those of the nicols, there will be complete darkness. At all intermediate points, the mineral will permit light to pass, the amount depending upon the angular posi-

tion of its vibration axes with reference to those of the nicols. It is at its maximum when the angle between the two is 45°.

We have seen (Case III, Art. 282) that if monochromatic light is used and the difference in path of the two waves is $N\lambda$, darkness will result. If we place, then, upon the stage of the microscope a section of a mineral which increases in thickness toward one end, we will have dark bands wherever the phasal difference is λ or a multiple thereof, and between these, by gradations, the greatest amount of light. We know that light of different colors is of different wave lengths, consequently if we have total interference for one color we may not have it for another. If, then, in the above experiments, white light be used instead of monochromatic, and the thickness of the section be such that the path difference is just a wave length for yellow light, it will be found that for red light, whose wave length is greater, the difference in path is less, and for blue light, with less wave length, it is greater. The consequence is, that instead of black bands appearing in a mineral wedge wherever the wave lengths of one color differ by $N\lambda$, we will have a series of colored bands, and at any point that color will predominate which is nearest $\frac{N\lambda}{2}$ in path difference.

284. Passage of Monochromatic Light through Two Nicol Prisms and a Mineral Section.—Let us assume that a beam of monochromatic light (*O*, Fig. 437) strikes the lower surface of a nicol prism. Its vibrations, which took place in every direction, will now be reduced to a single plane, that is, it will be polarized in the vibration plane of the nicol. The amount of light which passes through the nicol will not, however, be as great as that which entered it, since the component vibrating at right angles to the vibration plane of the polarizer will be totally reflected by the film of Canada balsam in the nicol, and will be lost. Since the calcite is a much denser substance than the air, the vibrations will be reduced both in amplitude and in wave length (*b'*).

Upon emerging into the air (*c'*), the amplitude of the waves increases, though it cannot reach that which it possessed before it entered the nicol, where a part of the light was lost. The wave length, however, again becomes exactly what it previously was in air (*a'*) and, of course, the color remains unchanged. When light travels in air it continues without change in the direction in which it started, consequently its vibrations are here parallel to a single plane, that of the vibration direction of the polarizer.

When the ray of light enters an anisotropic medium it is broken up into two rays (*d*) which vibrate in planes at right angles to each other. Since the medium is denser than air, the amplitude and the wave length are both reduced (*d'*).

When these two components, with vibrations in planes at right angles to

each other, leave the crystal, they emerge at different points, and if we had only to deal with a single ray, each component would pass on uninterruptedly. But it is not a single ray of light which we have to consider, but a beam made up of innumerable rays which entered the lower nicol, consequently the point of emergence of either of the two components of the ray O will also be the point of emergence of the other component of some other ray, as O_1. We have here, therefore, a case of two simple harmonic motions at right angles to each other, and usually of different amplitudes and in different phases,[1] combined with a uniform motion of translation in a direction at right angles to the plane of the resulting elliptical motion. As a consequence, the two components of the rays O and O_1 will unite to produce a movement of the particle along a helix (e') of elliptical cross-section (e). The light is **elliptically polarized.**[2]

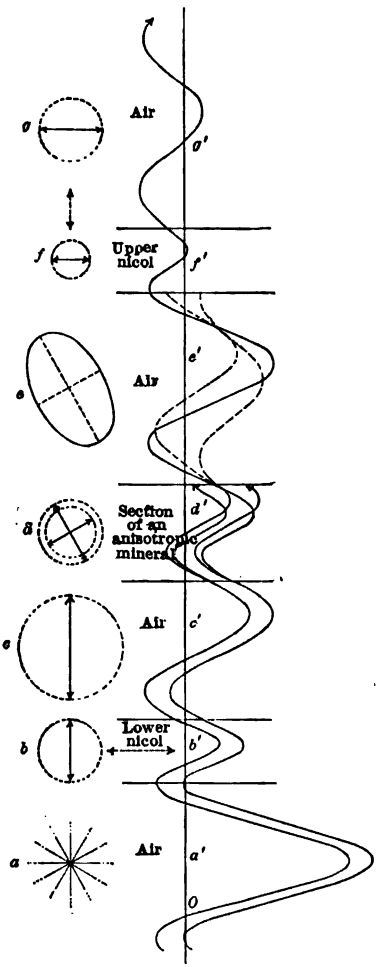


FIG. 437.—Passage of monochromatic light through two nicol prisms and a mineral section. (The wave is shown, in every case, rotated into the plane of the paper.) The vibration at a' diagramatically represents the intensity of all of the components of ordinary light vibrating in air, although actually vibrating in many planes, as at a.

The elliptically polarized beam, upon reaching the analyzer, is again plane polarized, this time in a direction at right angles to its former position (f). Both amplitude and wave length are reduced (f') on account of the density of the calcite. The amplitude will be less than the amplitude in the polarizer (b') since there is now not so much light entering, but the wave length (f') will be the same. Here again the component of the entering ray, which is at right angles to the vibration direction of the nicol, is reflected out and lost.

Upon emerging in air (g') the amplitude of the wave is again increased but, since part of the light was lost in the analyzer, it cannot have the amplitude it previously possessed. The wave length is the same as it was in air throughout (a', c', e'). The vibrations lie in a single plane, that of the analyzer.

[1] Art. 27.

[2] Art. 77.

285. The Intensity of the Emerging Light (After Fresnel).[1]—The intensity of the emerging light may be determined analytically as follows:

Let PP' (Fig. 438) be the vibration direction of the polarizer, and AA' that of the analyzer. To make the problem cover all cases, let the angle between polarizer and analyzer be any angle φ.

Let UU' and VV' be the vibration directions in an anisotropic mineral section, and let θ be the angle which UU' makes with the polarizer.

Let d ($=Od$) be the amount of light coming from the polarizer. On passing into the thin section it is broken up into the two rays x and y (Ox and Oy in the figure).

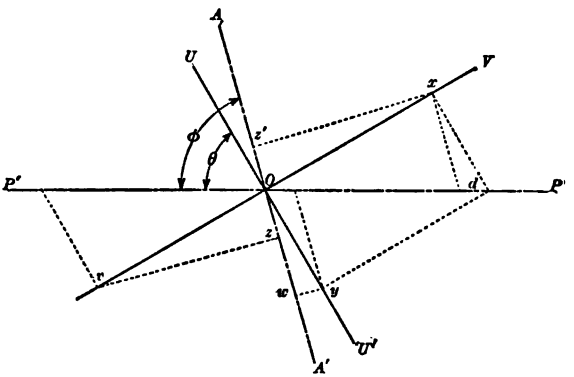


FIG. 438.

From Fig. 438 we have $\cos\theta=\frac{y}{d}$, $\sin\theta=\frac{x}{d}$, whereby $x=d\sin\theta$, and $y=d\cos\theta$. But from equation (7), Art. 25, we have $d=r\sin\frac{2\pi}{T}t$, in which d is the displacement, r the amplitude, t the time since the beginning of the movement, and T the time of one period. Substituting this value for d in the equation above, we have

$$x=r\sin\theta\sin\frac{2\pi}{T}t, \tag{1}$$

$$y=r\cos\theta\sin\frac{2\pi}{T}t. \tag{2}$$

Each of the two rays now moves with a different velocity, and independently, through the slide. If t_1 and t_2 are the periods of time required for the two rays to pass through, the value of the displacement, upon emergence, will be that given by equation (8), Art. 25, and equations (1) and (2) become

$$x'=r\sin\theta\sin\frac{2\pi(t-t_1)}{T}, \tag{3}$$

$$y'=r\cos\theta\sin\frac{2\pi(t-t_2)}{T}. \tag{4}$$

But t_1 and t_2 are equal to n_1M and n_2M (Eq. 1 and 2, Art. 280), therefore

$$x'=r\sin\theta\cdot\sin\frac{2\pi(t-n_1M)}{T}, \tag{5}$$

$$y'=r\cos\theta\cdot\sin\frac{2\pi(t-n_2M)}{T}. \tag{6}$$

These are the equations of the amplitude of the rays after leaving the mineral section.

[1] A. Fresnel: *Oeuvres complètes*, I, 615.

The rays now pass through air and reach the upper nicol, where they are again resolved into a single plane with components w and z, or w and z' (Ow, Oz, Oz' in the figure), depending upon whether the difference of phase is $\frac{N\lambda}{2}$ or $N\lambda$.

The equations are in the same form as before, as may be determined from the figure.

$$w = y' \cos(\theta-\varphi) = r\cos\theta \cdot \cos(\theta-\varphi) \cdot \sin\frac{2\pi(t-n_2M)}{T}, \tag{7}$$

$$z = x' \sin(\theta-\varphi) = r\sin\theta \cdot \sin(\theta-\varphi) \cdot \sin\frac{2\pi(t-n_1M)}{T}. \tag{8}$$

But these two vibrations are now polarized in the same plane, consequently they interfere. The amplitude A is equal to the amplitudes of the two components, therefore

$$A = w \pm z = r\cos\theta \cdot \cos(\theta-\varphi) \cdot \sin\frac{2\pi(t-n_2M)}{T} \pm r\sin\theta \cdot \sin(\theta-\varphi) \cdot \sin\frac{2\pi(t-n_1M)}{T}.$$

For simplicity, substitute $K = r\cos\theta \cdot \cos(\theta-\varphi)$, and $K' = r \cdot \sin\theta \cdot \sin(\theta-\varphi)$ and the equation becomes

$$A = K \sin\frac{2\pi(t-n_2M)}{T} \pm K' \sin\frac{2\pi(t-n_1M)}{T}. \tag{10}$$

By comparing equations (8) and (9), Art. 25, with equations (8) and (7) above, we see that r_1 and r_2 of the equation for the amplitude of the resultant of two harmonic motions (Eq. 4, Art. 28) become, after passing through the two nicols and the anisotropic medium, $r\cos\theta\cos(\theta-\varphi)$, and $r\sin\theta\sin(\theta-\varphi)$, consequently equation (4), Art. 28, may be written

$$A^2 = [r\cos\theta \cdot \cos(\theta-\varphi)]^2 + [r\sin\theta \cdot \sin(\theta-\varphi)]^2$$
$$+ 2[r\cos\theta \cdot \cos(\theta-\varphi)][r\sin\theta \cdot \sin(\theta-\varphi)] \cos\frac{2\pi(n_2-n_1)M}{T}. \tag{11}$$

From trigonometry we have

$$\cos 2\left[\frac{\pi(n_2-n_1)M}{T}\right] = 1 - 2\sin^2\left[\frac{\pi(n_2-n_1)M}{T}\right].$$

Developing equation (11), substituting these values, and inserting I for A^2, since the intensity of light is proportional to the square of its amplitude, we have:

$$\begin{aligned} I = A^2 &= r^2\cos^2(\theta-\varphi) \cdot \cos^2\theta + r^2\sin^2(\theta-\varphi) \cdot \sin^2\theta \\ &\quad + 2r^2\cos(\theta-\varphi) \cdot \sin(\theta-\varphi) \cdot \cos\theta\sin\theta \\ &\quad - r^2\sin^2(\theta-\varphi) \cdot \sin 2\theta \cdot \sin^2\left(\frac{\pi(n_2-n_1)M}{T}\right) \\ &= r^2[\cos(\theta-\varphi) \cdot \cos\theta - \sin(\theta-\varphi) \cdot \sin\theta]^2 \\ &\quad - [\sin 2(\theta-\varphi) \cdot \sin 2\theta \cdot \sin^2\left(\frac{\pi(n_2-n_1)M}{T}\right)] \\ &= r^2[\cos^2\varphi - \sin 2(\theta-\varphi) \cdot \sin 2\theta \cdot \sin^2\left(\frac{\pi(n_2-n_1)M}{T}\right)]. \end{aligned} \tag{12}$$

This is the equation for the intensity of light after passing through two nicol prisms and a mineral section.

From equation (10), Art. 25, we have

$$vT=\lambda.$$

If we consider the velocity of propagation (v) as unity, this equation becomes

$$T=\lambda. \tag{13}$$

That is, the time of oscillation is equal to the wave length. Substituting this value in equation (12) we have

$$I=r^2[\cos^2\varphi-\sin 2(\theta-\varphi)\cdot\sin 2\theta\cdot\sin^2\left(\frac{\pi(n_2-n_1)M}{\lambda}\right)]. \tag{14}$$

Considering this equation in its relation to the positions of the nicol prisms, we find that we have three cases.

Case I. The vibration directions of polarizer and analyzer make any angle (φ) with each other. In equations (13) and (14) the first term varies with the angle between the vibration directions of polarizer and analyzer, and the second depends upon the wave length of the light used.

If φ has any fixed value, and the thickness of the section M remains the same, the value of the intensity of the light will depend upon the value of θ, the angle which the vibration directions of the section make with that of the polarizer. When $\theta=0$, φ, $\frac{\pi}{2}t$, or $\varphi+\frac{\pi}{2}t$, the second member becomes zero, and the intensity of the light is given by

$$I=r^2\cos^2\varphi. \tag{15}$$

Case II. The vibration directions of polarizer and analyzer are perpendicular to each other. In this case $\varphi=90°=\frac{\pi}{2}$, Substitute this value in (14) and we have

$$I=r^2[-\sin 2\left(\theta-\frac{\pi}{2}\right)\cdot\sin 2\theta\cdot\sin^2\frac{\pi(n_2-n_1)M}{\lambda}].$$

But, by trigonometry, $\sin 2\left(\theta-\frac{\pi}{\lambda}\right)=-\sin 2\theta$, whereby

$$I=r^2\sin^2 2\theta\cdot\sin^2\left(\frac{\pi(n_2-n_1)M}{\lambda}\right). \tag{16}$$

Here again the value of the intensity depends upon the value of θ, the inclination of the principal sections of the mineral to those of polarizer and analyzer.

Equation (16) reaches its maximum when $\sin^2 2\theta=1$, that is when the principal vibration directions of the thin section make an angle of 45° with those of the nicols. Its minimum value is reached when $\sin^2 2\theta=0$, that is, when the principal vibration directions of mineral and nicols coincide. It follows, therefore, that the field must become dark four times on rotating the stage through 360°, and likewise bright the same number of times when the crystal is turned 45° from the positions of darkness.

As here considered, the wave length λ, the thickness of the section M, and the retardation $M(n_2-n_1)$, were taken as of fixed values. When they vary, the value of $\sin^2\left(\frac{\pi(n_2-n_1)M}{\lambda}\right)$ varies, consequently the intensity of the light also. When

$\sin^2\left(\frac{\pi(n_2-n_1)M}{\lambda}\right)=0$ the intensity is at its minimum, since then equation (16) becomes zero also, and the field is dark. This result is produced when $(n_2-n_1)M=2N\frac{\lambda}{2}$, where N is an integer. In other words, the field is dark when the phasal difference is a whole number, for by equation (4) Art. 281,

$$P=\frac{M(n_2-n_1)}{\lambda}=\frac{\frac{2N\lambda}{2}}{\lambda}=N, \tag{17}$$

a result like that derived by graphical methods, Case III, Art. 282.

The intensity is at its maximum when $\sin^2\left(\frac{\pi(n_2-n_1)M}{\lambda}\right)$ has its maximum value. This occurs when $(n_2-n_1)M=(2N+1)\frac{\lambda}{2}$, that is, when one of the light waves precedes the other by half a wave length, for

$$P=\frac{M(n_2-n_1)}{\lambda}=(2N+1)\frac{\lambda}{2}=\frac{2N+1}{2}, \tag{18}$$

which is the same result as that derived by graphical methods, Case IV, Art. 282.

Case III. The vibration directions of polarizer and analyzer are parallel. In this case $\varphi=0$, whereby $\cos^2\varphi$ equals unity, and equation (14) becomes

$$I=r^2\left[1-\sin^2 2\theta\cdot\sin^2\left(\frac{\pi(n_2-n_1)M}{\lambda}\right)\right]. \tag{19}$$

The light has its maximum intensity when $\theta=0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, or 2π, and its minimum value when $\theta=\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, or $\frac{7\pi}{4}$. That is to say, the light is at its maximum when the vibration directions are parallel or at right angles to those of the nicols, and at its minimum when they are at 45°.

The relation of $\sin^2\left(\frac{\pi(n_2-n_1)M}{\lambda}\right)$ to interference may be demonstrated as in the previous case. It will be found that the intensity of the light is at its maximum when $\frac{M(n_2-n_1)}{\lambda}=N$, and at its minimum when it equals $\frac{2N+1}{2}$, for in these positions $I=r^2(1-0)=r^2$, and $I=r^2(1-1)=0$, results which are the same as those obtained graphically in Cases I and II, Art. 282.

286. Two Superposed Mineral Plates.

I. Vibration directions are parallel.
a. Slow rays parallel,
b. Fast rays parallel.

The values of the vibration directions in any mineral section are expressed by a corresponding section through the ease-of-vibration- or the Fresnel ellipsoid. The form of this section is elliptical, consequently the fastest and slowest rays are represented by its principal axes. For brevity, the terms "fast- and slow-ray" will be used hereafter to express the directions of greatest and least ease of vibration

in any mineral section. These terms do not necessarily mean the maximum and minimum values in a mineral, but simply the maximum and minimum in the particular section under consideration.

The retardation produced by an anisotropic mineral is given by equation 3, Art. 280,

$$R = M(n_2 - n_1),$$

in which M is the thickness of the section, and n_2 and n_1, the maximum and minimum refractive indices in that section. If another thin section is placed above the first in such a position that their vibration directions are parallel, the effect will be that of adding or subtracting a certain amount of light to the former, depending upon whether the vibrations take place in the same or in opposite phase. Thus if M'' and M' be the thicknesses, and n''_2 and n''_1, and n'_2 and n'_1 (Fig. 439) the refractive indices of the two minerals, equation (16) will become

$$I = r^2 \sin^2 2\theta \cdot \sin^2\left(\frac{\pi\,[M'\,(n'_2 - n'_1) \pm M''\,(n''_2 - n''_1)]}{\lambda}\right). \qquad (23)$$

If the fast rays of the two minerals are parallel, the effect will be additive, if in opposite directions, subtractive, the two acting as a single mineral plate, as was first shown by Biot.[1] If $M''(n''_2 - n''_1) = M'(n'_2 - n'_1)$, the last part of equation (23) becomes $\sin^2 0 = 0$, consequently the value of $I = 0$, or darkness. If the retardations of the two are not the same, the result is an increase or a decrease in brightness, the amount depending upon the two values. If the retardation of one plate is known, it serves as a measure of that of the other, and also to determine the positions of its fast and slow rays.

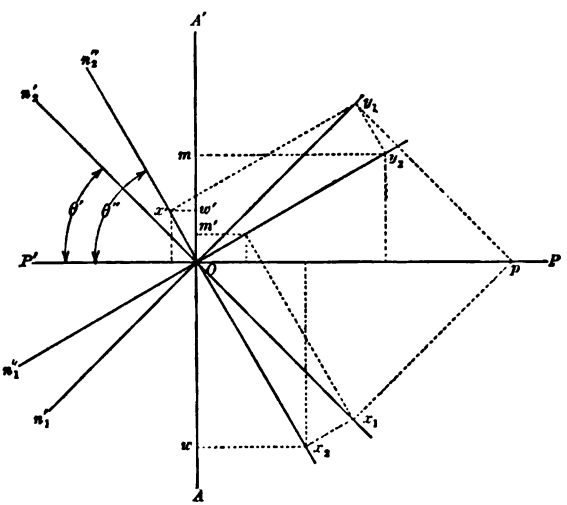


FIG. 439.—Intensity of light passing through two superposed mineral plates between crossed nicols.

II. Vibration directions of minerals are at other angles than 0° or 90° with those of the nicols.

Let PP', Fig. 439, be the vibration direction of the polarizer, and AA', that of the analyzer, and let the angle between them be 90°. Let n'_2 and n'_1, and n''_2 and n''_1 be the fast and slow rays of two mineral sections making angles of θ' and θ'', which are neither 0° nor 90°, with the polarizer. Let Op represent the amplitude of the ray emerging from the polarizer. Upon reaching the first mineral section $n'_2 n'_1$, it is broken up into two waves vibrating parallel to On'_2 and On'_1, and with amplitudes of Ox_1 and Oy_1. Each of these rays is broken up into two others in the second min-

[1] J. B. Biot: *Traité de physique.* Paris, 1816, IV, 419–422.

eral section $n''_2n''_1$; the ray Oy_1 into Oy_2 and Ox'_2, and Ox_1 into Oy'_2 and Ox_2. Upon the insertion of the analyzer, each of these four rays is again broken up into two, but of these, only those parallel to the vibration direction of the analyzer can pass through, those at right angles being reflected out, consequently the light reaching the eye is represented by $Om+Om'+Ow'-Ow$.

Analytically the same result may be obtained in a manner similar to that used for the determination of the intensity of the light emerging from two nicols and a single mineral section. The demonstration is long and of little importance in petrographic work except when the angles θ' and θ'' are 45° and 135°, in which case the two mineral sections are at right angles and the equation takes the form of equation (23). If the nicols are parallel and the minerals at an angle, the equation, similar to (19) is

$$I=r^2\left[1-\sin^2 2\theta\cdot\sin^2\left(\frac{\pi\,(M'\,(n'_2-n'_1)\pm M\,(n''_2-n''_1)}{\lambda}\right)\right].$$

If the minerals are rotated until their vibration directions coincide with those of the nicols, $\theta=0°$ or 90°, $\sin^2 2\theta=0$, and the equation becomes $I=r^2$.

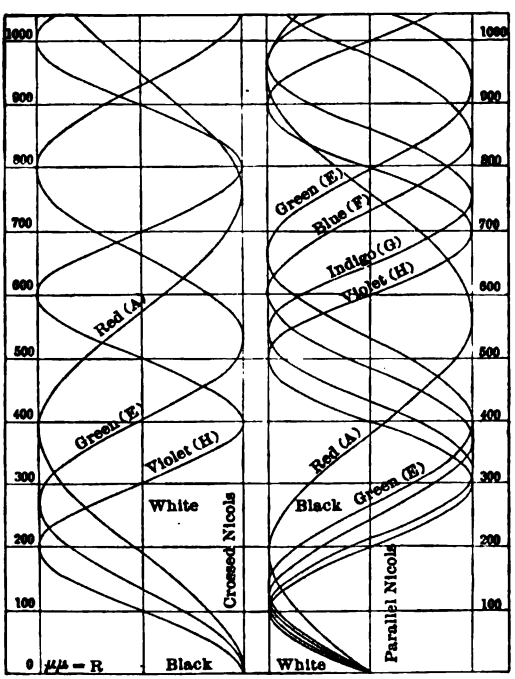


FIG. 440. FIG. 441.

FIGS. 440 and 441.—Wave lengths for different colors.

287. Examination by White Light. Interference Colors.—As shown by Case III, Art. 282, and Case II, Art. 285, when monochromatic light is used and the thickness of the mineral section is such that the two waves emerge with a retardation of $N\lambda$, the stage appears completely dark between crossed nicols. If, now, there be used white light, which is composed of many rays of different wave lengths (Figs. 440–441), the wave length of a certain color may be such as to produce darkness, but the other colors will pass through with greater or less intensity, and, as a result, they will produce an interference color. This may be demonstrated by equation 14, Art. 285, in which $\cos^2\varphi$, being independent of the wave length of light or of the thickness of the section, will be the same whether monochromatic or white light be used. In the general case, where the nicols are not crossed,

the result will be affected by $\sin 2\varphi \cdot \sin 2(\theta-\varphi) \cdot \sin^2\left(\frac{\pi(n_2-n_1)M}{\lambda}\right)$ of which the first part may be neglected, since these values affect chiefly the intensity of the color. They produce a slight dispersion of the bisectrices, but this is so small that the result will be but little affected by their omission. The second part of the term, as we saw in Case III, Art. 285, became o when $P=N$ (equation 17, Art. 285) for any definite color, that is, definite wave length of light. As a result, when white light is used, that color for which the thickness of section is the proper one will be extinguished, leaving the others to give color to the slide. It was shown by equation 18, which was derived from an equation for crossed nicols but which might just as well have been deduced from equation 14, that when for some definite color $P=\frac{2N+1}{2}$, the light is at its maximum; consequently, in the color resulting from white light, that color whose phasal difference is nearest this value will have the greatest influence upon it, and the one whose phasal difference equals N will have the least, for it will be totally extinguished. Suppose we start with a thin section which is of such a thickness that yellow light (D), whose wave length is $590\mu\mu$ (Art. 258), is completely extinguished. Its phasal difference must then be $N \cdot 590\mu\mu$. If white light be used, orange and green are a little more and less retarded, and red and blue still more and less. Their intensities are less, since they differ by some amount from $\frac{2N+1}{2}\lambda$, and the greater this difference the less effect will the color have upon the resulting. In the particular case taken, with a retardation of $590\mu\mu$, the resulting color is indigo, the complementary color of that extinguished. Again if $R=486\mu\mu$ (blue, F), the resulting complementary color is orange. If the phasal differences of all the colors approach that of the color which, in the particular section under consideration, is totally extinguished, then more and more are the colors extinguished until, with a retardation of $o\mu\mu$, darkness is produced. This takes place in isotropic crystals or basal section of uniaxial crystals, for in such the velocities, and consequently the retardations, are equal in all directions.

The color nearest white will appear when the least light is extinguished. This must occur when the phasal difference of each of the rays is nearest the mean of all values of $\frac{2N+1}{2}\lambda$, relative intensities of the various colors being taken into consideration. The value of this mean is about $250\mu\mu$, consequently when the retardation is of this amount, the interference color between crossed nicols is practically white (Fig. 440). With a retardation of $575\mu\mu$ the most intense light, near the D line, is extinguished, and the interference color remaining is a violet, known as the **sensitive tint, sensitive violet,** sometimes **"red" of the first order,** because a very slight change in

retardation in either direction produces a marked change in the interference color, red on one side and blue on the other.

Since the values of maximum and minimum intensity are exactly reversed when parallel nicols are used, the interference color is white when the retardation is zero, black when it is $250\mu\mu$, and elsewhere the color whose wave length equals the retardation. The colors are complementary to those produced by crossed nicols.

The effect of the superposition of several mineral plates with vibration directions parallel or at right angles may be determined by equation 23 and by comparison with the table in Art. 276. Thus if the retardation of one plate is $575\mu\mu$ and the interference color is violet, and of another plate $158\mu\mu$ and the color gray, the resultant of the two plates in parallel position is a retardation of $575\mu\mu+158\mu\mu=733\mu\mu$, or when the vibrations are at right angles $575\mu\mu-158\mu\mu=417\mu\mu$. In the first case the interference color is greenish blue, in the second yellowish brown.

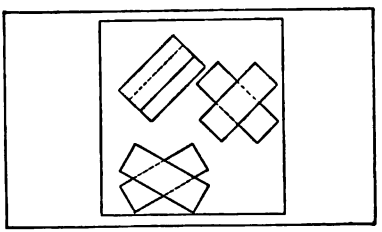


FIG. 442.—Arrangement of mica strips to show the effect of superposition in different positions.

The same effect as with minerals in parallel position is obtained by increasing the thickness of a section. Thus while the retardation in a section of quartz 0.03 mm. thick and cut parallel to crystallographic c, is $270\mu\mu$ and the interference color is a pale yellow, in a section 0.06 mm. thick it is $540\mu\mu$ and the interference color is red. Again a section of augite 0.02 mm. thick has a retardation of $500\mu\mu$ and a red interference color, while one of 0.06 mm. has a retardation of $1500\mu\mu$ and is of a third order yellowish red.

The effect of superposed plates may be beautifully shown by a plate constructed of thin mica strips, giving approximately a half wave length retardation, and arranged as shown in Fig. 442, partially overlapping in parallel position, crossed at 90°, and crossed at some other angle.

No matter what may be the inclination of the vibration directions of the crystal to those of the nicol, the interference color will remain the same since the retardation remains the same. A rotation of the stage will produce simply a change in the intensity of the color. The value of this intensity, being the sum of the intensities of all the emerging rays, may be represented by an equation similar in form to that of equation 14, Art. 285,

$$I = \Sigma r^2 \cos^2\varphi - \Sigma r^2 \sin 2(\theta-\varphi) \cdot \sin 2\theta \cdot \sin^2\left(\frac{\pi(n_2-n_1)M}{\lambda}\right), \qquad (20)$$

which is the general equation for the intensity of the emerging ray when white light is used.

With nicols crossed, the intensity equation is

$$I = \Sigma r^2 \sin^2 2\theta \cdot \sin^2 \left(\frac{\pi(n_2 - n_1)M}{\lambda}\right). \qquad (21)$$

With parallel nicols it is

$$I = \Sigma r^2 \pm \Sigma r^2 \sin^2 2\theta \cdot \sin^2 \left(\frac{\pi(n_2 - n_1)M}{\lambda}\right). \qquad (22)$$

With two superposed mineral plates having their vibration directions parallel, and their slow and fast rays parallel or at right angles, the interference colors will increase or decrease, since there is an increase or decrease in the phasal difference of the combined minerals, which here act as a single mineral section.

In the above it must be remembered that throughout, in the intensity equations, the same values for the refractive indices $(n_2 - n_1)$ have been assumed. It may be easily seen that if the difference between the indices increases, the value of the intensity increases, and *vice versa*. Now in anisotropic media the maximum difference between the refractive indices occurs when a thin section is cut parallel to the plane of the optic axes, consequently, in such sections, the maximum intensity of illumination appears. In basal sections of uniaxial, and in sections cut at right angles to the optic axes in biaxial minerals, the intensity is at its minimum.

288. Calculation of the Value of the Birefringence in any Section.—Knowing the value of the principal refractive indices of a crystal and the orientation of the section, it is possible to compute the value of the birefringence by the formula

$$\gamma' - \alpha' = (\gamma - \alpha) \sin\theta \sin\theta',$$

where θ and θ' are the angles between the normal to the given section and the two optic axes.

In 1819, Biot[1] found empirically that $(\gamma - \alpha) \sin\theta \sin\theta'$ was a constant for all directions, θ and θ' being the angles of inclination of the ray to the optic axes. The mathematical proof for this proposition was given by Neumann[2] in 1834.

The optical ellipsoid, in the most general case, namely in a biaxial crystal, is one of three axes, and has for its equation

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = 1, \qquad \text{(Eq. 1, Art. 63)}$$

or

$$a^2x^2 + b^2y^2 + c^2z^2 = 1. \qquad \text{(Eq. 2, Art. 63)}$$

Propagated along the normal to the section there will be two waves whose velocities are inversely proportional to γ' and α', the major and minor refractive indices

[1] J. B. Biot: *Mémoire sur les lois générales de la double réfraction et de la polarisation dans les corps régulièrement cristallisées.* Mém. Acad. France. Année, 1818, III (1820), 177–384, especially 230.

[2] F. E. Neumann: *Ueber die optischen Axen und die Farben zwei axiger Krystalle im polarisirten Licht.* Pogg. Ann. XXXIII (1834), 257–281.

in the section. The values γ' and α' will, therefore, represent the major and minor axes of the section of the ellipsoid parallel to the mineral section under consideration. If M is the length of a diameter conjugate to the other two (γ and α), and p is the perpendicular distance between the planes of the section and the tangent to the ellipsoid, we have by Cartesian geometry:

$$\gamma'^2+\alpha'^2+M^2=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} \tag{1}$$

and

$$\gamma'\alpha'p=\frac{1}{abc}. \tag{2}$$

The first equation may be written

$$\frac{\gamma'^2+\alpha'^2}{\gamma'^2\alpha'^2}=\frac{1}{\alpha'^2}+\frac{1}{\gamma'^2}=\left[\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-M^2\right]\frac{1}{\gamma'^2\alpha'^2}, \tag{3}$$

and the second

$$\frac{1}{\gamma'^2\alpha'^2}=(abcp)^2. \tag{4}$$

Substituting (4) in (3) we have

$$\frac{1}{\alpha'^2}+\frac{1}{\gamma'^2}=\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-M^2\right)(abcp)^2. \tag{5}$$

For simplicity in writing, let

$$v=\frac{1}{\alpha'^2},\quad m=\frac{1}{\gamma'^2},$$

$$K=v+m=\frac{1}{\alpha'^2}+\frac{1}{\gamma'^2},$$

$$P=\gamma'\alpha',\ L=(abcp)^2,$$

$$S=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-M^2. \tag{5a}$$

Equation (4) becomes $\frac{1}{P^2}=L$, and equation (5)

$$v+m=K=SL=\frac{S}{P^2}. \tag{6}$$

From (6) we have

$$(v+m)^2=v^2+2vm+m^2=K^2. \tag{7}$$

Subtracting $4vm$ from each member

$$v^2-2vm+m^2=(v-m)^2=K^2-4vm=K^2-\frac{4}{P^2}, \tag{8}$$

Extracting the square root

$$v-m=\sqrt{K^2-\frac{4}{P^2}}=\sqrt{\frac{S^2}{P^4}-\frac{4}{P^2}}, \tag{9}$$

From (9) we get

$$\left(\frac{1}{\alpha'^2}-\frac{1}{\gamma'^2}\right)^2=\left(\frac{1}{\alpha'^2}+\frac{1}{\gamma'^2}\right)^2-\frac{4}{\gamma'^2\alpha'^2}. \tag{10}$$

Now the normal equation of a plane through a point is

$$X\cos\chi+Y\cos i+Z\cos\zeta=p. \tag{11}$$

But the plane tangent to the ellipsoid $a^2x^2+b^2y^2+c^2z^2=1$ can be written

$$a^2xX+b^2yY+c^2zZ=1 \tag{12}$$

in which XYZ are current coordinates and xyz those of the point of contact.

From (11) and (12) we have

$$ax=\frac{\cos\chi}{ap},\quad by=\frac{\cos\iota}{bp},\quad cz=\frac{\cos\zeta}{cp}, \tag{13}$$

$$\left(\frac{\cos\chi}{a^2p}\right)^2+\left(\frac{\cos\iota}{b^2p}\right)^2+\left(\frac{\cos\zeta}{c^2p}\right)^2=1,$$

and

$$p=\sqrt{\frac{\cos^2\chi}{a^2}+\frac{\cos^2\iota}{b^2}+\frac{\cos^2\zeta}{c^2}}. \tag{14}$$

From (13) we have

$$x^2+y^2+z^2=\frac{1}{p^2}\left(\frac{\cos^2\chi}{a^4}+\frac{\cos^2\iota}{b^4}+\frac{\cos^2\zeta}{c^4}\right)=M^2. \tag{15}$$

Substituting the values of M^2 and p^2 and (15) and (14) in (5a),

$$\frac{1}{\alpha'^2}+\frac{1}{\gamma'^2}=SL=\frac{S}{P^2}.$$

But

$$S=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{\dfrac{\cos^2\chi}{a^4}+\dfrac{\cos^2\iota}{b^4}+\dfrac{\cos^2\zeta}{c^4}}{\dfrac{\cos^2\chi}{a^2}+\dfrac{\cos^2\iota}{b^2}+\dfrac{\cos^2\zeta}{c^2}}$$

$$=\frac{\dfrac{\cos^2\chi}{a^2}\left(\dfrac{1}{b^2}+\dfrac{1}{c^2}\right)+\dfrac{\cos^2\iota}{b^2}\left(\dfrac{1}{c^2}+\dfrac{1}{a^2}\right)+\dfrac{\cos^2\zeta}{c^2}\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)}{\dfrac{\cos^2\chi}{a^2}+\dfrac{\cos^2\iota}{b^2}+\dfrac{\cos^2\zeta}{c^2}} \tag{16}$$

$$P=\sqrt{\frac{1}{L}}=\frac{1}{abcp},$$

$$P=\frac{1}{abc}\cdot\frac{1}{\sqrt{\dfrac{\cos^2\chi}{a^2}+\dfrac{\cos^2\iota}{b^2}+\dfrac{\cos^2\zeta}{c^2}}}, \tag{17}$$

$$\frac{1}{\alpha'^2}+\frac{1}{\gamma'^2}=\frac{S}{P^2}=\cos^2\chi\,(b^2+c^2)+\cos^2\iota\,(c^2+a^2)+\cos^2\zeta\,(a^2+b^2). \tag{18}$$

Substitute the value of (17) in (4)

$$\frac{1}{\gamma'^2\alpha'^2}=\frac{1}{P^2}=b^2c^2\cos^2\chi+a^2c^2\cos^2\iota+a^2b^2\cos^2\zeta. \tag{19}$$

Equations (18) and (19) are expressed in terms of $\cos\chi$, $\cos\iota$, and $\cos\zeta$. They may be expressed in terms of directions of propagation along the optic axes. In a manner similar to that used for determining the value of $\tan^2 V$ in Art. 71, we may derive the formulæ for sine and cosine. They are

$$\cos^2 V=\frac{\dfrac{1}{\beta^2}-\dfrac{1}{\gamma^2}}{\dfrac{1}{\alpha^2}-\dfrac{1}{\gamma^2}}=\frac{b^2-c^2}{a^2-c^2}, \tag{20}$$

$$\sin^2 V=\frac{\frac{1}{\alpha^2}-\frac{1}{\beta^2}}{\frac{1}{\alpha^2}-\frac{1}{\gamma^2}}=\frac{a^2-b^2}{a^2-c^2}. \tag{20a}$$

Between two directions ($\cos\chi$, $\cos\iota$, $\cos\zeta$) and ($\cos\chi'$, $\cos\iota'$, $\cos\zeta'$) we have

$$\cos\theta=\cos\chi\cos\chi'+\cos\iota\cos\iota'+\cos\zeta\cos\zeta'. \tag{21}$$

When $\cos\chi'=\sin V$, $\cos\iota'=0$, and $\cos\zeta'=\cos V$, we have

$$\cos\theta=\cos\chi\sin V+\cos\zeta\cos V. \tag{22}$$

When $\cos\chi'=-\sin V$, $\cos\iota'=0$, and $\cos\zeta'=\cos V$, we have

$$\cos\theta'=-\cos\chi\sin V+\cos\zeta\cos V. \tag{23}$$

We also have

$$\cos^2\chi+\cos^2\iota+\cos^2\zeta=1. \tag{23a}$$

Solving Eq. (22) and (23) for $\cos\chi$, $\cos\iota$, and $\cos\zeta$, and squaring, we have

$$\cos\chi=\frac{\cos\theta-\cos\theta'}{2\sin V} \tag{24}$$

and

$$\cos\zeta=\frac{\cos\theta+\cos\theta'}{2\cos V}. \tag{25}$$

Squaring Eq. (24) and (25) and combining with Eq. (20),

$$\cos^2\zeta=\frac{(\cos\theta+\cos\theta')^2}{4}\cdot\frac{a^2-c^2}{b^2-c^2} \tag{26}$$

and

$$\cos^2\chi=\frac{(\cos\theta-\cos\theta')^2}{4}\cdot\frac{a^2-c^2}{a^2-b^2}. \tag{27}$$

Substituting values from Eq. (27), (23a), and (26) in (18), we obtain

$$\frac{1}{\alpha'^2}+\frac{1}{\gamma'^2}=a^2+c^2-\left(\frac{\cos\theta-\cos\theta'}{4}\right)^2(a^2-c^2)+\frac{(\cos\theta+\cos\theta')^2}{4}(a^2-c^2)$$

$$=a^2+c^2+(a^2-c^2)\cos\theta\cos\theta'. \tag{28}$$

Substituting the same values in (19) we have,

$$\frac{1}{\alpha'^2\gamma'^2}=\cos^2\chi b^2c^2+(1-\cos^2\chi-\cos^2\zeta)a^2c^2+\cos^2\zeta a^2b^2$$

$$=a^2c^2+(b^2-a^2)c^2\cos^2\chi+(b^2-c^2)a^2\cos^2\zeta$$

$$=a^2c^2-\frac{(a^2-b^2)c^2}{4(a^2-b^2)}(a^2-c^2)(\cos\theta-\cos\theta')^2+\frac{(b^2-c^2)a^2}{4(b^2-c^2)}(a^2-c^2)(\cos\theta+\cos\theta')^2$$

$$=a^2c^2-\frac{a^2-c^2}{4}c^2(\cos\theta-\cos\theta')^2+\frac{a^2-c^2}{4}a^2(\cos\theta+\cos\theta')^2$$

$$=a^2c^2-\frac{a^2c^2-c^4}{4}(\cos\theta-\cos\theta')^2+\frac{a^4-a^2c^2}{4}(\cos\theta+\cos\theta')^2$$

$$=a^2c^2-\frac{a^2c^2-c^4}{4}(\cos^2\theta-2\cos\theta\cos\theta'+\cos^2\theta')+\left(\frac{a^4-a^2c^2}{4}\right)(\cos^2\theta+2\cos\theta\cos\theta'$$

$$+\cos^2\theta')$$

$$= a^2c^2 + \left[\left(\frac{c^4}{4} - \frac{a^2c^2}{4}\right)(\cos^2\theta + \cos^2\theta') - \left(\frac{c^4 - a^2c^2}{2}\right)\cos\theta\cos\theta'\right]$$

$$+ \left[\left(\frac{a^4}{4} - \frac{a^2c^2}{4}\right)(\cos^2\theta + \cos^2\theta') + \left(\frac{a^4 - a^2c^2}{2}\right)\cos\theta\cos\theta'\right]$$

$$= a^2c^2 + \left(\frac{a^4 - 2a^2c^2 + c^4}{4}\right)(\cos^2\theta + \cos^2\theta') + \left(\frac{a^4 - c^4}{2}\right)\cos\theta\cos\theta'$$

$$= a^2c^2 + \frac{(a^2 - c^2)^2}{4}(\cos^2\theta + \cos^2\theta') + \frac{a^4 - c^4}{2}\cos\theta\cos\theta'. \quad (29)$$

Again, substituting in equation (10), we have

$$\left(\frac{1}{\alpha'^2} - \frac{1}{\gamma'^2}\right)^2 = (a^2 + c^2)^2 + (a^2 - c^2)^2(\cos^2\theta\cos^2\theta' - \cos^2\theta - \cos^2\theta') - 4a^2c^2. \quad (30)$$

From trigonometry we have

$$(1 - \cos^2\theta)(1 - \cos^2\theta') = 1 - \cos^2\theta - \cos^2\theta' + \cos^2\theta\cos^2\theta',$$

and this equation substituted in (30) gives

$$\left(\frac{1}{\alpha'^2} - \frac{1}{\gamma'^2}\right)^2 = (a^2 + c^2)^2 + (a^2 - c^2)[(1 - \cos^2\theta)(1 - \cos^2\theta') - 1] - 4a^2c^2$$

$$= (a^2 + c^2)^2 + (a^2 - c^2)^2(\sin^2\theta\sin^2\theta' - 1) - 4a^2c^2$$

$$= (a^2 - c^2)^2\sin^2\theta - \sin^2\theta'.$$

Extracting the square root we have

$$\left(\frac{1}{\alpha'^2} - \frac{1}{\gamma'^2}\right) = (a^2 - c^2)\sin\theta\sin\theta'. \quad (31)$$

But $a = \frac{1}{\alpha}$, and $c = \frac{1}{\gamma}$, whereby

$$\frac{1}{\alpha'^2} - \frac{1}{\gamma'^2} = \left(\frac{1}{\alpha^2} - \frac{1}{\gamma^2}\right)\sin\theta\sin\theta'. \quad (32)$$

Since the value of the birefringence $\gamma - \alpha$ is generally small, we may write with approximate accuracy

$$\gamma' - \alpha' = (\gamma - \alpha)\sin\theta\sin\theta'. \quad (33)$$

This is the desired equation for calculating the value of the birefringence of any section.

289. Lines of Equal Birefringence.—Curves of equal birefringence were first used by Michel-Lévy.[1] They may be readily determined by the equation $\gamma' - \alpha' = (\gamma - \alpha)\sin\theta\sin\theta'$,[2] and the results may be plotted by tracing the curve produced by the poles of the given sections. Such curves are of value in showing the relative accuracy of random sections of a mineral in comparison with sections of known orientation whose properties are known.

[1] A. Michel-Lévy: *Étude sur la détermination des feldspaths.* Première fascicule, Paris, 1894.

[2] Art. 288, Eq. 33.

The equation may be written

$$\frac{\gamma' - \alpha'}{\gamma - \alpha} = \sin\theta \sin\theta'.$$

The desired curve is represented by a given value of birefringence $(\gamma' - \alpha')$

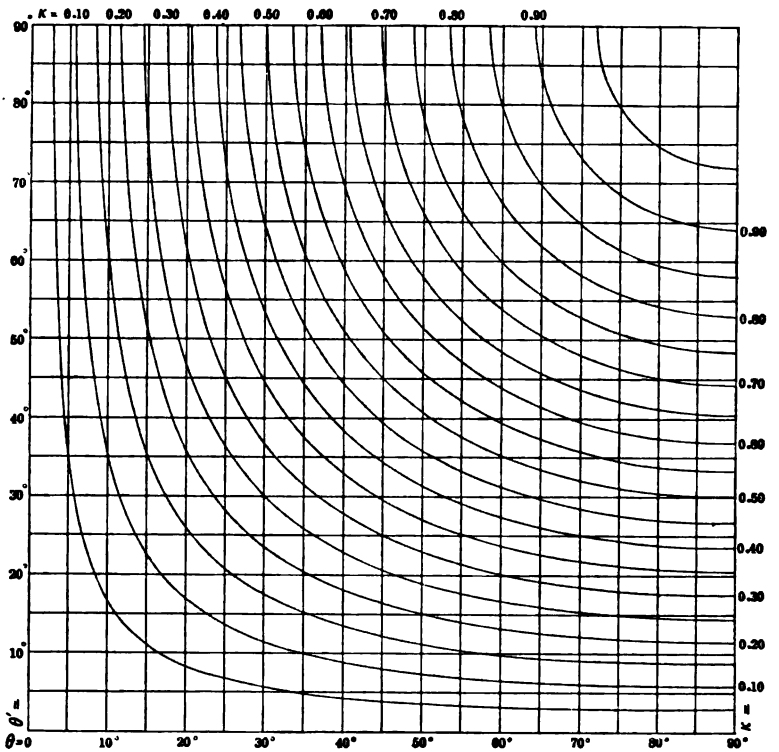


FIG. 443.—Diagram for computing the percentage of the maximum birefringence which appears in any section. (After Wright.[1])

which is therefore a constant, as is also $\gamma - \alpha$, for any given mineral. The equation therefore becomes

$$K = \sin\theta \sin\theta'$$

in which K is less than unity, since $\gamma - \alpha$ represents the maximum birefringence.

In Fig. 443 the abscissæ and ordinates represent values of θ and θ', and the curves the birefringence ratio $K = \frac{\gamma' - \alpha'}{\gamma - \alpha}$ for values of 0.1, 0.2, 0.3, . . .

[1] Duparc and Pearce, in their *Traité de technique minéralogique et pétrographique*, I, Leipzig, 1907, 229, give a similar diagram but use equal spaces for the sines of θ and θ' instead of equal spaces for the angles themselves.

1.0. The curves are equilateral hyperbolæ whose crests lie on a line making an angle of 45° with the coordinates.

By the use of this diagram the ratio of the birefringence of almost any section to the maximum birefringence may be determined if the angle between the normal to the section and the optic axes is known. An exception occurs when the section lies within the zone of circles tangent internally and externally. This is best shown by constructing curves, in stereographic projection, through the poles having the same percentage of the maximum birefringence, for while the stereographic projection is somewhat distorted toward the margin, the drawing will give a general idea of the actual appearance of the curves.

If to K, in the equation above, there be given definite values, such as 0.1, 0.2, 0.3, etc., and there be assigned to θ various values ranging from 0° to 90°, the corresponding values for θ' may be determined, or they may be taken directly from the diagram in Fig. 443, and from these values the curve may be plotted in stereographic projection. The method is as follows: Locate the optic axes in the projection, and with these as centers draw circles of proper radii from each. Thus, for $K=0.1$ we have the following values

$\theta=90°$	$\theta'=5.8°$
73.5	6.0
54.0	7.0
45.5	8.0
39.5	9.0
35.0	10.0
29.0	12.0
20.0	17.0
17.0	20.0
12.0	29.0
10.0	35.0
9.0	39.5
8.0	45.5
7.0	54.0
6.0	73.5
5.8	90.0

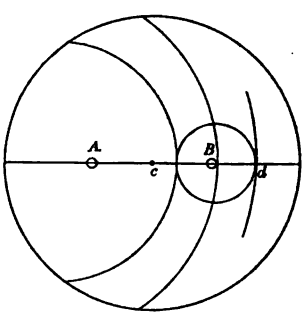


FIG. 444.

With the optic axis A (Fig. 444) as a center,[1] draw a circle of 90° radius (θ), and with B as a center a circle of 5.8° (θ'). The intersections of the two will give two points in every case except where the circles are tangent internally or externally, in which case only one point will occur (c or d, Fig. 444). Proceed likewise for $\theta=73°$, $\theta'=6°$, and so on, until enough points have been obtained to trace the curve. In a similar manner proceed with $K=0.2$, and so on.

The curves are determined much more easily by means of a stereographic

[1] It must be borne in mind that when one speaks of drawing a circle in stereographic projection, about a point as a center, that a point on the sphere is meant. The curves, although true circles, will *not* be concentric in the projection, although actually so on the sphere.

net, as was pointed out by Wulff.[1] He prepared a table, once for all, for various values of the birefringence ratio $\frac{\gamma'-\alpha'}{\gamma-\alpha}$ (Column I) and definite values

$\frac{\gamma'-\alpha'}{\gamma-\alpha}$	90	88	86	84	82	80	78	76	74	72	70	68	66	64	62	60	58	56	54	52	50	48	46	44	42	40	38	36	34	32	30	28	26	24	22	20
0.10	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	12	12	13	14	15	17
0.20	12	12	12	12	12	12	12	12	12	12	12	12	13	13	13	13	14	14	14	15	15	15	16	17	17	18	19	20	21	22	24	25				
0.30	18	18	18	18	18	18	18	18	18	18	19	19	19	19	20	20	21	21	22	22	23	24	25	26	27	28	29	31	32							
0.40	24	24	24	24	24	24	24	24	24	25	25	26	26	26	27	27	28	29	30	31	31	32	34	35	37	38										
0.50	30	30	30	30	30	31	31	31	31	32	32	33	33	34	34	35	36	37	38	39	41	42	44													
0.60	37	37	37	37	37	38	38	38	39	39	40	40	41	41	43	44	45	46	48	50																
0.70	44	44	45	45	45	45	46	46	47	47	48	49	50	51	52	54	56																			
0.75	49	49	49	49	49	50	50	51	51	52	53	54	55	56	58	60																				
0.80	53	53	53	53	54	54	55	55	56	57	58	60	61	63																						
0.85	58	58	58	59	59	60	60	61	62	63	65	66																								
0.90	64	64	64	65	65	66	67	68	69	71																										
0.95	72	72	72	73	74	75	77																													

of α. The values for α', in even degrees, are given at the intersections of horizontal and vertical lines through these two values.

Further, once for all, upon a piece of tracing paper, the vertical small circles of half the Wulff net are drawn from one pole to the equator. This tracing is placed concentrically over a Wulff net (Fig. 32) in such a position that its pole lies on the periphery at a distance of the true axial angle ($2V$) from the pole of the net, and is fastened in this position by means of thumb tacks. There will now appear, in the desired quadrant, a double net of vertical small circles whose intersections will give the angles α and α' from the points of emergence of the optic axes on the periphery of the circle. By placing above the double net a clean sheet of tracing paper, the desired curves may be drawn through the proper intersections as given in the above table. In this way the curves of equal double refraction are projected, for a single quadrant, in a plane parallel to the plane of the optic axes. The other three quadrants are symmetrical with the first, and may be reproduced by tracing it (Fig. 445). The true values of the curves may be obtained by multiplying their ratio values, as given in the table, by the value of the maximum double refraction of the mineral in question. The projection plane may be changed to any other plane desired by the method given in Art. 16, problem 9.

The curves in Figs. 445 and 446, which are those of albite ($Ab_{98}An_2$, with $2V=77°$), were constructed in this manner. Analyzed independently, the fringes form closed circles when $K<\sin^2 V$ (Fig. 444, curve 0.2): when

[1] Georg Wulff: *Untersuchungen im Gebiete der optischen Eigenschaften isomorpher Krystalle.* Zeitschr. f. Kryst., XXXVI (1901), 20–22.

$K = \sin^2 V$ they become tangent at the center over the acute bisectrix and form a "figure eight." When $\sin^2 V > K < \cos^2 V$ the curves do not pass between the optic axes in the acute axial angle, but form a closed curve around the two axes. When $K = \cos^2 V$, the curve becomes tangent at the obtuse bisectrix; it still forms a closed curve and extends around both optic axes. When $K > \cos^2 V$ the circles do not become tangent and the curves are open.

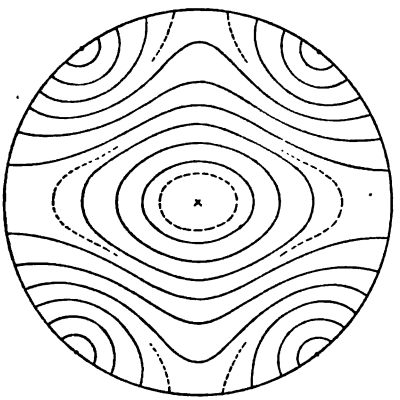


FIG. 445.—Lines of equal birefringence in a section of albite cut parallel to the plane of the optic axes. (After Wulff.)

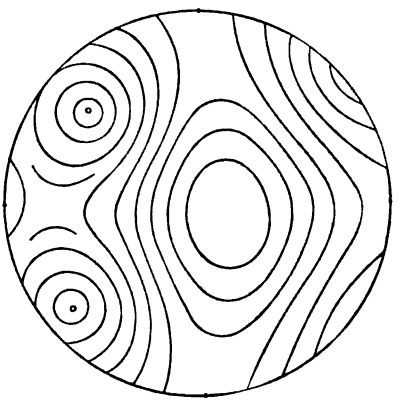


FIG. 446.—Lines of equal birefringence in a section of albite cut parallel to 001. (After Wulff.)

From these different cases it is seen that when $K < \sin^2 V$, for certain values of θ, the curves cross the plane of the optic axes in both the acute and the obtuse optic axial angles; $K = \sin^2 V$ is the limiting case. When $K < \cos^2 V$ but $> \sin^2 V$ the curves cross the plane of the optic axes in the obtuse optic axial angle, and $K = \cos^2 V$ is the limiting case. The minimum birefringence is given by sections cut at right angles to an optic axis, and the maximum birefringence by sections parallel to the plane of the optic axes.

290. Abnormal Birefringence.—Owing to the fact that the retardation of rays of different colors is not exactly the same for all, the resulting interference color is not the pure complementary color of that extinguished. This may be seen by inspecting the following table of retardations in calcite for various colors of the spectrum.

Color	Fraunhofer line	Wave length	ω	ϵ	$\omega - \epsilon$
Red	*A*	759.40	1.650	1.483	0.167
Red	*B*	686.74	1.653	1.484	0.169
Yellow	*D*	589.60	1.658	1.486	0.172
Blue	*F*	486.15	1.668	1.491	0.177
Violet	*H*	396.81	1.683	1.498	0.185

The double refraction for the color at one end of the spectrum is considerably less than that at the other and, as a consequence, the resulting interference color will not be normal. When the double refraction for the red is less than that for the violet, as it is in calcite, Becke called this abnormal color **supernormal** (*übernormal*); when the reverse is the case, **subnormal** (*unternormal*).

Another cause of abnormal colors is the fact that in certain minerals the birefringence is zero for certain wave lengths. For example, fuggerite[1] ($Ca_3Al_2Si_2O_{10}$), at one end of the spectrum, is positive ($\epsilon > \omega$), at the other, negative ($\omega > \epsilon$), and for sodium light it is dark ($\omega = \epsilon$). By white light, owing to the extinguishing of the yellow, the complementary color, deep blue, appears. The same color, naturally, will appear no matter what may be the thickness of the slide, the color being only deeper in thick sections. That the color is abnormal may be seen by inserting a Johannsen wedge, or any other accessory plate which will compensate for less than 1/4 wave length. Instead of producing darkness, as it ordinarily would at the point of compensation, a pale brownish color appears. The blue of the second order, which somewhat resembles the abnormal color, is reduced to a bright orange by the same retardation.

The abnormal blue color is likewise shown by melilite, vesuvianite,[2] and chlorite. Certain other minerals show different abnormal colors, depending upon the wave length which is totally extinguished in them.

Another cause for abnormal interference colors is the dispersion of the directions of vibration in monoclinic and triclinic crystals. Since all of the colors are not extinguished at the same time, an abnormal color results. Zoisite is an example.

A third cause for abnormal colors may be found in the fact that part of the light from an illuminating system of large aperture does not pass through the crystal in strictly parallel directions, but at an angle. Traveling thus different paths, the amount of the retardation will be different, and a mixed interference color results.

A fourth cause for abnormal colors is the modification produced by the color of the mineral itself. Thus chlorite, when of a deep green, may show a greenish interference color instead of the normal blue, and that of biotite or hornblende may appear to be that of the mineral itself.

[1] E. Weinschenk: *Fuggerit, ein neues Mineral aus dem Fassathal.* Zeitschr. f. Kryst., XXVII (1896–7), 577–582.

[2] C. Hlawatsch: *Bestimmung der Doppelbrechung für verschiedene Farben an einigen Mineralien.* T. M. P. M., XXI (1902), 107–156.

CHAPTER XXIV

DETERMINATION OF THE VIBRATION DIRECTIONS IN MINERAL PLATES

291. Optical Character of the Elongation.—It has already been pointed out[1] that when two anisotropic minerals are superposed, the resulting interference color is their algebraic sum. This principle is made use of in determining the directions of the fast and slow rays in mineral plates.

If an anisotropic mineral plate, in which the vibration directions are unknown, is placed upon the stage of the microscope between crossed nicols, and it is rotated until no light is transmitted, its vibration directions lie parallel to those of the polarizer and analyzer.[2] If it is rotated still farther, until its vibration directions make angles of 45° with its former position, it will be in the position of maximum illumination. If, now, there is placed above it, also in its position of maximum illumination, a mineral plate in which the vibration directions are known, it may be seen, readily, that if the interference color rises in the scale, the vibration directions of the unknown mineral are parallel to those of the known, and if it sinks, at right angles. Various mineral plates, with the vibration direction of the slow ray (usually) marked by arrows,[3] are provided with petrographic microscopes. The most common are the quarter-wave plate, gypsum plate, and quartz wedge.

The determination of the fast and slow rays in a crystal section may or may not be of value in its determination. If the **orientation,** that is the relation of the vibration directions to crystallographic directions, is known in uniaxial crystals, the determination of the fast or slow ray determines the optical character of the mineral itself, for if crystallographic *c* is the fast ray, the mineral is negative, if it is the slow ray, positive.[4] In biaxial crystals the optical character of the mineral may be determined by the orientation in any section if the positions of the optic axes are known.[5]

But crystals have characteristic cleavages, consequently the fragments found in rock sections are commonly bounded by cleavage planes. The

[1] Art. 286 *supra.*

[2] Art. 283 and Art. 285, Case II, *supra.*

[3] The arrow so marked ———→*c*, does not mean that the slow ray travels to the right, but that its plane of vibration is parallel to the shaft, thus ←———→. This explanation may seem absurdly unnecessary, but apparently is not, judging from questions asked by students.

[4] Art. 51.

[5] Art. 75.

determination of the vibration directions in such pieces may separate two similar minerals, for in one the fast ray may be parallel to the cleavage and in the other at right angles to it. Minerals in which such cleavages are common usually occur in lath-like fragments in rock sections, and this characteristic extension in one direction is spoken of as the **elongation of the mineral** and includes both cleavage elongation and prismatic elongation of natural crystals. If the fast ray coincides with the long direction, the elongation is said to be *negative*, if the slow ray coincides, *positive*.[1]

ACCESSORIES USED FOR THE DETERMINATION OF THE VIBRATION DIRECTIONS OF A MINERAL

292. Kinds of Accessories.—Only the simplest forms of the many accessories which may be used for the determination of vibration directions are given below. Many others[2] are described under the methods for the determination of birefringence and extinction angles.

The simple forms may be grouped into two classes.

I. Simple plane-parallel plates.
   a. Quarter undulation mica plate.
   b. Unit retardation plate.

II. Wedges.
   a. Simple quartz or gypsum wedge.
   b. Fedorow mica comparator.
   c. Wright combination wedge.
   d. Johannsen quartz-mica wedge.

I. SIMPLE PLANE PARALLEL PLATES

293. Quarter Undulation Mica Plate.—The quarter undulation mica plate, also called quarter wave-, quarter order mica-, or $1/4\ \lambda$ plate, is made of such a thickness that $M(n_2-n_1)=1/4\ \lambda$,[3] whereby one vibration will be retarded a quarter of a wave length behind the other, and the transmitted wave will be elliptically polarized.

The fact that two mineral sections superposed at right angles to each other show a reduction in the interference tint, was discovered by Arago[4] in 1811,

[1] The signs of the optical character of the *elongation* and of the optical character of the *mineral* may be remembered by connecting them thus: When the *c* axis of a uniaxial or the acute bisectrix of a biaxial mineral is the fast ray, the *mineral* is negative; when the elongation is parallel to the fast ray, the *elongation* is negative.

[2] Various accessories are also described in:

G. Valentin: *Die Untersuchungen der Pflanzen- und der Thiergewebe im polarisirten Lichte.* 1861.*

Moigno: *Répertoire d'optique moderne*, 1850, Tome IV, 1592 et seq.*

[3] Equation 3, Art. 280, *supra.*

[4] F. Arago: *Mémoire sur une modification remarquable qu'éprouvent les rayons lumineux dans leur passage à travers certains corps diaphanes, et sur uelques autres nouveaux phénomènes d'optique.* Mém. Acad. France. Année 1811, Pt. I. XII (1812), 93–134.

but Biot[1] was the first to suggest that the vibration directions in an unknown mineral could be determined by comparison with those of one which is known. He used thin plates of mica, gypsum, quartz, and other substances whose vibration directions were determined. He did not specify any particular thickness of plate, but had a series of different thicknesses, and chose whichever compensated with the unknown mineral. If the mineral under examination had the same optical character as the plate, he called it attractive, if the opposite, repulsive. To such minerals Brewster[2] gave the names positive and negative.

The use of thin plates of definite thicknesses was probably introduced by Airy[3] in 1831. He showed that light would be circularly polarized by plates of 1/4, 3/4, 5/4, etc., retardation. De Senarmont,[4] in 1851, first applied a half-wave plate to the determination of the three vibration axes of crystals. A quarter undulation plate was used by Bravais,[5] in 1855, and since then it has been in common use as an accessory in petrographic microscopic work.

The most convenient mineral from which a quarter undulation plate can be constructed is muscovite. It cleaves in plates which may be made of almost any degree of thinness, and, since these plates differ but 2° from being perpendicular to the acute bisectrix of the optic axial angle, this bisectrix emerges in the center of the field. Since muscovite is negative, the bisectrix is the fast ray a. The other vibration directions may be determined by examining the mineral plate in convergent polarized light.[6] In the interference figure thus obtained, the slowest ray c will vibrate in the direction of the line connecting the points of emergence of the optic axes, that is, the points of rotation of the black bars; b is the direction at right angles to c and also lies in the cleavage flake.

[1] J. B. Biot: *Mémoire sur une nouvelle application de la théorie des oscillations de la lumière.* Lu à l'Institute 27 dec. 1813. Mém. Acad. France, Année 1812, XIII, Paris, 1816, Pt. II, 1–18.

Idem: *Traité de physique.* Paris, 1816, IV, 420–422, 543–566.

[2] David Brewster: *On the laws of polarization and double refraction in regularly crystallized bodies.* Phil. Trans. Roy. Soc. London, CVIII (1818), 199–273, in particular 219.

[3] G. B. Airy: *On the nature of the light in the two rays produced by the double refraction of quartz.* Read Feb. 21, 1831. Cambridge Phil. Soc., IV (1833), 79–123.

Idem: *Addition to a paper "On the nature of the light in the two rays produced by the double refraction of quartz."* Read April 18, 1831. Cambridge Phil. Soc., IV (1833), 199–208.

Idem: *Ueber die Natur des Lichtes in den beiden durch die Doppelbrechung des Bergkrystalls hervorgebrachten Strahlen.* Pogg. Ann., XXIII (1831), 204–280.

[4] H. de Senarmont: *Recherches sur les propriétés optiques biréfringentes des corps isomorphes.* Ann. d. chim. et phys., XXXIII (1851), 391–401.

[5] A. Bravais: *Beschreibung eines neuen Polariskops und Untersuchung über die schwachen Doppelbrechungen.* Pogg. Ann., XCVI (1855), 395–414.

Idem: *D'un nouveau polariscope et recherches sur les doubles réfractions peu énergiques.* Ann. d. chim. et phys., XLIII (1855), 129–149.

[6] See Chapter XXIX.

To prepare a quarter undulation mica plate, select a clear piece of muscovite and split it into very thin plates by inserting a pin between the lamellæ. It is not possible to obtain large lamellæ uniformly thin throughout, and it is therefore advisable to examine them between crossed nicols and scratch lines upon the surface around the areas of like interference colors. The cleavage plate may then be cut apart on the contour lines and pieces of like thickness kept separate for different purposes. For the quarter undulation plate select such pieces as give a pale neutral gray interference color, and whose first interference ring makes a perfect ellipse around both the eyes.[1] They may be tested further by comparison, by compensation, with a standard $1/4\lambda$ plate. They should be of such thickness that by sodium light the retardation is just a quarter of a wave length.

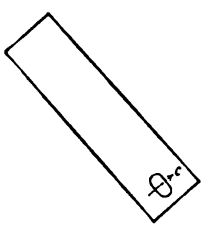


FIG. 447.—Quarter undulation mica plate.

From plates so prepared, rectangular pieces should be cut, parallel or at right angles to the vibration direction of the slow ray, the directions being determined by examination of the interference figure. The films, finally, should be mounted in Canada balsam between glass plates. The vibration direction of the slow ray should be indicated by an arrow scratched on the glass. As usually mounted, the c direction lies directly across the slip (Fig. 447). In some microscopes the slot for the accessories is parallel to the cross-hairs, therefore the c direction of the mica must lie at 45° with the long direction. When cut in the former of the two ways, it is easy to remember that the slow vibration is parallel to the short, and the fast vibration parallel to the long edge of the plate.

The thickness of a mica plate necessary to produce a retardation of $1/4\lambda$ may be computed from equation 4, Art. 280,

$$P = M(n_2 - n_1).$$

Since the section is cut at right angles to the acute bisectrix (a), it contains the axes b and c with indices n_2 and n_1, equal to γ and β. If these are 1.603 and 1.595 in the specimen of mica used, we have, for sodium light,

$$\frac{1}{4} = \frac{M\ (1.603 - 1.595)}{0.000589},$$

whereby

$$M = \frac{0.000589}{4 \times 0.008} = 0.0184 \text{ mm.}$$

To determine the fast and slow vibration directions, the process is as given above. Turn the mineral, between crossed nicols, 45° off the position of darkness, and insert the mica plate in the slot provided for it. If the interference color of the mineral is increased by $1/4\lambda$ the slow rays of the two are parallel, if it decreases, they are at right angles.

Further uses for the mica plate are given in Art. 404.

[1] See Art. 360 and Fig. 561.

294. Unit Retardation Plate.—A plate whose retardation is equal to 575$\mu\mu$, the wave length of rays near the *D* line, extinguishes the intense yellow rays, and the resulting color is the sensitive violet.[1] If such a plate is cut from mica it must be four times as thick as the quarter undulation plate just described. Since mica is rarely entirely colorless, such a plate generally has a yellow tinge, and for that reason gypsum usually is, though quartz may be, used. Gypsum is monoclinic, the angle $\beta = 80°42'$, $b = \mathfrak{b}$, and $c : \mathfrak{c} = 53°$ in the obtuse angle (Fig. 448), 010 cleavage good, 111 and 100 distinct. In the 010 cleavage flakes lies the plane of the optic axes, and the $\mathfrak{b}$ direction is perpendicular to it. The first order violet plate may be cut with its long direction parallel either to $\mathfrak{a}$ or $\mathfrak{c}$. To avoid confusion it is better that the elongation of all of the accessories be the same. The retardation corresponds practically to a wave length of the mean of white light, and the plate may therefore be spoken of as the **unit retardation plate.** It is usually called the Violet of the first order, "Red" of the first order, or Sensitive plate.

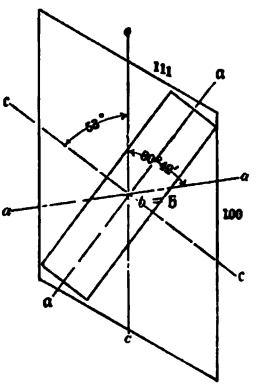


FIG. 448.—Orientation of the unit retardation plate.

The thickness may be calculated as before.

If $\gamma - \alpha = 0.0095$, $\lambda = M(\gamma - \alpha)$, $M = \frac{0.000589}{0.0059} = 0.062$ mm.

If quartz is used, it should be cut to a plane parallel to the optic axis and of a thickness $M = \frac{.000589}{.009} = 0.0655$ mm.

The first use made of a unit retardation plate seems to have been by Biot[2] in 1813 and the term *teinte sensible* was introduced by him.

For the determination of extinction angles, slight double refractions, and the optical character of minerals by means of this plate, see Arts. 319, 334, and 405. For the determination of vibration directions in mineral plates, the method is exactly the same as that just described for the mica plate.

II. RETARDATION WEDGES

295. Simple Quartz or Gypsum Wedge.—Instead of using a plane-parallel plate, it is often convenient to use one of a wedge-shape. When such a plate is inserted in the microscope between crossed nicols, it will not be uniformly colored, but will show parallel bands (Fig. 449) corresponding to the whole

[1] *Teinte sensible* of Biot, *teinte de passage.*

[2] J. B. Biot: *Mémoire sur une nouvelle application de la théorie des oscillations de la lumière.* Lu à l'Institute 27 dec., 1813. Mém. Acad. France. Année 1812, Paris, XIII (1816), pt. ii, 1–18.

Idem: *Traité de physique*, Paris, 1816, IV, 420–422, 543–566.

range of Newton's colors, and varying from nearly darkness to colors of the third, fourth, or higher orders. The explanation is, of course, that the increased thickness of wedge, as it is shoved into the field, causes more and more retardation of the transmitted rays. When such an accessory is inserted, with its thin edge foremost, above a mineral plate, there will be a gradual reduction of the interference colors to zero when the vibration directions of the two are at right angles. At the point where the retardations of the two are equal, a black band will appear. It is the point of **compensation.**

A quartz wedge must be carefully cut to avoid breaking the thin edge, and it should be mounted between glasses to protect it. The smaller the angle of the wedge, the broader will be the interference bands. It is very desirable to have several wedges; one with broad bands from the first to the third order, and one from the third to the seventh. The orientation of the vibration directions in the wedges should be the same as that in the mica and gypsum plates, to avoid confusion.

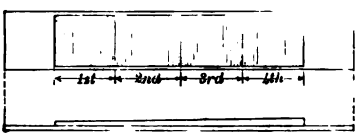


FIG. 449.—Simple quartz or gypsum wedge.

Like the mica plate, the quartz wedge was first used by Biot.[1] It was later made use of by de Senarmont,[2] but subsequently seems to have been forgotten until Sorby[3] announced it as new in 1877.

296. Fedorow Mica Comparator.—The Fedorow mica comparator is built up of sixteen rectangular quarter undulation mica plates, each 2 mm. shorter than the preceding. It is described in full below.[4] For the determination of the optical character of the elongation it is used like a quartz wedge.

297. Wright Combination Wedge.—The difficulty with ordinary quartz wedges is that it is impossible to grind the front edge sufficiently thin to give a dark band and as a result, upon insertion, the color rises abruptly to about a quarter order. To overcome this objection, Wright[5] combined a quartz wedge (*b*, Fig. 450), having its fast ray parallel to the long direction of the

[1] J. B. Biot: *Mémoire sur les propriétés physiques que les molécules lumineuses acquièrent en traversant les cristaux doués de la double réfraction.* Lu 22 mai, 1814. Mém. Acad. France, Année 1812. Paris, 1814. 31–38.

Idem: *Traité de physique.* Paris, 1816, IV, 420–422, 543–566.

[2] H. de Senarmont: *Op. cit.*, 1851, 401.

[3] H. C. Sorby: *On a new arrangement for distinguishing the axes of doubly refracting substances.* Mon. Microsc. Jour., XVIII (1877), 209–211.

[4] Art. 308.

[5] Fred. Eugene Wright: *Die foyaitisch-theralitischen Eruptivgesteine der Insel Cabo Frio, Rio de Janeiro, Brasilien.* T.M.P.M., XX (1901), footnote, p. 275.

Idem: *A new combination wedge for use with the petrographical microscope.* Jour. Geol., X (1902), 33–35.

wedge, with a second order green selenite plate (*c*) in which the fast ray vibrates at right angles to this direction. By this arrangement compensation is produced at about the center of the wedge where a dark band appears, to the right and left of which the interference colors rise. Minerals seen through the dark bar will have the same interference colors as though the wedge were not there, but on shoving the wedge either way the colors gradu-

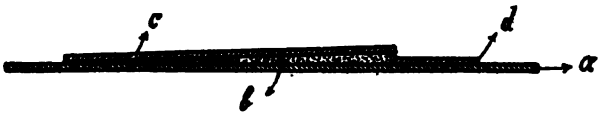


FIG. 450.—Wright's combination wedge. (Fuess.)

ally rise. At one end of this accessory, for convenience, a first order red (*a*) is added. As originally described this gypsum plate was separated from the wedge by an open space, and not as shown in the figure.

The combination wedge was later made of a quartz wedge and a quartz plate, and was improved[1] by making the upper and lower faces parallel (Fig. 451) thus causing no displacement of the image. It likewise had engraved upon the upper surface a scale divided into 0.1 mm., the wedge being so calculated that the reading gave directly the difference in $\mu\mu$ in the retardation of the wave. In order that the divisions of the scale may be seen, this wedge must be inserted in the focal plane of the ocular.

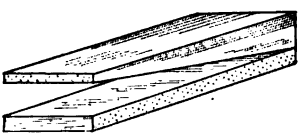


FIG. 451.—Improved combination wedge.

298. Johannsen Quartz-mica Wedge.—The interference colors of the Wright wedge rise abruptly to the second order, no matter which end is inserted first, and then fall to zero. In the wedge described by Johannsen,[2] which is made on a similar principle, this does not occur, and the transition from the interference color of the mineral to that of the wedge is imperceptible. The wedge (Fig. 452) consists of a carriage, exactly fitting the slot above the objective, and permanently retained in the tube of the microscope by means of two end screws like those holding the Bertrand lens bar in the Fuess microscope. At one end is a square of gypsum (*a*) giving red of the first order; *b* is an opening, and *c* is a first to fourth order quartz wedge underlaid by a mica plate,

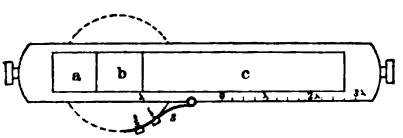


FIG. 452.—Johannsen quartz-mica wedge.

[1] Fred. Eugene Wright: *A new ocular for use with the petrographic microscope.* Amer. Jour. Sci., XXIX (1910), 416-417.

[2] Albert Johannsen: *Some simple improvements for a petrographical microscope.* Amer. Jour. Sci., XXIX (1910), 436.

the two minerals having their c directions at right angles to each other. The thickness of the mica is such that it *exactly compensates* the end of the wedge at the edge of the opening *b*, and as a result, when the wedge is shoved forward, it begins at total darkness, just as though the wedge were infinitely thin at this end, and gradually increases to the fourth order. A spring *s*, attached to the side of the microscope tube or within it, presses against the carriage and produces enough friction to hold it wherever placed. When the opening *b* is centered, the spring drops into a rounded notch as shown.

For the determination of the character of the elongation of a mineral, the wedge might be further improved by making it in two parts, like the Evans double wedge,[1] one with its long direction parallel to the *c* axis, and the other at right angles to it. If ground to the same slope, both would begin at absolute darkness. Since the slow ray would be vibrating parallel to the long direction in one, and at right angles to it in the other, when placed above a mineral section in the position of maximum illumination, the former would compensate with minerals having negative elongation and would show a dark bar, and the latter would compensate with those having positive elongation and show the bar. If, further, a scale were engraved on the upper surface, showing directly the values of the retardations, it would be of still greater value. To read the divisions on the scale, if inserted in the usual slot above the objective, it would only be necessary to insert the Bertrand lens. If inserted in the focal plane of the ocular, as in the Seidentopf[2] compensator, the divisions, likewise, could be directly read.

[1] Art. 315.
[2] Art. 316.

CHAPTER XXV

DETERMINATION OF THE ORDER OF BIREFRINGENCE

299. Birefringence.—It has been shown[1] that the retardation in a doubly refracting mineral is expressed by $R=M(n_2-n_1)$, and that the interference color produced, when white light is used, depends upon this value; n_2 and n_1, in the equation, being the refractive indices in two directions at right angles to each other. As the thickness of the section or the difference between n_2 and n_1 increases, so does the color. The thickness of section may be taken as of any value. If it is considered unity, the resulting value for R, that is n_2-n_1, depends only upon the kind of mineral and upon the orientation of the section with respect to the principal vibration directions. If n_2 and n_1 are taken as the values of the indices of refraction along the maximum and minimum ease of vibration directions in any mineral, the difference between them will be a measure of the maximum double refraction or **birefringence** of that mineral. Just as is the refractive index, so also is the value for the maximum birefringence characteristic for any mineral. It is expressed in positive uniaxial crystals by $\epsilon-\omega$, and in negative by $\omega-\epsilon$. In biaxial crystals it is expressed by $\gamma-\alpha$, but besides this maximum value there are, of course, two other characteristic values, namely, $\gamma-\beta$ and $\beta-\alpha$.

The value of the maximum birefringence of a mineral may be determined by computation from the formula just given if the refractive indices and the thickness of the section are known. It may also be calculated if the orientation of the section and its birefringence or indices are known.[2]

In practice, the double refraction of a mineral is usually determined by measuring the thickness of the section and determining the point of compensation.[3] The simplest accessory for this purpose is the compensating wedge already described.

300. Compensating Wedge for the Determination of Birefringence.—The method for determining the birefringence of a mineral by means of the quartz or gypsum wedge follows directly from the method given above for determining the vibration directions of a crystal. If an unknown mineral is placed upon the stage of the microscope, between crossed nicols, and it is

[1] Art, 280, *supra.*

[2] Art. 288, *supra.*

[3] For the effect of dispersion on double refraction see:

C. Hlawatsch: *Bestimmung der Doppelbrechung für verschiedene Farben an einigen Mineralien.* T. M. P. M., XXI (1902), 107–156.

turned 45° off extinction, it will be in its position of maximum illumination. A quartz wedge is now inserted above the mineral, and the change of colors noted. If they ascend in the scale,[1] that is, pass through yellow to red to violet to blue to green to yellow, etc., the vibration directions of mineral and wedge are clearly parallel. The mineral is, therefore, rotated through 90° and the wedge again inserted. The order of the change of colors will now be reversed, and if the section be not too thick or the birefringence of the mineral too high, a point of compensation will be reached. Beyond the dark bar of compensation the colors will again appear, but in ascending order. When compensation occurs, the wedge should be held in place and the mineral removed from the stage. The interference color shown by the wedge should now be the same as that which originally was shown by the mineral, except so far as the latter may have been made abnormal by color, dispersion, etc. As the wedge is slowly withdrawn, the sequence of colors may be noted and, by counting the number of times a color recurs and making comparsion with a color chart,[2] its exact position in the scale may be determined. The value obtained, however, is that of the retardation, and not the true value of the birefringence, for this is influenced by the thickness of the section, as may be seen from the equation, $R = M(n_2 - n_1)$. To determine the thickness M, recourse may be had to the method of the Duc de Chaulnes or to any of the other methods suggested in Art. 208. In de Chaulnes method, however, it is necessary to know the refractive index of the mineral. This may be unknown in the mineral under observation for birefringence, but in a rock section, adjacent to the unknown mineral, there probably is some mineral which is known. In the known mineral, then, the determination of thickness may be made. By dividing the value of the retardation, as obtained with the quartz wedge, by the thickness, a retardation value for unity may be obtained, and from this, by comparison with a table, the value of the birefringence of the mineral.

301. Michel-Lévy Chart of Birefringences (1888).—The best table for the comparison of interference colors is that devised by Michel-Lévy[3] and shown in outline in Fig. 453. In the original chart there are shown, from left to right, colors as nearly as possible like those produced by increased retardation, the values of which $[M(n_2 - n_1)]$ are shown in millionths of

[1] Cf. Newton's scale, Arts. 276–277.

[2] Art. 301. The colored plate of birefringences, originally given by Lévy and Lacroix, has been reproduced frequently and may be found in Rosenbusch-Wülfing, Duparc and Pearce, Iddings, or Johannsen. Cf. Fig. 451.

[3] A. Michel-Lévy et Alf. Lacroix: *Les minéraux des roches.* Paris, 1888, plate 1.

See also A. Michel-Lévy: *Mesure du pouvoir biréfringent des minéraux en plaque mince.* Bull. soc. min. France, VI (1883), 143–161.

Idem: *Note sur la biréfringence de quelques minéraux; application à l'étude des roches en plaques minces.* Ibidem, VII (1884), 43–47.

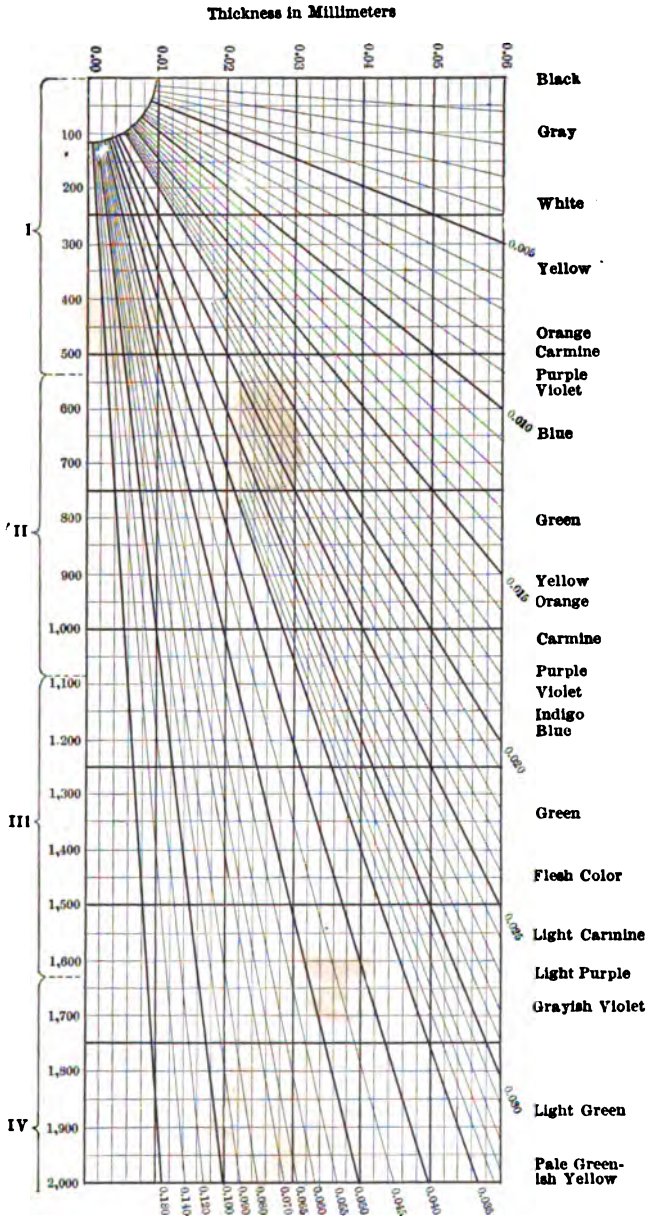


FIG. 453.—Outline of Michel-Lévy's chart of birefringences, the positions of the colors modified according to the Kraft scale for a clear sky.

millimeters by the abscissæ. The ordinates represent thickness of section (M). The value of unit birefringence, $n_2 - n_1$ (that is, $\gamma - \alpha$ or $\omega - \epsilon$), remains constant for any mineral, but as the section increases in thickness so does the retardation increase, as may be seen from the retardation equation. The diagonal lines in the diagram represent, therefore, the retardations produced by sections of different thicknesses.

From this chart one may determine not only the order of birefringence of a mineral, but the thickness of a section as well, provided some mineral contained in the slide is known. For example, in a granite the fragments of quartz are easily recognized. Among these, note the highest interference color. If the slide contains numerous fragments it is probable that this is a section parallel to the optic axis, and its birefringence has the maximum value, 0.009. In the chart this value is given by a diagonal line. Follow it down toward the lower left corner until it intersects the vertical line giving the interference color shown in the slide. The ordinate at the point of intersection represents the thickness of the slide, and its value may be found by following out the horizontal line, through the intersection, to the scale at the left. The value there found is the thickness of the slide in millimeters.

To determine the birefringence of an unknown mineral the method is as follows: Determine the thickness of the section by any of the methods given in Art. 208, or by means of the birefringence of some known mineral by the method just indicated. It is advisable to make use of a known mineral fragment lying as near as possible to the unknown, since there may be a slight difference in the thickness of different parts of the slide. No hesitation should be felt, however, in using this method for fear that the section may be unequally ground, for differences in thickness can be recognized readily by the variation in the interference colors in different parts. Having determined the thickness of the slide, determine the highest interference color in any fragment of the unknown mineral. Take the intersection of the horizontal line of thickness in the chart with this color. The diagonal line passing through this point of intersection indicates the birefringence of the unknown mineral.

For example, in a slide of a basalt there are many fragments of labradorite whose maximum birefringence is 0.008. If its highest interference color in the rock slice is pale straw color, the thickness of the slide is 0.034. An unknown mineral in the same rock slice has an interference color of blue of the third order. The diagonal line crossing the point of intersection is 0.035, which is the value of the maximum birefringence of meionite, humite, and olivine. From other characteristics of the minerals we can easily separate these three and determine the unknown mineral as olivine.

TABLE OF MAXIMUM BIREFRINGENCES

Rutile	0.287	Hedenbergite	0.019
Micaceous hematite	0.28	Lawsonite	0.019
Siderite	0.238	Glaucophane	0.018
Magnesite	0.202	Monticellite	0.017
Dolomite	0.179	Spodumene	0.016
Calcite	0.172	Common hornblende	0.016
Brookite	0.158	Mizzonite	0.015
Aragonite	0.156	Wollastonite	0.015
Titanite	0.145	Anorthite	0.013
Cassiterite	0.096	Serpentine	0.013
Anatase	0.073	Dipyr	0.013
Basaltic hornblende	0.072	Hypersthene	0.013
Zircon	0.062	Cornerupine	0.013
Grünerite	0.056	Natrolite	0.012
Astrophyllite	0.055	Disthene	0.012
Fayalite	0.050	Johnstrupite	0.012
Ægirite	0.050	Mosandrite	0.012
Talc	0.050	Hydronephelite	0.012
Diaspore	0.048	Laumontite	0.012
Monazite	0.045	Andalusite	0.011
Anhydrite	0.044	Antigorite	0.011
Datolite	0.044	Clinochlore	0.011
Phlogopite	0.044	Dumortierite	0.011
Biotite	0.040	Gypsum	0.010
Lavenite	0.040	Axinite	0.010
Muscovite	0.038	Staurolite	0.010
Pectolite	0.038	Ottrelite	0.010
Lazulite	0.036	Epistilbite	0.010
Olivine	0.035	Albite	0.010
Humite	0.035	**Quartz**	**0.009**
Meionite	0.035	Corundum	0.009
Prehnite	0.033	Enstatite	0.009
Titanolivine	0.033	Bronzite	0.009
Pistacite	0.032	Cordierite	0.009
Chondrodite	0.032	Topaz	0.009
Orthite	0.032	Zoisite	0.009
Diopside	0.029	Labradorite	0.008
Jadeite	0.029	Kaolin	0.008
Ægirite-augite	0.029	Clinozoisite	0.008
Cancrinite	0.028	Scolecite	0.007
Thomsonite	0.028	Heulandite	0.007
Actinolite	0.027	Orthoclase	0.006
Tremolite	0.026	Gehlenite	0.006
Wöhlerite	0.026	Ænigmatite	0.006
Rosenbuschite	0.026	Stilbite	0.006
Tourmaline	0.025	Sapphirine	0.005
Augite	0.025	Melilite	0.005
Anthophyllite	0.024	Nephelite	0.005
Hydrargillite	0.023	Riebeckite	0.004
Carpholite	0.022	Apatite	0.004
Sillimanite	0.022	Eucolite	0.003
Brucite	0.021	Phillipsite	0.003
Gedrite	0.021	Eudialite	0.002
Barkevikite	0.021	Tridymite	0.002
Alunite	0.020	Vesuvianite	0.002
Melinophane	c.020	Pennine	0.002
Pargasite	0.020	Leucite	0.001

302. Babinet Compensator.—One of the most delicate instruments for determining the birefringence of a mineral is the Babinet compensator.

Unfortunately the methods for determining the thickness of a section are not so delicate as that for determining the birefringence by this instrument, and since this is a factor in obtaining the result, the advantage is somewhat lessened.

The Babinet[1] compensator consists essentially of a Ramsden ocular, beneath which are arranged two right angled quartz wedges with equal slopes and with their inclined faces toward each other (Fig. 454). One of these wedges is movable by means of a micrometer screw, and one is stationary. The vibration directions of the two wedges lie at right angles to each other,

FIG. 454.—Section through the wedges of a Babinet compensator.

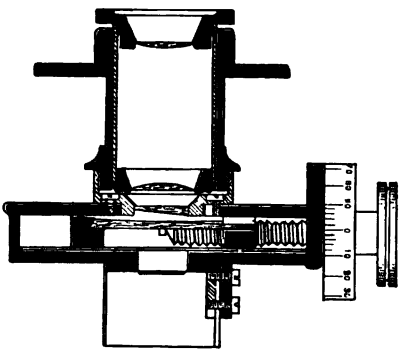


FIG. 455.—Babinet compensator. 2/3 natural size. (Fuess.)

one being cut with the long direction parallel to the optic axis, and one at right angles to it. In the Fuess instrument (Fig. 455), the lower wedge, which has a length of 25 mm., is the one which is movable. Upon the stationary wedge there is engraved a cross, whose arms make an angle of 30° with each other, and whose center is on the axis of the microscope.

[1] M. J. Jamin: *Mémoire sur la réflexion a la surface des corps transparents.* Ann. d. chim. et phys., XXIX (1850), 263–304, especially 271–274.

A. Bravais: *Beschreibung eines neuen Polariskops und Untersuchung über die schwachen Doppelbrechungen.* Pogg. Ann., XCVI (1855), 395–414, especially pages 404–409.

Idem: *D'un nouveau polariscope et recherches sur les doubles réfractions peu énergiques.* Ann. d. chim. et phys., XLIII (1855), 129–149.

G. Quincke: *Optische Experimental-Untersuchungen. II. Ueber die elliptische Polarisation des bei totaler Reflexion eingedrungenen oder Zurückgeworfenen Lichtes.* Pogg. Ann., CXXVII (1866), 203–212.

J. Macé de Lépinay: *Recherches expérimentales sur la double réfraction accidentelle.* Ann. d. chim. et phys., XIX (1880), 5–90.

Karl E. Franz Schmidt: *Zur Theorie des Babinet'schen Compensators.* Wiedem. Ann., XXXV (1888), 360–369.

Idem: *Zur Konstruktion des Babinet'schen Kompensators.* Zeitschr. f. Instrum., XI (1891), 439–444.

J. Macé de Lépinay: *Sur la localisation des franges des lames cristallines.* Ann. d. Phys., X (1891), 204–213.*

C. Leiss: *Die optischen Instrumente, etc.*, Leipzig. 1899, 223.

Thomas Preston: *The theory of light.* London, 3d ed., 1901, 410–415.

F. Becke: Denkschr. Akad. Wiss. Wien, LXXV (1904), 58–*

Rosenbusch-Wülfing: *Mikroskopische Physiographie*, Stuttgart, 4 Aufl., 1904, I_1, 284–289.

Duparc et Pearce: *Traité de technique min. et pétrog.* Leipzig, 1907, 206–211.

A. E. H. Tutton: *Crystallography.* London, 1911, 859–862.

This cross serves for a starting point from which to measure the displacement, which may be read to 0.005 mm. by means of the vernier.

The Babinet compensator is inserted in the tube of the microscope instead of an ocular, and is so placed that the axes of the quartz wedges make angles of 45° with the vibration plane of the polarizer. The analyzer in the tube of the microscope is not inserted, but a cap nicol is placed above the eyepiece. In such a position, and with the vernier set at zero, a black bar appears in the center of the field and, on either side of it, a series of colored bars in white light, or black bars separated by white spaces in monochromatic light. These bars are caused by the separation into two rays of the polarized light which enters from below at right angles to the wedge. One of these rays has vibrations parallel to the vibration direction of the lower wedge, the other vibrates at right angles to it. Upon passing into the upper wedge, the vibration directions remain the same but the velocities are different, the slow ray of the first becoming the fast ray of the second, and *vice versa.* At the center of the wedge, where the thickness of each is the same, the sum of the two vibrations in opposite directions will be zero, for $R=M(n_2-n_1)-M(n_2-n_1)=0$. If the lower wedge is moved so that its thickness is M_1 the equation becomes $R_1=(M_1-M)\ (n_2-n_1)$.

Since the scale and vernier are graduated to millimeters, it is necessary to determine the relation between the retardation and the lateral displacement. This may be accomplished very simply by setting the cross-hairs on the dark band by white light and then, by monochromatic light, measuring the distance through which the wedge must be moved to cause the cross-hairs to coincide with one of the adjacent dark bars. This displacement represents 1λ for whatever light was used. If this was sodium light, then the number of divisions through which the drum D was turned corresponds to $589\mu\mu$, and each division to $\frac{589}{D}=K$, the constant for the instrument with sodium light. Measurements should be made for the value of K between all the dark bars, and if the instrument is properly made, these values should be the same. If they are not, a curve may be drawn to represent the value of retardation for one division of the drum at every point of the wedge.

To determine the birefringence of any mineral with the Babinet compensator, the instrument should be set up as described above, and the black bar be made to correspond with the cross. The vernier should read zero at this point. If the graduations are in millimeters it is of little importance whether this reading is correct or not, for the displacement may be determined by the difference. If, however, the graduations are in $\mu\mu$, the zero value should correspond. When a mineral section is placed upon the stage and it is turned 45° off extinction, the dark bar will be displaced to the right or left, depending upon whether the vibration directions are parallel

or at right angles to those of the comparator.[1] To bring the black bar back to the cross, a number of turns of the drum are necessary. From the calibration previously made, the amount of this displacement in $\mu\mu$ may be determined. The value thus obtained represents the retardation of the light in passing through the mineral plate. The actual value of the birefringence may be determined from this retardation and the known thickness of the slice in the manner described for the quartz wedge.[2]

303. Von Chrustschoff Twin Compensator (1896).—The von Chrustschoff[3] compensator is a modification of that of Babinet. Instead of a simple wedge, it is made up of two pairs (Fig. 456), the orientation of the vibration directions being such as would occur if a simple Babinet pair were cut longitudinally and one pair rotated 180° in altitude so that the bottom becomes the top. In this position the two parts of the upper pair and the two parts of the lower pair are cemented together, making artificial twin wedges. When set at zero, the black bar is continuous across the two. If, however, a mineral is placed beneath it, one-half of the wedge will reduce the retardation while the other half will add to it. The black bar will thus be separated into two bars equally distant from the center, one-half the distance between them representing the retardation A scale engraved on the upper surface, along the twinning line, permits a coarse direct reading to 0.1 mm. displacement. The fine adjustment is by means of a screw, which may be calibrated in a manner similar to the Babinet. The results obtained with this instrument are said to be accurate to the fourth decimal place.

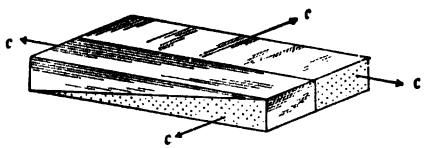


FIG. 456.—Von Chrustschoff twin compensator.

As constructed by Fuess,[4] the two upper wedges are short and stationary, the lower movable in separate mountings and so arranged that with one movement of the screw the two slide with equal displacements in opposite directions. Two scales are provided; one with divisions of 0.1 mm. is engraved directly above the wedge and appears in the field of the microscope, another, for accurate measurements, is given by the screw micrometer which reads to 0.001 mm. The actual amount of the displacement is, of course, double that given by the micrometer, since each wedge has traveled the distance recorded.

[1] For the effect of dispersion produced by the Babinet comparator on the dark bar, see C. Hlawatsch: *Bestimmung der Doppelbrechung für verschiedene Farben an einigen Mineralien.* T. M. P. M., XXI (1902), 107–156.

[2] Art. 300, *supra.*

[3] K. von Chrustschoff: Abh. d. kais. russ. min. Gesell., Ser. II, XXXIV (1896), 165–169.* Review *Ueber einen Zwillingscompensator.* Zeitschr. f. Kryst., XXX (1899), 389.

[4] C. Leiss: *Die optischen instrumente, etc.*, Leipzig, 1899, 223–224.

304. Michel-Lévy Comparator (1883).—The Michel-Lévy[1] comparator (Fig. 457) is another device for determining the double refraction of minerals. It differs in principle from the preceding in that the determination is made by comparison and not by compensation and is, perhaps, quite as good for very small fragments of colorless minerals.

The internal arrangement of the instrument is shown in the cross-section Fig. 458. *E–F* is an ocular, in the place of whose diaphragm there is inserted a prism *P*, silvered on the slanting face with the exception of a small circular opening in the center. Over this clear space, which is on the axis of the microscope, a second prism *P'*, cut from a cylinder of glass, is cemented by Canada balsam. There thus passes to the eye along the axis of the microscope a beam of light coming from the rock section. At the same time the periphery of the field is illuminated by the light which is reflected from the mirror *M* and passes through the prism *C*, a quartz wedge *A*, a diaphragm *D*, and the two nicol prisms *N'* and *N* whose vibration planes lie at 45° to those of the wedge. The latter may be moved across the field of view by means of the screw shown in Fig. 457, and the amount of the movement read from the scale and vernier. Between the second nicol *N* and the prism *P*, the lens *B* converts the light into parallel rays which are reflected from the silvered back of the prism *P* to the eye. If the two nicols, *N* and *N'*, are crossed, the periphery of the field of view of the microscope will show interference colors produced by the quartz wedge, increasing in the scale as the wedge is moved forward. On looking through the microscope, then, the center of the field will be colored by the mineral, which should be turned

FIG. 457.—Michel-Lévy comparator. (Nachet.)

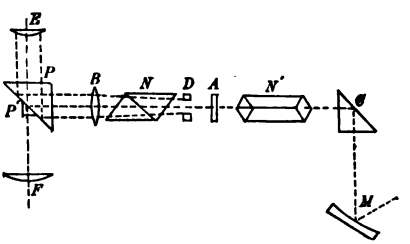


FIG. 458.—Section through the Michel-Lévy comparator.

[1] A. Michel-Lévy: *Mesure du pouvoir biréfringent des minéraux en plaque mince.* Bull. soc. min. France, VI (1883), 143–161.

Lévy et Lacroix: *Les minéraux des roches.* Paris, 1888, 54–59.

R. Fuess: *Quarzkeilcomparator nach Michel-Lévy.* Neues Jahrb. B.B., VII (1889), 77–79.

C. Leiss: *Die optischen Instrumente, etc.* Leipzig, 1899, 224.

45° off extinction, and the border by the quartz wedge of the comparator. To determine the color of the former all that is necessary is to move the wedge until the boundaries disappear and the colors are the same. If the interference color of the mineral is less than the first order yellow, it is advisable to increase it by placing a quarter undulation or unit retardation plate above it, in parallel position, allowing for this increase in computing the result.

The instrument, as made by Fuess, differs somewhat from that described above in that both nicols may be rotated and the quartz wedge thrown out of the field. The latter improvement is important in equalizing, prior to an observation, the illumination through the microscope and through the comparator. When the wedge is removed, the periphery of the field will appear dark. By closing the iris diaphragm below the stage, more or less, the amount of light in the center of the field may be reduced to the same amount as that reflected through the wedge, so that the whole field will appear equally dark.

The calibration of the instrument is performed in a manner similar to the calibration of the Babinet compensator. Two readings are taken, one for the violet of the first order with a retardation of 575 $\mu\mu$, and one for the second order with a retardation of 1128 $\mu\mu$. If t and t' represent the readings on the scale, one division d will be represented by the equation

$$d = \frac{1128 - 575}{t' - t} \mu\mu.$$

PROBLEMS

Calibrate the Michel-Lévy comparator.

Determine the interference color shown by a section of quartz cut parallel to the optic axis.

Determine the actual birefringence of an unknown mineral in a rock section containing a known mineral.

305. Fedorow Method for Determining Low Interference Colors (1892). —We saw[1] that the interference color of a mineral plate was different between parallel and between crossed nicols. Fedorow[2] made use of this difference in determining the value of interference colors of the lower first order. These colors, low gray and white, are recognizable with difficulty between crossed nicols but are readily distinguishable when the nicols are parallel, as may be seen from the table given in Arts. 308, 276, or 277.

[1] Arts. 276–277. See also Art. 286.

[2] E. von Fedorow: *Mikroskopische Beobachtung bei paralleler Lage der Nicols.* Neues Jahrb., 1892 (II), 69–70.

Idem: *Zur Bestimmung der Feldspäthe und des Quarzes in Dünnschliffen.* Zeitschr. f. Kryst., XXIV (1894–5), 131.

306. Cesaro Wedge (1893).—Cesàro's[1] wedge does not differ in its essentials from the earlier wedges, but the determinations are made by compensation, not to darkness, but to the first order violet. This, according to Cesàro, is most sensitive between parallel nicols when it has a retardation of 281 $\mu\mu$. A scale, indicating the displacement, is attached outside the ocular, and, being divided into millimeters, must be calibrated. The drum vernier reads to twentieths of millimeters.

307. Amann Birefractometer (1895).—Another instrument, based on compensation, is that of Amann.[2] This consists essentially of a quartz wedge inserted in the focal plane of a Huygens' ocular and movable by means of a screw. The upper surface of the scale is divided into millimeters, and the subdivisions are read by means of the vernier drum. As constructed by Fuess (Fig. 459), the quartz wedge *K* covers but half the field of view. It is attached to the slides *ss*, and moved by the drum *k*. The glass plate *o*, to the lower side of which the wedge is fastened, is engraved with a scale divided to 0.2 mm. The long side of the wedge and two parallel lines engraved upon a thin glass serve as cross-hairs. *J* is a lever to control an iris diaphragm.

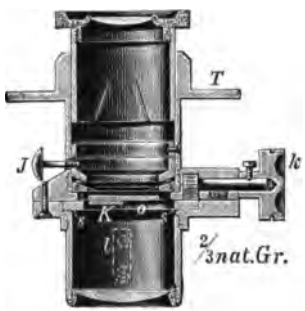


FIG. 459.—Amann birefractometer. 2/3 natural size. (Fuess.)

To determine the birefringence of a mineral, the ocular is inserted in the tube of the microscope in the 45° position and a cap nicol is placed over it, resting on the shoulder *T*. The mineral to be determined is rotated to 45° off extinction and with its fast ray at right angles to that of the wedge. The latter is screwed forward to compensation and, from previous calibration, the double refraction is determined. For mineral fragments or crystals around the periphery of a rock slice, this refractometer may be used as a comparator, since the wedge covers but half the field. With ordinary light it may be used as a screw micrometer ocular for measuring distances.

308. Von Fedorow Comparator (1895).—The von Fedorow[3] comparator

[1] G. Cesàro: *Sur une méthode simple pour mesurer le retard des minéraux en lames minces.* Bull. Accad. Roy. Belgique, Cl. Sci., XXVI (1893), 208–227.

[2] J. Amann: *Le biréfractomètre ou oculaire-comparateur.* Zeitschr. f. wiss. Mikrosk., XI (1894), 440–454.

C. Leiss: *Compensator-Ocular nach J. Amann.* Neues Jahrb., B.B., X (1895–6), 425–428.

C. Leiss: *Die optischen Instrumente, etc.*, Leipzig, 1899, 226.

[3] E. von Fedorow: *Ueber einen Glimmercomparator.* Zeitschr. f. Kryst., XXV (1895), 349–351.

Idem: *Calibrirung der Glimmercomparatoren.* Ibidem, XXVI (1896), 251–254.

Idem: *Die Feldspäthe des Bogoslowsk'schen Bergreviers.* Ibidem, XXIX (1897–98), 611–613.

C. Leiss: *Die optischen Instrumente, etc.* Leipzig, 1899, 211.

consists of sixteen rectangular, quarter undulation mica plates, each 2 mm. shorter than the preceding, and cemented with Canada balsam into a step-like "wedge" (Fig. 460). There is thus formed a compensator ranging in values from 1/4 wave length to four wave lengths retardation, each step of 1/4 λ being called by von Fedorow a Levy (written *L*). The comparator is used in a manner similar to a quartz wedge. After compensating, the mineral is removed and the number of the step determined. If the Bertrand lens is inserted and the nicols left crossed, the separating lines between the plates may be distinctly seen. The wedge may be purchased with the value of the retardation of each step engraved upon the cover-glass, thus avoiding the necessity of counting the steps.

FIG. 460.—Von Fedorow mica compensator. Vertical scale greatly exaggerated.

The retardation and interference color of each step between crossed and parallel nicols are as follows:

Order	Crossed nicols	Retardation	Order	Parallel nicols
1	Gray	127μμ	½	Brownish yellow
1	Pure white	255	½	Dark violet-brown
1	Orange-yellow	382	1½	Sky-blue
1	Orange-red	511	1½	Light yellowish
2	Blue	637	1½	Canary-yellow
2	Green	765	1½	Yellowish orange
2	Yellow	892	2½	Intense violet-blue
2	Orange-red	1020	2½	Leek-green
3	Indigo	1147	2½	Chrome-yellow
3	Smaragdite-green	1275	2½	Light orange
3	Lemon-yellow	1402	3½	Pure violet
3	Orange	1530	3½	Pure green
4	Violet-red	1657	3½	Light yellowish
4	Grass-green	1785	3½	Light orange
4	Greenish yellow	1912	4½	Light reddish blue.
4	Rose	2040	4½	Light greenish

By means of this comparator it is possible not only to determine the birefringence of a mineral to one Levy (1/4 λ), that is, to the retardation of a single step, but even to 1/8 Levy (1/32 λ).

For example, if one reduces a mineral to darkness by means of this compensator and finds one step dark and the step above and the step below equally illuminated, then the difference of phase is exactly the value of the step $N\frac{\lambda}{4}$. If two adjacent steps are equally bright, the value is intermediate between the two, and a half step must be added, $\frac{N\lambda}{4}+1/8\lambda$ (or $N \cdot L+1/2\,L$). If two adjacent steps are not quite equally dark, one can estimate the value

by comparison with the two beyond the dark pair, as 1 to 3, or 1/4 of one step (1/4 *L*) difference.

A method of increasing the delicacy of the determinations was given by von Fedorow[1] in 1898. He prepared, first, 1/4 Levy (1/16 λ) retardation plates by splitting mica into very thin lamellæ, and from those having like interference colors, he selected four that just compensated a quarter undulation mica plate when superimposed in parallel positions or sixteen that compensated a first order red. From such 1/4 *L* micas, two small rectangles were cut as shown in Fig. 461, in which the dotted lines represent the steps of a von Fedorow compensator and the heavy lines the mica accessory plate. The width of each plate is one-half that of the comparator, and the length 4 mm., so that each covers two steps. The two micas are cemented between cover-glasses, one with its vibration directions parallel, the

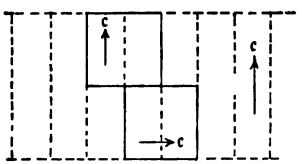


FIG. 461.

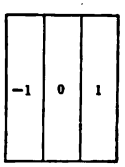


FIG. 462.

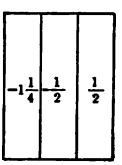


FIG. 463.

FIG. 461.—Von Fedorow mica accessory plate.
FIG. 462.—A mineral differs by 1 *L* from a step of the compensator.
FIG. 463.—A 2 1/2 *L* mineral differs by 1/2 *L* from one step of the compensator.

other at right angles, to that of the comparator. By this means it is possible to make readings to 1/8 *L* (1/32 λ) with ease and accuracy.

In using this comparator it is placed above a mineral in opposite phase, and the latter is reduced, by compensation, as nearly as possible to zero. The mica accessory plate is then placed above the comparator, with its center above the darkest step, and it is determined whether a step becomes completely dark, or whether two sections have equal illumination.

For example, let a mineral of 2*L* (1/2 λ) retardation be placed upon the stage of the microscope with its vibration directions at right angles to those of the comparator. The two will compensate when the second step of the comparator is placed above it. If the mineral be considered negative and the wedge positive, the first step will equal $1L-2L=-1L$, and the third $3L-2L=1L$ (Fig. 462). That is, the steps on either side of the one which becomes dark are equally illuminated.

If the mineral differs by half a Levy from any step of the comparator, this is shown by equal illumination of two adjacent steps (Fig. 463). Thus a 2 1/2 *L* mineral will make the first step $1\ L-2\ 1/2\ L=-1\ 1/2\ L$, the second $2\ L-2\ 1/2\ L=-1/2\ L$, the third $3\ L-2\ 1/2\ L=1/2\ L$.

So far without the mica accessory plate. If this be placed above the comparator, the values in Levys of the first three steps with no mineral on the stage will be as shown in Fig. 464.

Suppose a mineral, differing by 1/4 *L* from one of the steps, for example the second (therefore = 2 1/4 *L*), be placed on the stage with its vibration directions

[1] *Op. cit.*, 1897–8.

at right angles to those of the comparator. The values become 1 1/4 L − 2 1/4 L = −1 L, 2 1/4 L − 2 1/4 L = 0, 3 L − 2 1/4 L = 3/4 L, 1 L − 2 1/4 L = −1 1/4 L, 1 3/4 L − 2 1/4 L = −1/2 L, 2 3/4 L − 2 1/4 L = 1/2 L. These are shown in Fig. 465. A difference of 3/4 L, of course, will be but a difference of 1/4 L in the opposite direction. This is shown in Fig. 466, where a 1 3/4 L mineral was used.

A difference of 1/8 L is shown in Fig. 467, with a 2 1/8 L mineral. Here no section is reduced to zero but two spaces are uniformly lighted.

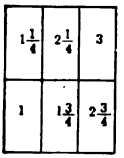


FIG. 464.—No mineral on the stage. Mica accessory plate overlying the first three steps of the compensator.

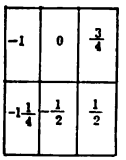


FIG. 465.—A difference of 1/4 L.

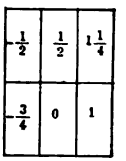


FIG. 466.—A difference of 3/4 L.

A 2 3/8 L mineral giving a 3/8 L difference is shown in Fig. 468.

A 1 5/8 L mineral giving a 5/8 L difference is shown in Fig. 469.

A 1 7/8 L mineral giving a 7/8 L difference is shown in Fig. 470.

These different cases may be summarized thus. Calling the upper row of sections positive and the lower row negative, we have:

One Levy difference is shown by darkness of a full step (Fig. 462).

One-half Levy difference is shown by equal illumination of two adjacent steps (Fig. 463).

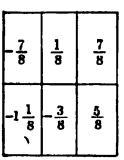


FIG. 467.

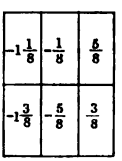


FIG. 468.

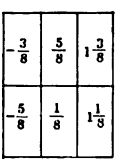


FIG. 469.

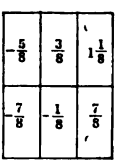


FIG. 470.

FIGS. 467 to 470.—Differences of 1/8, 3/8, 5/8 and 7/8 L.

1/4 L difference, by darkness of section + 2 and equal illumination of − 2 and − 3.

3/4 L difference by darkness of section − 2 and equal illumination of + 1 and + 2.

1/8 L difference by equal illumination of + 1 and + 3.

3/8 L difference by equal illumination of + 3 and − 2.

5/8 L difference by equal illumination of + 2 and − 1.

7/8 L difference by equal illumination of − 1 and − 3.

It is thus possible to get definite reductions to 1/8 L (1/32 λ), and one may even clearly see a change of 1/64 λ, in which case the illuminations of the proper sections are not quite equal.

It is very important, in making these delicate measurements, to be sure that the steps of the comparator and the mica accession plate are truly 1/4 λ and 1/16 λ, which may be done by comparing two mica wedges by compensation.

If the original comparator were made in steps of 0.2 λ instead of 0.25 λ, the divisions could be written in simple decimals.

309. Salomon's Method for Computing the Value of $\omega-\epsilon$ in Uniaxial Minerals (1869).—In connection with the determination of the refractive indices of unknown minerals by means of comparison with those of quartz, Salomon suggested a method by which the order of birefringence and the thickness of section of uniaxial minerals may be accurately computed. It is described in full in Art. 242. The method may be applied, not only to quartz, but to any uniaxial mineral.

310. Wallerant's Method for Measuring Slight Double Refraction (1899).—Interference colors, lower than first order yellow, may be hard to distinguish. Wallerant[1] devised a method by which the color may be doubled, and, in consequence, more easily measured. He placed a horizontal mirror under the section, and reflected the light along the axis of the microscope by means of a small glass plate inserted at an angle of 45° in the opening from which the Bertrand lens had been removed. These reflected rays were polarized by the analyzer, passed through the crystal twice, once before and once after reflection from the mirror, and returned through the analyzer to the eye. The color seen, therefore, was the same as that of a section twice as thick between *parallel* nicols, and could be determined readily by the Michel-Lévy comparator or any other method.

311. Nikitin's Method (1900).—Von Fedorow's universal stage may be used in the determination of birefringence, as was shown by von Fedorow and by Nikitin. The method is described in Art. 443.

312. Joly's Method (1901).—A method, very similar to that of Wallerant, was used by Joly.[2] Instead of reflecting the light twice through the analyzer he used a third nicol outside the tube. After passing through this nicol, the light was reflected, by means of a prism above the objective, to a polished speculum metal or silver mirror beneath the thin section. In this way, double the interference color was seen between crossed nicols. Joly suggested placing the section with cover-glass down so that the rock slice is as near as possible to the mirror. By this method it is possible, also, to double the colors of minerals with slight pleochroism.

313. Wright Combination Wedge (1901).—The Wright quartz-gypsum wedge has been described in Art. 297. The method for determining the double refraction is the same as with the ordinary quartz wedge.

314. Evans Simple Quartz Wedge (1905).—Evans,[3] in 1905, proposed

[1] Fréd. Wallerant: *Note sur la mesure des birefringences des minéraux en lames minces.* Bull. soc. min. France, XX (1897), 172–3.

[2] J. Joly: *On an improved method of identifying crystals in rock-sections by the use of birefringence.* Proc. Roy. Soc., Dublin, IX (1901), 485–494.

[3] John W. Evans: *On some new forms of quartz-wedges and their uses.* Mineralog. Mag., XIV (1905), 87–92.

two new quartz wedges. The first was simply an ordinary quartz wedge of larger angle than usual. It was about 1 1/2 mm. thick at the thick end, and gave twenty-eight orders of interference colors. On the upper surface a scale was engraved, the relative retardation of two adjacent divisions differing by 1000 $\mu\mu$. The calibration was effected in sodium light, first with crossed nicols to obtain the dark bands corresponding to even half-wave lengths, and then between parallel nicols for the odd half-wave lengths.

315. Evans Double Quartz Wedge (1905).—Evan's second wedge consists of two quartz wedges placed close together, one with its length parallel to the optic axis of the quartz, the other with its breadth parallel to the same direction. The two wedges are ground down to the same slope whereby, since the vibration directions are at right angles to each other, they extinguish simultaneously between crossed nicols. In the 45° position the bands of color extending across the wedges are the same in both. If, however, a mineral is placed on the stage of the microscope and it is rotated 45° off extinction, one wedge will show the black compensation bar while the other will show colors of increased retardation, double that of the crystal plate.

This wedge is very convenient, since it is not necessary to experiment first 45° to the right and then to the left to get the mineral into position for compensation. In determining extinction angles it is to be noted that at extinction the bands pass across the two without break.

316. Seidentopf Quartz Wedge Compensator (1906).—The Seidentopf[1] compensator (Fig. 471) consists of a Ramsden ocular in whose focal plane there may be placed movable quartz wedges. To reduce the length, three are provided, one from 0 to the 2d order, another from the 2d to the 8th, and a third from the 8th to the 39th. The wedges are simple except the first, which consists of a superimposed pair with their optic axes at right angles to each other, thus giving, at a certain point, exact compensation. The upper surfaces are graduated so that the retardation may be read directly from the scale; the first and second to 0.1 μ and estimated to 0.01 μ, the third to 0.2 μ and estimated to 0.04 μ.

FIG. 471.—Seidentopf quartz-wedge compensator. 1/2 natural size. (Fuess.)

[1] H. Seidentopf: *Mikroskop-Okular mit Quarzkeil-Kompensator.* Centralbl. f. Min. etc., 1906, 745–746.

317. Wright Double Combination Wedge (1908).—The Wright[1] double combination wedge (Fig. 472) is made by cutting in halves, longitudinally, a single combination wedge whose line of compensation is at the middle, and rotating one-half through 180° in azimuth. By this means the wedge is divided into four parts. In each half, like in the single wedge, the retardation effect of the wedge predominates at one end and the underlying plate at the other. Since their vibration directions lie at right angles to each other, this produces, in the double wedge, retardation as shown in the figure, the predominating vibration directions being shown by the shading.

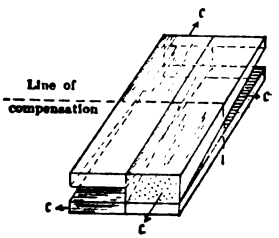


FIG. 472.—Wright double combination wedge.

FIG. 473.—Nikitin compensator. (Fuess.)

318. Nikitin Quartz Compensator (1910).—Another attachment for determining low values of birefringence is that proposed by Nikitin.[2] It consists (Fig. 473) of a plate of quartz cut so that its normal makes an angle of 25° with the optic axis, and is inserted in a carrier which fits into the slot above the objective. The quartz plate may be rotated by means of a milled head, and is so arranged that the optic axis moves in a plane at right angles to the pivot. Upon inserting this accessory in the microscope between crossed nicols, the stage will appear dark when the pointer is at 0. A rotation of any amount from this position will produce an interference color, violet of the first order appearing, as shown in the figure, when the scale indicates 60°, in which position the optic axis of the quartz is inclined 35° to the axis of the microscope. Further rotation will produce higher colors.

In the instrument shown, the quartz plate has a thickness of 0.07 mm., and all the colors of the first order may be obtained. By using minerals of greater birefringence for the plate, it would be possible to increase the orders of colors. The maximum error of observation with this instrument is not over 4 $\mu\mu$.

[1] Fred Eugene Wright: *On the measurement of extinction angles in the thin section.* Amer. Jour. Sci., XXVI (1908), 370.

Idem: *The methods of petrographic-microscopic research.* Washington, 1911, 134–135.

The illustrations of this wedge given in the above publications are incorrect, and differ from the letter-press descriptions.

[2] W. Nikitin: *Drehbarer Compensator für Mikroskope.* Zeitschr. f. Kryst., XLVII (1910), 378–379.

CHAPTER XXVI

DETERMINATION OF VERY SLIGHT DOUBLE REFRACTION

319. Biot's Sensitive Violet (1813).—For the determination of very slight double refraction, the usual accessory employed is the sensitive violet. By examining the scale of interference colors,[1] it may be seen that a very slight retardation produces a decided change in color, both with crossed nicols, giving a retardation of $575\mu\mu$, or with parallel nicols, giving a retardation of $281\mu\mu$. The method of observation is to insert, above the apparently isotropic mineral, a unit retardation plate, and note whether there is a change in the interference color when the stage is rotated.

Besides the gypsum plate of Biot already described[2] the following have been used:

320. Biot Quartz Plate (1813).—The quartz plate proposed by Biot[3] consists of a section of quartz cut at right angles to the optic axis. Owing to the thinness of the plate, the rotary polarization is not noticeable, and the section will appear dark between crossed nicols. If, however, a mineral showing slight double refraction is placed on the stage, the plate will appear colored. This plate was further developed by Klein (Art. 324).

FIG. 474.—Savart plate.

321. Savart Plate (before 1835).—An extremely delicate test for small amounts of polarized light is Savart's[4] polariscope. This consists of two plates of quartz or calcite, cut at 45° to the optic axis, superposed with their principal sections at right angles to each other, and cemented with Canada balsam. When such a plate is mounted in front of an analyzer, nothing is seen, but if the entering light be ever so slightly polarized, parallel bands bisecting the angle between the principal sections of the plate immediately appear (Fig. 474). These are known as **Savart's bands,** and increase in strength as the plane of polarization approaches the direction of the bands them-

[1] Arts. 276–277.

[2] Art. 294.

[3] J. B. Biot: *Mémoire sur un nouveaux genre d'oscillation que les molécules de la lumière éprouvent en traversant certains cristaux.* Lu à l'Institute, 3 Nov. 1813. Mém. Acad. France. Année 1812, XIII, pt 1 (1814), 1–371.

Idem: *Précis élémentaire de physique expérimentale.* Paris, 1824, II, 572.

[4] Original reference not found. It was in use as early as 1835.

selves, that is, bisects the angle between the principal sections of the component plates. The two plates should be of exactly the same thickness, and are best prepared by using the two halves of a single preparation.

The instrument is extremely sensitive and is capable of detecting the polarization of light reflected from the sky. It might be used with advantage for some purposes in petrographic work.

322. Soleil Bi-quartz Plate (1845).—The Soleil[1] double quartz plate is based on the principle of the Biot quartz plate. It consists of two adjacent, equally thick, right- and left-handed quartz plates, cut accurately at right angles to the optic axis. This plate, as such, is not much used in petrographic work, but in many saccharimeters it is the testing plate. If rotary polarization occurs in the substance under examination, the rays are turned in one direction by one half of the plate and in the other by the other half, consequently different interference colors appear. With no mineral plate on the stage and with parallel nicols, the two halves appear equally illuminated when the rotation is exactly 90° or 180°, monochromatic light being used. This occurs for sodium light when the quartz is 4.1 mm. thick,[2] since 1 mm. of quartz produces a rotation of 21.7°. With a thickness of 8.25 mm. the plane of polarization will be rotated 180° and parallel nicols will give the violet "teinte sensible."

323. Bravais Twinned Mica Plate (1851).—This is described in full in Art. 335.

324. Klein Quartz Plate (1874).—If a thick plate of quartz, cut at right angles to the optic axis, is inserted between crossed nicols, there will appear an interference color which will increase or decrease in the scale upon rotating the upper nicol, the color depending upon the amount of rotation. If a mineral plate is placed between the quartz plate and the polarizer, the resulting interference color will be a combination of the two. Klein[3] took advantage of this power of quartz and constructed a plate, 3.75 mm. in thickness, which is very useful for detecting slight double refraction, slight differences

[1] Henri Soleil: *Note sur un moyen de faciliter les expériences de polarisation rotatoire.* Comptes Rendus, XX (1845), 1805–1808.

Idem: *Nouvel appareil propre à la mesure des déviations dans les expériences de polarisation rotatoire.* Ibidem, XXI (1845), 426–430.

Idem: *Note sur un perfectionnement apporté au pointage du saccharimètre.* Ibidem, XXIV (1847), 973–975.

M. M. Arago, Regnault et Babinet: *Rapport sur le saccharimètre de M. Soliel.* Ibidem, XXVI (1848), 162–168.

Jules Duboscq et Henri Soleil: *Note sur un nouveau compensateur pour le saccharimètre.* Ibidem, XXXI (1850), 248–250.

H. Landolt: *Das optische Drehungsvermögen.* Braunschweig, 2 Aufl., 1898, 295.

[2] Cf. Art. 78.

[3] Carl Klein: *Mineralogische Mittheilungen IV.* Neues Jahrb., 1874, 9.

in the extinction angles of twinned plates, and polarization of light reflected from opaque, metallic, anisotropic surfaces.[1] Very slight differences in the orientation of the vibration directions produce different interference colors, no matter what may be the amount of the rotation of the analyzer. For colorless minerals, the most sensitive tint is the violet *teinte sensible*, which is produced with the indicated thickness of plate when the nicols are parallel. For a colored mineral the tone to which it is most sensitive should be selected. This plate is sometimes called the Biot-Klein plate (Cf. Art. 320).

325. Bertrand Ocular (1877).—This is described in Article 339.

326. Calderon Ocular (1878).—This is described in Article 340.

327. Traube Bi-mica Plate (1898).—This is described in Article 343.

328. Brace Half-shade Elliptical Polarizer and Compensator (1904).—Brace[2] made use of an extremely thin mica flake, as thin as 0.00017 mm., which he inserted in the focal plane of the ocular, covering only one-half the field. He claims that this apparatus is two hundred times as sensitive as the Bravais plate, and is capable of detecting retardations of $6.10^{-5}\lambda$.

329. Sommerfeldt Twinned Gypsum Plate (1907).—This is described in Article 345.

330. Königsberger Ocular (1908).—Königsberger[3] constructed an ocular in which the sensitive plate is composed of four mica plates, as thin as possible, "crossed in pairs," with vibration directions at 45° to those of polarizer and analyzer. The apparatus is sensitive to a difference of $1.10^{-4}\lambda$, and by it the double refraction induced in a piece of glass when lightly pressed between two fingers may be detected. As made by Fuess, the mica plate in the Königsberger ocular consists of two pieces only.

331. Half-shade Plates.—To determine the rotating power of a substance by monochromatic light, half-shade plates are generally used. These consist of transparent plates so cut that at a certain position of the analyzer, depending upon the rotating power of the substance under examination, a uniform darkening of the plate takes place. On account of the half light transmitted at this point, such devices are called half-shade plates.[4] In

[1] E. A. Wülfing: *Ueber die empfindlichen Farben und über ihre Anwendung bei der Erkennung schwach doppelbrechender Medien.* Sitzb. Akad. Wiss. Heidelberg, 1910, 24te Abhandl., pp. 16.

[2] D. B. Brace: *A half-shade elliptical polarizer and compensator.* Physical Review, XVIII (1904), 70–88. See also Phil. Mag., VII (1904), 323.

[3] Joh. Königsberger: *Vorrichtung zur Erkennung und Messung geringster Doppelbrechung.* Centralbl. f. Min., etc., 1908, 729–730.

[4] Halbschattenapparate, polarimètres à pénombre.

For principles of construction of half-shade plates see H. Landolt: *Die optische Drehungsvermögen, Braunschweig,* 1898, 300–302.

petrographic work few of these instruments are used. They are widely used, however, to determine the rotating power of liquids, as in saccharimeters and polarimeters.

The first half-shade plate was probably constructed by Jellet[1] in 1860, although the principle had been used earlier by Bravais and others. Among half-shade apparatus are those of Laurent,[2] Lippich,[3] Lommel,[4] Glan,[5] Landolt,[6] Wiedemann,[7] Lummer,[8] Macé de Lépinay,[9] Brace,[10] Nakamura,[11] and Wright.[12] The reader is referred to the original literature. The instrument of Brace is described in Art. 328; those of Wiedemann, Macé de Lépinay, and Wright in the next chapter.

[1] Rev. Prof. Jellett: *On a new instrument for determining the plane of polarization.* Rept. Brit. Asso. Adv. Sci., Trans. of the Sections, Oxford meeting, 1860, 13.

[2] L. Laurent: *Sur l'orientation précise de la section principale des Nicols, dans les appareils de polarisation.* Comptes Rendus, LXXXVI (1878), 662–664.

Idem: *Sur le saccharimètre Laurent.* Ibidem, LXXXIX (1879), 665–666.

F. Lippich: *Ueber die Vergleichbarkeit polarimetrischer Messungen.* Zeitschr. f. Instrum. XII (1892), 333–342.

H. Landolt: *Die optische Drehungsvermögen.* Braunschweig, 2 Aufl., 1898, 308–314.

[3] F. Lippich: *Zur Theorie der Halbschattenpolarimeter.* Sitzb. Akad. Wiss. Wien, XCIX (ii), 1890, 695.

Idem: *Ueber die Vergleichbarkeit polarimetrischer Messungen.* Zeitschr. f. Instrum., XII (1892), 333–342.

Idem: Lotus, N. F. II (1880). *

Idem: *Ueber ein neues Halbschattenpolarimeter.* Zeitschr. f. Instrum., II (1882), 167–174.

Idem: *Ueber eine Verbesserung an Halbschattenpolarisatoren.* Ibidem, XIV (1894), 326–327.

Idem: *Dreitheiliger Halbschatten-Polarisator.* Sitzb. Akad. Wiss. Wien, CV (ii A), 1896, 317–361.

O. Lummer: *Neues Kontrast-Polarimeter.* Zeitschr. f. Instrum., XVI (1896), 209–211.

[4] E. Lommel: *Neue Methode zur Messung der Drehung der Polarisationsebene für die Fraunhofer'schen Linien.* Sitzb. Akad. Wiss. München, XVIII (1888), 321–324.

Idem: Same title. Zeitschr. f. Instrum., IX (1889), 227.

[5] Paul Glan: *Ein Spektrosaccharimeter.* Wiedem. Ann., XLIII (1891), 441–448.

[6] H. Landolt: *Ueber eine veränderte Form des Polarisationsapparates für chemische Zwecke.* Ber. deutsch. chem. Gesell., XXVIII (1895), 3102.

[7] See Article 342.

[8] O. Lummer: *Ueber ein neues Halbschattenprinzip.* Zeitschr. f. Instrum., XV (1895), 293–294.

[9] See Article 344.[2]

[10] See Article 328.

[11] S. Nakamura: *Ueber einen Quarzhalbschattenapparat.* Centralbl. f. Min., etc., 1905 267–279.

[12] See Article 347.

CHAPTER XXVII

PRACTICAL METHODS FOR THE DETERMINATION OF EXTINCTION ANGLES

332. Relation of the Optical Ellipsoid to the Crystallographic Axes. Parallel and Inclined Extinction.—It has been demonstrated geometrically[1] and analytically[2] that when the principal vibration directions of a crystal correspond in direction with the principal planes of the nicol prisms, the light is extinguished and the field becomes dark. This, of course, occurs four times on rotating the stage through 360°, and these positions are called the **positions of extinction.** We have seen, also, that in isometric crystals the ease of vibration is the same in every direction, consequently the field remains dark between crossed nicols during a complete rotation of the stage. In uniaxial crystals the vibration ease is the same in every direction in basal sections, consequently these likewise remain dark on rotating the stage. In sections of uniaxial minerals at right angles to the base, double refraction occurs, and the field darkens only when the trace of the basal plane and the direction at right angles to it are parallel to the cross-hairs. Since in uniaxial crystals the principal vibration axes are parallel to the crystallographic axes, this extinction will take place when crystallographic c is parallel to the cross-hairs. In sections intermediate between the basal section and the section containing crystallographic c, the field will likewise become dark four times, namely, when the trace of the plane of the base or the one containing crystallographic c is parallel to the cross-hairs. In orthorhombic crystals (Fig. 475) the vibration axes, likewise, coincide with the crystallographic axes, consequently when traces of the planes containing these lines are parallel to the cross-hairs, the field becomes dark. Unlike uniaxial crystals, the basal plane here extinguishes four times.

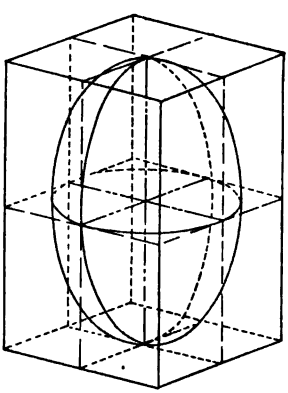


FIG. 475.—Orthorhombic system. Relation between crystallographic lines and the optical ellipsoid.

We thus see that in tetragonal, hexagonal, and orthorhombic crystals the

[1] Art. 283.

[2] Arts. 285 and 287.

extinction lines are parallel to the crystallographic axes. Now in most crystals there are cleavage lines which are parallel to the crystallographic axes, and when these lines lie parallel to the cross-hairs, the vibration directions also lie parallel, and the field becomes dark. Such extinctions are said to be **parallel** (Fig. 476). The cross-hairs may correspond to a, b, or c, or to

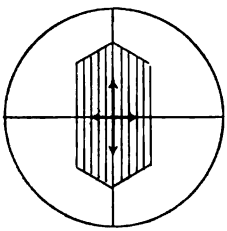


FIG. 476.—Parallel extinction.

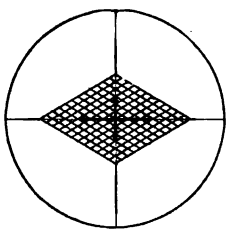


FIG. 477.—Symmetrical extinction.

some intermediate vibration direction, but we cannot determine which, unless the orientation of the crystal is known. We can simply determine that the vibrations in one direction are faster than in the other. The relation of the extinction lines to prismatic cleavage in uniaxial or orthorhombic crystals will, in certain sections, be **symmetrical** (Fig. 477). In other sections,

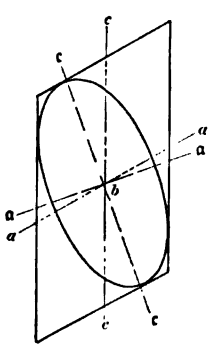


FIG. 478.

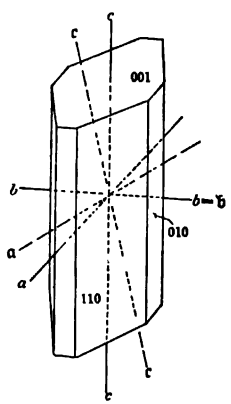


FIG. 479.

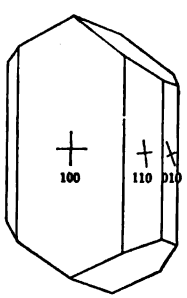


FIG. 480.

FIGS. 478 and 479.—Extinction angles in a monoclinic crystal.
FIG. 480.—Augite showing angles of extinction on the different faces of a zone.

however, these extinctions will appear parallel, and the *mineral* is said to have parallel extinction.

In monoclinic crystals there is but one plane of symmetry, and the optical ellipsoid will have but a single axis coinciding with a crystallographic axis, namely, the one at right angles to this plane, or crystallographic *b* (Figs. 478–479). The other axes will lie anywhere in the plane of *a* and *c*. In general, neither corresponds with a vibration axis, but it is possible, of

course, for one to do so, but not for both, since the angle between *a* and *c* is not a right angle, and that between the vibration axes is. Extinction in such crystals is said to be **inclined,** although in sections in the 100–001 zone (Figs. 479–480) it will appear parallel.

In triclinic crystals the vibration axes do not coincide with the crystallographic axes, although in special cases, of course, a single one may do so.

The angle between the extinction direction and a crystallographic axis, shown by cleavage lines or crystal edges, is called the **extinction angle of the face.** Thus in Fig. 480, which is a crystal of augite, the extinction angle on 010 is 45°, on 110 it is 36°, and on 100 it is 0°. The maximum extinction angle between any crystallographic axis and the nearest vibration axis is usually taken as the **extinction angle of the mineral.** The relation between the angles on different faces will be considered in full in Chapter XXVIII.

333. Methods for Measuring.—Before making accurate determinations of extinction angles, it is very essential that the microscope be in adjustment, the principal planes of the two nicols exactly at right angles to each other and parallel to the cross-hairs. If special oculars are used, these, also, should be tested to see that their cross lines are parallel to the principal planes of the nicols. For the methods of adjustment see Arts. 199–202.

Except for extremely accurate measurements, extinction angles are determined by white light. If a crystal is to be measured, oriented sections should be cut and the maximum extinction angle determined. With random sections, such as occur in a rock slice, many should be examined, and the maximum angle taken as the extinction angle of the crystal, or a universal stage should be used and the angle determined by the methods given in Art. 440.

The usual method of determining extinction angles is to rotate the mineral, between crossed nicols, to the position of maximum darkness. This operation should be repeated half a dozen times, and then agan the same number of times with the stage rotated 180° from its former position. An average of the twelve readings will give the extinction angle. Increasing the number of readings will decrease the error of observation. Thus Max Schuster,[1] in his determinations of the extinction angles of the plagioclase feldspars, made 80 readings on each side of the twinning line, 80 to determine the position parallel to the 001-010 edge, then turned the slide cover-glass down and made a like number of readings!

Owing to the fact that the eye cannot always accurately determine the position of maximum darkness, especially in sections showing but slight birefringence, various accessories have been devised, most of them depending

[1] Max Schuster: *Ueber die optische Orientierung der Plagioklase.* T. M. P. M., III (1881), 117–284, in particular 146–147.

for their efficiency upon the *teinte sensible*, or upon the multiplying effect of twinned plates upon birefringence.

334. Unit Retardation Plate.—If a unit retardation plate, such as has already been fully described,[1] is placed above a mineral rotated to the position of extinction, the sensitive violet will appear just as though no mineral were upon the stage. If the mineral be very slightly rotated, however, the very small amount of retardation produced in the transmitted rays will produce a decided difference in the interference color, red in one direction and blue in the other. Applied to the determination of extinction angles this plate is serviceable only when the mineral is colorless and of not too high birefringence. It can be used, also, only on isolated fragments, on grains adjacent to the Canada balsam around the periphery of a rock section, or adjacent to an absolutely isotropic mineral; this because it is necessary to rotate the mineral until the interference colors of field and grain are exactly the same.

335. Bravais Twinned Mica Plate (1851).—Probably the first twinned plate used was that of Bravais.[2] He took a mica plate, 1/9 mm. in thickness, and thus having a violet interference color and giving exactly one wave length retardation by yellow light, and cut it along a line making an angle of 45° with the principal section (Fig. 481). One part was now turned through 180° in altitude so that the upper surface became the lower, and the parts were cemented on glass in the position shown in the figure. Between crossed nicols the two parts show the same interference color if the light strikes the lower surface at right angles. If, however, a mineral plate is placed on the stage of the microscope, one half the Bravais plate will add to its interference color and the other half will subtract from it, the effect being to show, between the two halves, double the actual retardation. When the mineral has been rotated until the two parts of the Bravais plate are uniformly colored, it is in its position of extinction.

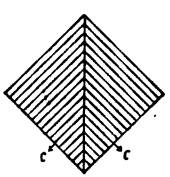


FIG. 481.—Bravais plate.

This, and all twinned plates, should be carefully tested. The angle which each part makes with the bisecting line must be absolutely the same, otherwise there will be an error of reading equal to half the difference between them.

[1] Art. 294, *supra*.

[2] A. Bravais: *Déscription d'un nouveau polariscope et recherches sur les doubles réfractions peu énergiques*. Comptes Rendus, XXXII (1851), 112–116.

Idem: *D'un nouveau polariscope, et recherches sur les doubles réfractions peu énergiques*. Ann. chim. et phys., XLIII (1855), 129–149.

Idem: *Beschreibung eines neuen Polariskops und Untersuchung über die schwachen Doppelbrechungen*. Pogg. Ann., XCVI (1855), 395–414.

336. Kobell Stauroscope (1855).—The Kobell[1] stauroscope consists of a calcite plate cut at right angles to the optic axis and inserted between the mineral and the analyzer. In the original polariscope with which this was used, the polarizing plate was a black glass mirror and the analyzer a tourmaline plate. The instrument was so arranged that the interference cross of the calcite could be seen on looking through the eye lens. With no mineral on the stage, the cross appeared undisturbed, but with an anisotropic mineral inserted in any position except that of extinction, the cross was more or less rotated.

337. Klein Quartz Plate (1874).—The Klein quartz plate, described in Art. 324, may be used to determine extinction angles in isolated mineral grains or minerals adjacent to isotropic media, hence around the periphery of a rock section. The method is similar to that used with the unit retardation plate except that the upper nicol is rotated until the desired sensitive tint is obtained, and its vibration plane, therefore, in general is not at right angles to that of the lower nicol.

338. Bertrand Ocular (1877).—In the Bertrand[2] ocular, use is made of the rotating power of sections of quartz cut at right angles to the optic axis. It differs from the Soleil double plate in that it is made up of a double pair of dextro-rotary and levo-rotary quartz plates, instead of a simple pair. The four pieces, each 2.5 mm. in thickness, are so placed that the two right-handed and the two left-handed quartzes lie in opposite quadrants (Fig. 482). They are so inserted in the focal plane of the ocular that their separating lines are exactly parallel to the vibration directions of the nicols, and thus serve as cross-hairs. The tube analyzer is removed and a cap nicol is placed above the ocular. When the nicols are crossed, the four quadrants are of a uniform pale blue color, since the vibration directions of all make the same angle with the vibration directions of the nicols. When a doubly refracting mineral section is placed upon the stage of the microscope, the opposite quadrants add to or subtract from its retardation, except when the mineral is in its position of extinction, and they become differently colored. Extremely small variations from the parallel position can thus be determined. According to Wright[3] its sensitiveness for colorless minerals is such that

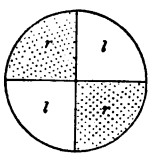


FIG. 482.—Plate in Bertrand ocular.

[1] Fr. v. Kobell: *Optisch-krystallographische Beobachtungen und über ein neues Polariskop. Stauroskop.* Pogg. Ann., XCV (1855), 320–332.

[2] E. Bertrand: *Vorrichtung zur Bestimmung der Schwingungsrichtung doppelbrechender Krystalle im Mikroskop.* Zeitschr. f. Kryst., I (1877), 69.

Idem: *De l'application du microscope à l'étude de la minéralogie.* Bull. soc. min. France, I (1878), 22–28, especially 27.

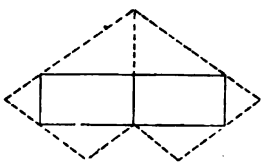
Rosenbusch-Wülfing: *Mikroskopische Physiographie*, I_1, 1904, 250–251.

[3] Fred. Eugene Wright: *The methods of petrographic-microscopic research.* Washington, 1911, 146.

angles may be read to between 0.1° and 0.5°, and a wave retardation of ±0.005 can be recognized.

Schraf[1] suggested that the lenses in an ocular fitted with a Bertrand plate be separated farther than usual in order that its influence upon the refraction be eliminated, and the resolving power of the microscope remain the same. When so made, the instrument may be used as an ordinary ocular, the lines between the four quadrants serving as cross-hairs.

339. Calderon Plate (1878).—The Calderon[2] plate, like that of Bertrand, is used with a cap nicol. The sensitive plate, lying in the focal plane of the ocular, consists of an artificial twin of calcite which is constructed by sawing a rhombohedron of Iceland spar along the short diagonal, removing a wedge-shaped piece from each cut plane, and cementing the remainder on these new surfaces. The projecting and reentrant angles are removed by grinding to two parallel faces (Fig. 483), leaving a plane-parallel plate of such a thickness that the interference color is "white of the higher orders." With crossed nicols the two halves appear alike, but upon inserting a doubly refracting mineral the two are unequally illuminated except when the mineral is in the position of extinction. Calderon claims an accuracy to 2 minutes.



483.—Calderon plate.

An objection to this plate is that the formation of a second image by double refraction is very annoying. In the oculars prepared for petrographic microscopes the field is generally small.

340. Von Fedorow's Method by Means of the Universal Stage (1892).—The method for the determination of the extinction angle of a crystal from the extinction angle of the section under examination, by means of the universal stage, is described in Chapters XXXV and XXXVI.

341. Wiedemann Double Double-Quartz Wedge. (Before 1895).—The Wiedemann[3] double double-quartz wedge, though designed for the determination of rotary polarization for different colors, may be used for the determination of extinction angles. The instrument consists of two wedges (Fig. 484), each of which is itself made up of a pair of dextro- and levogyrate quartz wedges. They are cut with

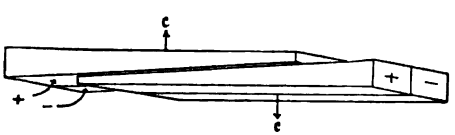


FIG. 484.—Wiedemann double-double quartz wedge.

[1] A. Schraf: *Ueber die Verwendung der Bertrand'schen Quarzplatte zu mikrostauroskopischen Beobachtungen.* Zeitschr. f. Kryst. VIII (1884), 81–82.

[2] L. Calderon: *Ueber einige Modificationen des Groth'schen Universalapparates und über eine neue Stauroskopvorrichtung.* Zeitschr. f. Kryst., II (1878), 68–73.

[3] Gustav Wiedemann: *Die Lehre von der Elektricität*, 2 Aufl., III, Braunschweig, 1895, 1051–1052.*

their bases at right angles to the optic axes, and are so superposed that the wedges of like rotation are on the same side. The amount of rotation is increased by varying the thickness on the axis of the microscope.

342. Stöber Quartz Double Plate (1897).—Identical with the Bravais, except that it is made from two quartz plates cut parallel to the optic axis, 0.064 mm. thick, and giving violet of the second order, is Stöber's[1] quartz double plate. The quartz is cemented between two round cover glasses, and the plate thus prepared is inserted as near as possible to the cross-hairs in the focal plane of an ocular and in such a position that the artificial twinning line is parallel to one of them.

343. Traube Bi-mica Plate (1898).—Similar to the Calderon plate, but much easier to construct since no grinding is necessary, is the Traube[2] bi-mica plate. Two rectangular strips (Fig. 485) are cut from a quarter undulation mica flake in such a direction that their axial planes make angles of 3 1/2° with the long edges. The two strips are cemented between glass so that the double extinction angle is 7°. The complete plate is placed in the focal plane of the ocular, and is used in the manner of the Calderon.

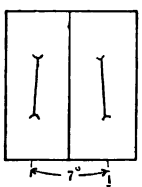


FIG. 485.—Traube bi-mica plate.

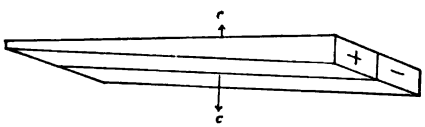


FIG. 486.—Macé de Lépinay half-shade plate.

344. Macé de Lépinay Half-shade Plate (1900).—The Macé de Lépinay[3] half-shade plate is nothing more than the half of a Wiedemann wedge. It consists of a double-quartz wedge (Fig. 486), one dextrogyrate and one levogyrate, cut at right angles to the optic axis and varying in thickness from 0.06 mm. to 0.12 mm. The base of the wedge is turned toward the analyzer and is placed as close to it as possible. The slanting surface, however, causes a slight deflection of the light. Schönrock[4] suggested that this

[1] F. Stöber: *Ueber eine empfindliche Quarzdoppelplatte.* Zeitschr. f. Kryst., XXIX (1897–9), 22–24.

[2] Hermann Traube: *Eine einfache Glimmerdoppelplatte zu stauroskopischen Bestimmung.* Neues Jahrb., 1898 (I), 251.

[3] L. Macé de Lépinay: *Sur un nouvel Analyseur a pénombres.* Jour. de phys., IX (1900), footnote 267, 585–588, 644.

Idem: Same title. Comptes Rendus, CXXXI (1900), 832–834.

Idem: *Détermination des constantes optiques du quartz pour la radiation verte du mercure. Leur application aux mesures d'épaisseurs par la méthode de Mouton.* Jour. de phys. IX (1900), 644–652.

[4] O. Schönrock: *Neuer Halbschattenanalysator.* Zeitschr. f. Instrum., XXI (1901), 90–93.

might be overcome by using two wedges of different thicknesses, which makes of it, however, a Wiedemann double double-quartz wedge.

345. Sommerfeldt Twinned Gypsum Plate (1907).—The cheap device proposed by Sommerfeldt[1] for determining whether nicols are absolutely at right angles to each other may well be used for the determination of extinction angles. He used a cleavage plate of a twinned gypsum crystal in which the trace of the twinning plane is a straight line (Fig. 487). The two individuals appear equally illuminated between crossed nicols when the twinning line is parallel or at 45° to the cross-hairs.

If an anisotropic mineral plate is inserted, the two parts of the field become differently colored unless the mineral is exactly at extinction.

The present writer has used, for a number of years, a wedge made from a twinned gypsum crystal. It is cut with its long direction parallel to the twinning line, and shows colors from gray of the first to pink of the fourth order.

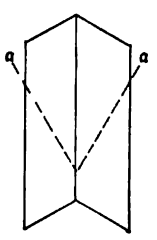


FIG. 487.—Sommerfeldt twinned gypsum plate.

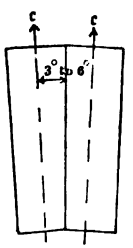


FIG. 488.—Wright artificially twinned quartz plate.

346. Wright Artificially Twinned Quartz Plate (1908).—The twinned quartz plate, suggested by Wright,[2] is similar to Sommerfeldt's plate, but is made from quartz. This is cut parallel to *c*, and with one lateral edge ground down until it makes an angle of from 3° to 6° with this axis (Fig. 488). The plate is cut across transversely, and the two inclined edges are cemented together. Wright suggests that such plates may be made as quarter undulation plates, first order violet plates, or even in the form of wedges.

[1] Ernst Sommerfeldt: *Eine einfache Methode zur Justierung der Nikols am mineralogischen Mikroskop.* Zeitschr. wiss. Mikrosk., XXIV (1907), 24–25.

See also Max Berek: *Die Bestimmung von Auslöschungsrichtungen doppelbrechender inaktiver Krystallplatten mit Hilfe von Halbschattenvorrichtungen im einfarbigen Lichte.* Neues Jahrb., B.B., XXXIII (1912), 583–661.

[2] Fred. Eugene Wright: *On the measurement of extinction angles.* Amer. Jour. Sci., XXVI (1908), 374.

Idem: *The methods of petrographic-microscopic research.* Washington, 1911, 136–137.

347. Wright Bi-quartz Wedge Plate (1908).—The Wright[1] bi-quartz wedge plate consists of two quartz wedges, one dextrogyrate and one levogyrate, each underlaid by a plane-parallel quartz plate of opposite sign (Fig. 489), thus producing zero rotation where the plates are of the same thickness, much in the manner of the Wright single combination wedge. This wedge is inserted in the focal plane of an ocular. It divides the field into halves of equal illumination when the stage is bare or when a mineral placed thereon has its extinction directions parallel to the principal planes of the nicols. A very slight rotation produces a difference in the amount of the illumination in the two parts, the most marked difference being found by inserting or withdrawing the wedge, more or less. To avoid tilting the wedge, and thus permitting light to pass through in a direction other than parallel to the optic axis, it is set in a carriage which slides snugly in an ocular very similar in appearance to that shown in Fig. 471.

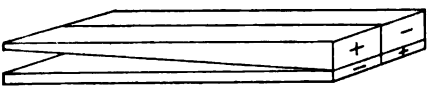


FIG. 489.—Wright bi-quartz-wedge plate.

[1] Fred. Eugene Wright: *On the measurement of extinction angles.* Amer. Jour. Sci., XXVI (1908), 377.

Idem: *The bi-quartz wedge plate applied to polarimeters and saccharimeters.* Ibidem, 391–398.

O. Schönrock: *Keilförmiger Biquarz für Polarisationsapparate und Saccharimeter.* Zeitschr. f. Instrum., XXX (1910), 198–199.

Fred. Eugene Wright: *The methods of petrographic-microscopic research.* Washington, 1911, footnote, 141.

CHAPTER XXVIII

CALCULATION OF EXTINCTION ANGLES IN RANDOM THIN SECTIONS

348. Zones.—A zone has been defined as being made up of all sections which are parallel to the same line, called the axis of the zone, but not parallel to each other. Thus the 100, 101, 001 faces lie in a single zone, as do also 100, 110, and 010.

We have seen that the extinction angles in the 100, 110, 010 zone of crystals of the monoclinic system vary from zero on 100 to a definite value on 010. In the zone 100, 101, 001, the value remains zero throughout, just as in every zone of uniaxial crystals. In triclinic crystals the values vary in every zone from a minimum, exceptionally zero, to a maximum. From an examination of all cases, the rule may be stated that parallel extinction occurs in all the planes of a zone whose axis coincides with an axis of symmetry.

349. Calculation of Extinction Angles for any Face of the 100–010 Zone of a Monoclinic Crystal.—To determine the traces of the vibration planes on any face of a crystal, use may be made of Fresnel's law which states that in any section of a biaxial crystal (*abm*, Fig. 494), the direction of extinction (*md*) is at the intersection of the plane of the section (*abm*) with the plane (*mdDM*) bisecting the angle between the two planes (*mbBM* and *maAM*) containing the optic axes (*MB*, *MA*) and a line at right angles to the section (*mM*).

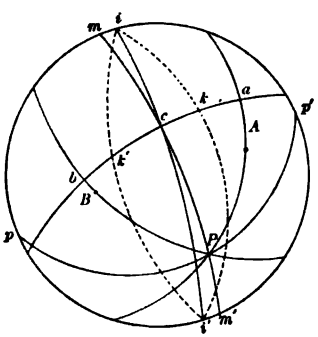


FIG. 490.

That the plane bisecting this angle is one of the vibration directions, and the plane at right angles to it is another, may be proved very simply by means of a stereographic projection. Let *A* and *B* (Fig. 490) be the projection of the optic axes, and *P* the projection of the normal to the section upon which the extinction is to be measured. Draw two circles, *iki'* and *ik'i'*, polar to *B* and *A*. They therefore represent the intersections of the circular sections of the optical ellipsoid with the sphere of projection and lie at right angles to the optic axes *A* and *B*. Draw *k'k*, the trace of the plane of which *P* represents the normal, and draw *PB* and *PA*, two planes through the points *PA* and *PB* and the center of the sphere. Let *a* and *b* be the points at which the traces of the planes *PA* and *PB* cut the plane *k'k*. Since the traces of planes at right angles to lines lie 90° from the piercing points of these

lines, Pb amd Bk are 90° apart, therefore $bk = 90°$. Ak' and Pa are 90° apart, therefore also $ak' = 90°$, whereby $bk = ak'$ and $bk' = ak$. Since iki' and $ik'i'$ are the traces of the circular sections of the optical ellipsoid, their bisecting plane ici' contains the bisectrix, and the distance $k'c$ must equal kc, whereby $k'c + k'b = kc + ka$, and $bc = ca$. The bisecting plane Pc, therefore, passes through an axis of the ellipsoid which determines one of the vibration directions. The other is at right angles to the first and is shown in pPp'.

Returning to the problem of determining the trace of the vibration plane on any face in the 100–010 zone of a monoclinic crystal: Let Fig. 491 represent a 010 section from such a crystal. This corresponds with its symmetry plane. Let MC

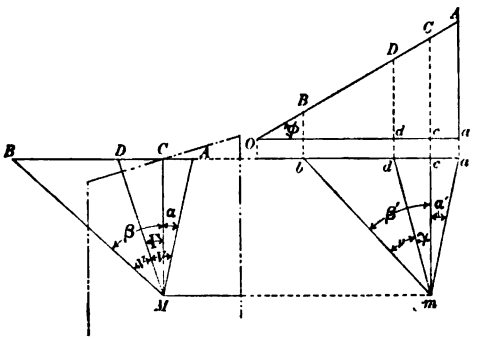


FIG. 491.

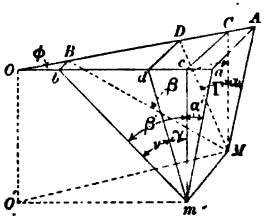


FIG. 492.
FIG. 493.

FIG. 494.

be the crystallographic axis, and MA and MB the optic axes, then MD, which bisects the angle BMA, is the acute bisectrix. It therefore is a vibration and an extinction direction.

In Figs. 492 and 494 let OA be the trace of the 010 plane, and let it be so placed that the new section plane Oa, upon which the extinction is to be determined, lies at right angles to the line of sight. OA, therefore, will form an angle of φ (AOa) with Oa, Aa being the direction of the line of sight; and the intercepts on the new plane ($Odca$, Figs. 492, 493, and 494) will represent the distances as they now appear. In this plane the vertical distance mc (Figs. 493–494) appears in its true length ($MC = mc$) since it is parallel to the line about which the plane was rotated. All other lines will appear foreshortened, and all angles will be of less than natural size.

If we let β' = the angle bmc, $\beta = BMC$, $\alpha' = amc$, and $\alpha = AMC$, we have, from the figure,

$$\tan \beta' = \frac{bc}{cm} = \frac{bc}{CM} = \frac{bc}{BC} \cdot \frac{BC}{CM} = \cos \varphi \tan \beta. \qquad (1)$$

Likewise $$\tan \alpha' = \cos \varphi \tan \alpha. \qquad (2)$$

If dmc, the extinction on the new plane, $= \gamma$, and $bma = bmd + dma = 2\nu$

$$\beta' - \gamma = bmd = \nu, \qquad (3)$$

$$\beta' + \alpha' = bma = 2\nu. \qquad (4)$$

Multiply (3) by two, and unite with (4)

$$2\beta' - 2\gamma = \beta' + \alpha',$$

$$2\gamma = \beta' - \alpha'. \qquad (5)$$

By trigonometry, from the equation of the tangent of the difference between two angles we have

$$\tan(\beta'-\alpha')=\frac{\tan\beta'-\tan\alpha'}{1+\tan\beta'\tan\alpha'}.$$

Substitute values from (1), (2), and (5),

$$\tan 2\gamma=\frac{\cos\varphi\tan\beta-\cos\varphi\tan\alpha}{1+\tan\alpha\tan\beta\cos^2\varphi}. \tag{6}$$

But $\beta=V+\Gamma$, and $\alpha=U-\Gamma$.

Substitute these values in (6),

$$\tan 2\gamma=\frac{\cos\varphi\,(\tan(V+\Gamma)-\tan(V-\Gamma)}{1+\cos^2\varphi\tan(V+\Gamma)\tan(V-\Gamma)}.^1 \tag{7}$$

From this equation the extinction angle (γ) *in any section in the prismatic zone of monoclinic crystals whose value for* V *is less than* Γ, *may be determined when the inclination* (φ) *of the section to the* 010 *plane, the maximum extinction angle* (Γ), *and the optic axial angle* ($2V$) *are known.*

Equation 7 may be written in terms of cotangents—

$$\cot 2\gamma=\frac{\cot(V+\Gamma)\cos(V-\Gamma)+\cos^2\varphi}{\cos\varphi[\cot(V-\Gamma)-\cot(V+\Gamma)]}.$$

If the extinction angle is greater than V(that is $\Gamma>V$), equations (4) and (5) become

$$\beta'-\alpha'=2\nu, \tag{9}$$

and

$$2\gamma=\beta'+\alpha'. \tag{10}$$

Equation (6) now the equation of the tangent of the sum of two angles, becomes

$$\tan 2\gamma=\frac{\cos\varphi\,[\tan(V+\Gamma)+\tan(V-\Gamma)]}{1+\cos^2\varphi\tan(V+\Gamma)\tan(V-\Gamma)}. \tag{11}$$

G. Cesàro (*Sur une méthode simple pour chercherla variation ce l'angle d'extinction dans les différentes faces d'une même zone.* Mém. Acad. royale de Belgique, 3d paper, LIV (1896), 26 pp. Read July 7, 1894) gives an equation expressed in terms of the distance $OA=a$ (Fig. 495), $OB=b$, $OC=c$, and the angle $RCO=\beta$. A and B are the points of emergence of the optic axes, IV the bisectrix of the angle in the new plane, and $VIC=x$.

$$\tan 2x=\frac{(a+b)\sin\beta+2c\cos\beta}{1-(a\sin\beta+c\cos\beta)(b\sin\beta+c\cos\beta)} \tag{A}$$

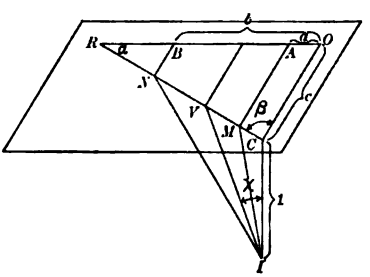


FIG. 495.

This equation is the general equation for crystals of any system. If the axis of the zone lies in the plane of the optic axes, $c=0$, and

$$\tan 2x=\frac{(a+b)\cos(90^\circ-\beta)}{1-ab\cos^2(90^\circ-\beta)} \tag{B}$$

which is the equation for the 100–010 zone.

If the axis of the zone lies in either of the other two principal sections, $b=-a$, and

$$\tan 2x=\frac{2c\cos\beta}{1+a^2\sin^2\beta-c^2\cos^2\beta} \tag{C}$$

which is the equation for the 001–010 zone.

These equations are extremely simple after the values of a and b have been determined.

[1] The same equation, somewhat differently derived, is given by Rosenbusch-Wülfing (*Mikroskopische Physiographie*, I_1, 4th ed., 1904, 253–254).

It must be remembered that $V-\Gamma$ will be negative in this case, since $\Gamma > V$, therefore, since the tangent of a negative angle is equal to minus the tangent of the same positive angle,

$$\tan (V-\Gamma) = -\tan (\Gamma - V),$$

and the equation may be written

$$\tan 2\gamma = \frac{\cos \varphi [\tan (V+\Gamma) - \tan (\Gamma - V)]}{1-\cos^2 \varphi \tan (V+\Gamma) \tan (\Gamma - V)}. \qquad (12)$$

This is the equation for extinction angles (γ) in any section of a monoclinic crystal whose value for $\Gamma < V$.

Two practical applications may be made of these formulæ: (*a*) The extinction angles in cleavage flakes of minerals of a group, such as the amphiboles or pyroxenes, may be computed; (*b*) The extinction angles in any section of a zone of a mineral may be determined.

As an example of the first we may take the pyroxenes, whose cleavage angle (2φ) is 92° 54′. In the following table[1] the first column gives values of Γ, the extinction angle on 010 for the particular pyroxene under examination; the other columns give the extinction angles on 110 for different values of the optic angle.

EXTINCTION ANGLES ON 110 CLEAVAGE PLATES OF PYROXENES

Γ on 010	Values of $2V$ for		
	50°	60°	70°
35°	29½	30½	31½
36	30¼	31¼	32¼
37	31¼	32¼	33¼
38	32¼	33¼	34¼
39	33¼	34¼	35¼
40	34¼	35¼	36¼
41	35	36¼	37¼
42	36	37¼	38¼
43	37	38¼	39¼
44	38	39¼	40¼
45	39	40¼	41¼
46	40	41½	42½
47	41	42½	44
48	42	43½	45
49	43½	44½	46
50	44½	45½	47
51	45½	46½	48
52	46½	47½	49½
53	47½	48½	50½
54	48½	49½	51½

[1] Alfred Harker: *Extinction angles in cleavage-flakes.* Mineralog. Mag., X (1893), 239–240.

R. A. Daly: *On the optical characters of the vertical zone of amphiboles and pyroxenes, etc.* Proc. Amer. Acad., XXXIV (1899), 313.

As an example of the second application, we may use diopside with $2V = 60°$, Γ on $010 = 39°$. The values computed from formula (12) are given in the following table and are graphically shown in Fig. 496.

EXTINCTION ANGLES IN THE 100–010 ZONE

φ	γ
0°	39° 0′
10°	38° 47′
20°	38° 06′
30°	36° 57′
40°	35° 09′
50°	32° 29′
60°	28° 11′
70°	22° 24′
80°	12° 57′
90°	0°

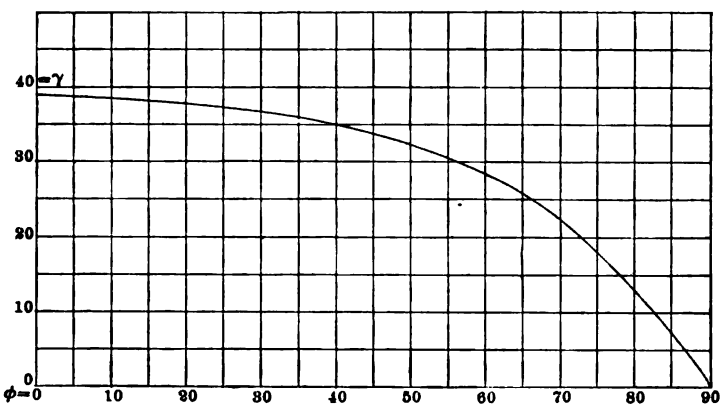


FIG. 496.—Curve showing extinction angles in the 100-010 zone of diopside.

350. Calculation of the Extinction Angle for any Face of any Zone of any Crystal.—This, the most general case of extinction angles in zones, was first worked out by Michel-Lévy[1] in 1877. Other formulæ are given by Ferro[2]

[1] A. Michel-Lévy: *De l'emploi du microscope polarisant a lumière parallèle.* Ann. d. Mines, XII (1877), 392–471.

Review by H. Bücking: *Ueber die Schwingungsrichtungen zweiaxiger Krystallplatten und deren Abhängigkeit von der Richtung der Platten.* Zeitschr. f. Kryst., III (1878–9), 217–231.

A. Michel-Lévy et Alf. Lacroix: *Les minéraux des roches.* Paris, 1888, 9–40.

[2] A. A. Ferro: Rivista di mineraligia, etc., Padua, XX (1898), 1–11, 11–14.*

Idem: Atti Soc. Ligustica di Sc. Natur., Genova, IX (1898), 143– , 230– *.

Review of two preceding papers, Zeitschr. f. Kryst., XXXII (1899–1900), 532.

and by de Sousa-Brandão.[1] That of Michel-Lévy is

$$\cot 2\gamma = \frac{[\cos\mu\cos\upsilon - \sin\mu\sin\upsilon\cos^2 v] + [\sin\mu\sin\upsilon]\sin^2\varphi}{[\cos v\sin(\mu+\upsilon)]\cos\varphi - [\sin v\sin(\mu-\upsilon)]\sin\varphi}.$$

in which γ = extinction angle in the section under examination,

φ = the angle giving the position of this section, measured from a plane passing through the axis of the zone and the bisectrix of the acute optic angle,

μ = angle between the axis of the zone and one optic axis,

υ = angle between the axis of the zone and the other optic axis,

$2v$ = angle on the plane under examination made by its intersection with the two planes passing through the optic axes and the axis of the zone.

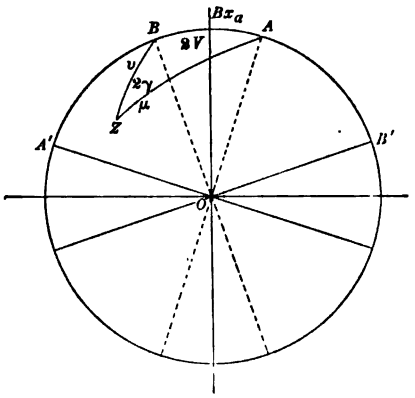


FIG. 497.

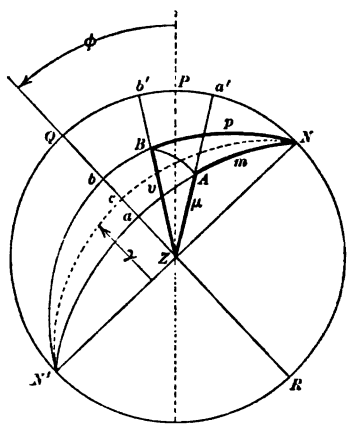


FIG. 498.

Let the plane of the paper, Fig. 497, represent the plane of the optic axes, OA and OB, of a sphere having a radius of unity. Z is the piercing point on the sphere of the axis of the zone in which the extinction angles are to be determined. υ and μ are the distances between Z and the points where the optic axes emerge from the sphere (ZB and ZA). Let $2V \lesseqgtr 90°$, and $\upsilon+\mu \lesseqgtr 180°$. Let $2V+\mu+\upsilon=2p$, then in the spherical triangle ABZ

$$\cos 2V = \cos\mu\cos\upsilon + \sin\mu\sin\upsilon\cos 2v,$$

and

$$\tan v = \sqrt{\frac{\sin(p-\mu)\sin(p-\upsilon)}{\sin p\sin(p-2V)}}. \qquad (1)$$

If the axis of the zone lies in the plane of the optic axes,

$$v = 0° \text{ or } 90°,\ \mu \pm \upsilon = 2V.$$

[1] Vicente de Sousa-Brandão: *Sur la détermination de l'angle des axes optiques dans les minéraux des roches.* Communicações da direcção dos Serviços geologicos, Lissabon, IV (1900), 35–40.

Idem: *Sur la détermination de la position des axes optiques au moyen des directions d'extinction.* Ibidem, 41–56.

When the axis of the zone lies in the plane of the bisectrix b and the axis of least ease of vibration e,

$$\mu = 180° - \upsilon \quad \text{or} \quad \mu + \upsilon = 180.°$$

Let the angle between that section of the zone in which the extinction is to be determined (QR, Fig. 498) and the plane of the bisectrix (ZP) be φ. Take as the plane of the drawing the plane at right angles to the axis of the zone Z. It contains the normal N. Draw circles (appearing as straight lines in the projection of the figure) through ZB and ZA, cutting the circumference of the sphere at b' and a'. Draw great circles through NB and NA, cutting QR at b and a, and draw a plane bisecting BNA. The trace of this bisecting plane (Nc) cuts QR at c, and its intersection with the QR plane is the direction of extinction.[1] If the angle $cZ = \gamma$, then from the figure,

$$cZ = \gamma = \frac{aZ + bZ}{2}.$$

But

$$aZ = ZNA = 90° - ANa',$$

and in the spherical right triangle ANa'

$$\tan ANa' = \frac{\tan Aa'}{\sin Na'} = \frac{\cot \mu}{\cos (\varphi + \upsilon)},$$

from which

$$\tan aZ = \tan \mu \cos (\varphi + \upsilon).$$

In the same manner

$$\tan bZ = \tan \upsilon \cos (\varphi - \upsilon),$$

whereby, finally

$$\cot 2\gamma = \cot (aZ + bZ) = \frac{1 - \tan \mu \tan \upsilon \cos (\varphi + \upsilon) \cos (\varphi - \upsilon)}{\tan \mu \cos (\varphi + \upsilon) + \tan \upsilon \cos (\varphi - \upsilon)}, \tag{2}$$

or, developing the value of the tangents into cosine values,

$$\cot 2\gamma = \frac{(\cos \mu \cos \upsilon - \sin \mu \sin \upsilon \cos^2 v) + (\sin \mu \sin \upsilon) \sin^2 \varphi}{[\cos v \sin (\mu + \upsilon)] \cos \varphi - [\sin v \sin (\mu - \upsilon)] \sin \varphi}. \tag{3}$$

The value of v may be determined from equation (1) and inserted in (3). If we substitute the following letters for the values that remain constant in the same zone,

$$A = \cos \mu \cos \upsilon - \sin \mu \sin \upsilon \cos^2 v,$$
$$B = \sin \mu \sin \upsilon,$$
$$C = \cos v \sin (\mu + \upsilon),$$
$$D = \sin v \sin (\mu - \upsilon).$$

the equation becomes

$$\cot 2\gamma = \frac{A + B \sin^2 \varphi}{C \cos \varphi - D \sin \varphi}. \tag{4}$$

From this equation may be calculated the extinction angle for any section in the same zone if the angles μ, υ, and φ are known.

[1] Cf. Art. 349.

351. Graphical Methods for the Determination of Extinction Angles on any Plane.—The extinction angles on different faces of a zone may be determined much more easily by means of stereographic projection than by analytical methods, as was well shown by Michel-Lévy[1] in 1894. The method has been used by many writers since then, among others by von Fedorow, Wulff, Viola, and especially by Duparc and Pearce,[2] who give the following.

We know, from the law first determined experimentally by Biot[3] and proved theoretically by Fresnel,[4] that the direction of extinction on a section is that in which the section is cut by the trace of the plane bisecting the angle between the planes through the optic axes and the normal to the section. If, then, *A* and *B*, Fig. 499, are the poles of the two optic axes, and *P* is that of any section whose trace is *Dacb*, the traces *PA* and *PB* will represent the planes through the optic axes, and *Pc* and *Pd*, bisecting *bPa* and *bPa'*, the bisecting planes whose intersections with the plane *acbd* are the lines of extinction. In the stereographic projection, *c* and *d* will be the poles of these extinctions and the arcs from *c* and *d* to *x*, the point of emergence of some crystallographic line (*Ox*, Fig. 449), will represent the extinction angle on that plane.

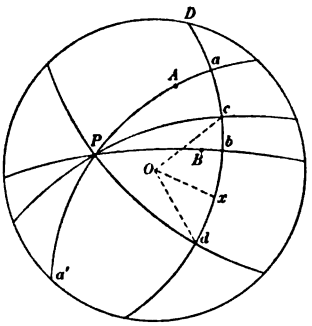


FIG. 499.

The construction is very simple if the axis of the zone lies at right angles to the plane of the drawing, Fig. 500. In this case the great circles through the axis are represented by straight lines, since all the planes of that zone are perpendicular to the plane of projection. The axis of the zone is in the center. The method for changing the plane of the projection is given in Article 16, Problem 9.

The extinction angle on any plane, such as *SS'* (Fig. 500) of the zone of which *Z* is the axis, may be determined, according to Fresnel's law,[5] by passing through the normal to that plane (*NN'*), two planes, each containing

[1] A. Michel-Lévy: *Étude sur la détermination des feldspaths.* I, Paris, 1894, 16–18.

[2] F. Duparc und F. Pearce: *Ueber die Auslöschungswinkel der Flächen einer Zone.* Zeitschr. f. Kryst., XLII (1905–7), 34–46.

Idem: *Traité de technique minéralogique et pétrographique. Leipzig*, 1907, 249–260.

[3] J. B. Biot: *Mémoire sur les lois générales de la double réfraction et de la polarisation, dans les corps régulièrement cristallisées.* Mém. Acad. France, Année 1818, III (1820), 177–384.

[4] A. Fresnel: *Mémoire sur la double réfraction.* Ibidem, VII (1827), 45–176.

Idem: *Ueber die doppelte Strahlenbrechung.* Translation of preceding. Pogg. Ann., XXIII (1831), 372–434, 494–560. In particular 542–545.

[5] Cf. Arts. 349 and 350.

the normal—consequently lying at right angle to the plane SS'—and one of the optic axes (A or B). If, now, a plane is passed through N, bisecting the angle between the two planes containing the optic axes, its trace on SS' will represent the extinction angle on that plane. In the stereographic projection these planes are represented by great circles passing through NN' and A or B.

To determine the position of the bisecting plane, two methods may be used. The projection of the optic angle on the plane SS' is measured by the arc LR, and may be read directly if a stereographic net is used. Half the angle ($LE=ER$) gives the point E, which represents the position of the line of extinction. The angle ZE is the required angle of extinction. If no stereographic net is used, the value may be obtained by drawing lines through N and the points L and R where the great circles $N'AN$ and $N'BN$ cut SS' and continuing them to the circumference of the circle at L' and R'. The arc $L'R'$ measures the projection ($=2v$) of the optic axial angle ($=2V$) on SS', and its bisectrix NE' gives the angle of extinction, measured by the arc $N'E'$ or ZE. The projection of the other extinction direction likewise lies on SS' and at a distance of 90° from the first ($E'F'=90°$). Its projection is at F.

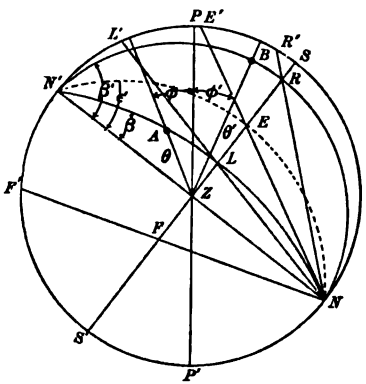


FIG. 500.

The extinction angles of all possible planes in the zone about Z can be determined by rotating the plane SS' about the axis Z, whereby a series of points, analogous to E and F, representing the points of emergence of the extinction line for all planes in the zone, will be obtained (Fig. 501). These points will lie on two curves, and will be so placed that on every plane through the zonal axis they will be separated by 90°. The lengths of the arcs connecting any point with the point Z represents the extinction angle in the plane whose trace coincides with the arc. In one direction the extinction will be toward the slowest ray and in the other toward the fastest.

Several cases may occur, depending upon the position of the axis of the zone.[1]

I. If the axis of the zone occupies a random position in relation to the position of the optic axis, the extinction curve, for a value of $2V=60°$, is as shown in Fig. 501. This may be developed on rectangular coordinates as shown in Fig. 502, in which, as in the preceding figure, the solid line indicates extinction angles from the zonal axis to the acute bisectrix (Bx_a) and the

[1] See Duparc and Pearce, *Op. cit.*, for a mathematical discussion of the various curves produced.

dotted line to the obtuse (Bx_o), the plane SS' being taken as the initial point.

II. If the axis of the zone lies in the plane which passes through the acute

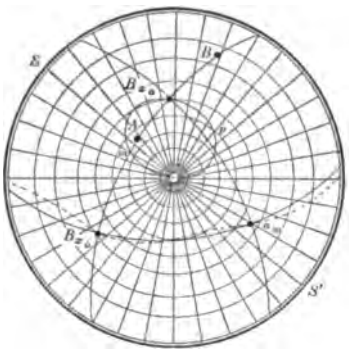


FIG. 501.—Extinction curve for $2V = 60°$, axis of zone in random position.

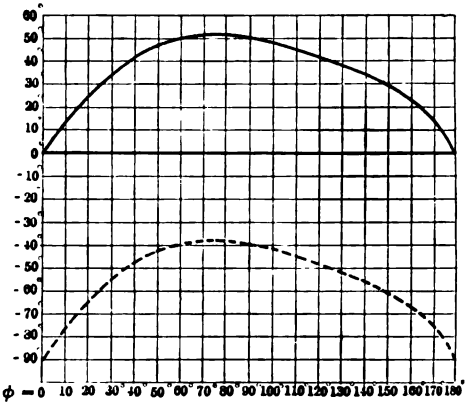


FIG. 502.—Extinction angles, derived from the stereographic projection of preceding figure, developed on rectangular coordinates.

bisectrix and the direction of mean ease of vibration (b), the extinction angles are as shown in Fig. 503. The position of the axis of mean ease of vibration

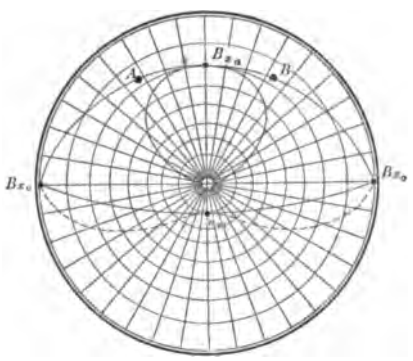


FIG. 503.—Extinction curve when the axis of the zone lies in the plane which passes through the acute bisectrix and the direction of mean ease of vibration.

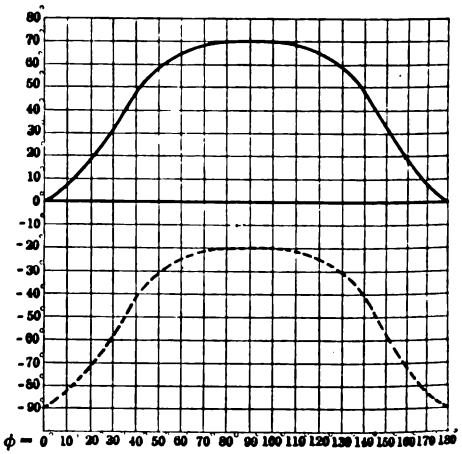


FIG. 504.—Extinction angles, derived from the stereographic projection of preceding figure, developed on rectangular coordinates.

is determined by the point of intersection of the two planes whose poles are the two bisectrices (Bx_a and Bx_o) of the optic axial angles. Developing

the curve on rectangular coordinates, and using the trace of the Bx_a–Bx_o plane as the initial line, we have the curve shown in Fig. 504.

III. The axis of the zone lies in the plane passing through the axis of inter-

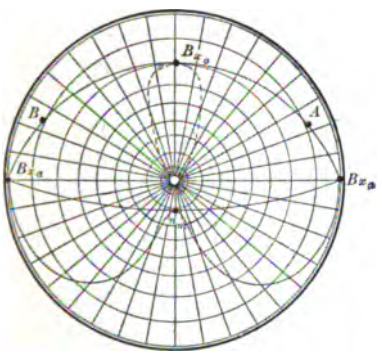


FIG. 505.—Extinction curve when the axis of the zone lies in the plane passing through the intermediate ease of vibration and the obtuse bisectrix.

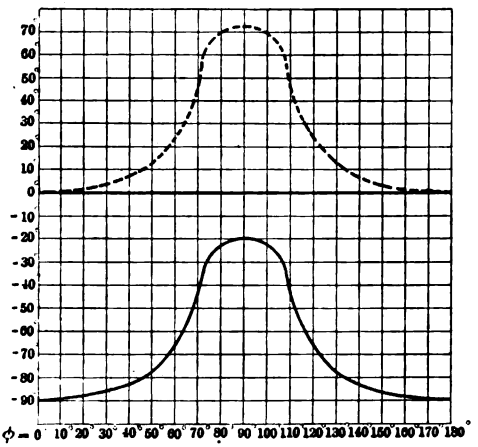


FIG. 506.—Extinction angles, derived from the stereographic projection of preceding figure, developed on rectangular coordinates.

mediate ease of vibration and the obtuse bisectrix. The extinction angles are as shown in Figs. 505 and 506.

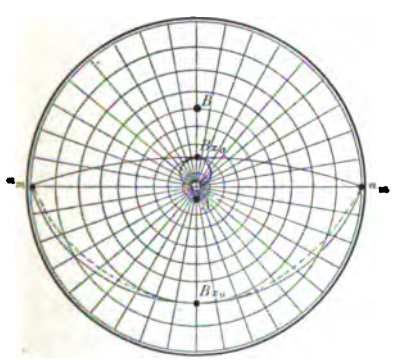


FIG. 507.

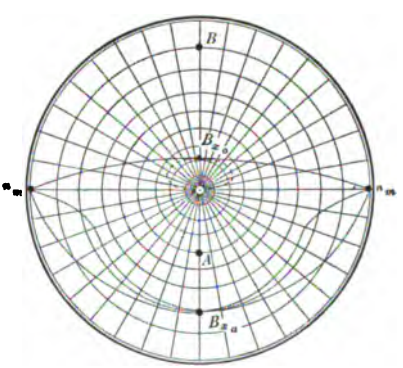


FIG. 508.

FIGS. 507 and 508.—Extinction curve when the axis of the zone lies in the plane of the optic axis. Fig. 507. The zonal axis falls in the quadrant containing the acute bisectrix. Fig. 508. The zonal axis falls in the quadrant containing the obtuse bisectrix.

IV. The axis of the zone lies in the plane of the optic axes. There are two cases, (a) the zonal axis falls in the quadrant containing the acute bisectrix (Fig. 507), and (b) it falls in the quadrant of the obtuse bisectrix (Fig. 508).

PROBLEMS

Construct, on rectangular coordinates, the extinction curves shown in Figs. 507 and 508.

Construct, first in stereographic projection, then on rectangular coordinates, the extinction curve for the 100–001 zone of diopside.

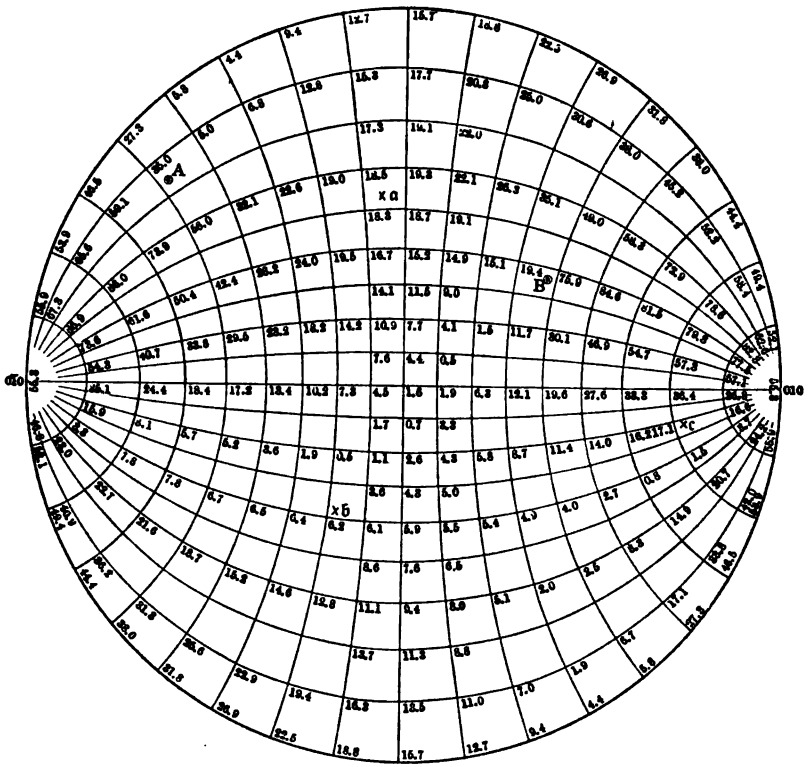


FIG. 509.—Extinction angles in andesine ($Ab_{58}An_{37}$), shown in stereographic projection at the poles of the faces.

352. Extinction Diagram and Curves of Equal Extinction.—Instead of making a diagram showing the different extinction angles in a zone by means of the piercing points of the extinction lines, we can make a diagram which gives all possible extinctions in a crystal. These extinction angles may be shown, in stereographic projection, by indicating their values at the poles of the different planes, usually at the intersection of every tenth parallel and meridian (Fig. 509).[1] The values for the extinction angles in the 100–010 zone will thus be given around the periphery of the projection circle, and will correspond in value to the angles shown, by the previous

[1] After Rosenbusch: *Mikroskopische Physiographie*, 4 Aufl., 1905, I_2, plate XVII.

construction, in Fig. 510. The extinction angles in the 100–001 zone are shown along the vertical diameter (Fig. 509), and those in the 010–001 zone along the horizontal diameter. By connecting equal values, we obtain curves of equal extinction. The lines, in other words, represent the emergence of the poles of all the planes in which the extinction angles are equal

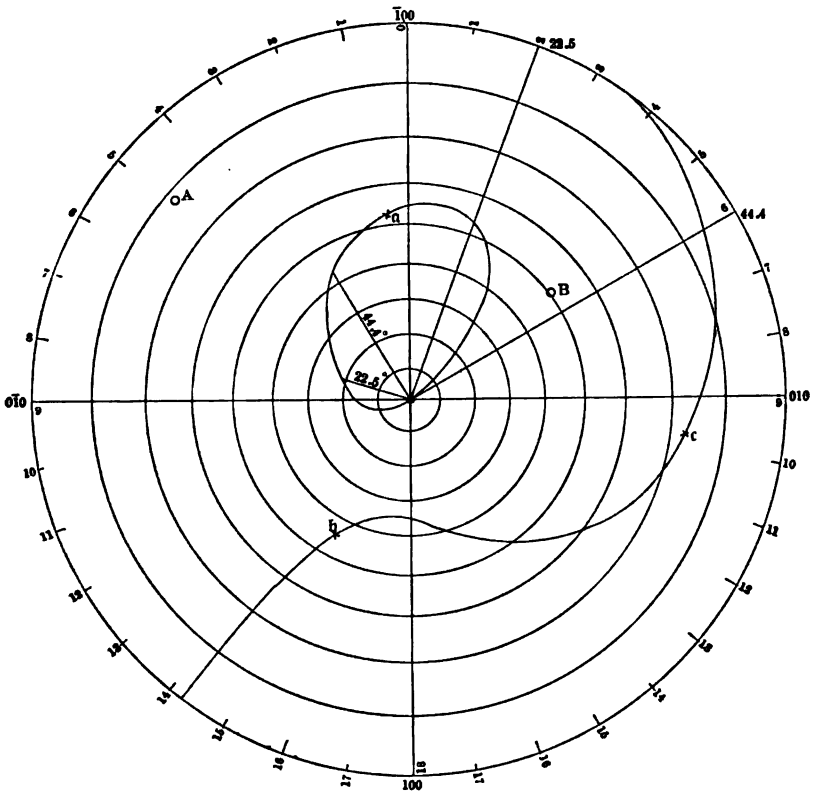


FIG. 510.—Construction for determining the extinction angles in andesine.

(Fig. 511). Practical use is made of these curves in the study of certain minerals, notably the plagioclase feldspars.[1] They are also used in the von Fedorow[2] method for determining the optic axial angle.

PROBLEMS

From the diagram of extinction angles, Fig. 509, draw, in rectangular coordinates, the extinction angles in the 100–001 zone.

Make a diagram of equal extinction angles for diopside.

[1] Michel-Lévy: *Étude sur la détermination des feldspaths dans les plaques minces.* Paris, 1894, I, planches I–VII.

[2] See Arts. 427 et seq.

353. Influence of Dispersion upon Extinction Angles.—The property of dispersion, possessed to a greater or less degree by all crystals, has its

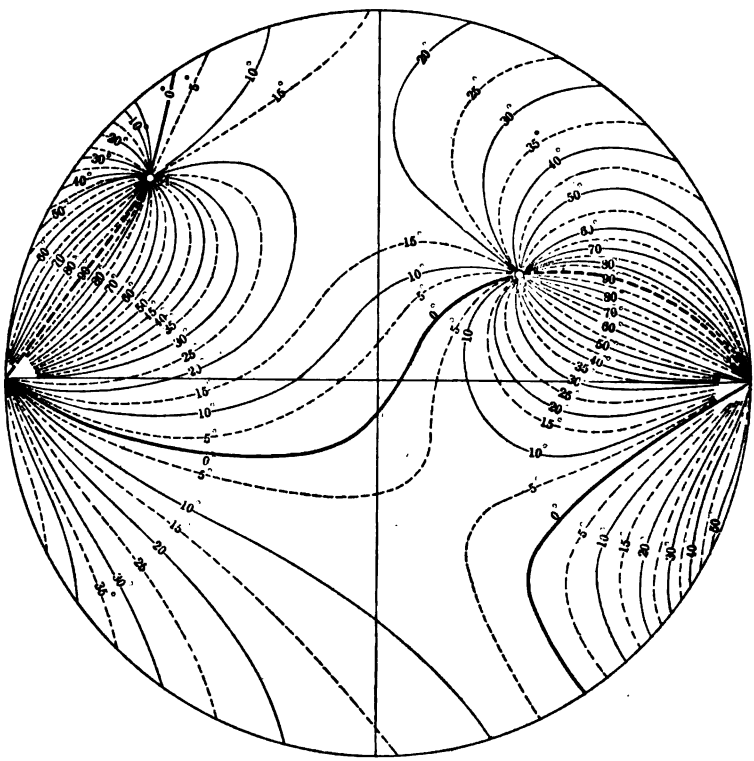


FIG. 511.—Curves of equal extinction in andesine. ($Ab_{68}An_{32}$).

influence upon the angles of extinction. The optical ellipsoid is slightly differently oriented for different colors (Fig. 512), therefore its axes will lie in different positions and, consequently, will show slightly different extinction angles for different colored light. As a result, when white light is used, there will be no position of total darkness in certain crystals which possess high dispersion, since the rays do not all extinguish together. If monochromatic light be used, the extinction angles will be slightly different for different colors.

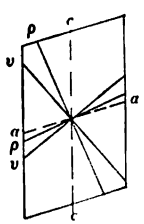


FIG. 512.—Dispersion of the bisectrices in a monoclinic crystal.

This dispersion of the extinction lines is called *dispersion of the bisectrices*, since the extinction lines coincide with the bisectrices of the optic angles.

CHAPTER XXIX

OBSERVATIONS BY CONVERGENT LIGHT

354. Polariscope, Conoscope.—Another series of tests may be made upon minerals by observing the phenomena produced in them by means of convergent polarized light, by whose interference, under certain conditions, there will be produced a figure. Instruments fitted for such observations, and consisting of polarizer and analyzer, and strongly converging lenses above and below the object stage, are called polariscopes[1] or conoscopes.[2] Usually the magnifying power of such instruments is not great, and they are used for observations on large mineral slices. Being rarely used for making observations in petrographic determinations, they will not be described here. The petrographic microscope, however, may be converted into a conoscope by using a medium or high-power objective and inserting, below the stage, a converging-lens system (Figs. 255–260). Such lenses were originally inserted in metal caps which were placed over the upper end of the polarizer. This necessitated the removal of the thin section or the withdrawal and replacement of the polarizer, an awkward proceeding with some microscopes. At the present time most makers insert the condensing system on pivots or sliders,[3] the object being to be able to change rapidly from parallel to convergent light.[4] Czapski[5] suggested that it is possible to change from parallel to convergent light by simply stopping down, by means of a diaphragm, the light coming from below. He says that although it reduces the amount of light, it is possible, by this means, to obtain as good interference figures as when the condensing lenses are inserted. An objection to this method is that the size of the field is greatly reduced.

The passage of the light through a microscope arranged as a conoscope is shown in Fig. 513. The light, reflected from the mirror, is plane polarized on passing through the lower nicol. It converges in a cone of wide angle

[1] G. Kirchhoff: *Ueber den Winkel der optischen Axen des Aragonits für die verschiedenen Fraunhofer'schen Linien.* Pogg. Ann., CVIII (1859), 567–575.

P. Groth: *Ueber Apparate und Beobachtungsmethoden für krystallographisch-optische Untersuchungen.* Pogg. Ann., CXLIV (1871), 34–55.

[2] Gustav Tschermak.*

[3] Art. 118, *supra.*

[4] H. Laspeyres: *Vorrichtung am Mikroskope zur raschen Umwandlung paralleler Lichtstrahlen in convergente.* Zeitschr. f. Kryst., XXI (1902), 256–257.

[5] S. Czapski: *Ueber Einrichtungen behufs schnellen Ueberganges vom parallelen zum convergenten Lichte und die Beobachtung der Axenbilder von sehr kleinen Krystallen in Polarisations-Mikroskopen.* Zeitschr. f. Kryst., XXII (1893–94), 158–162.

from below the object, passes through the objective and the analyzer, and forms a real image K_3 in the tube, a short distance above the analyzer. As ordinarily arranged, this image cannot be seen through the ocular, since it does not lie in its focal plane. It may be seen, however, if there is inserted

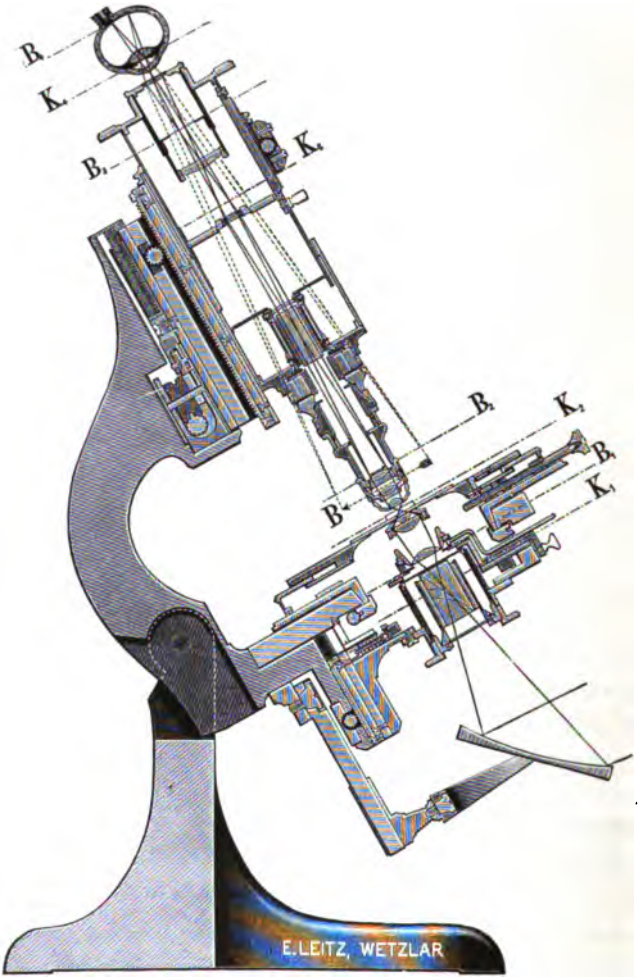


FIG. 513.—Passage of light through a microscope arranged as a conoscope. (Leitz.)

an accessory lens, as shown in the figure, making, with the ocular above it, a weak compound microscope in itself. The image may be seen, likewise, by removing the ocular entirely and looking down the tube, or it may be observed by placing a hand lens at the proper distance above the second image which is formed in the Ramsden disk above the ocular. The various methods will be described in detail below.[1]

[1] Arts. 389–401.

355. Interference Figures.—The kind of image formed by the conoscope depends upon the crystal system of the mineral under examination and upon the orientation of the section. It consists of curves and bars (Figs. 520, 522, 528, etc.), either black, or black and colored, depending upon whether monochromatic or white light is used. By means of these images, called interference figures since the bars and colors are produced by the interference of the rays which have traversed the crystal in different directions, it is possible to separate uniaxial from biaxial crystals, to determine the direction of the optic axes, the angle between them in biaxial crystals, the directions of the fastest and the slowest rays in the crystal, the character of the dispersion, and the orientation of the section. It is also possible to determine by them the amount of the retardation, consequently, if its thickness is known, the value of the birefringence of the mineral.

By convergent light we may divide all crystals into three groups, two of of which may again be subdivided:

Isotropic crystals.
Uniaxial crystals—positive—negative.
Biaxial crystals—positive—negative.

ISOTROPIC CRYSTALS

356. Random Sections.—We saw that in parallel polarized light, between crossed nicols, an isotropic crystal remained dark during a complete rotation of the stage. Upon altering the light from parallel to convergent, no change takes place in the appearance of the field of view. The light, passing through with equal ease in every direction, has no effect upon the plane of polarization of the light entering from below, consequently it is completely cut off by the analyzer and darkness results. The mineral under examination must be either amorphous or belong to the isometric system.

In practice the light is never completely polarized, for a beam of plane polarized light, falling at a considerable inclination upon an isotropic substance, such as glass, suffers a certain amount of polarization, and, as a consequence, the emerging light no longer vibrates in a single plane. The greater the inclination of the rays, the greater the polarization of the emerging light, whereby, as was shown by Rinne,[1] the light is polarized in directions at right angles to the radii at the edges of lenses, especially those of short focal lengths. The planes of polarization of these rays are thus represented by the radiating lines in Fig. 514.

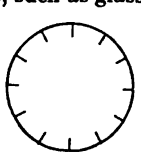


FIG. 514—Polarization of light by lenses.

[1] F. Rinne: *Bemerkung über die Polarisationswirkung von Linsenrändern.* Centralbl. f. Min., etc., 1900, 88–89.

See also G. Cesàro: *Étude de la rotation imprimée au plan de polarisation du faisceau lumineux venant du polariseur, par les lentilles du microscope à lumière convergente.* Bull. Acad. Roy. Belgique Cl. d. Sci., 1906, 459–492.

The effect of this polarization by the lenses may be seen if an isotropic mineral or glass is examined for its interference figure. With most microscopes there will appear a broad, apparently uniaxial, cross, usually of weak positive character. The same cross will appear if no mineral lies upon the stage, wherefore care must be taken in regard to this figure so that it may cause no confusion. If the interference figure of a biaxial crystal, having a large optic axial angle, such as muscovite, adularia, etc., and cut at right angles to the acute bisectrix, be examined, it will be found that if the upper nicol is removed, the interference figure will still be seen around the edges of the field, though somewhat dimmer than before.

ANISOTROPIC CRYSTALS

UNIAXIAL CRYSTALS

357. Section Perpendicular to the Optic Axis.— Let us consider, first, a basal section of a uniaxial crystal. From the optical ellipsoid we know that such a section lies at right angles to the optic axis, and that the rays passing through it vibrate with the same ease in every direction. Rays passing

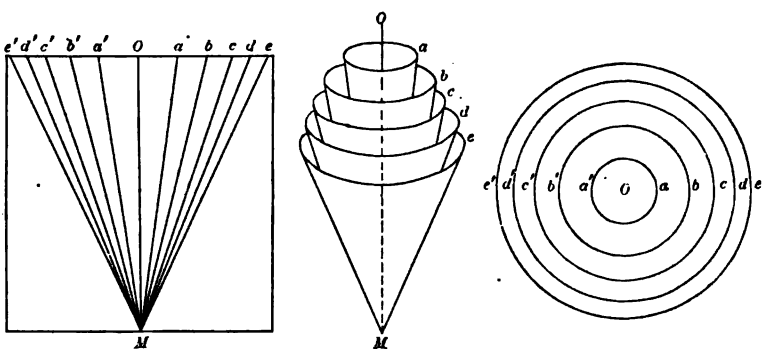


FIG. 515. FIG. 516. FIG. 517.

FIGS. 515 to 517.—Interference of convergent, polarized light in uniaxial crystals. (After Miers.)

through it in any other direction are doubly refracted and, consequently, interfere. Suppose a cone of monochromatic light passes through a crystal section (Fig. 515) cut at right angles to the optic axis *MO*. The ray *MO* has its vibrations equal in all directions and perpendicular to this axis. It passes through without interference. At this point of emergence (*O*, Fig. 517) the section will appear dark. The ray *Ma*, passing through the crystal at an angle with the optic axis, is doubly refracted. Let the inclination of this ray be such that, owing to its greater path difference and to the greater difference of refractive indices of the two rays in this direction, the retardation is exactly one wave length. We saw[1] that rays with a retardation of $N\lambda$ were extinguished between crossed nicols, consequently at the points *a* and *a'*, where the retardation is one wave length, there will be darkness. In a

[1] Art. 282, *supra*.

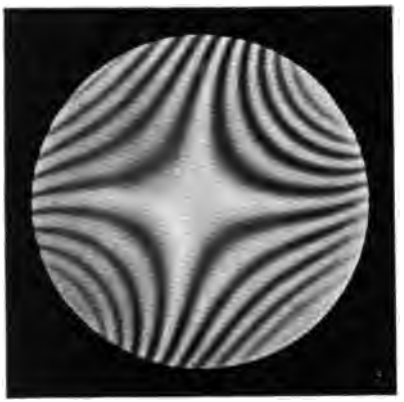


FIG. 518.

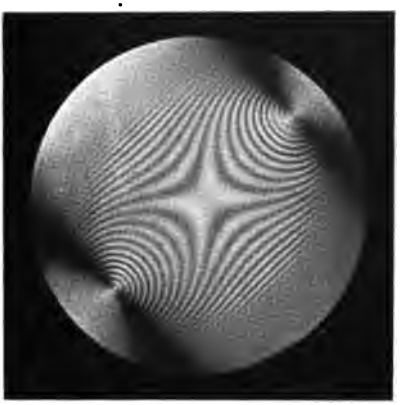


FIG. 519.

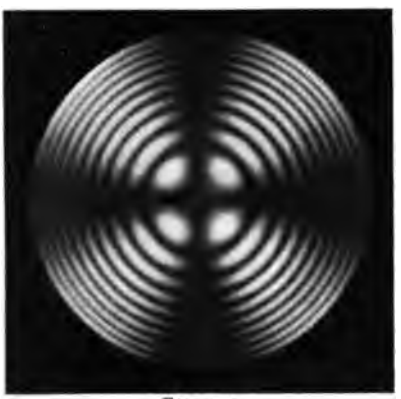


FIG. 520.

FIG. 521.

FIG. 522.

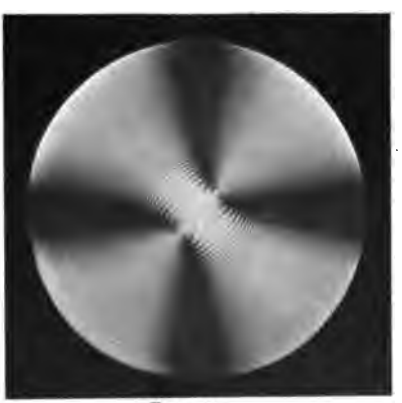


FIG. 523.

FIG. 518.—Dependence of the size of an interference figure upon the numerical aperture of the condenser. Topaz, section at right angles to the acute bisectrix, placed in diagonal position between crossed nicols. N. A. of condenser 0.636.

FIG. 519.—Ditto. N. A. 1.168.

FIG. 520.—Dependence of the curves of equal retardation upon the color of light used. Calcite, 1/4 mm. thick, cut at right angles to the optic axis. Between crossed nicols. Wave length of source of light, 620–720$\mu\mu$ (blue).

FIG. 521.—Ditto. Wave length of source of light, 410–450$\mu\mu$ (yellowish orange).

FIG. 522.—Ditto. Cerussite, biaxial. Wave length of source of light, 620–720$\mu\mu$ (blue).

FIG. 523.—Ditto. Wave length of source of light, 410–450$\mu\mu$ (yellowish orange).

(*Facing Page* 416.)

cone of light, such as we are considering, there will be an equal retardation of one wave length everywhere at a distance of *Oa* from the optic axis, whereby a circle of darkness, *aa'*, Figs. 516–517, will appear at that distance. Another ray *Ob* has a retardation of 2λ, consequently *b* and *b'*, Fig. 515, or the circle *bb'*, Figs. 516–517, will appear dark. Other rays will interfere with phasal differences of 3λ, 4λ, 5λ, etc., forming, thus, concentric rings between which there will be rings of light.

If a different color of light from that used in obtaining the above rings were used, we should find, owing to the difference in their wave lengths, that the distances *Oa*, *Ob*, *Oc*, etc., would be different, consequently the rings would be farther apart with blue light (Figs. 520 and 522), and closer together with yellow (Figs. 521 and 523).

The number of rings which may be seen in the field of the microscope likewise depends upon the value of the double refraction of the mineral (Figs. 590–591), the numerical aperture of the condenser (Figs. 518–519), and the thickness of the section (Figs. 591–592). Thus a section of quartz 1.0 mm. in thickness (Fig. 590) will show two isochromatic curves, while a section of calcite of the same thickness (Fig. 591) will show a great many. If the quartz section is increased in thickness, a point will be reached where the same number of rings is seen. Knowing the thickness of the section, it is, consequently, possible to determine approximately the maximum birefringence of a mineral from a section which, perhaps, show no double refraction at all.

If white light were used instead of monochromatic, bands of color would appear everywhere instead of darkness and light, each color representing the complementary one of that which was extinguished at that point, the color being the same as that which would be produced by the retardation in a section of the same thickness cut at right angles to the direction of the ray. Thus the color at *a* (Fig. 515) will be the same as that which would be produced by a plate of a thickness *Ma*, cut at an angle *OMa* with the optic axis, and viewed between crossed nicols in parallel polarized light.

But besides these dark (or colored) rings, there appear, also, in the interference figures of uniaxial crystals (Figs. 520–521), certain dark bars,[1] due to the relation of the differently orientated vibration directions to the principal planes of the nicols. These curves of like vibration directions are called **isogyres.** They may be explained graphically as follows:

Let a certain amount of light, represented by the horizontal line[2] *ab*, Fig. 524, enter the crystal plate. Since the ray is now passing through at an angle with the optic axis, the ease of vibration is not the same in every direction, but the light vibrates in planes at right angles to each other. Let these two

[1] Cf. Art. 371.

[2] The line actually represents the square root of the intensity, as we shall see later. This applies to Fig. 525 also.

directions be *ac* and *ad*. Resolving the original intensity of the light into these two directions, we have *ac* and *ad* as the values. Leaving the crystal, the light passes to the upper nicol, which has its vibration direction at right angles to the polarizer. The rays *ac* and *ad* are each resolved into two rays, parallel and at right angles to the analyzer; the former components, *ae* and *af*, pass through, but the latter are annihilated. There reach the eye, therefore, only the rays *ae* and *af*, and their sum represents the intensity of the light reaching the eye through the crystal at that point. Applying a similar construction to various rays in Fig. 525, we see that all points on any radial line from *O* have the same intensity. This intensity is at its maximum on the line making an angle of 45° with the vibration directions of the two nicols, and is equal to zero when parallel to them. As a result, there will be a gradation of light from the maximum intensity in the diagonal positions, to darkness when the vibration planes are parallel to the principal sections of the nicols.

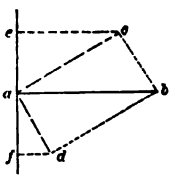


FIG. 524.

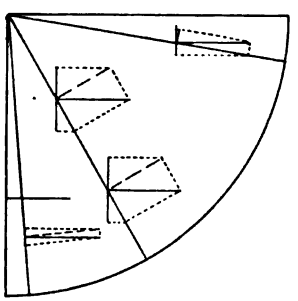


FIG. 525.—Intensity of emerging light and the cause of the appearance of the isogyres in a uniaxial interference figure.

If the nicols are placed in parallel position, the maximum amount of light will pass along the vibration planes, and instead of a dark cross (Fig. 526), there will be one of light (Fig. 527). The greatest darkness in the interference figure will be on the diagonals.

Analytically we have, for the equation of the intensity of light at any point in a crystal plate between crossed nicols,

$$I = r^2 \sin^2 2\theta \sin^2 \frac{\pi}{\lambda} M(n_2 - n_1) \qquad \text{(Eq. 16, Art. 285)},$$

where r is the amplitude of vibration, θ the angle between the vibration planes of the mineral and the polarizer, and $M(n_2 - n_1)$ the retardation. The amplitude, the wave length, and the retardation remain the same for any circle around the point *O*, whereby the equation may be written

$$I = K \sin^2 2\theta.$$

The intensity, therefore, depends directly upon the value of $\sin^2 2\theta$. The maximum value for the sine of an angle is unity, consequently the maximum intensity of the light is where $\sin^2 2\theta = 1$. From this equation we obtain $2\theta = 90°$, or $\theta = 45°$. The minimum intensity occurs where $\sin^2 2\theta = 0$ which occurs when $2\theta = 0$ or 180°, and $\theta = 0$ or 90°. These results are the same as those derived from the graphical method above.

358. Section Oblique to the Optic Axis.—If the axis of the crystal is perpendicular to the section, the center of the cross is on the axis of the microscope (Fig. 526), and no matter whether the slide is displaced laterally

FIG. 526.

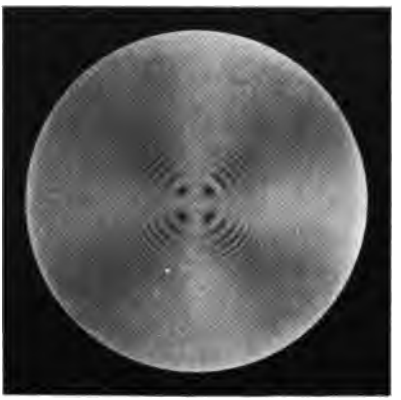


FIG. 527.

FIG. 528.

FIG. 529.

FIG. 530.

FIG. 531.

FIG. 526.—Interference figure of calcite. Plate 1/2 mm. thick, cut at right angles to the optic axis. In sodium light with nicols crossed.

FIG. 527.—Ditto. In white light with nicols parallel.

FIG. 528.—Calcite plate, section cut at an angle of 80° with the optic axis. Nicols crossed, optic axis of crystal lying in the vibration plane of the analyzer; sodium light.

FIG. 529.—Ditto in diagonal position.

FIG. 530.—Calcite plate. Section cut at an angle of 67 1/2° with the optic axis. Nicols crossed, optic axis lying in diagonal position; sodium light.

FIG. 531.—Calcite plate, section cut parallel to the optic axis. In sodium light, between crossed nicols. Optic axis in diagonal position.

(*Facing Page* 418.)

or the stage rotated, neither bars nor circles of the interference figure show change in position. If, however, the axis of the crystal is inclined to the plane of the section, the case becomes somewhat different. The line along which there is no double refraction is now no longer in the center, but is displaced to one side. On either side, at unequal distances, are the positions

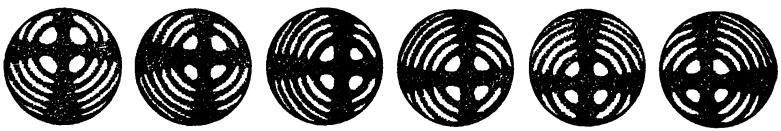


FIG. 532. FIG. 533. FIG. 534. FIG. 535. FIG. 536. FIG. 537.

FIGS. 532 to 537.—Uniaxial interference figure. Section inclined to the optic axis, which emerges between the center and the edge of the field of view.

of one wave length retardation. The figure seen under the microscope (Fig. 528) is now not symmetrically placed in the center of the field but lies to one side, and the rings *aa'*, *bb'*, etc., form curves, which are not perfectly true circles, around it. As the stage of the microscope is rotated, the optic axis describes a cone. The interference figure has moved, by a rotation to

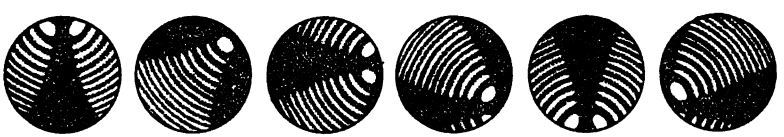


FIG. 538. FIG. 539. FIG. 540. FIG. 541. FIG. 542. FIG. 543.

FIGS. 538 to 543.—Uniaxial interference figure. Section inclined to the optic axis, which emerges beyond the field of view.

the right, successively through the positions shown in Figs. 532 to 537 (Cf. also Figs. 528–529). If the inclination of the axis is still greater, so that it emerges beyond the field of view, the successive positions are as shown in Figs. 538 to 543 (Cf. also Fig. 530).

When the inclination is slight, *the bars remain practically parallel to the vibration planes of the nicols*, and the center of the cross moves in the same direction as that in which the stage was rotated. If the inclination of the optic axis is great, the bar may appear somewhat curved (Fig. 530).[1]

359. Sections Parallel to the Optic Axis.[2]—Sections parallel to the optic axis show hyperbolæ which are very similar to those seen in biaxial figures except that they do not appear in the field until the stage is nearly in the 90° position; then they move in, from a cross, which is seldom sharp, and very rapidly move out again in the direction of the principal axis. During the greater part of the rotation, therefore, they do not appear in the field

[1] Cf. Art. 372.

[2] Cf. Art. 373

of the microscope. In Fig. 531, which is that of an interference figure in a section parallel to the optic axis, the bars are quite sharp, the section being very thick.

The optic axis lies in the direction toward which the hyperbolæ leave the field. Its position may also be determined by the appearance of the interference color. If the stage is turned 45° from the position in which the bars form a cross, there will appear a certain interference tint in the center of the field. Outward from this center there will be a fall in the color scale toward the quadrants in which the optic axis lies, and a rise in the other two quadrants. This holds good for both positive and negative crystals (Figs. 544–545). In thick plates, or in minerals having high double refraction, this rise or fall in the interference colors is observable only at the center; beyond that, there is a uniform rise.

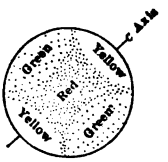


FIG. 544.

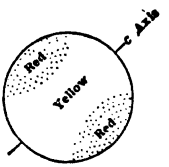


FIG. 545.

FIGS. 544 and 545.—Interference figures of uniaxial minerals cut parallel to the optic axes. Fig. 544. Quartz (+). Fig. 545. Apatite (−).

BIAXIAL CRYSTALS

360. Sections Cut at Right Angles to the Acute Bisectrix.[1]—Vibrations take place with equal ease in every direction about the optic axes of a biaxial crystal; in consequence there will be no double refraction at their points of emergence, since there is no interference. At these points (*OO*, Fig. 546) the section will appear dark. At some distance in every direction from the optic axes, there will be points at which the retardation is exactly one wave length. These points lie in an imaginary cone having an oval base (*Ma*, Fig. 546). Another cone will be the surface of 2λ retardation (*Mb*), another of 3λ, and so on. Since there are two optic axes, there will be two such sets of cones, and around them the ovals unite to form lemniscate curves, whereby, in monochromatic light, there will be produced an interference figure (Fig. 548) showing two dark spots surrounded by dark oval or lemniscate curves. These dark spots, which show the points of emergence of the optic axes, are often called "eyes." The term **melatope** (μελα, dark, black; and τόπος, place) is here suggested instead. As in uniaxial figures, and for the same reason, so in these, also, the lemniscate curves become curves of color in white light.

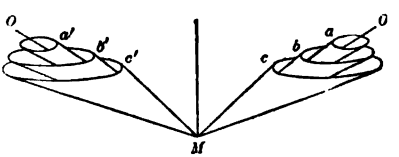


FIG. 546.—Cones of equal retardation in biaxial interference figures.

[1] Cf. Arts. 368 and 374.

FIG. 547.

FIG. 548.

FIG. 549.

FIG. 550.

FIG. 551.

FIG. 552.

FIG. 547.—Interference figure of titanite. Plate cut at right angles to the acute bisectrix. Between crossed nicols by sodium light, 22 1/2° rotation from the position parallel to the principal sections of the nicols.

FIG. 548.—Ditto. Rotated 45°.

FIG. 549.—Ditto. Rotated 90°.

FIG. 550.—Cane sugar. Plate parallel to 100. Interference figure by sodium light, in diagonal position.

FIG. 551.—Diopside. Plate parallel to 100 in sodium light. Diagonal position.

FIG. 552.—Euclase. Cleavage plate parallel to the axial plane (010). Interference figure by sodium light, in diagonal position.

(*Facing Page* 420.)

Besides these curves of equal retardation (isochromatic curves), there appear, also, certain dark bars or brushes called **isogyres** (ἴσος, equal; γυρος, circle). On rotating the stage, the cross (Fig. 549), which appears when the vibration directions in the mineral are parallel to those of the nicols, dissolves into two hyperbolæ whose poles are the loci of the optic axes. These bars revolve in the opposite direction from the stage (Figs. 547–549). The smaller the axial angle, the nearer together will be the loci of the optic axes,[1] until, as a limiting case, the form is that of the uniaxial interference cross.

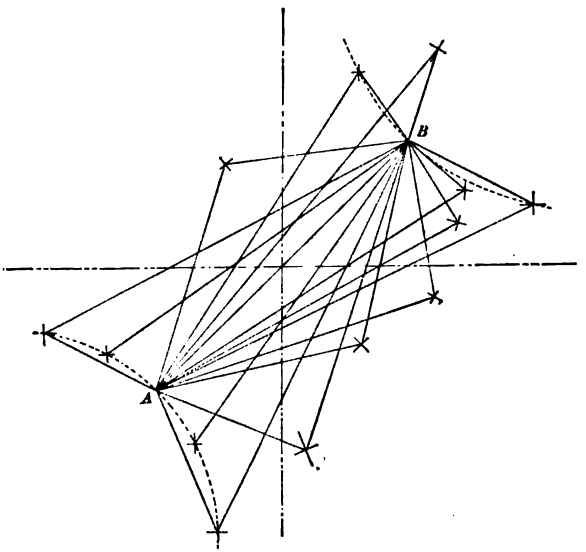


FIG. 553.—Vibration directions of light producing a biaxial interference figure.

The explanation is analogous to that given for the dark cross in basal sections of uniaxial crystals. In the latter the two directions of vibration, into which the ray entering from below was broken up, were those of the radii and the tangents. In a biaxial crystal the vibration directions are likewise normals and tangents to the advancing wave fronts. The loci of the optic axes are the foci of ellipses formed by the advancing wave front, and the normal to this wave, at any point, is the bisectrix of the angle between the two lines connecting this point and the foci (Fig. 553). Having determined the vibration directions for every point, the dark brush can be readily determined. Applying the construction of Fig. 525 to Fig. 553, we see that there will be darkness wherever the vibration directions of the crystal are parallel to the principal planes of the nicols. The variation of the positions of these dark brushes upon rotating the stage is well brought out in a diagram given

[1] Cf. Figs. 556 and 560. In the former the apparent axial angle is 30°, in the latter 80°.

by ten Siethoff[1] (Fig. 554). In this figure the vibration directions for many rays are shown by small crosses. If the diagram is placed upon a rectangular table, whose sides may be taken to represent the principal planes of the nicol prisms (and consequently the cross hairs of the microscope), and it is rotated in azimuth, the form of the interference figure at any instant may be seen by observing the small crosses whose arms are parallel to the sides of the table. A rotation of the diagram through 67 1/2° will bring about the consecutive changes of the figure shown in Figs. 547–549. Ten Siethoff's diagram also brings out clearly the fact that when the plane of the optic axes is parallel to one of the nicols, the dark cross has one broad and one narrow bar (Cf. Figs. 549, 555 and 557), the width of the former depending upon the optic angle. In using this diagram, the parallel crosses may be seen best by placing the eye at one side and but a few centimeters above the plane of the paper, or by laying over it a transparent piece of celluloid ruled into rectangles.

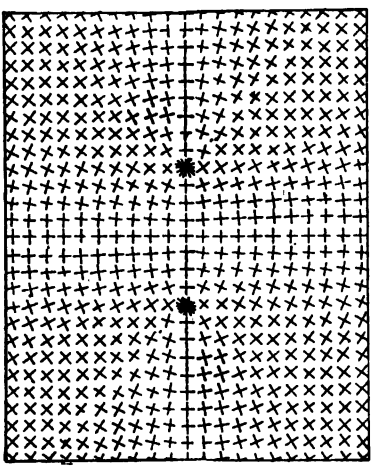


FIG. 554.—Ten Siethoff's diagram showing vibration directions in the interference figures of biaxial crystals.

The number of isochromatic rings seen in a biaxial interference figure, around each axis, depends, as it does in uniaxial figures, upon the strength of the double refraction of the mineral, and upon the thickness of the section (Figs. 555 and 557). The number of complete rings corresponds to the number of wave lengths retardation. This may be seen clearly by examining the interference figures produced by sheets of mica of different thicknesses, especially well by the different steps of a von Fedorow wedge. The first step, which has a retardation of 1/4 λ, gives a figure (Fig. 561) in which one lemniscate curve completely surrounds the melatopes and a second partial ellipse shows beyond it. The second step, having a retardation of 1/2 λ, shows the lemniscate curves closing up toward the acute bisectrix (Fig. 562). The curves approach each other still more in the third step with 3/4 λ retardation, and, when the retardation is a single wave length (Fig. 563), the first curve just unites at the center and forms a figure eight, one loop around each optic axis. The sixth step (Fig. 564) gives a retardation of 1 1/2 λ. Here the first curve is divided into two, one forming a closed curve around each axis, and the second forming a lemniscate about the two. The eighth step shows a retardation of two wave lengths and presents two complete

[1] E. G. A. ten Siethoff: *Eine einfache Construction des sogen. Interferenzkreuzes der zweiaxigen Krystalle.* Centralbl. f. Min., etc., 1900, 267–269.

FIG. 555.

FIG. 556.

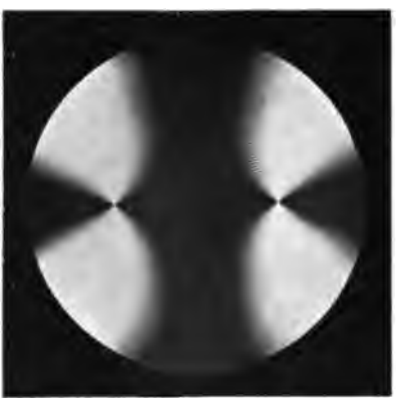


FIG. 557.

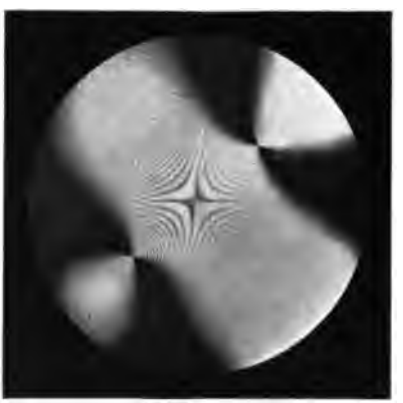


FIG. 558.

FIG. 559.

FIG. 560.

FIG. 555.—Aragonite. Plate cut at right angles to the acute bisectrix in sodium light between crossed nicols. Plate 1/2 mm. thick. Parallel position.

FIG. 556.—Ditto. In diagonal position.

FIG. 557.—Ditto. Plate 2 mm. thick. Parallel position.

FIG. 558.—Ditto. Plate 2 mm. thick. Diagonal position.

FIG. 559.—Muscovite. Plate at right angles to the acute bisectrix. Sodium light, nicols crossed. Parallel position. Retardation two wave lengths.

FIG. 560.—Ditto. Diagonal position.

(*Facing Page* 422.)

rings, one within the other, about each axis. (Fig. 565. See also Figs. 559–560.) The inner ring is approximately a circle, while the outer is like that obtained with a retardation of one wave length. The tenth step gives 2 1/2 λ retardation, and the interference figure is made up of two closed rings about each axis; the two pairs enclosed by lemniscate curves (Fig. 566). It is to be noted that there is no change in the distance between the melatopes,[1] the axial angle, of course, remaining the same.

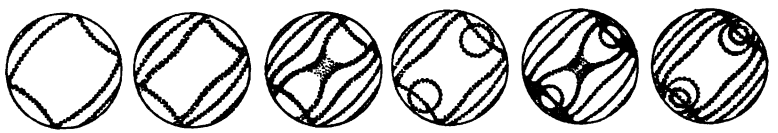


FIG. 561. FIG. 562. FIG. 563. FIG. 564. FIG. 565. FIG. 566.

FIGS. 561 to 566.—Interference figures in mica wedge, showing retardations of 1/4, 1/2, 1, 1 1/2, 2, and 2 1/2 wave lengths.

361. Sections Cut at Right Angles to the Obtuse Bisectrix.[2]—When the axial angle is nearly 90°, the interference figure produced in a section cut at right angles to the obtuse bisectrix resembles that in a section cut at right angles to the acute bisectrix. The melatopes, however, will not appear in the field of view, since the angular aperture of the condensers of most microscopes will permit the full figure to appear only when $2V$ is less than about 60°.

The fact that the isogyres always have their convex sides toward the acute bisectrix *when the plane of the optic axes forms an angle of* 45° *with the principal sections of the nicols* cannot be used to determine whether the acute or obtuse bisectrix is in the field of view, since neither brush appears in the field in this position. When the obtuse optic angle is large, it is generally possible to recognize it by the fact that the brushes remain in the field of the microscope but a short time upon rotating the stage, coming in when the rotation of the stage has brought the plane of the optic axes and the principal section of the nicols close together, and disappearing immediately after that position has been passed. The method of determining the value of the optic axial angle by this means is discussed in full in Art. 416.

362. Sections Inclined to the Bisectrices.[3]—More and more of one melatope and less of the other is seen as the section is more and more inclined (Figs. 550–551). The convex side of the hyperbola is always turned toward the acute bisectrix when the mineral is turned in the 45° position, and the arm rotates in a direction opposite to that in which the stage is turned (Figs. 567–586). When the melatope lies near the edge of the field of view, the

[1] Cf. Figs. 556 and 558.

[2] Cf. Art. 375.

[3] Cf. Art. 378, *infra*.

appearances in uniaxial (Fig. 530) and biaxial (Fig. 551) crystals are very similar.

363. Sections at Right Angles to an Optic Axis.[1]—Sections cut at right angles to an optic axis show nearly circular, concentric curves crossed by

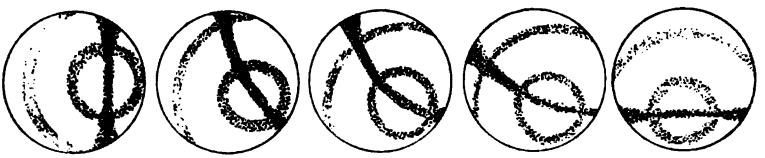


FIG. 567. FIG. 568. FIG. 569. FIG. 570. FIG. 571.

FIGS. 567 to 571.—Biaxial interference figure. Section somewhat inclined to the plane of the optic axes. One optic axis emerges within the field of view, the acute bisectrix just beyond.

FIG. 572. FIG. 573. FIG. 574. FIG. 575. FIG. 576.

FIGS. 572 to 576.—Biaxial interference figure. Section inclined at a greater angle to the plane of the optic axes than in preceding case. The optic axes and the bisectrices emerge beyond the field of view.

FIG. 577. FIG. 578. FIG. 579. FIG. 580. FIG. 581.

FIGS. 577 to 581.—Biaxial interference figure. Section at right angles to the plane of the optic axes. One optic axis emerges within the field of view, the acute bisectrix emerges just beyond. The isogyre is straight when it passes through the center of the field.

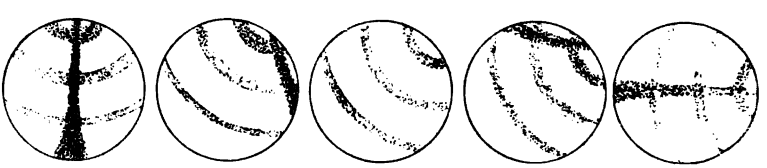


FIG. 582. FIG. 583. FIG. 584. FIG. 585. FIG. 586.

FIGS. 582 to 586.—Biaxial interference figure. Section at right angles to the plane of the optic axes. The optic axes and the bisectrices emerge beyond the field of view. The isogyre is straight when it passes through the center of the field.

a single dark bar, which is straight whenever it is parallel to the planes of vibration of the nicols (Fig. 587). Upon rotating the stage, this bar generally changes to a slightly curved hyperbola (Fig. 588) with its convex side toward

[1] Cf. Art. 377.

FIG. 587.

FIG. 588.

FIG. 589.

FIG. 590.

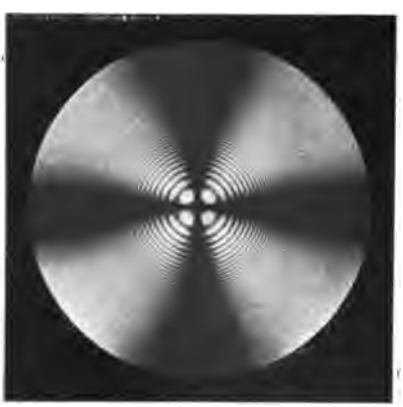


FIG. 591.

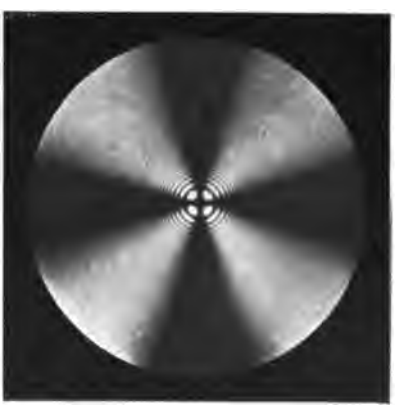


FIG. 592.

FIG. 587.—Topaz. Section cut at right angles to an optic axis. Nicols crossed. Parallel position.
FIG. 588.—Ditto. Section thicker than preceding. Diagonal position. 2V approximately 60°.
FIG. 589.—Andalusite. Plate at right angles to an optic axis. Diagonal position. $2V = 83° 30'$.
FIG. 590.—Quartz. Plate at right angles to the optic axis. 1 mm. thick.
FIG. 591.—Calcite. Plate 1 mm. thick, cut at right angles to the optic axis.
FIG. 592.—Calcite. Plate 3 mm. thick.

(*Facing Page* 424.)

the acute bisectrix. The amount of curvature depends upon the value of the optic axial angle. The smaller the angle, the greater the curvature. Figs. 587–588 show interference figures of topaz, with $2V$ approximately equal to 60°. When $2V$ equals 90° the bar is straight. It is generally impossible to recognize the curvature when $2V$ is greater than 80°, as for example in andalusite with $2V$ equal to 83° 30′ (Fig. 589). Sometimes a bar will appear approximately straight on one side and concave on the other (Fig. 588). In such cases the straight side is toward the acute bisectrix.

Since light is dispersed in all biaxial crystals, a section can be actually at right angles to an optic axis only for a given color. The dispersion is generally so slight, however, that it may be overlooked, and one will see, in white light, a series of colored rings whose tints will differ from the pure colors of Newton's scale more and more with increasing dispersion.

364. Sections Parallel to the Plane of the Optic Axes.[1]—Sections cut parallel to the plane of the optic axes (perpendicular to the optic normal b) may be recognized in parallel polarized light by the fact that they show the highest interference colors of any section of that mineral. In convergent light they show figures (Fig. 552) similar to those shown by uniaxial crystals (Fig. 531) parallel to the optic axis. Upon rotating the stage, the hyperbolæ come in from the sides very rapidly, darken the field, and with very little farther rotation immediately disappear in the direction of the acute bisectrix. When the field is dark the axes a and c are parallel to the cross hairs.

Becke[2] has shown that the acute bisectrix in this section is the line uniting the quadrants containing the lower colors. In negative minerals it is a, and in positive, c. When the axial angle approaches 90° the color variation becomes indistinct; when $2V=90°$ it disappears.[3]

PROBLEM

Use the gypsum plate as a mineral section and determine, by this method, the direction of c.

LOCATING THE POINT OF EMERGENCE OF AN OPTIC AXIS

365. Uniaxial Crystals.—If the point of emergence of the optic axis of a uniaxial crystal lies within the field of the microscope, its position is readily determinable by the fact that it lies at the intersection of the dark bars. The inclination of the optic axis to the axis of the microscope, consequently the inclination of the section, may be determined by measuring, from the

[1] Cf. Art. 376.

[2] F. Becke: *Unterscheidung von optisch + und − zweiaxigen Mineralien mit dem Mikroskop.* T. M. P. M., XVI (1896–97), 181.

[3] Compare the method given for the same determination in uniaxial crystals in Art. 359.

center of the field, the distance to the point of emergence, or by measuring half the distance between the cross in two positions 180° apart, computing the angular value by Mallard's formula, and reducing to the true angle of inclination by the formula $\sin E = n \sin V$.

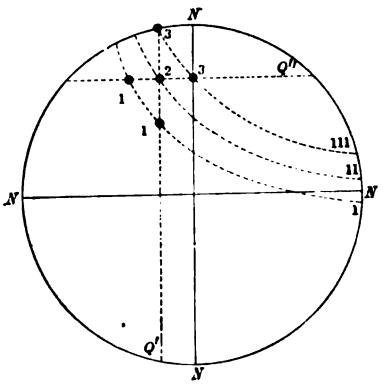


FIG. 593.

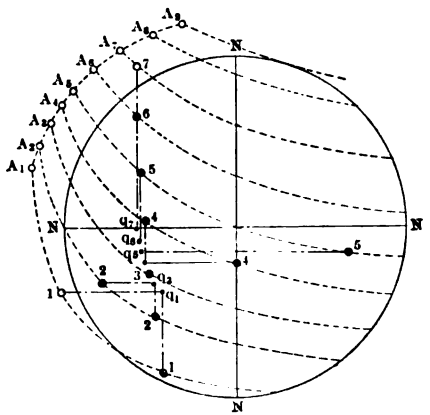


FIG. 594.

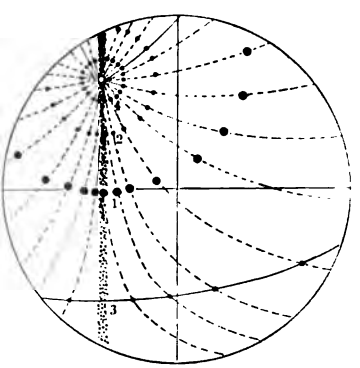


FIG. 595.

366. Biaxial Crystals.—The points of emergence of the optic axes in biaxial crystals may be determined by locating the point of rotation as in the method suggested by Becke.[1] Another method is that of Viola[2] which is based upon the fact that the directions of vibration in the isogyres are parallel to the vibration planes of the nicols. If a section of a biaxial mineral, giving an interference figure showing the point of emergence of one of the optic axes, is placed upon the stage of the microscope, and above it is placed a section of quartz giving a uniaxial interference figure, the only points of darkness will be where the isogyres intersect, since only here will the rays reaching the eye be parallel to the vibration planes of the nicols, and one will see two black dots surrounded, in white light, by colored curves.

If I (Fig. 593) is the isogyre from the lower thin section, and $Q'Q''$ that from the thin section of quartz, 1 and 1 will be the two black spots which appear where the two intersect. If II is the biaxial isogyre, the

[1] F. Becke: *Bestimmung kalkreicher Plagioklase durch die Interferenzbilder von Zwillingen.* T. M. P. M., XIV (1894–95), 415–442. Cf. Art. 418. *infra.*

[2] C. Viola: *Methode zur Bestimmung der Lage der optischen Axen in Dünnschliffen.* T. M. P. M., XV (1896), 481–486.

intersection at 2 forms a single spot. If the isogyre is at III, there will again appear two spots, 3, 3.

If the stage of the microscope is rotated, the points of emergence of the optic axes of both the biaxial mineral and the quartz likewise rotate, but retain their relative positions. Thus, in Fig. 594, upon rotating the stage, the black dots 1, 1 become 2, 2, then coincide in 3, separate to 4, 4, 5, 5, etc., farther and farther apart as the stage is revolved. At the same time the progressive positions of the melatope of the quartz are q_1, q_2, q_3, q_4, etc. At 3, where but a single dark spot appears, it is evident that the optic axis of the quartz coincides with the biaxial isogyre.

The phenomenon appears much simpler if the nicols are rotated instead of the stage. Let A, Fig. 595, be the point of emergence of the optic axis of the biaxial mineral. If only the biaxial mineral section lies on the stage, the isogyre will appear as a straight line when the plane of the optic axes lies parallel to the vibration direction of one of the nicols. If there is now placed above the biaxial mineral a quartz plate in such a position that the point of emergence of its optic axis lies on this line, the straight bar, parallel to one of the nicols, will still appear, since along that line, in both minerals, the light is extinguished. Let the center of the quartz cross appear at 1. If the nicols are rotated simultaneously, the isogyre of the biaxial mineral will assume successively the positions shown by the dotted lines. The center of the uniaxial cross of the quartz will retain its position, but the bars will revolve so that they remain constantly parallel to the nicols. As a result, the points of intersection of the two figures will appear as two black dots (in Fig. 595 one of the dots lies beyond the field), which will rotate about A as the nicols are turned. The spots will lie nearer to A than 2 or farther away than 3 according to whether the axis of the quartz lies nearer or more distant from A. If the axis of the quartz corresponds with the axis of the biaxial mineral, only a single black dot will appear, and it will retain its position upon rotating the nicols.

To determine the positions of the melatopes of a biaxial mineral as well as their angular distances from the axis of the microscope, Viola had cut a series of ten thin sections of quartz, each differing by 10° from the preceding in its inclination to the optic axis. These quartz sections were so mounted on a glass slip that the c axis of all lay in the same plane. For the determination of the position of the optic axis of a biaxial mineral they are inserted successively above it, but always in such a position that the dark bar falls within the field. As each different slice appears, the nicols are slightly rotated, and notice is taken as to whether the dark spot moves or is stationary. When it is stationary the optic axes of the two must coincide, and the unknown optic axis forms an angle with the axis of the microscope equal to the known angle of the quartz.

Usually no quartz slice over the biaxial interference figure will produce

a spot absolutely stationary, and it will then be possible to determine only the angle as lying intermediate between those of two known quartz sections. Closer approximation may be reached if quartz slices cut at 5° intervals are used.

If a rotating stage is used, instead of a microscope with simultaneously rotating nicols, the black spot of coincidence, of course, always rotates, but the distance of this spot from the center remains constant.

CHAPTER XXX

ISOTAQUES, SKIODROMES, AND ISOGYRES

367. Isotaques or Curves of Equal Velocity.—The positions of the isogyres in random sections may be determined by a method suggested by Becke,[1] who made use of the curves of equal velocity, named by him **isotaques** (ἴσος, equal, ταχός, swift, or ἰσο-ταχής, equally swift). These had long previously been worked out by Beer,[2] who showed that it is possible, by following the same law as that by which an ellipse is constructed about its focii—namely that the sum of the distances of any point from the focii is a constant—to draw on the surface of a sphere two systems of curves about two points. If 2α is the major axis of the spherical ellipse, and φ and φ' are the angles between the focii and the point, then $\varphi+\varphi'=2\alpha=$ a constant.

If two series of spherical ellipses are constructed upon the surface of the sphere about the points of emergence of the optic axes, one series will have for its center the acute bisectrix, and the other, the obtuse. Together the two series will cover the sphere with a network of lines, intersecting at right angles, by the aid of which the vibration directions at any point of the surface may be easily determined.

Beer showed that all lines drawn from the center of the sphere to points on the same spherical ellipse represent the direction of propagation of rays having the same velocity. In other words, the ellipses represent curves of equal velocities. Also, the vibration directions of these rays lie at right angles to their respective ellipses, whereby the tangents at the points of intersection

[1] F. Becke: *Optische Untersuchungsmethoden.* Denkschr. Akad. Wiss. Wien, LXXV (1904), 41 pp.*

Idem: *Die Skiodromen. Ein Hilfsmittel bei der Ableitung der Interferenzbilder.* T. M. P. M., XXIV (1905), 1–34.

J. Beckenkamp: Review of preceding two. Zeitschr. f. Kryst., XLII (1906–7), 644–648.

J. W. E[vans]: Review ditto. Mineralog. Mag., XIV (1907), 276–280.

Ernst Sommerfeldt: *Ueber die Bedeutung der Skiodromen für die Krystalloptik.* T. M. P. M., XXVII (1908), 285–292.

John W. Evans: *Notes on skiodromes and isogyres.* Mineralog. Mag., XIV (1907), 230–234. (The movement of the isogyres with fixed stage and movable nicols is described.)

[2] August Beer: *Ueber eine neue Art die Gesetze der Fortpflanzung und Polarisation des Lichtes in optisch zweiaxigen Medien darzustellen.* Grunert's Arch., Th. XVI (1851), 223–229.

Idem: *Einleitung in die höhere Optik.* Braunschweig, 1st Aufl., 1853, 401–402; 2 Aufl., 1882, 309, 373.

of the two series, being at right angles to each other, represent the vibration directions of both rays propagated along the radius of the sphere to that point.

The ellipses of one series were called by ecke *equatorial ellipses* (Geschwindigkeits-ellipsen erster Art, by Beer). They surround the acute optic angle ($2V$) and have values such that $\varphi + \varphi' \gtrless 2V \lesseqgtr 180°$.

The ellipses of the other series were called by Becke *meridian ellipses* (Geschwindigkeits-ellipsen zweiter Art, by Beer). They surround the obtuse optic angle ($180°-2V$) and have values such that

$$\varphi + (180°-\varphi') = 2\alpha' \gtrless 180° - 2V \lesseqgtr 180°.$$

In uniaxial crystals, where $2V = O$, the equatorial ellipses become parallels, and the meridian ellipses meridians.

Becke further distinguishes

α ellipses, or those whose tangents represent the vibration direction of the fastest ray,

γ ellipses, or those whose tangents represent the vibration direction of the slowest ray,

whereby, in optically positive (+) crystals

meridian ellipses are γ ellipses,
equatorial ellipses are α ellipses;

and in optically negative (−) crystals

meridian ellipses are α ellipses,
equatorial ellipses are γ ellipses.

In the following figures, the α ellipses are shown by broken-, and the γ ellipses by dotted lines.

368. Skiodromes.—The isotaques, or curves of equal velocity, may be well shown in stereographic projection. Becke, however, has given them by preference in orthographic projection; and they are thus reproduced here. To such projections of the isotaques Becke has given the name **skiodromes** (*σκιά*, a shadow; *δρόμος*, course).

Analytically, the construction of the curves is given in the work cited above.[1] Here only the resulting values are brought together.

Let 2α = the sum of the angles determining the equatorial ellipses,
$2\alpha'$ = the sum of the angles determining the meridian ellipses,
a = the major axis of any equatorial ellipse,
b = the minor axis of any equatorial ellipse,
a' = the major axis of any meridian ellipse,
b' = the minor axis of any meridian ellipse,
$2V$ = the acute optic axial angle.

[1] Denkschriften, etc. *Op. cit.*

1. *Sections perpendicular to the acute bisectrix (lying in the xy plane)* (Fig. 596).

a. Equatorial skiodromes give ellipses wherein

$$\text{the major axis } a \text{ (parallel to } x) = \sin \alpha, \tag{1}$$

$$\text{the minor axis } b \text{ (parallel to } y) = \frac{\sqrt{\cos^2 V - \cos^2 \alpha}}{\cos V}. \tag{2}$$

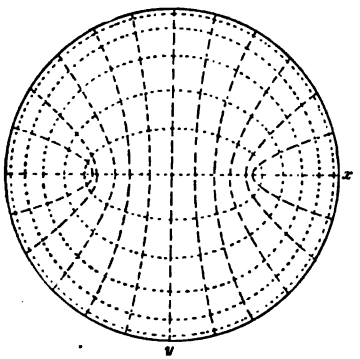


FIG. 596.

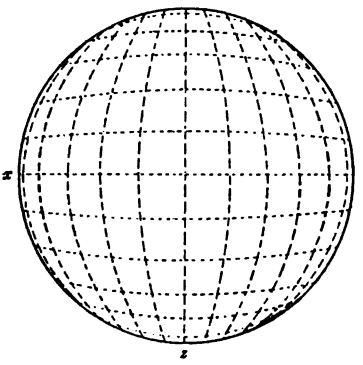


FIG. 597.

FIG. 596.—Skiodrome of a negative, optically biaxial crystal. Projection of a section at right angles to the acute bisectrix. Broken lines, skiodromes of the fast rays (α skiodromes), dotted lines, skiodromes of the slow rays (γ skiodromes). $2V = 60°$.

FIG. 597.—Skiodrome of a negative, biaxial crystal. Projection of a section parallel to the plane of the optic axes. $2V = 60°$.

b. Meridian skiodromes give hyperbolæ wherein

$$\text{the real axis } a' \text{ (parallel to } x) = \cos \alpha', \tag{3}$$

$$\text{the imaginary axis } b' \text{ (parallel to } y) = \frac{\sqrt{\sin^2 V - \cos^2 \alpha}}{\cos V} \tag{4}$$

2. *Sections parallel to the axial plane (lying in the xz plane)* (Fig. 597).

Equatorial skiodromes give partial ellipses wherein

$$\text{the major axis } a \text{ (parallel to } x) = \frac{\sin \alpha}{\sin V}, \tag{5}$$

$$\text{the minor axis } c \text{ (parallel to } z) = \frac{\cos \alpha}{\cos V}. \tag{6}$$

The meridional skiodromes give partial ellipses in which

$$\text{the major axis } c' \text{ (parallel to } z) = \frac{\sin \alpha'}{\cos V}, \tag{7}$$

$$\text{the minor axis } a' \text{ (parallel to } x) = \frac{\cos \alpha'}{\sin V}. \tag{8}$$

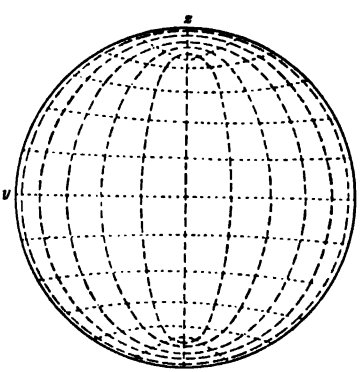


FIG. 598.—Skiodrome of a negative biaxial crystal. Projection of a section perpendicular to the obtuse bisectrix. $2V = 60°$.

3. *Sections perpendicular to the obtuse bisectrix (lying in the yz plane)* (Fig. 598). Equatorial skiodromes give hyperbolæ wherein

$$\text{the true axis } c \text{ (parallel to } z) = \cos \alpha, \tag{9}$$

$$\text{the imaginary axis } b \text{ (parallel to } y) = \frac{\sqrt{\cos^2 V - \cos^2 \alpha}}{\sin V}. \tag{10}$$

The meridian skiodromes give ellipses in which

$$\text{the major axis } c' \text{ (parallel to } z) = \sin \alpha', \tag{11}$$

$$\text{the minor axis } b' \text{ (parallel to } y) = \frac{\sqrt{\sin^2 V - \cos^2 \alpha'}}{\sin V}. \tag{12}$$

To construct the skiodromes it is necessary to assume successive values for the constants 2α and $2\alpha'$. In practice it was found more convenient to use as variables, not α and α', but the angle which the short axis of the spherical ellipse subtends at the center of the sphere. If this value is represented by β, we obtain, from the relationships

$$\cos \alpha = \cos V \cos \beta, \tag{13}$$

and

$$\cos \alpha' = \sin V \cos \beta, \tag{14}$$

the following values for our equations.

(1) Equatorial skiodromes

$$a = \sqrt{1 - \cos^2 V \cos^2 \beta}, \tag{1a}$$

$$b = \sin \beta. \tag{2a}$$

Meridian skiodromes

$$a' = \sin V \cos \beta, \tag{3a}$$

$$b' = \tan V \sin \beta. \tag{4a}$$

(2) Equatorial skiodromes

$$a = \sqrt{\frac{1 - \cos^2 V \cos^2 \beta}{\sin V}}, \tag{5a}$$

$$c = \cos \beta. \tag{6a}$$

Meridian skiodromes

$$c' = \sqrt{\frac{1 \sin^2 V \cos^2 \beta}{\cos V}}, \tag{7a}$$

$$a' = \cos \beta. \tag{8a}$$

(3) Equatorial skiodromes

$$c = \cos V \cos \beta, \tag{9a}$$

$$b = \cos V \sin \beta. \tag{10a}$$

Meridian skiodromes

$$c' = \sqrt{1 - \sin^2 V \cos^2 \beta}, \tag{11a}$$

$$b' = \sin \beta. \tag{12a}$$

The values of the equations in terms of the angles V and β may be obtained more rapidly by first computing the value of α or α' from equations (13) and (14) and substituting the results obtained in equations 1, 2, 5, 6, 7, 8, 11, and 12. Equations 3a, 4a, 9a, and 10a may be used as they are.

The skiodromes shown in Figs. 596, 597, and 598 were constructed from these equations; $2V$ being taken as 60°, and β as 15°, 30°, 45°, 60° and 75°. If the dotted skiodromes in the figures represent the vibration directions of the slow rays (γ skiodromes) and the broken-line skiodromes the fast rays (α skiodromes), the crystal represented is negative. If the dotted skiodromes represent the fast and the broken-line skiodromes the slow, the crystal is positive. Figures 599 and 600 show the skiodromes of sections at right angles to the acute bisectrix and parallel to the plane of the optic axes when $2V = 90°$.

The skiodromes, deduced as above, cover the field of view with curves which intersect at right angles everywhere except near the edge of the projection of the sphere. Here the light is no longer plane, but elliptically

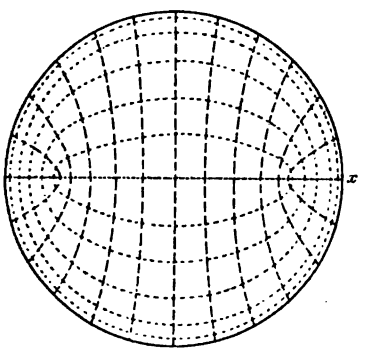


FIG. 599.—Skiodrome of a biaxial crystal. $2V = 90°$. Projection of a section perpendicular to a bisectrix.

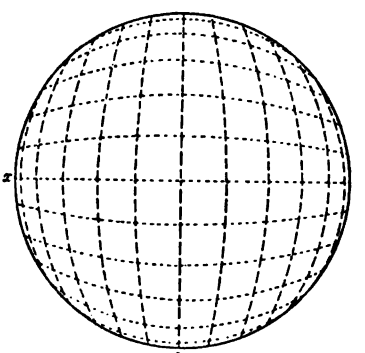


FIG. 600.—Skiodrome of a biaxial crystal. $2V = 90°$. Projection of a section parallel to the plane of the optic axes.

polarized, and in the projection the lines cross each other obliquely, the obliquity increasing with distance from the center. The part of the field in the center, which is seen in the conoscope, depends for its size upon the mean refractive index of the substance under examination and the numerical aperture of the conoscope. If $2A$ = the numerical aperture and n the mean refractive index of the crystal, $\frac{A}{n}$ = the radius of the field of view.

369. To Construct the Skiodromes for a Random Section.—After the skiodromes for the symmetrical sections have been constructed, it is a simple matter to construct those for random sections. Thus if it is desired to construct a new projection on a plane whose pole in the orthographic projection is M (Fig. 601), it is necessary to rotate M first to O. The angle through which this point has been rotated is shown in its true value in the section to the left, $mO'o$. All other points in the original projection, such as A and B, must be rotated through an equal angle. A vertical plane through

A has for its trace FE, and in rotated position appears as the circle fae, on which a represents the present position of A. The point a is now to be rotated through an angle equal to $mO'o$ or $aO'a'$, which is most simply con-

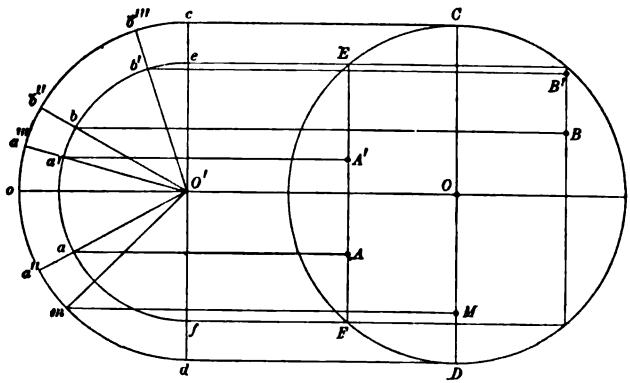


FIG. 601.—Construction for rotating the plane of projection in orthographic projection.

structed by laying off from a'' to a''' a chord equal to mo. All other points on the original skiodrome are transposed in a similar manner.

370. Deduction of the Isogyres from the Skiodromes.—Every point in the field of view where a tangent or a normal to a skiodrome lies parallel to the principal section of one of the crossed nicols belongs to an isogyre.

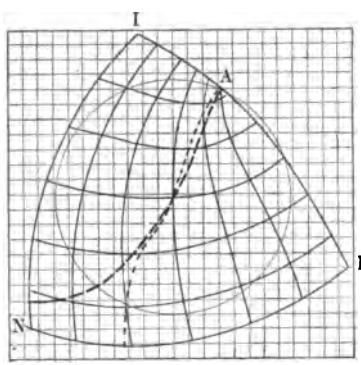


FIG. 602.—Skiodrome of an octant of an inclined section of a biaxial mineral placed beneath a nicol net. I is the acute bisectrix, II the obtuse bisectrix, N the normal, A an optic axis. The circle represents the aperture of the conoscope (ca. 30°). The partial isogyre of the meridian skiodrome is shown by broken lines, that of the equatorial skiodrome by dots.

To determine these points, Becke used a sheet of transparent paper on which were drawn rectangular coordinates, and which, since the lines represent the vibration directions of the nicol prisms, may be called a **nicol net.** When such a sheet is placed over the skiodromes (Fig. 602), it is easy to determine and connect, by a continuous line, the points where the two are parallel. If the skiodrome net is rotated below the nicol net, in the same way as is the mineral section between crossed nicols, the positions of tangency will change in the same manner as the isogyres under the microscope.

As may be seen from Fig. 602, which shows, beneath a nicol net, the skiodrome of an octant of an inclined section cut from a biaxial mineral, there appear two isogyres, one derived from

the meridian and one from the equatorial skiodrome. They coincide in the center of the field and at the optic axis, and there the isogyre forms a sharply defined dark bar. Near the edge of the field the skiodromes cut each other obliquely, and the vibrations are not at right angles, therefore the isogyres from the two series do not extinguish simultaneously, and the two curves widely diverge. As a result, instead of a sharp bar, a diffused brush appears. Practically, it is sufficient to draw the mean line to represent the resulting isogyre.

The equations for the isogyres have been determined by Hilton.[1] Let ρ be the radius of the sphere. Let the gnomonic projection (not orthographic) of the axes X and Y be parallel to the direction of the nicols and directly through the center,

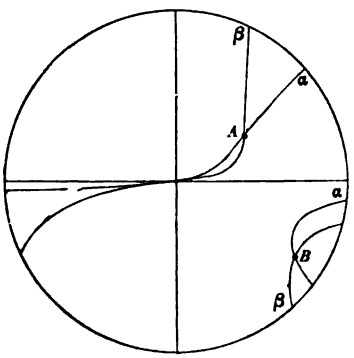


FIG. 603.

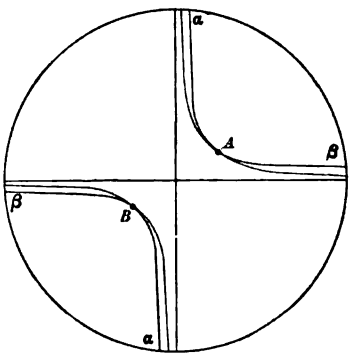


FIG. 604.

FIGS. 603 and 604.—Isogyres constructed by Hilton's equation.

and let the coordinates of the projections of the optic axes be x_1y_1 and x_2y_2. The equations of the two partial isogyres, which are the loci of all points where the projection of the tangents to the spherical ellipse are parallel to the vibration plane of the nicols (X, Y), will be

$$x[(y^2-\rho^2)(y_1+y_2)+2y(\rho^2-y_1y_2)]=(y^2+\rho^2)\,[y(x_1+x_2)-(x_1y_2+x_2y_1)] \qquad (\alpha)$$

and

$$y[(x^2-\rho^2)(x_1+x_2)+2x(\rho^2-x_1x_2)]=(x^2+\rho^2)[x(y_1+y_2)-(x_1y_2+x_2y_1)]. \qquad (\beta)$$

Figs. 603 and 604 show the curves for two positions of the plane of the optic axes, as developed by these equations.

(I) SKIODROMES OF UNIAXIAL CRYSTALS

371. Sections Cut at Right Angles to the Optic Axis.—In sections cut at right angles to the optic axis (Fig. 605; Cf. Fig. 526), the isogyres form a dark cross along the vertical and horizontal isotaques, the only lines in the skiodrome which are parallel to the vibration directions of the nicols.

[1] H. Hilton: *Ueber die dunklen Büschel von Dünnschliffen im convergenten Lichte.* Zeitschr. f. Kryst., XLII (1906–7), 277–8.

Review of above in Mineralog. Mag., XIV (1907), 282.

372. Sections Inclined to the Optic Axis.—In sections inclined to the optic axis (Fig. 606; Cf. Fig. 528), the center of the cross is displaced to one side. Upon rotating the stage, the center moves in the same direction (Figs. 532–537). If the inclination of the section is slight, all meridian skiodromes may be regarded as straight lines and all equatorial skiodromes as concentric circles. Upon rotating the stage, the isogyres, consequently, are displaced to positions parallel to the principal sections of the nicols, and approximately parallel to their original positions. They remain practically straight bars.

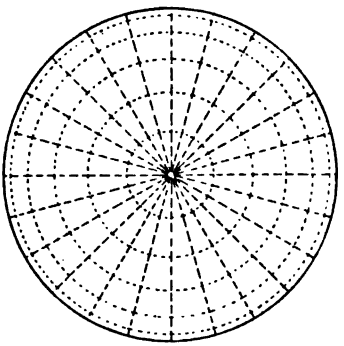


FIG. 605.—Skiodrome of a negative uniaxial crystal in a section cut at right angles to the optic axis.

If the inclination of the section is great, the meridian skiodromes are perceptibly curved, and the equatorial skiodromes are no longer concentric. As a consequence, on rotating the stage, the black bar is less rapidly displaced at the end where it moves in the same direction as that in which the stage is rotated (the *homodrome* end, ὁμος, the same, δρόμος, course, path) than in the other (the *antidrome* end, ἀντί, against), and it appears to swing back and forth.[1] It does not remain parallel to itself, therefore, during the rotation.

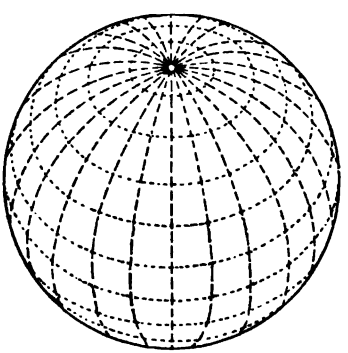


FIG. 606.—Skiodrome of a negative uniaxial crystal. Section inclined to the optic axis.

In a uniaxial crystal cut at an angle with the optic axis, *at some position during the rotation, the isogyre forms a straight bar, symmetrically dividing the field into halves and lying parallel to the principal section of one of the nicols* (Figs. 532 and 538).

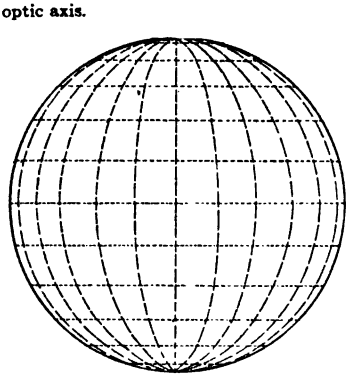


FIG. 607.—Skiodrome of a negative uniaxial crystal. Section parallel to the optic axis.

373. Sections Parallel to the Optic Axis.—In sections parallel to the optic axis (Fig. 607, Cf. Fig. 531), the meridian skiodromes appear as flattened curves, concave toward the center, and extending from pole to pole; the equatorial skiodromes, as parallel lines. If the principal sections lie parallel to the vibration planes of one of the nicols, the interference figure appears as a broad, black

[1] Cf. Art. 378 for biaxial crystals with a single bar.

cross, the outer edges of the four quadrants showing a small amount of light. A very slight rotation of the nicols will immediately disturb the parallel position of the equatorial skiodromes, consequently the entire field will be weakly illuminated. At the same time the meridian skiodromes will cause a pair of shadowy hyperbolæ to appear, which, however, disappear on very little more rotation.

PROBLEMS

Construct a nicol net on transparent paper, and draw the isogyres for 0°, 30°, 60°, and 90° rotation of the mineral section shown in Figs. 605, 606, and 607.

Construct the isogyres formed by rotating the two nicols simultaneously through the same angles as before, leaving the mineral stationary. Compare the results.

Examine (a) basal section, (b) inclined section, (c) section parallel to crystallographic *c* of quartz and of calcite.

II. SKIODROMES OF BIAXIAL CRYSTALS

A. SECTIONS PERPENDICULAR TO THE PRINCIPAL VIBRATION AXES

374. Sections Perpendicular to the Acute Bisectrix.—In sections perpendicular to the acute bisectrix (Fig. 596; Cf. Figs. 547–549),[1] the isogyres form a dark cross when the vibration axes are parallel to the principal sections of the nicols (Fig. 549). Of the two dark bars, the one passing through the points of emergence of the optic axes is called the axial-bar or axial-isogyre, and is much more sharply defined than the bar at right angles to it. The latter is called the central-bar or central-isogyre, and is more or less diffused, the width increasing with increasing axial angle. When the stage is rotated, the dark cross separates into two hyperbolæ (Fig. 548), half the central bar uniting with half the axial bar to form each. The end which belongs to the axial bar, however, is distinguished from that which belongs to the central bar by the fact that it is homodrome while the latter is antidrome. The velocity of the homodrome end depends upon the location of the point of emergence of the optic axis. If this lies outside the field of view of the microscope, the homodrome end moves more rapidly than the rotation of the stage; if it lies exactly on the periphery, the velocities are the same; and if it lies within the field, it moves more slowly. In every case, however, the movement is in the same direction as the stage.

375. Sections Perpendicular to the Obtuse Bisectrix.—The angular aperture of an ordinary petrographic microscope will permit the points of emergence of both optic axes to be seen when the apparent axial angle ($2E$) is less than 90°, consequently neither melatope can be seen in sections cut at right angles to the obtuse bisectrix (Fig. 598).[2] When the true axial angle

[1] Cf. Art. 360.

[2] Cf. Art. 361.

($2V$) is large and approaches 90°, the interference figure seen in sections cut at right angles to the obtuse bisectrix differs very little from that seen in sections perpendicular to the acute bisectrix. Which bisectrix is present may best be determined by the fact that the hyperbolæ of the interference figure around the obtuse bisectrix disappear from the field with less rotation of the stage than do those around the acute bisectrix.[1]

376. Sections Perpendicular to the Optic Normal.—The isogyres formed in sections perpendicular, or nearly perpendicular, to the optic normal b (Fig. 597; Cf. Fig. 552) are very indistinct,[2] the part of the skiodrome seen in the field of the microscope presenting a network of lines with practically rectangular intersections. All such sections are characterized by the rapid lighting up of the field upon a very slight rotation of the stage, and the formation of two indistinct, shadowy hyperbolæ, which move off with but little more rotation.

PROBLEMS

With the nicol net, construct the isogyres for 0°, 30°, 60°, and 90° rotation of the sections indicated in Figs. 596, 597, and 598.

Compare the isogyres formed by rotating the nicols through the same angles, leaving the mineral section stationary.

Examine the interference figures in (a) the 1/4 λ mica plate, (b) the 100 face of topaz or enstatite, (c) the gypsum unit retardation plate.

B. Sections Perpendicular to an Optical Plane of Symmetry

377. Sections Perpendicular to the Plane of the Optic Axes.—Of all planes perpendicular to the plane of the optic axes (Figs. 608–609; Cf. Figs. 587–589), the most important are those nearly or quite at right angles to an optic axis.[3] Fig. 608 is the skiodrome of an optically negative crystal, and Fig. 609 of a neutral crystal with an axial angle of 90°. In each case there appears but a single bar, which remains in the field of view during a complete rotation. Both ends are antidrome.

When the principal vibration directions of the crystal and the nicols are parallel, the isogyre is a straight bar which is parallel to the principal section of one of the nicols. If the section is cut exactly at right angles to the plane of the optic axes, the bar, when straight, divides the field symmetrically; if the section is somewhat inclined, the bar, when it straightens out, does not cross the center (Figs. 567, 571, 572, 576).

[1] Cf. the Michel-Lévy (Art. 417) and the Becke (Art. 421) methods for measuring $2E$.

[2] Cf. Art. 364.

[3] Cf. Art. 363.

The most advantageous position for the study of these sections is when the crystal is placed in the diagonal position. When the true optic axial angle is less than 90°, the isogyre takes the form of a hyperbola with the apex of the convex side pointing toward the acute bisectrix.

The smaller the axial angle, the sharper will be the curve, being a right angle when $2V = 0°$,—that is, in a uniaxial crystal. If $2V$ becomes greater, the curvature of the isogyre becomes less and it flattens out more and more until, when $2V = 90°$ (Figs. 589 and 609), it is a straight bar which lies at 45° to the plane of the optic axes when the crystal is in the diagonal position. When $2V$ is greater than 90°, the curve bends in the opposite direction, that is, the acute axial angle becomes the obtuse, and *vice versa*.

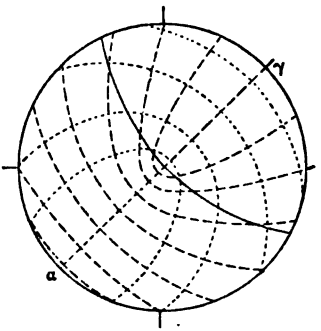


FIG. 608.—Skiodrome of a negative biaxial crystal, section at right angles to one of the optic axes. Only that portion of the construction sphere which represents the field of view of the conoscope is shown. Note that the convex side of the isogyre lies on the side toward the acute bisectrix.

Sections perpendicular to the plane of the optic axes, but intermediate in position between those perpendicular to a bisectrix and perpendicular to an optic axis, show a single isogyre which is straight when it is parallel to the principal section of one of the nicols. The homodrome end shows the direction of the optic axis, and the antidrome end that of the bisectrix. In no case, except that of sections exactly at right angles to the plane of the optic axes (Figs. 577, 581, 582, 586), does the straight isogyre symmetrically divide the field, but lies to one side of the middle. In other positions the bar is curved, and the homodrome end moves less rapidly than the antidrome.

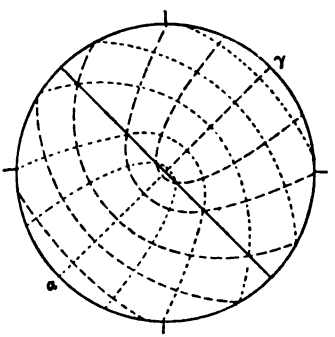


FIG. 609.—Skiodrome of a neutral biaxial crystal ($2V = 90°$) in a section cut at right angles to an optic axis. The isogyre is a straight bar.

C. INCLINED SECTIONS

378. Random Sections.—Among random sections[1] of biaxial minerals in thin rock-slices, the most common, of course, are those that are inclined to the optical symmetry planes, so that, in parallel position, the center of the dark cross, in general, will not be in the center of the field of view,

[1] Fouque et Lévy: *Minéralogie micrographique*, Paris, 1879, 102. Lévy et Lacroix: *Les minéraux des roches*, Paris, 1888. F. Becke: *Zur Unterscheidung ein- und zweiachsiger Krystalle im Konoskop*. T. M. P. M., XXVII (1908), 177–178.

Cf. Art. 362, *supra*.

but will lie to one side, and the isogyre will not divide the field symmetrically. The bar is straight and lies in the center of the field and parallel to the principal sections of the nicols only when the section is perpendicular to a symmetry plane (Figs. 577, 581, 582, 586). More often the section lies to one side, consequently inclined to all three symmetry planes of the optical ellipsoid. In such a case, *the isogyre will cross the center of the field of view at an angle to the vibration planes of the nicols* (Figs. 569, 575, 610.) *In its straight position it will lie to one side of the center* (Figs. 567, 571, 572, 576). When $2V=90°$, the bar will be straight in every position, and when it crosses the center it will make an angle of 45° (Fig. 611). The separation from uniaxial crystals may be made by noting that upon rotating the stage, the homodrome end will move *more* rapidly than the antidrome.[1]

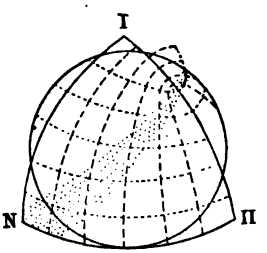


FIG. 610.—Skiodrome of a negative biaxial crystal ($2V=60°$). Inclined section, between the axis and the normal. Isogyre curved.

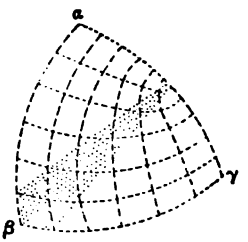


FIG. 611.—Skiodrome of a neutral biaxial crystal ($2V=90°$). Inclined section, between axis and normal. The isogyre forms a straight bar.

PROBLEM

Examine inclined sections of augite and of olivine showing a single bar, and note the difference between the isogyres seen here and those seen in an inclined section of quartz.

379. Equations for the Isogyres or Neutral Curves.—Analytically the isogyres in biaxial crystals may be explained as follows:

(*a*) *Sections at Right Angles to the Acute Bisectrix. The Line Connecting the Points of Emergence of the Optic Axes Forms an Angle* (β) *with the Principal Section of One of the Nicols.*—Let Fig. 612 represent the isogyres seen in the interference figure of a biaxial mineral cut at right angles to the acute bisectrix and turned to a diagonal position, so that the line $O'O$, connecting the points of emergence of the optic axes, makes an angle of β with the principal plane of the polarizer $P'P$. The point R, lying on the neutral curve, is dark, consequently the vibration directions of the ray CR, emerging at R, are bR and aR. According to the Fresnel construction, the vibration direction of any ray CR lies in the plane which bisects the angle $O'RO$ in space,[2] that is, it is the intersection of the two planes

[1] Cf. Art. 372 for uniaxial crystals.

[2] See Art. 349.

ORC and $O'RC$, each of which contains the ray CR and an optic axis (CO or CO'). For small angles we may use the orthographic projection (Fig. 612) instead of the angle in space, whereby the trace of the vibration plane is the line Ra which bisects the angle $O'RO$. Let the coordinates of the point of emergence of the optic axis O be x' and y', and those of the point R be x and y.

From the figure we have

$$\cot Rha = \cot ROd = \frac{dO}{dR} = \frac{x'-x}{y-y'}$$

$$\cot Rfa = \cot RO'k = \frac{O'k}{Rk} = \frac{x'+x}{y+y'}.$$

$Rha = Rfa$, since Ra is at right angles to PP' and bisects the angle $O'RO$, therefore

$$\frac{x'-x}{y-y'} = \frac{x+x'}{y+y'},$$

whereby

$$x'y' = xy. \tag{1}$$

This is the equation of a hyperbola passing through O and O' and referred to its asymptotes. The locus of R, therefore, is a rectangular hyperbola whose asymptotes are parallel and at right angles to the principal sections of the polarizer. The curve represents the position of all points whose vibration directions are parallel to that of the polarizer.

In a similar manner, for $\beta = 90°$, two other hyperbolic branches will be found, also passing through O and O', and having for their asymptotes lines parallel and at right angles to the principal section of the analyzer. These curves represent the positions of all points whose vibration directions are parallel to that of the analyzer.

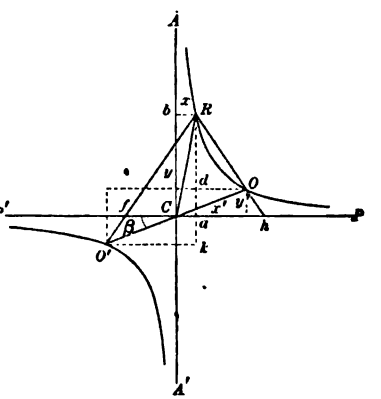


FIG. 612.

The isogyres, therefore, will consist of two rectangular hyperbolæ (four hyperbolic branches), two branches passing through O and two through O'. The bars will appear dark when the nicol prisms are crossed, since the hyperbola, representing the light passing through the polarizer and parallel to its principal section, is exactly covered by the hyperbola representing the light passing through the analyzer and parallel to its principal section. The two being at right angles, all light is extinguished.

When the nicols are parallel, the hyperbolæ will appear light, since along these lines all light is transmitted without change.

(*b*) *Sections at Right Angles to the Acute Bisectrix. Line Connecting the Melatopes Parallel to the Principal Section of Polarizer or Analyzer.*—If the line OO' (Fig. 612) coincides with PP' or AA', $\beta = 0$, y' becomes 0, and equation (1) becomes

$$xy = 0. \tag{2}$$

The hyperbola is reduced to its asymptotes and forms a cross (Fig. 549).

CHAPTER XXXI

DISPERSION OF LIGHT IN CRYSTALS

380. Normal and Anomalous Dispersion.—When a beam of white light is refracted by a transparent medium, it is separated into rays of different wave lengths, consequently, of different colors. For example, in passing obliquely from air to glass, the beam of white light (W, Fig. 613) is separated or **dispersed**[1] into colored rays following the order of the spectrum. Ordinarily the ray bent least from the direct path (having the greatest angle of refraction) is the red, and the one bent most, the violet, but in certain substances the order is different, as was first discovered by Talbot[2] about 1840, and rediscovered by Leroux[3] in 1860. To this phenomenon the name **anomalous dispersion** is given, although there is actually nothing anomalous in the phenomenon. Schuster[4] proposed the term **selective dispersion.**

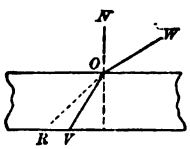


FIG. 613.—Dispersion of light.

As for the cause of normal dispersion, it follows, in isotropic media, very simply from the formula

$$n = \frac{V}{V'},$$

where V' is the velocity of the light of a certain wave length in the given medium. Thus red waves are longer than blue, consequently, of two such

[1] For the theoretical discussion of dispersion see:

L. Lorenz: *Theorie der Dispersion.* Wiedem. Ann., XX (1883), 1–21.

E. Lommel: *Das Gesetz der Rotationsdispersion.* Ibidem, XX (1883), 578–592.

Paul Drude: *The theory of optics.* Translated by Mann and Millikan. New York, 1902, Chapt. V.

Thomas Preston: *The theory of light.* London, 3d ed., 1901, 406–408; 429–430; 477–478; 485–488.

Arthur Schuster: *An introduction to the theory of optics.* London, 1904, Chapt. XI.

Robert W. Wood: *Physical Optics.* New York, 1905, Chapt. V.

A. Winkelmann: *Handbuch der Physik. VI, Optik.* Leipzig, 2 Aufl., 1906, 618–636, 1316–1333.

[2] H. F. Talbot: *Note on some anomalous spectra.* Proc. Roy. Soc., Edinburgh, VII (1872), 408–410.

P. G. Tait: *On anomalous spectra.* Ibidem, 410–412.

Idem: *Light.* Edinburgh, 2d ed., 1889, 171–72.

[3] F. P. Leroux: *Dispersion anomale de la vapeur d'iode.* Comptes Rendus., LV (1862), 126–128.

Idem: *Anomale Dispersion des Ioddampfes.* Pogg. Ann., CXVII (1862), 659–660.

[4] *Loc. cit.*

rays derived from the same beam of white light and therefore having their vibration periods alike, the red will travel farther in a given time. It will therefore differ less than the blue from the distance traveled by the same ray in air and will be less deflected (Fig. 613).

In crystals other than those that are isotropic, the difference in the refractive indices, consequently of the velocity of the light in different directions, has its effect upon the dispersion. For example, in a uniaxial crystal whose value for ω is not very different from that for ϵ, it may be that ϵ is the direction of vibration of the slow ray for light of one color while for another color it is the direction of the fast ray. As a consequence the crystal is positive for the first light and negative for the second. For some intermediate color the crystal must appear isotropic.

In biaxial crystals the effect of dispersion is not so simply shown, the phenomenon depending not only upon the different values of the refractive indices in different directions, but also upon the crystal symmetry. The result is that there may be a dispersion of the optic axial angle, of the bisectrices, or of the axial plane.

DISPERSION IN ORTHORHOMBIC CRYSTALS

381. Dispersion of the Optic Axes.—The dispersion of the optic axial angle, usually called the dispersion of the optic axes, depends upon the fact that the three refractive indices α, β, and γ, are different for different colored rays, consequently V, in the formula

$$\tan^2 V_f = \frac{\alpha^2(\gamma^2-\beta^2)}{\gamma^2(\beta^2-\alpha^2)}, \qquad \text{(Eq. 19, Art. 71)}$$

has different values. In practice it is customary to express this difference by indicating the relation between the two extreme rays. Thus $\rho > v$ means that the angle for the red rays (ρ) is greater than that for the violet (v). The reverse relationship is expressed by $\rho < v$ (Figs. 614–615).

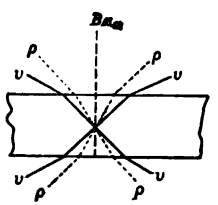


FIG. 614.—Dispersion of the optic axes. $\rho < v$.

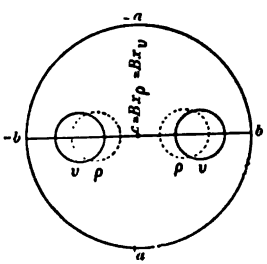


FIG. 615.—Dispersion of the optic axes. $\rho < v$.

Dispersion of the optic axes is found in orthorhombic crystals. The three principal vibration axes always coincide with the three crystallographic axes for all colors, but not necessarily do the same crystallographic axes coincide with the same vibration axes for different colors. Any one of the crystallographic axes may be parallel to the maximum, minimum, or intermediate vibration axis for a certain color of light, but for another wave length it may be parallel to a different vibration axis. The symmetry of the dispersion depends

upon the symmetry of the crystal. Since the principal vibration directions coincide with the crystallographic axes, the symmetry of the crystal will prevent any dispersion of the bisectrices. The interference figures will be symmetrical to two planes which are at right angles to each other, pass through the acute bisectrix, and contain the axes of the crystal. One plane is the plane of the optic axes, the other, the plane at right angles to it.

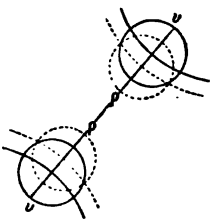


FIG. 616.

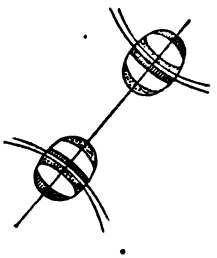


FIG. 617.

FIGS. 616 and 617.—Interference figure produced by dispersion of the optic axes. $\rho < v$.

The effect on the interference figure is shown diagrammatically in Figs. 615–617. Under the microscope it will be seen that the color whose dispersion is least always lies immediately adjacent to the hyperbolæ on the concave sides (Fig. 617). The curves around the melatopes will be of equal size, and the colors in the two will be symmetrical with respect to the plane of the optic axes. They will also be symmetrical to the plane through the bisectrix and at right angles to the plane of the optic axes.

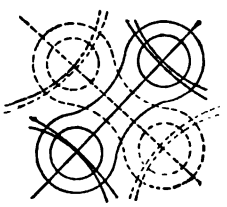


FIG. 618.—Crossed axial plane dispersion.

382. Crossed Axial Plane Dispersion.—In a few crystals the refractive indices for light of different colors vary so unequally that, for example, the axis of least ease of vibration changes position with that of intermediate ease. As a consequence the plane of the optic axes may come to lie at right angles to its former position and we have crossed axial plane dispersion[1] (Fig. 618). At some intermediate position, that is with some intermediate color, the crystal must be uniaxial. For example, in brookite (Figs. 619–624) we have for monochromatic light:

[1] A. E. H. Tutton: *Allgemeine Erklärung des Phänomens der Dispersion in gekreuzten Axenebenen.* Zeitschr. f. Kryst., XLII (1907), 554–557.

Idem: *The optical constants of gypsum at different temperatures, etc.* Proc. Roy. Soc. London, (A), LXXXI (1908), 40–57.

Idem: *Crystallography and practical crystal measurement.* London, 1911, 784–798.

FIG. 619.

FIG. 620.

FIG. 621.

FIG. 622.

FIG. 623.

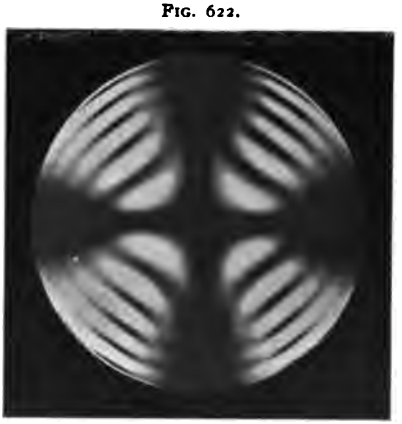


FIG. 624.

FIGS. 619–624.—Crossed axial plane dispersion in brookite. Plate cut at right angles to the acute bisectrix, normal position.

FIG. 619.—Wave length of source of light 645–620$\mu\mu$. Plane of the optic axes 001.

FIG. 620.—Wave length, 590–575$\mu\mu$. Plane of the optic axes 001.

FIG. 621.—Wave length, 575–555$\mu\mu$. Uniaxial.

FIG. 622.—Wave length, 560–540$\mu\mu$. Plane of the optic axes 010.

FIG. 623.—Wave length, 540–520$\mu\mu$. Plane of the optic axes 010.

FIG. 624.—Brookite by white light. Crossing of planes of the optic axes. (*Facing Page* 444.)

Light	Wave length $\mu\mu$	Color	2E	Plane of optic axes
Li	670	Red	58° 0′	001 (Fig. 619)
Na	589	Yellow	38° 10′	001 (Fig. 620)
	555	Yellowish green	0°	a axis (Fig. 621)
Tl	535	Green	21° 40′	010 (Fig. 623)
	525	Green	33° 0′	010

When white light is used, the interference figure is complicated (Fig. 624), and is the resultant of the superposed figures derived from the different colored rays.

PROBLEM

Examine the interference figures of thomsonite ($\rho < v$, strong), olivine ($\rho < v$, distinct); anhydrite ($\rho < v$, strong); sillimanite ($\rho > v$, strong).

Examine the interference figure of a (100) section of brookite ($\rho > v$) by red, yellowish green, green, and white light, and note the difference in the appearance of the interference figures.

DISPERSION IN MONOCLINIC CRYSTALS

383. Dispersion of the Bisectrices.—In the monoclinic system there is but a single plane of symmetry, namely, the plane perpendicular to the b axis. This axis is the direction of one of the principal vibration axes or axes of the optical ellipsoid. It has, therefore, the same position for all colors of light, but the other two axes may be dispersed differently in the plane of symmetry. This dispersion is known as the dispersion of the bisectrices. It is always accompanied by a dispersion of the optic axes. Three cases may occur:

384. Case I. Inclined Dispersion (of Both Bisectrices).—When the b axis of the crystal coincides with the β axis of the optical ellipsoid, the plane

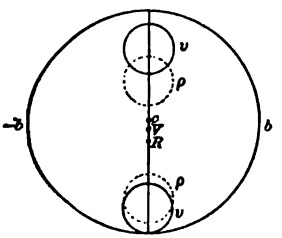


FIG. 625.

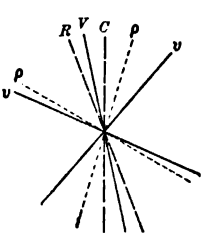


FIG. 626.

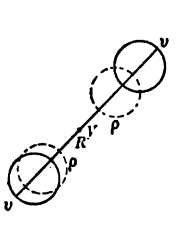


FIG. 627.

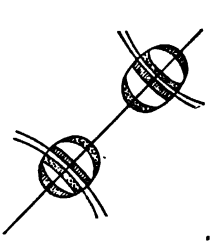


FIG. 628.

FIGS. 625 to 628.—Inclined dispersion in monoclinic crystals. $\rho < v$.

of the optic axes (plane of α and γ) coincides with the symmetry plane (010). Dispersion in such crystals, being symmetrical only to the plane, is produced by the greater or less displacement of the axes of the optical ellipsoid for different colors, consequently the bisectrices of the optic angles for different

colors differ by some angle in this plane (Figs. 625–628). Such dispersion was called *dispersion inclinée* by Des Cloizeaux.[1] Since it is always accompanied by a dispersion of the optic axes, the interference figure produced is no longer symmetrical with respect to a plane at right angles to the plane of the optic axes, although it must necessarily remain so with respect to the latter plane (Figs. 625 and 627). The isochromatic curves around the melatopes, produced by the displacement of axes having larger or smaller angles between them, will be larger and more elongated at one melatope than at the other. The colors, likewise, will be more intense, and their sequence different. If the dispersion of the bisectrices, as well as that of the axes, is great, the relations of the red and the violet to the hyperbolæ will be reversed; in one the red will lie on the concave side, in the other the violet. Usually, however, the dispersion is too small to reverse the order of the colors, although the intensity may be different, for example, in gypsum (Fig. 628).

PROBLEM

Examine the interference figure of gypsum ($\rho < \upsilon$) and of diopside ($\rho > \upsilon$), first by white light, then with color screens.

385. Case II. Horizontal Dispersion (of the Acute Bisectrix).—When β and the acute bisectrix lie in the plane of symmetry, and the third axis of the optic ellipsoid coincides with crystallographic *b*, the plane of the optic axes

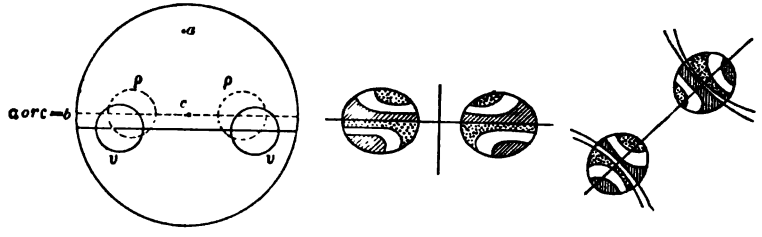


FIG. 629. FIG. 630. FIG. 631.

FIGS. 629 to 631.—Horizontal dispersion of the acute bisectrix. $\rho < \upsilon$.

will lie at right angles to the symmetry plane for all colors, but the inclination of this plane to the vertical axis may differ for different colors (Fig. 629). The figures, produced by the illumination of the mineral by the spectral colors in order, will successively occupy parallel positions farther and farther removed from the starting point (Figs. 632–637). This dispersion, consequently, is called horizontal dispersion (*dispersion horizontale*, Des Cloi-

[1] A. Des Cloizeaux: *Sur l'emploi des propriétés optiques biréfringentes pour la détermination des espèces cristallisées.* Ann d. Mines, XIV (1858), 341–342.

Idem: *Mémoire sur l'emploi du microscope polarisant et sur l'étude des propriétés optiques biréfringentes propres a déterminer le système cristallin dans les cristaux naturel ou artificiels.* Ibidem, VI (1864), 565–569.

FIG. 632.

FIG. 633.

FIG. 634.

FIG. 635.

FIG. 636.

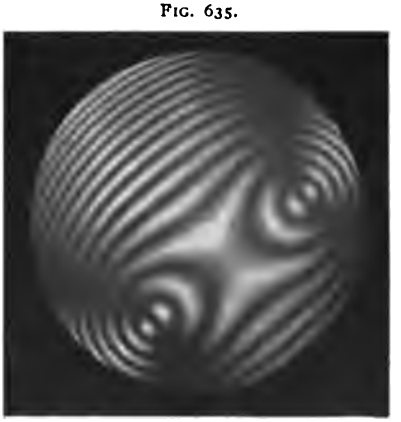


FIG. 637.

FIGS. 632 637.—Effect of horizontal dispersion in monoclinic crystals. Interference figure of rubidium platino cyanide. Diagonal position.

FIG. 632.—Wave length of source of light 450–460$\mu\mu$.

FIG. 633.—Wave length 480–490$\mu\mu$.

FIG. 634.—Wave length 520–530$\mu\mu$.

FIG. 635.—Wave length 546.1$\mu\mu$ (Green, Hg light).

FIG. 636.—Wave length 560–570$\mu\mu$.

FIG. 637.—Wave length 600–620$\mu\mu$.

(*Facing Page* 446.)

zeaux).[1] The interference figure in white light, the resultant of those for all the colors, will still be symmetrical with respect to the plane of symmetry, but not in a direction at right angles to it (Figs. 630–631). This dispersion is most clearly seen when the interference figure is placed with its principal sections parallel to those of the nicols (Fig. 630). The dark bar, passing through the two melatopes, will be bordered at the top and bottom by different colors.

PROBLEM

Note the horizontal dispersion shown by the interference figure of sanidine; of muscovite.

386. Case III. Crossed Dispersion (of the Obtuse Bisectrix).—When the acute bisectrix ($=\alpha$ or γ) coincides with crystallographic b, it retains its position because the refraction is the same for light of all colors. The obtuse bisectrix, and β and γ or α, lie in the symmetry plane, whereby, in this case,

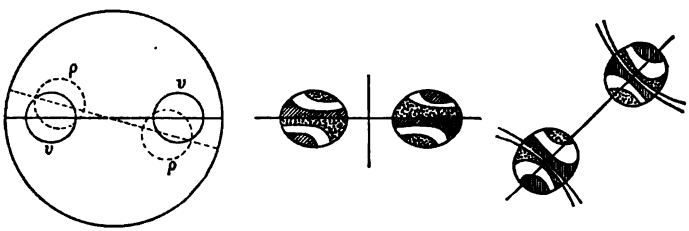


FIG. 638. FIG. 639. FIG. 640.

FIGS. 638 to 640.—Crossed dispersion of the obtuse bisectrix.

the obtuse bisectrix and the optic angle are dispersed. This dispersion was called *dispersion tournnate* or *dispersion croisée* by Des Cloizeaux,[2] since by successive monochromatic illuminations in the order of the spectrum, the interference figure will successively occupy the positions between $\rho\rho$ and vv in Fig. 638, and will appear to rotate about the b axis. In white light the distribution of color in the two melatopes differs from right to left and from top to bottom (Figs. 639–640). The colors occur in inverse order, so that if the figure is rotated through 180° about the acute bisectrix, each melatope will occupy the position previously held by the other, and the color distribution will be exactly the same as before. When the interference figure is in parallel position, so that its bars coincide with the principal planes of the nicols, the black cross is unsymmetrically bordered by colored bands (Fig. 639). That is, the upper left and lower right sides are bordered by one color, and the upper right and lower left by another.

[1] *Op. cit.*
[2] *Op. cit.*

PROBLEM

Examine a crystal of borax between crossed nicols.

DISPERSION IN TRICLINIC CRYSTALS

387. Unsymmetrical Dispersion.—In triclinic crystals the shape of the optical ellipsoid (indicatrix) as well as its position in the crystal, differs for different colors, whereby the three principal vibration axes are all more or less dispersed. An interference figure from such a section of such a crystal cut at right angles to the acute bisectrix will show, in white light, a totally unsymmetrical figure (Figs. 641–642).

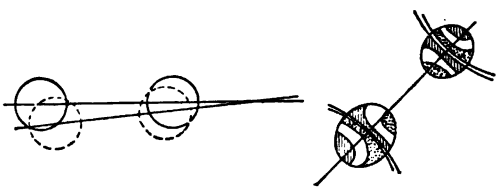


FIG. 641. FIG. 642.

FIGS. 641 and 642.—Unsymmetrical dispersion in triclinic crystals.

388. Effect of Temperature Change on Dispersion.—The indices of refraction of transparent substances are generally decreased by an increase in temperature,[1] therefore the axes of the optical ellipsoid are changed and, consequently, the optic axial angle. This change may be so great that the axial plane for one temperature may lie at 90° to that at another, as was first observed by Brewster[2] in glauberite. In such crystals, at some intermediate temperature, the interference figure is uniaxial; for example, in gypsum between 105° and 115°, the value differing slightly in different crystals.

[1] A. E. H. Tutton: *Crystallography and practical crystal measurement.* London, 1911, Chapter XLVIII. Also plate III, p. 790.

Idem: *Op. cit. supra,* in Art. 382.

E. H. Kraus and L. J. Youngs: *Ueber die Aenderungen des optischen Achsenwinkels in Gips mit der Temperatur.* Neues Jahrb., 1912 (I), 123–146.

Edward H. Kraus: *Die Aenderungen des optischen Axenwinkels im Glauberit mit der Temperatur.* Zeitschr. f. Kryst., LII (1913), 321–326.

[2] Sir David Brewster: *On the action of heat in changing the number and nature of the optical or resultant axes of glauberite.* Phil. Mag., (3), I (1832), 417–420.

CHAPTER XXXII

THE PETROGRAPHIC MICROSCOPE AS A CONOSCOPE AND THE METHODS FOR OBSERVING INTERFERENCE FIGURES

389. Observing Interference Figures with the Microscope.—A general description of the petrographic microscope as a conoscope was given in Article 354, and it was there mentioned that the image of the interference figure is formed in the tube too far below the ocular to be seen through it without the insertion of an accessory lens (Fig. 511). The fact that it could there be seen was long unrecognized, for although interference figures had been obtained and studied in large crystal sections by means of the Nörrenberg polarization apparatus, and later by Groth's[1] conoscope, no method for their observation under the microscope was given until Lasaulx published his description in 1878.

390. Lasaulx Method (1878).—Lasaulx[2] recognized the fact that interference figures could be seen by simply removing the ocular. He found that a high-power objective and a stronger system of condensing lenses than up to that time had been placed above the polarizer, would increase the size of the figures. Such condensing lenses are now universally attached to petrographic microscopes and Lasaulx's method of observing interference figures is still commonly used. It is simply necessary to remove the ocular and look down the tube at the mineral between crossed nicols. The objection to this method of observation is that by removing the ocular, the cross-hairs or micrometer scale are removed also, and no means remains for measuring the positions of the points of emergence of the optic axes. Various kinds of oculars have been invented to overcome this, and these will be described in the chapter dealing with the measurement of the optic axial angle. The difficulty with most of these devices is that while the interference figure is magnified, it is also made less sharp.

391. Bertrand Method (1878).—Independently of Lasaulx, Bertrand[3]

[1] P. Groth: *Ueber Apparate und Beobachtungsmethoden für krystallographisch-optische Untersuchungen.* Pogg. Ann. CXLIV (1872), footnote, p. 38.

[2] A. v. Lasaulx: *Ueber die Verwendung des Mikroskopes als Polarisationsinstrument im convergenten Lichte und ein neues Mikroskop zu mineralogischen Zwecken.* Neues Jahrb., 1878 (March 7), 377–380.

[3] Emile Bertrand: *De l'application du microscope à l'étude de la minéralogie.* Bull. Soc. Min. France, I (1878), 22–28, 93–96.

Idem: *Sur l'examen des minéraux en lumière polarisée convergente.* Ibidem, VIII (1885), 29–31.

Idem: *Nouvelle disposition du microscope permettant de mesurer l'écartement des axes optiques et les indices de réfraction.* Ibidem, VIII (1885), 377–383.

L. Calderon: Review of articles in Bull. Soc. Min. France, I (1878), 22–28, 96–97, in Zeitschr. f. Kryst., III (1879), 642–644.

in the same year, described a method by which interference figures not only could be seen, but could be measured. So far back as 1844, Amici[1] had used an auxiliary lens in the conoscope for observing and magnifying the interference figures of mineral sections. The same principle was used by Bertrand in the polarizing microscope. He placed, beneath the ocular, an achromatic lens, which, with the eyepiece, formed a weak independent microscope, and was so arranged that it could be focussed upon the primary interference figure lying within the tube (Fig. 511). The image appeared enlarged, and, since the ocular was used, the distance between the points of emergence of the optic axes could be measured. Above the polarizer he used two very short focus lenses to produce strongly convergent light, thus enlarging the figure.

In his second paper, Bertrand[2] improved his lens by fastening it in a slider so that it could be inserted or removed quickly from the axis of the microscope. While the interference figures produced by the Bertrand lens are not quite so sharp as those seen by the Lasaulx method, yet the convenience of observation, and the ease of measuring are of such great advantage, that all modern microscopes are fitted with them. Such a lens should be permanently attached to the microscope in a slider similar to that carrying the analyzer, whereby it may be quickly inserted or removed. It should be so arranged in a sliding sleeve that its distance from different oculars may be regulated to bring the interference figure sharply into focus without the necessity of raising the ocular in the tube, an improvement due to the suggestion of Laspeyres.[3]

The Bertrand lens is sometimes called the Amici-Bertrand lens, incorrectly, according to Klein,[4] for the Amici lens had been in use for thirty-four years without it having occurred to any one before Bertrand, to apply it to the polarizing microscope.

392. Klein Method (1876) 1878.—In 1876, according to Cohen,[5] Klein had discovered a similar method, although he did not publish it until 1878.[6]

[1] M. Amici: *Note sur un appareil de polarisation.* Ann. d. Chim. et Phys., XII (1844)' 114–120.

Translation of same: *Ueber einen Polarisations-Apparat von Hrn. Amici.* Pogg. Ann., LXIV (1845), 472–475.

[2] Emile Bertrand: *Op cit.*, I (1878), 96–97.

[3] H. Laspeyres: *Mineralogische Bemerkungen. Interferenzbilder des Piemontit.* Zeitschr. f. Kryst., IV (1880), 460–464.

[4] C. Klein: *Ueber das Arbeiten mit dem in ein Polarisations instrument umgewandelten Polarisationsmikroskop und über eine dabei in Betracht kommende vereinfachte Methode zur Bestimmung des Charakters der Doppelbrechung.* Sitzb. Akad. Wiss., Berlin, 1893 (I), footnote 3, p. 224.

[5] E. Cohen: *Zusammenstellung petrographischer Untersuchungsmethoden.* 1st ed., 1884, footnote 1, page 12.

[6] C. Klein: *Ueber den Feldspath im Basalt vom Hohen Hagen bei Göttingen, etc.* Nachr. Gesell. Wiss. Göttingen, 1878, 461.*

Instead of using a lens within the tube, he found that the interference figure could be seen by holding a hand lens at the proper focus above the Ramsden disk of the ocular, or even by simply placing the eye at or beyond the distance of distinct vision, and looking down upon the Ramsden disk. Later[1] he mentioned another simple method by means of which the interference figure might be magnified, namely, by placing directly on the regular eyepiece another Huyghens' ocular. By this method the cross-hairs or micrometer scale remain in the field, and the interference figure may be measured. The method is not applicable to all microscopes since the focal plane of the upper ocular, which should fall exactly in the Ramsden disk, does not always do so. If it falls below, the upper ocular may be slightly raised; if it falls above, the ocular cannot be used unless it is reversed, whereby the cross-hairs disappear from view.

393. Laspeyres Method (1880).—Laspeyres[2] attached his Bertrand lens to the bottom of the inner tube which regulates the tube length. By this means the Betrand lens and the ocular move together. This is practically the arrangement adopted in modern petrographic microscopes with the exception that the distance between the ocular and the Bertrand lens can be regulated to permit the use of oculars of different magnifications.

Laspeyres also used a combination of the Lasaulx and the Bertrand methods, removing the ocular but using the Bertrand lens. He claimed that this arrangement gave him a large, sharp, and highly magnified field, although it did not permit the use of cross-hairs. He obtained different magnifications by raising or lowering the Bertrand lens, and obtained in the field, with a Hartnack microscope, both melatopes in a crystal of topaz ($2E=120°$). This combination method does not work equally well with every microscope, the effectiveness depending largely upon the character of the Bertrand lens.

In the same article, Laspeyres compared the Lasaulx, Bertrand, and Klein methods, and found that the first and third gave the sharpest figures, though small in size, but did not permit the use of cross-hairs; the Bertrand method gave greater magnification and preserved the cross-hairs, but the image was diminished in brightness and sharpness.

394. Bertrand Method (1880).—Bertrand[3] made the discovery, in 1880, that in the bubbles which are found so frequently in the balsam between the mineral section and the cover-glass, one may see an interference figure of the mineral lying below it. The bubble acts as a lens of short focal length and takes the place of the objective, while the ocular and objective combined act as do the Bertrand lens and the ocular in the usual Bertrand method.

[1] C. Klein: *Ueber das Arbeiten, etc.*, page 226, *Cit. supra.*

[2] H. Laspeyres: *Mineralogische Bemerkungen. Interferenzbilder des Piemontit.* Zeitschr. f. Kryst., IV (1880), 460–461.

[3] Emile Bertrand: *De l'application du microscope à l'étude de la minéralogie.* Bull. Soc. Min. France, III (1880), 93–96.

395. Schroeder Van der Kolk Method (1891).—The method just mentioned was carried a step farther by Schroeder van der Kolk.[1] He placed a drop of glycerine upon the cover-glass of the slide to be examined, and stirred it rapidly with a thin rod so that it became filled with very small bubbles. Over this he placed a cover-glass and, between crossed nicols and with a medium power objective (Zeiss B, or Hartnack 4), he focussed upon the bubbles. The tube was then depressed a very little, since the bubbles themselves act as biconcave lenses, and there appeared in each bubble a fine interference figure. Though the images are small, the optical character can be determined readily. Even the dispersion in a flake of muscovite can be seen if it is examined without a cover-glass.

This method of observing interference figures possesses, in certain cases, very great advantages. Since no change is made in the arrangement of the microscope, it is possible to observe the mineral and the interference figures at the same time, and likewise to see, simultaneously, the interference figures of a number of grains, whereby their optical orientation can be compared. The method is especially of advantage in examining fragments which are too small to produce, by any other method, even with the use of a diaphragm, interference figures undisturbed by the surrounding grains. Schroeder van der Kolk found, further, that mineral fragments must be somewhat larger than, the bubble to show the figures clearly, and that the farther the bubble is removed from the mineral, the larger must the latter be to give a good figure. It is, therefore, of advantage, for very small grains, to remove the cover-glass of the rock-slice. Bubbles as small as 0.002 mm. in diameter can be used.

To avoid the necessity each time of preparing anew a glycerine foam Schroeder v. d. Kolk used the following simple device: He placed a drop of Canada balsam on an object-glass and, if necessary, cooked it. After producing foam by rapid stirring, he placed a cover-glass upon it, and the instrument was complete. To use it he placed it, cover-glass downward, upon the rock-section, and shoved it into such positions that bubbles appeared over the desired spots.

In place of bubbles small drops of a highly refracting fluid can be used, but in that case the tube of the microscope must be raised instead of lowered to get the image into focus. To make these bubbles of very small size a little α monobromnaphthaline is placed on an object-glass. Above it, at a distance of 1 cm., the section to be examined is held with the cover-glass down. The α monobromnaphthalene is heated, and the distillation product condenses, in fine bubbles, on the cover-glass of the mineral slice, especially if it is cooled somewhat by placing a few drops of water on the top surface.

[1] J. L. C. Schroeder van der Kolk: *Ueber eine Methode zur Beobachtung der optischen Interferenzerscheinungen im convergenten polarisirten Lichte, insbesondere in Gesteinsschliffen.* Zeitschr. f. wiss. Mikrosk., VIII (1891), 459–461.

396. Czapski Ocular (1893).—Czapski,[1] in 1893, made an improvement in the method of observing interference figures by the Lasaulx method, by inserting a diaphragm in the focal plane of a Ramsden eyepiece (Fig. 643). A shoulder holds the ocular in such a position above the eyepiece of the microscope, that the diaphragm *i* lies in the plane of the Ramsden disk. Upon removing the ocular itself, the interference figure of the central grain only appears, undisturbed by the minerals surrounding it.

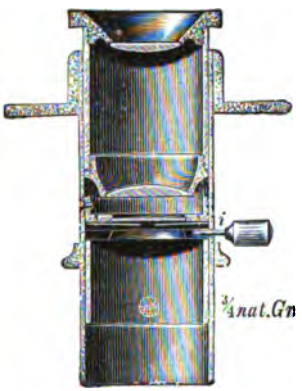


FIG. 643.—Czapski ocular. 3/4 natural size. (Fuess.)

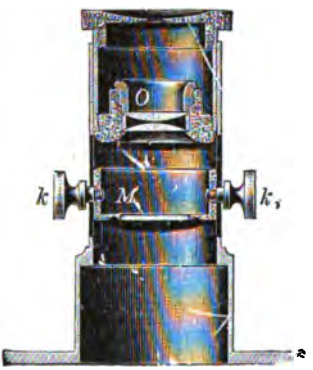


FIG. 644.—Becke-Klein magnifier. 3/4 natural size. (Fuess.)

397. Becke-Klein Magnifier (1895).—Becke[2] combined Klein's method of observing interference figures with Czapski's, by which means sharp and magnified figures may be obtained, and the cross-hairs or micrometer scale in the ocular retained. Above a Czapski ocular with iris diaphragm (Fig. 643) is placed a magnifying glass (*O*, Fig. 644) in a casing, and so adjusted, that when placed on the shoulder of the Czapski ocular, the diaphragm of the latter appears in sharp focus. The micrometer scale *M* is then adjusted by means of the screws *k* and k_1, until it lies, without parallax, in the plane of the image. To this apparatus Becke gave the name of Klein magnifier (Klein'sche Lupe) since Klein[3] first made use of a lens above the Ramsden disk for examining interference figures.

398. Lenk-Lasaulx Method (1895).—To preserve the sharpness of the

[1] S. Czapski: *Ueber Einrichtungen behufs schnellen Ueberganges vom parallelen zum convergenten Lichte und die Beobachtung der Axenbilder von sehr kleinen Krystallen in Polarisations-Mikroskopen.* Zeitschr. f. Kryst., XXII (1893), 158–162.
C. Leiss: *Die optischen Instrumente, etc.*, Leipzig, 1899, 218.

[2] F. Becke: *Klein'sche Lupe mit Mikrometer.* T. M. P. M., XIV (1894–95), 375–378.
C. Leiss: *Die optischen Instrumente, etc.*, Leipzig, 1899, 218–219.

[3] See Art. 392.

interference figure and still be able to measure it, Lenk[1] used the Lasaulx method but inserted a micrometer scale engraved on glass between the lenses of the *objective* (Fuess No. 7) and in its upper focal plane (or above the upper lens in the Zeiss DD objective). The divisions of the micrometer scale may be made clearer, without reducing the sharpness of the interference figure, by means of a long focus lens placed in the tube of the microscope.

399. Sommerfeldt Condenser (1895).—For the examination of mineral grains which are so minute that it is necessary to use a very small stop to cut out interference from adjacent minerals, thereby reducing the light too much to give a figure capable of examination by the Bertrand lens, Sommerfeldt[2] proposed a new condenser made on the principle of that of ten Siethoff.[3] On the flat side of the lower lens of a condensing system of three plano-convex lenses is added a scale whose image appears in the same plane as the interference figure, as seen by the Lasaulx method. The optic axial angle of sections cut at right angles to the acute bisectrix, therefore, may be measured by this means in the same manner as though the Bertrand lens were used.

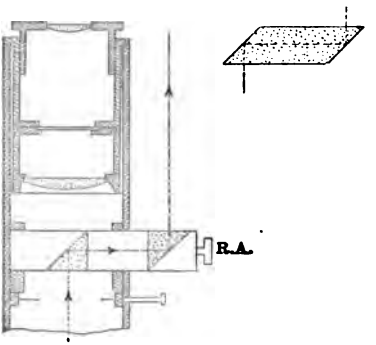


FIG. 645.—Wright reflecting prism.

400. Wright-Lasaulx Method (1906).—To avoid the necessity of removing the ocular for the observation of interference figures by means of the Lasaulx method, Wright[4] suggested the use of the reflecting apparatus shown in Fig. 645. By means of two prisms set in a slider, the light is reflected to the side of the tube where the interference figure may be seen without the removal of the ocular. The loss of light by reflection is not great enough appreciably to decrease the brilliancy of the interference figure. A specially ground rectangular prism of the form shown at the right was suggested by Wright[5] as an improvement, requiring to be adjusted but once, and needing no special arrangement of bearings for the prisms.

401. Johannsen Auxiliary Lens (1912).—By means of a small auxiliary

[1] H. Lenk: *Messung des Winkels der optischen Axen im Mikroskope.* Zeitschr. f. Kryst., XXV (1895–96), 379–380.

C. Leiss: *Einrichtung zur Axenwinkelmessung am Mikroskop nach H. Lenk.* Neues Jahrb. B. B., X (1895–96), 429–430.

[2] E. Sommerfeldt: *Die mikroskopische Achsenwinkelbestimmung bei sehr kleinen Kristallpräparaten.* Zeitschr. f. wiss. Mikrosk., XXII (1905), 356–362.

[3] Art. 254.

[4] Fred. Eugene Wright: *A modification of the Lasaulx method for observing interference figures under the microscope.* Amer. Jour. Sci., XXII (1906), 19–20.

[5] Idem: *In litteris*, Nov. 13, 1912.

lens placed directly upon the cover-glass of the mineral under examination, Johannsen[1] was able to obtain interference figures of extremely small grains, and at the same time observe the mineral by parallel light. The method depends on the same principle as does that of Schroeder van der Kolk,[2] but the lens is constructed of glass. Johannsen heated a small glass rod in the flame of a Bunsen burner, and drew it out into a thin thread, which was again heated and again drawn out to hair-like thinness. After breaking the glass into pieces 3 or 4 cm. in length, their extremities were held an instant in the edge of the flame, whereby truly spherical globules were produced at each end. After preparing a number of these spherical lenses, they were examined under the microscope, and all that were not perfect, or which contained bubbles, were rejected. Likewise only those which had a diameter of less than 0.1 mm. were retained. If such a lens is placed directly in contact with the cover-glass of the mineral to be examined, and the microscope arranged with crossed nicols, ocular, and a medium or low-power objective (No. 0 to 4, Fuess), there will appear in it a small but perfect interference figure. The microscope should be focussed upon the glass sphere, after which the tube should be slightly raised. A condensing lens is not necessary, but without it part of the figure is cut off by the dark border. The optical character and dispersion can be determined as well by this method as by the use of a Bertrand lens, and the figure is decidedly sharper. By its means it is possible to examine the interference figures, undisturbed by surrounding minerals, of grains smaller than is possible by the Lasaulx, Klein, or Bertrand methods, and it possesses the further advantage that the mineral and the interference figure can be seen at the same time. By shifting the lens, the optical orientation of all of the grains in a section can be determined. When used with a von Fedorow stage, it is possible to examine interference figures with low-power objectives, a considerable advantage in roughly determining the orientation of the mineral and, consequently, the setting of the stage.

To permit the rapid examination of a slide, the rod to which the glass globule is attached may be fastened with a bit of soft modeling wax to the edge of the stage at such an angle that the lens rests against the cover-glass of the mineral section. The latter may now be shifted around the stage as much as desired, bringing, successively, the different mineral constituents under the lens, which remains undisturbed in the center of the field. Another method is to attach the rod of the lens, by means of wax, to the rim of a cork or wood ring, allowing the lens to project toward the center, with the rod so tilted that the lens rests on the same plane as the bottom of the ring. This method protects the delicate glass rod better than the first, but is not quite

[1] Albert Johannsen: *An accessory lens for observing interference figures of small mineral grains.* Jour. Geol., XXI (1913) 96–98.

[2] Art. 395, *supra.*

so convenient, since both rock section and cork must be moved when it is desired to place different minerals in the center of the field.

402. Orientation of Image in Relation to Object.—In determining the direction of inclination of the optic axes or the bisectrix, it is important to know the orientation of the interference figure in relation to the object itself or to its image. Klein[1] tabulated the results for the Lassaulx, Klein, and Bertrand methods. The table is here extended to include several others.

Method	Position of image in relation to object	Position of interference figure in relation to object	Position of interference figure in relation to image
Microscope as ordinarily used	Inverted		
Lasaulx method		Parallel	Inverted.
Klein method		Inverted	Parallel.
Bertrand (1878)		Inverted	Parallel.
Laspeyres		Inverted	Parallel.
Schroeder v. d. Kolk		Parallel	Inverted.
Becke-Klein		Inverted	Parallel.
Lenk-Lasaulx		Parallel	Inverted.
Sommerfeldt-Lasaulx		Parallel	Inverted.
Wright-Lasaulx		Parallel	Inverted.
Johannsen		Inverted	Parallel.

[1] C. Klein: *Ueber das Arbeiten mit dem in ein Polarisations instrument umgewandelten Polarisationsmikroskop und über eine dabei in Betracht kommende vereinfachte Methode zur Bestimmung des Charakters der Doppelbrechung.* Sitzb. Akad. Wiss. Berlin, 1893 (I), 227.

E. Weinschenk: Review of above. Zeitschr. f. Kryst., XXV (1896), 607–609.

CHAPTER XXXIII

DETERMINATION OF THE OPTICAL CHARACTER OF A CRYSTAL BY MEANS OF ITS INTERFERENCE FIGURE

403. Positive and Negative Minerals.—Biot,[1] in 1814, discovered that quartz and beryl acted differently, in polarized light, toward minerals whose action was known, and distinguished them as *attractive* and *repulsive.*[2] For these terms Brewster[3] substituted *positive* and *negative;* terms which have continued in use to the present time. As mentioned in Arts. 51 and 75, uniaxial crystals are considered positive when crystallographic *c* is the direction of vibration of the slow ray, and biaxial crystals are considered positive when the slow ray (c) vibrates parallel to the acute bisectrix.

Various accessories may be used in making the determination of the optical character of a crystal, the most common being the mica plate, the gypsum plate, and the quartz wedge. The history of the introduction of these accessories was fully given above;[4] here their application to the determination of the optical characters of crystals, by means of their interference figures, will be considered.

Uniaxial Crystals

404. Quarter Undulation Mica Plate.—Although the quarter undulation mica plate was introduced as a compensator by Airy,[5] in 1831, it was first applied to the determination of the optical character of crystals by Dove[6] in 1837.

If a mica plate is placed above a mineral giving a uniaxial interference cross, and in such a position that its vibration directions lie at 45° to those of the nicols, it will be found that the center of the cross is broken apart and

[1] J. B. Biot: *Mémoire sur la découverte d'une propriété nouvelle dont jouissent les forces polarisantes de certains cristaux.* (Read April 25, 1814.) Mém. Acad. France, XIII, Année 1812, pt. ii, Paris, 1814, 19–26.

Idem: *Traité de physique.* Paris, 1816, IV, 420–422, 543–566.

[2] Idem: *Addition au Mémoire, Sur les deux genres de polarisation exercés par les cristaux doués de la double réfraction.* (Read May 15, 1814.) Ibidem, 27–30, especially 30.

[3] David Brewster: *On the laws of polarization and double refraction in regularly crystallized bodies.* Phil. Trans. Roy. Soc. London, CVIII (1818), 199–273, especially 219.

[4] Arts. 293, 294, and 295.

[5] Art. 293.

[6] H. W. Dove: *Ueber den Unterschied positiver und negativer einaxiger Krystalle bei circularer und bei elliptischer Polarisation.* Pogg. Ann., XL (1837), 457–462.

two dark spots appear, their positions depending upon the optical character of the mineral under examination. The phenomenon observed may be explained as follows:

In Fig. 646 the two rays emerging at O are traveling with the same velocity, consequently there is no difference of phase between them. This spot, therefore, will appear dark.[1] At a', where the retardation is $1/4\,\lambda$, the extraordinary ray will be vibrating in the plane Oa', and the ordinary ray, perpendicular to this plane. If the crystal is negative, the extraordinary ray will be the fast ray ($\epsilon < \omega$, $E > O$), and the ordinary ray will emerge one-fourth of a wave length behind it. In the diagram, the ease of vibration in this quadrant may be represented by an ellipse in which the diameters represent both the vibration directions and the ease of vibration of the fast and slow rays. The retardation is not shown in the figure. At another point a of the interference figure, the extraordinary ray will be vibrating in the plane Oa. The velocity ellipse, therefore, in this quadrant of the interference

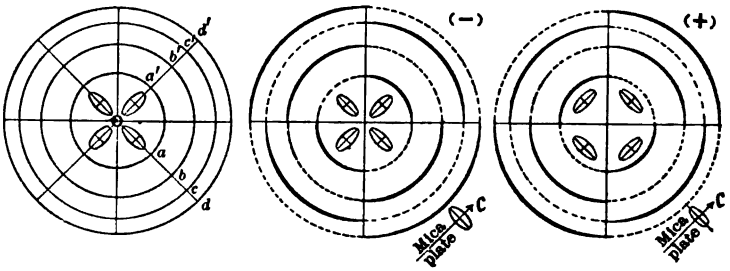


FIG. 646. FIG. 647. FIG. 648.

FIGS. 646 to 648.—Positive and negative uniaxial interference figures.

figure, will lie at right angles to that in the northeast quadrant. In the southwest quadrant, the vibration directions are parallel to those in the northeast, and those in the northwest, parallel to those in the southeast.

If, now, a quarter undulation mica plate, with its slow ray vibrating in a direction at right angles to its long edge (Fig. 647), is placed over the mineral producing the interference figure, the retardation, at the point where it was originally zero, is now added to that of the mica plate, and the sum of $1/4\,\lambda$ and o equals $1/4\,\lambda$.[2] At a, where the retardation was $1/4\,\lambda$, it becomes $1/2\,\lambda$, since the vibration directions in that quadrant are parallel to those in the mica. At a', however, where the retardation was the same, it now becomes zero, since the mica plate, also $1/4\,\lambda$ but with vibrations in opposite directions, has produced exact compensation at this point. In a similar manner b, originally $1/2\,\lambda$, becomes $3/4\,\lambda$; c becomes λ; and so on; while b', originally $1/2\,\lambda$, becomes $1/4\,\lambda$; c' becomes $3/4\,\lambda$; and so forth.

[1] Art 357.
[2] Art. 286.

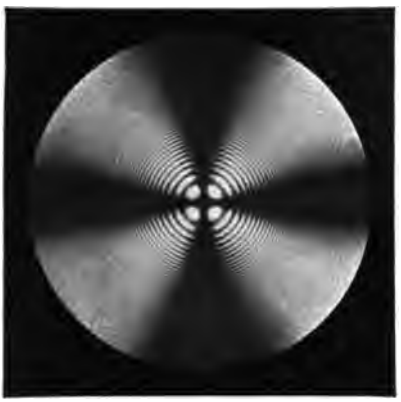


FIG. 649.

FIG. 650

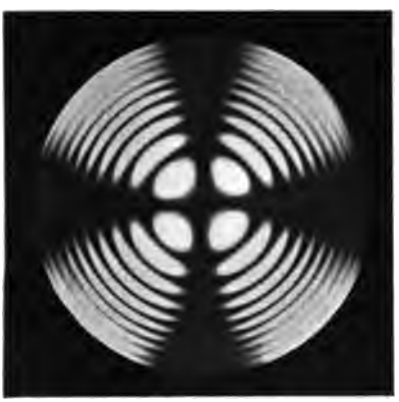


FIG. 651.

FIG. 652.

FIG. 649.—Interference figure of calcite, plate cut at right angles to the optic axis.

FIG. 650.—Ditto, combined with a 1/4 undulation mica plate whose slow vibration direction lies N.E.–S.W. Negative character of the calcite shown by the position of the black dots.

FIG. 651.—Interference figure of zircon in plate cut at right angles to the optic axis; sodium light.

FIG. 652.—Ditto, combined with a mica plate whose slow vibration direction lies N.E.–S.W Positive character of the zircon shown.

(*Facing Page* 458.)

As a result of this addition and subtraction of retardations, the originally symmetrical, negative interference figure (Fig. 649) will appear as shown in Figs. 650 and 647, with two dark spots near the center, lying along the direction of the slow ray of the quarter undulation plate.

In the same manner it may be shown that in a positive uniaxial crystal (Fig. 651), the two dark spots will lie along the direction of the fastest ray of the mica plate (Figs. 648 and 652).

The above description applies, of course, only when the slow vibration direction of the mica lies at right angles to the long direction of the plate, and it is inserted along the northwest-southeast direction. If it is cut with the slow ray parallel to the long edge, the phenomenon will be reversed.

PROBLEMS

Examine the interference figure of calcite; of zircon. Work out theoretically, in the same manner as above, the location of the black spots, using a mica plate cut with the slow ray parallel to the long direction.

405. Unit Retardation Plate.—When a gypsum plate, giving red or violet of the first order, is placed over a uniaxial interference figure, a change, similar to that produced by the mica plate, will take place. Instead of the addition or subtraction of a quarter wave length, however, there is now a change of one wave length. The gypsum plate itself gives a red or violet interference color, consequently the dark center and the isogyres of the interference figure, having no influence upon the overlying plate, become red. In a positive crystal, and with a gypsum plate whose slow ray vibrates at right angles to the long direction of the plate, the position corresponding to a', Fig. 648, originally $1/4\lambda$ becomes $1\ 1/4\lambda$, and a becomes $3/4\lambda$. By an examination of Fig. 453 or the tables in Arts. 276–277, it will be seen that an increase of a quarter of a wave length retardation will change the first order red to blue, and a decrease of a quarter wave length will change it to yellow. The interference figure, shown in Fig. 653, will, therefore, show blue spots immediately adjacent to the red center in the northeast and southwest quadrants, and yellow spots in the northwest and southeast. The phenomenon of color is usually much more pronounced than that of the dark spots of the mica plate, and it is, therefore, generally advisable to use the gypsum plate for interference figures produced by minerals having low birefringence. For those having high birefrigence, the phenomena produced by the quartz wedge are most easily recognized.

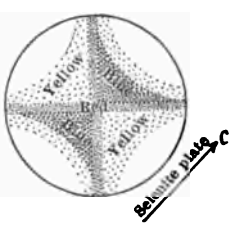


FIG. 653.—Determination of the optical character of a uniaxial crystal, cut at right angles to the optic axis., by means of the gypsum plate. Quartz (+).

In negative uniaxial crystals, the phenomenon observed is reversed, and

the blue spots will lie in the northwest and southeast quadrants. If the gypsum plate used has its long direction parallel to the slow ray, the appearances seen in positive and negative crystals are, of course, reversed.

The first use made of a gypsum plate for the determination of the optical character of interference figures was by Brewster,[1] who observed, in 1818, the different colors which appeared in the alternate quadrants. No definite thickness of plate was used, however, until 1835, when Brewster[2] introduced the II order red. The gypsum plate seems to have fallen into disuse and was not revived until 1887, when a I order red was among the accessories used by Rosenbusch, although he did not publish it until 1892.[3] It was discovered, independently, by Rinne[4] in 1891.

PROBLEM

Examine the interference figures of nephelite and quartz with a unit retardation plate.

406. The Quartz or Gypsum Wedge.—Comparable in every way to the phenomena observed with the mica and the gypsum plates, are those seen with a quartz or gypsum wedge. Instead, however, of showing a single rise of color, there will be a progressive change, due to the increasing thickness of the wedge as it is pushed forward. As a result, the interference colors will appear to move toward the center in two opposite quadrants, and away from it in the others. The directions of movement may be worked out in the same manner as was done for the mica plate. If the wedge has its slow vibration direction perpendicular to the long edge, and it is pushed from southeast to northwest, with its thin end foremost, above a positive mineral, the colors will appear to move away from the center in the northwest and southeast quadrants, and to-

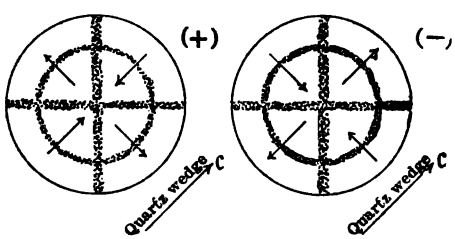


FIG. 654. FIG. 655.

FIGS. 654 and 655.—Determination of the optical character of a uniaxial crystal, cut at right angles to the optic axis, by means of the quartz wedge.

[1] David Brewster: *On the laws of polarization and double refraction in regularly crystallized bodies.* Phil. Trans. Roy. Soc. London, CVIII (1818), 199–273, in particular 219–220.

[2] Idem: *Optics*, 1835, 197.*

[3] H. Rosenbusch: *Mikroskopische Physiographie d. Mineralien.* Stuttgart, 3 Aufl., 1892, 189–190.

[4] F. Rinne: *Ueber eine einfache Methode den Charakter der Doppelbrechung im convergenten polarisirten Lichte zu bestimmen.* Neues Jahrb., 1891 (II), 21–27.

Idem: *Notiz über die Bestimmung des Charakters der Doppelbrechung im convergenten polarisirten Lichte mit Hülfe des Gypsblättchen vom Roth I Ordnung.* Centralbl. f. Min., etc., 1901, 653–655.

ward it in the others (Fig. 654). If the crystal is negative, the reverse movement takes place (Fig. 655). With a wedge cut with its slow ray parallel to the long direction, the phenomenon for the positive crystal is the same as given for the negative crystal above, and *vice versa*.

The quartz wedge was introduced by Biot[1] in 1814, and has been more or less used ever since.

PROBLEM

Examine the interference figures of quartz and calcite for optical character, using the quartz wedge.

Demonstrate that the movement takes place in the opposite directions from that given in the text, when the long direction of the wedge is parallel to the slow ray.

407. Uniaxial Crystals. Inclined Sections.—Inclined sections have exactly the same effect upon the accessories as do sections cut at right angles

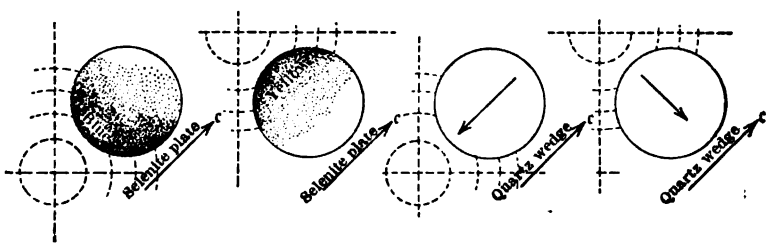


FIG. 656. FIG. 657. FIG. 658. FIG. 659.

FIGS. 656 and 657.—Determination of the optical character of a uniaxial crystal, section inclined to the axis, by means of the gypsum plate.

FIGS. 658 and 659.—Determination of the optical character of a uniaxial crystal, section inclined to the axis, by means of the quartz wedge.

to crystallographic *c*, and it is only necessary to complete, in imagination, the partial interference figure seen in the section. Thus Figs. 656 and 657 show, respectively, the northeast and the southeast quadrant of a positive uniaxial crystal as affected by a gypsum plate (I order red). With the quartz wedge, the movement of the colors, in the same quadrants, will be as shown in Figs. 658 and 659, the movement being much clearer in the southeast quadrant, which is, consequently, the better one to use in this determination. With negative crystals, or with the fast and slow directions of the accessories reversed, the phenomena are reversed.

PROBLEM

Examine inclined sections of quartz and calcite by means of the gypsum plate and quartz wedge, and determine their optical characters.

[1] J. B. Biot: *Mémoire sur les propriétés physiques que les molécules lumineuses acquirent en traversant les cristaux doués de la double réfraction.* Lu 22 Mai, 1814. Mém. Acad. France, Année 1812. Paris, 1814, 31–38.

Idem: *Traité de physique.* Paris, 1816, IV, 420–422, 543–566.

408. Sections Parallel to the Optic Axis.—To determine the optical character of sections cut parallel to the optic axis, the position of this axis should first be determined by means of the interference colors,[1] which descend in the scale outward from the center along its direction. At right angles to it the colors rise. Having determined the direction of the optic axis (crystallographic *c*), the optical sign of the elongation, which here also is the sign of the mineral, may be determined in parallel polarized light.

BIAXIAL CRYSTALS

409. Mica Plate, Gypsum Plate, and Quartz Wedge.—Following the same method of reasoning as that used in developing the phenomena seen in uniaxial crystals, we may determine what will take place in those that are biaxial.

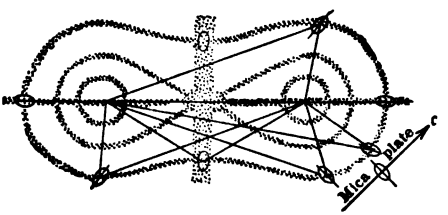


FIG. 660.

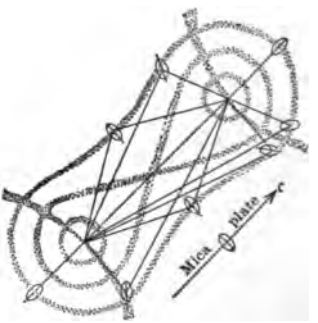


FIG. 661.

FIGS. 660 and 661.—Vibration directions and location of the isogyres in biaxial interference figures.

Let Figs. 660 and 661 represent the interference figures of a biaxial crystal, seen, respectively, in parallel and in diagonal positions. The directions of

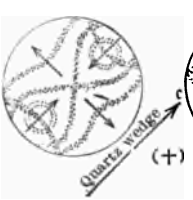


FIG. 662.

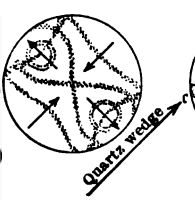


FIG. 663.

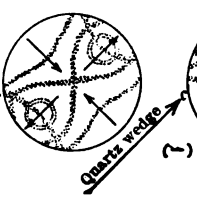


FIG. 664.

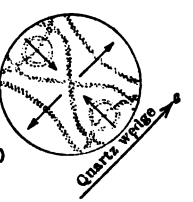


FIG. 665.

FIGS. 662 to 665.—Movement of the colors upon inserting a quartz wedge above the interference figures of positive and negative minerals.

vibration and transmission of the rays are shown as developed above.[2] If the crystal is negative, the acute bisectrix is the fast ray (α), consequently

[1] Art. 359.

[2] Arts. 360 and 374.

the velocity ellipses of rays traveling in different directions appear as shown in the figures. If, now, a quarter undulation or unit retardation plate, or a wedge, is inserted—vibration directions as before—there will be an increase in the color scale in that part of the figure in which the vibration directions are parallel, and a decrease where they are at right angles. The resulting movement is exactly the same as that which takes place in uniaxial inter-

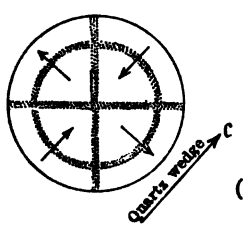


FIG. 666.

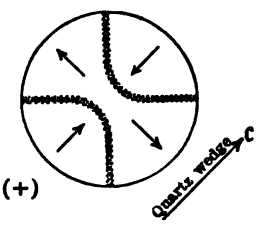


FIG. 667.

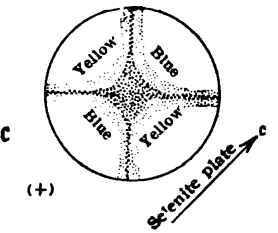


FIG. 668.

FIGS. 666 and 667.—Comparison of the movement of the colors in positive uniaxial and biaxial minerals upon the insertion of a quartz wedge above the interference figures.

FIG. 668.—Biaxial interference figure in parallel position, combined with a gypsum plate.

ference figures, as may be seen by an inspection of Figs. 662–665. As a matter of fact, a uniaxial crystal is only the special case of a biaxial crystal in which the optic angle is equal to zero, and the two may be considered together. This is clearly brought out by Figs. 666–667 which show the movement produced, by a quartz wedge, in the colors, respectively, of a uniaxial crystal and of a biaxial crystal placed with their vibration directions nearly parallel to the nicols. Fig. 668 shows the blue and yellow spots produced by the unit retardation plate[1] in a biaxial crystal similarly placed.

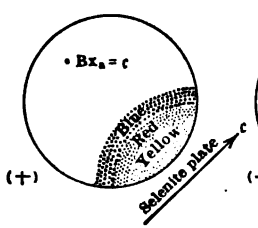


FIG. 669.

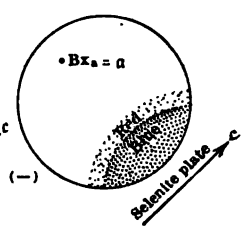


FIG. 670.

FIGS. 669 and 670.—Determination of the optical character of biaxial crystals, in sections showing the emergence of one optic axis, by means of the gypsum plate.

Becke[2] showed that inclined sections may be determined in the same way if one takes note of the position of the acute bisectrix, which always lies on the convex side of the hyperbola when the crystal is rotated to its diagonal position. Thus in Fig. 669, which is of a positive mineral, the acute bisectrix lies to the northwest, whereby, if the gypsum plate is used, the color to the southeast of the red will be yellow. In Fig. 670, which is negative, the color southeast of the red will be blue.

[1] F. Rinne: *Op. cit.*

[12] F. Becke: *Die Skiodromen.* T. M. P. M., XXIV (1905), 31–34.

With a wedge, the movement in inclined sections is exactly the same as it is in uniaxial crystals. One has only to keep in mind the appearance which the interference figure would have if it were complete. Various cases are illustrated in Figs. 671–673.

If one neglects determining the position of the acute bisectrix, confusion will result, for, except for the curvature of the isogyre, the left melatope, for example, of a positive mineral (Fig. 671) and the right melatope of a negative mineral (Fig. 672) are alike, and *vice versa*.

When the point of emergence of the optic axis lies beyond the field of the microscope, it is impossible to determine the curve of the isogyre when the crystal is turned to the diagonal position (Fig. 673). It is, consequently, impossible to determine the location of the acute bisectrix by this method.

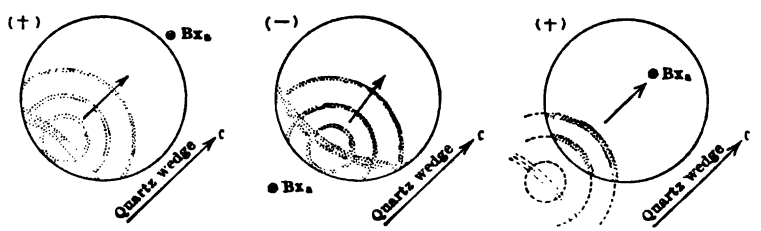


FIG. 671. FIG. 672. FIG. 673.

FIGS. 671 to 673.—Movement of the colors in the interference figures of biaxial crystals combined with the quartz wedge. The section is inclined to the optic axis.

Since the phenomena seen in a negative mineral showing the acute bisectrix are exactly the same as those seen in a positive mineral showing the obtuse bisectrix, determinations of the optical character in such sections are of no value. The determination whether the optic angle is greater or less than 90°, consequently whether the acute or obtuse bisectrix lies in the field, will be discussed below.[1] If this is determinable, the optical character of the crystal can be determined from a figure inclined as much as that shown in Fig. 673.

Sections of biaxial crystals cut parallel to the plane of the optic axes give interference figures as shown in Fig. 552. In such the determination of the optical character is easy, as was shown by Becke.[2] The method has already been given in reference to uniaxial interference figures.[3] The line uniting the quadrants containing the lowest colors is the direction of the acute bisectrix, which is a in negative and c in positive crystals. When the axial

[1] Chapter XXXIV, *infra*..

[2] F. Becke: *Unterscheidung von optisch + und − zweiaxigen Mineralien mit dem Mikrokonoskop (als Konoskop gebrauchtes Mikroskop)*. T. M. P. M., XVI (1896), 181.

Idem: *Die Skiodromen*. T. M. P. M., XXIV (1905), 32.

[3] Arts. 359 and 408.

angle is approximately 90°, the color variation becomes indistinct, when $V = 90°$, it disappears.

PROBLEMS

Examine for optical character, the interference figures of muscovite, olivine, gypsum, hornblende, titanite.

Examine interference figures given by the unit retardation plate, or by cleavage flakes along the best cleavage of enstatite (+), and hypersthene (−).

CHAPTER XXXIV

MEASUREMENT OF THE OPTIC AXIAL ANGLE BY CONVERGENT POLARIZED LIGHT

410. Introductory.—In a previous chapter[1] the relation between the optic axes and the axial angle was discussed, and it was there shown that

$$\sin E = n \sin V,$$

where E is one-half the apparent optic angle ($Ao'B$, Fig. 674), V one-half the true optic angle (aob), and n the mean refractive index of the crystal. It was also shown (Eq. 19, Art. 71) that

$$\tan^2 V_f = \frac{\alpha^2(\gamma^2-\beta^2)}{\gamma^2(\beta^2-\alpha^2)}$$

where V_f is one-half the axial angle whose bisectrix is the fast ray, and α, β, and γ, the refractive indices of the substance under examination.

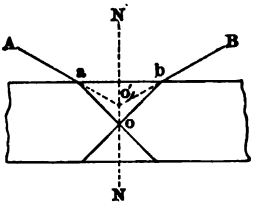


FIG. 674.—Relation between true and apparent axial angle.

To measure the optic axial angle, one might pivot the crystal at O and rotate it until the line $o'B$ coincided with the axis of the microscope. If a reading were taken, on a graduated arc, at that position, and the crystal rotated about the same axis until the line Ao' coincided, and another reading taken, the resulting angle $Ao'B$ would be the apparent axial angle ($2E$). The true value could then be determined from the first equation given above. Petrographic microscopes are not ordinarily adapted to measuring angles of rotation in the plane of the axis of the microscope, although special apparatus have been devised for this purpose.[2] In axial angle instruments, the method of rotation is the one usually employed, but such measurements belong to the province of crystallography rather than to petrography.

Another method of determining the axial angle follows from the second equation, and it is clear that if we can accurately determine the values of the three principal indices of refraction, we can compute the value of V. For these measurements, also, the ordinary petrographic microscope is not well adapted; the process requiring the orientation of the crystal in certain definite positions.

[1] Art. 72.

[2] Chapter XXXV, *infra*.

The usual methods for the determination of the axial angle, by means of the petrographic microscope, are based upon an examination of the interference figures produced by minerals. The results, however, cannot be quite so exact as measurements made with specially designed instruments, owing to the fact that interference figures do not lie in a plane but in a curved surface, and it is therefore impossible to focus sharply upon the center and the edges at the same time. Neither is it possible exactly to determine the points of emergence of the optic axes, since they are represented by rather broad bars and not by sharply defined points. Nevertheless, in spite of these drawbacks, the measurements made by the microscope in convergent polarized light need not vary more than a few degrees from the true values, provided the instrument is capable of accurate work and proper precautions are taken in making the determinations.[1]

For the identification of minerals, the optic angle may be measured under the microscope by white light. For accurate determinations, however, it is necessary to use monochromatic light, since there may be a considerable difference in the angle for light of different wave lengths. This is well shown by the interference figures of rubidium platino cyanide (Figs. 632-637). (See also Figs. 522–523.)

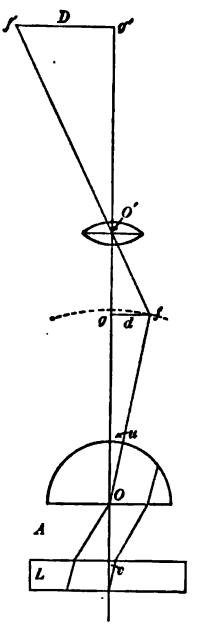


FIG. 675.—Diagram illustrating Mallard's formula.

411. Mallard Method for Sections showing the Points of Emergence of Both Optic Axes (1882).—The most accurate methods for determining the value of the axial angle under the microscope are based upon the work of Mallard,[2] who found that half the distance between the points of emergence of the optic axes (*D*, Fig. 675) is proportional to the sine of half the angle between them.

Let parallel rays enter the crystal section (*L*, Fig. 675) at an angle of v with the normal. The rays are refracted when they pass into the air *A*, and enter the lower lens *O* of the objective at a new angle u. If $Of = f$ = the focal distance of the objective, and $fg = d$, then

$$d = f \sin u. \qquad (1)$$

[1] Cf. S. Czapski: *Die dioptischen Bedingungen der Messung von Axenwinkeln mittelst des Polarisationsmikroskops.* Neues Jahrb., B.B., VII (1891), 506–515.

[2] Er. Mallard: *Sur la mesure de l'angle des axes optiques.* Bull. Soc. Min. France, V (1882), 77–87.

See also B. Hecht: *Ueber die Bestimmung des Winkels der optischen Axen an Platten deren Normale nicht mit der Halbirungslinie des Winkels der optischen Axen zusammenfällt.* Neues Jahrb., 1887 (I), 250–261.

Since the relative positions of the objective, Bertrand lens, and ocular remain constant, the distance d bears almost exactly a constant ratio to the corresponding distance D, seen in the interference figure, therefore

$$D = \kappa d$$

where κ is a constant. Substitute this value in equation (1),

$$\frac{D}{\kappa} = f \sin u.$$

But u may be replaced by E, the apparent optic angle in air, whereby

$$D = \kappa f \sin E.$$

Since κ and f are constant for the same combination of lenses, they may be replaced by K, and the equation becomes

$$D = K \sin E \qquad (2)^1$$

in which D is half the distance between the points of emergence of the optic axes, K a constant which may be determined for any combination of lenses, and E one-half the optic axial angle in air.

Equation (2) is known as Mallard's formula. The constant K, known as **Mallard's constant,** may be determined very simply for each lens combination of a given microscope by using as a test plate a mineral of known axial angle,[2] measuring the distance between the points of emergence of the optic axes ($2D$), and substituting this value in the formula. For example, using a flake of muscovite with a known angle of $2E = 71° 15'$, the distance between the melatopes ($2D$), with a Fuess No. 7 objective and a No. 2 ocular, was found to be 29, whereby

$$K = \frac{14.5}{\sin 35° 37.5'} = \frac{14.5}{0.5825} = 24.9+.$$

If possible, determinations should be made on a number of minerals with known axial angles. The mean value for K may be taken as a reliable value for the constant for that particular microscope and lens combination.

412. Becke's Graphical Solution of sin E = n sin V (1894).—Instead of calculating the value of $2E$ for each individual case, Becke[3] constructed, once for all for a certain microscope and lens combination, curves whose abscissæ represented the divisions of the micrometer scale of the eyepiece, and whose

[1] This formula has been tested by several writers by comparing the calculated values with those obtained by experiment, and the agreement, in general, is close over the entire field. See Rosenbusch-Wülfing: *Mikroskopische Physiographie*, 4 ed., 1904, 330, and F. E. Wright: *Measurement of the optic axial angle*, Amer. Jour. Sci., XXIV (1907), 327–331.

[2] Such test plates may be obtained from most dealers in petrographic microscopes and accessories.

[3] F. Becke: *Klein'sche Lupe mit Mikrometer.* T.M.P.M., XIV (1894), 375–378.

ordinates represented the apparent axial angles for indices of 1.5, 1.6, and 1.7. A specimen of a diagram of such curves is shown in Fig. 676, plotted with $K=25$. Different curves must be constructed, of course, for each different microscope and lens combination used. From such a diagram, by interpolation, the true axial angle for any refractive index may be determined.

Another diagram to express the relation $\sin E = n \sin V$, is shown, as constructed by von Fedorow,[1] in Fig. 698.

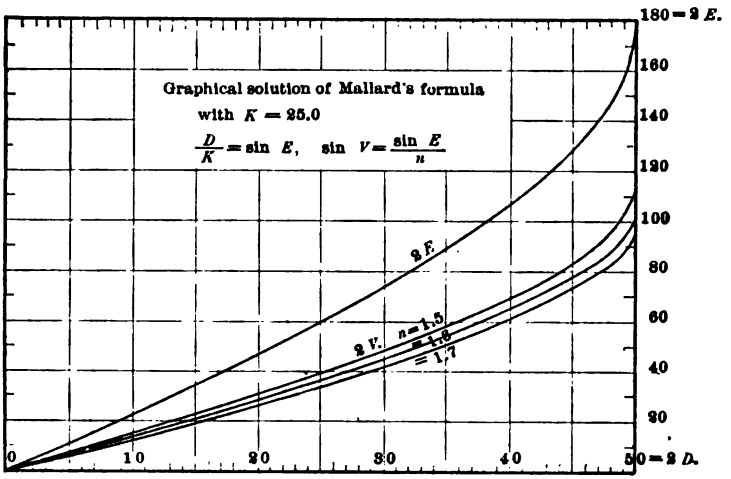


FIG. 676.—Graphical solution of Mallard's formula.

413. Schwarzmann Axial Angle Scale (1896).—Another method for determining the values of $2E$ according to Mallard's formula is by means of a slide rule, upon the movable part of which the position for each lens system may be marked. Slide rules, however, are not common adjuncts to a petro-

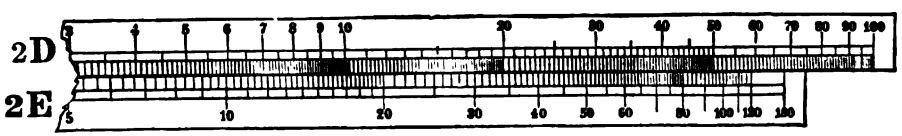


FIG. 677.—Schwartzmann's axial angle scale. (Fuess.)

graphical laboratory, wherefore Schwarzmann[2] presented a scale, based on logarithmic principles, from which the values of $2E$ may be read directly.

[1] E. von Fedorow: *Universal Methode und Feldspathstudien.* Zeitschr. f. Kryst., XXVI (1896), 246 and Fig. 3, pl. IV. The same diagram, drawn to a larger scale, is given by Wright: *Methods, etc.*, pl. VII.

[2] Max Schwarzmann: *Hilfsmittel um die Ausrechnung der Mallard'schen Formel zu ersparen.* Neues Jahrb., 1896 (I), 52–56, pl. II.

C. Leiss: *Die optischen Instrumente* etc., Leipzig, 1899, 189–190.

The scale consists of two parts, of which one ($2D$, Fig. 677)[1] gives micrometer divisions and the other the values of $2E$. To set the scale for any particular microscope and lens combination, the number of divisions ($2D$) in the micrometer eyepiece, corresponding to the distance between the melatopes in a mineral whose axial angle is known, is determined. (For accurate measurements the determinations should be made by monochromatic light.) The second scale is placed below the first so that the known axial angle corresponds with the determined number of micrometer divisions between the two melatopes. As a check, it is desirable to use several minerals of known optic angles before determining the relative positions of the two parts. For convenience in use, a scale may be prepared, for each microscope and lens combination, by pasting the two scales in proper position on a single card.

In the scale shown in the illustration, for example, the proper setting may have been obtained by noting that the optic angle of aragonite, with $2E = 30° 15'$ by Na light, corresponded to 17.7 divisions of the micrometer, and placing the two scales in position with these values corresponding. If, now, barite is to be tested, and the distance between the hyperbolæ in the diagonal position is found to be 35.7 divisions, the value of $2E$ is $63° 15'$.

The Schwarzmann scale may be used, further, to determine the value of $2V$ if the value of $2E$ is known. Since $n \sin V = \sin E$, $\log \sin V = \log \sin E - \log n$; where n equals the mean refractive index (β) of the mineral. It is therefore only necessary to lay off to the left, from the mark for $2E$, the distance between 1 and n. For example, the mean refractive index of barite is 1.638. The distance between 1.638 and 1.0 is determined from the upper scale and is laid off to the left from $63° 15'$ ($=2E$), and the value $37°$ ($=2V$) is obtained. If the scales are arranged to slide as an ordinary slide rule, it would be necessary simply to place $63° 15'$ below 1.638 and to read the angle beneath the value 1.0.

414. Schwarzmann Ocular (1896).—Schwarzmann[2] further suggested the convenience of having an ocular directly graduated to values of $2E$ instead of the usual uniform divisions, the values being those of sin E but marked from either side of the 0 point with the values of $2E$. Such an ocular naturally could be used only with one particular microscope and lens combination. If the acute bisectrix did not fall exactly at the zero point, but one side had a value of $2E'$ and the other $2E'+d$, the value of $2E$ would be approximately $2E'+\frac{d}{2}$. For example, if a piece of barite with $2E = 63°$ were not truly centered, so that one side read 58° and the other 68°, we would have

$$2E = 58° + \frac{10°}{2} = 63°.$$

[1] In the original article the distance between the two eyes is given as D. $2D$ is here used to correspond with the method of counting the divisions in the Mallard formula.

[2] Max Schwarzmann: *Op. cit.*, 55–56.

415. De Souza-Brandão Axial Angle Diagram (1903).—Another type of protractor for determining values of $2E$ from Mallard's formula was devised

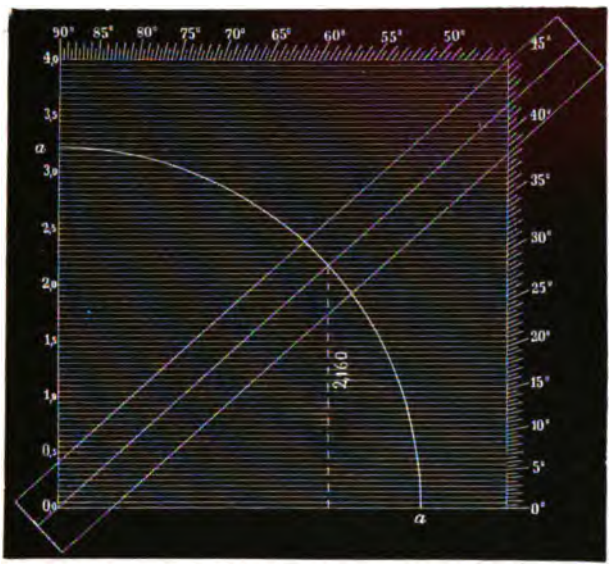


FIG. 678.—De Souza-Brandão axial angle diagram. (Fuess.)

by de Souza-Brandão.[1] It consists of a rectangular diagram (Fig. 678), 15 by 15 cm., upon which the ordinates represent D in the formula $D = K \sin E$, and the angular graduations represent E. To prepare the protractor for use, a quadrant of a circle aa is drawn with the lower left corner as a center and with the observed value of K as a radius. (In the illustration $K = 3.225$.) The instrument is now ready for use. To determine any axial angle, a ruler, preferably one of celluloid with a central mark as shown in the figure, is extended from the lower left corner through the point where the value for half the distance between the melatopes (in this

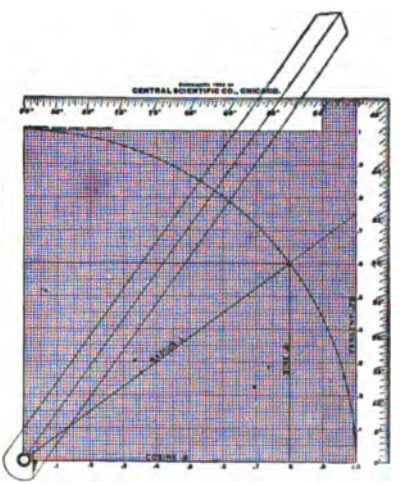


FIG. 679.—Trigonometer. (Central Scientific Co.)

[1] V. de Souza-Brandão: *O novo microscopio da commissão do serviço geologiço.* Communicaçoes da Commissão do Serviço Geologico de Portugal, V (1903–1904), 118–250, in particular 197–199.

case 2.160) cuts the circle. The extension of this line to the protractor edge gives the value for E. In the figure, $E=42°$, whereby the apparent optic axial angle in air ($2E$) is 84°. This diagram possesses the advantage that any number of circles may be drawn, each representing a different microscope or lens combination.

The trigonometer shown in Fig. 679 may be used for the same purpose.

416. Michel-Lévy Method for Sections Perpendicular to a Bisectrix (1888).—In a section cut at right angles to one of the bisectrices of a mineral whose optic angle is so large that the points of emergence of the optic axes lie beyond the field of view ($2E > \pm 85°$), it is impossible, by inspection, to determine whether the acute or the obtuse bisectrix appears. To make the determination, Michel-Lévy[1] devised a method by means of which it is possible to obtain a fairly accurate value for $2E$.

A glass plate, with a few concentric circles engraved upon it, is inserted in the tube of the microscope between the analyzer and the objective and in such a position that the lines appear in the plane of the interference figure. The section, whose angle is to be determined, is now placed on the stage in parallel position, and the amount of rotation necessary to bring the isogyres from the form of a black cross to the point of tangency to a given circle, is determined. The values thus obtained are substituted in the formula

$$\sin O = \frac{\sin E}{n}\sqrt{\frac{1}{\sin 2\varphi}},$$

in which E is half the apparent axial angle of a known mineral, used as a measure of the circle of reference, n the refractive index of the glass of the objective, and φ the amount of rotation of the stage necessary to bring the isogyres of the unknown mineral from the crossed position to the point of tangency.

Since the value of n is usually unknown, Wright[2] devised a formula in which Mallard's constant is introduced in its place. This formula is based on the equation for the hyperbolic isogyres of a biaxial interference figure

$$xy = x'y'. \quad \text{(Eq. 1, Art. 379.)} \tag{1}$$

In the 45° position, $x=y$, and the equation becomes

$$x^2 = x'y'. \tag{2}$$

In polar coordinates $x=\rho \cos \varphi$, $y=\rho \sin \varphi$, $x'=r \cos \varphi$, and $y'=r \sin \varphi$, whereby (2) becomes

$$\rho^2 \cos^2 \varphi = r^2 \cos \varphi \sin \varphi. \tag{3}$$

But $2 \cos \varphi \sin \varphi = \sin 2\varphi$ and (3) becomes

[1] Lévy et Lacroix: *Les minéraux des roches*. Paris, 1888, 94–95.

[2] Fred. Eugene Wright: *The determination of the optical character of birefracting minerals*. Amer. Jour. Sci., XX (1905), 288–9.

$$2\rho^2\cos^2\varphi = r^2\sin 2\varphi.$$

When $x=y$, $\cos^2\varphi=(1/2\sqrt{2})^2=1/2$, and the equation reads

$$\rho^2=r^2\sin 2\varphi. \tag{4}$$

Substituting $r=D$, in Mallard's formula, we have

$$r=K\sin E.$$

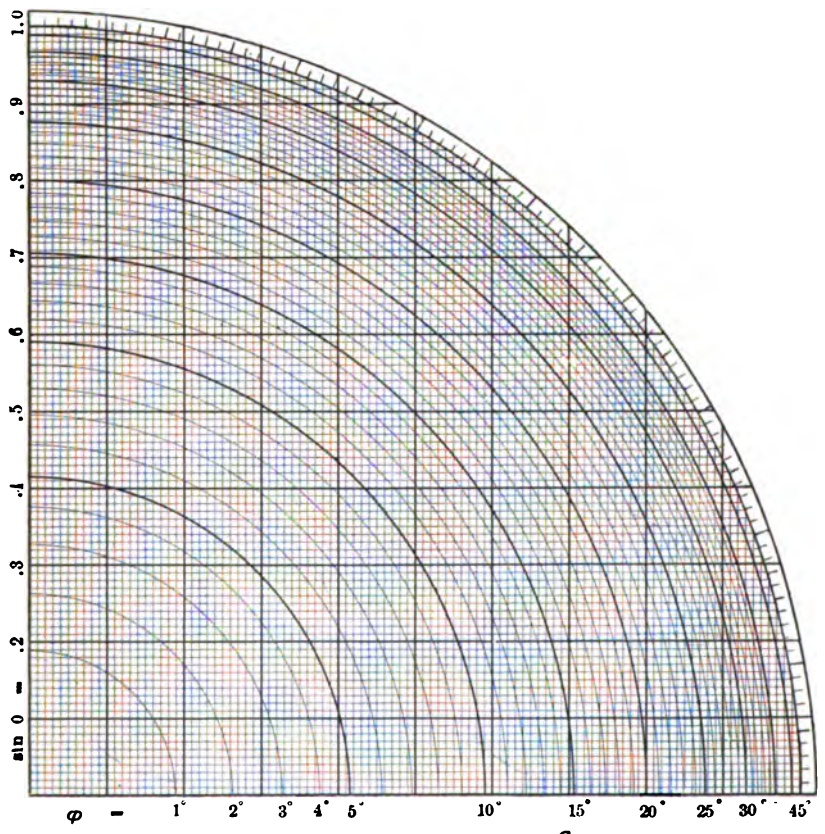


FIG. 680.—Diagram for solving the equation $\sin E=\frac{C}{\sqrt{\sin 2\phi}}$. (After Wright.)

Likewise in the circle used as a standard of measurement

$$\rho=K\sin O.$$

Substituting in (4)

$$K^2\sin^2 O=K^2\sin^2 E\cdot\sin 2\varphi$$

or

$$\sin E=\frac{\sin O}{\sqrt{\sin 2\varphi}}=\frac{C}{\sqrt{\sin 2\varphi}}, \tag{5}$$

where $\sin O$ $(=C)$ is the constant of the circle used, and is to be determined,

once for all, by the Mallard method, by making use of a mineral of known axial angle, φ is the angle of revolution of the stage, and E is one-half the apparent axial angle of the mineral to be determined.

If the circle of reference is taken so that its diameter is equal to the distance between the melatopes in a mineral whose value of $2E$ is known, it must be tangent to the circle in the diagonal position when $\varphi = 45°$. In this case $\sqrt{\sin 2\varphi} = 1$, and sin E of the known mineral = sin $O = C$, a constant.

The equation does not hold when sin $2\varphi <$ sin^2 O, for in that case sin E in equation (5) will be greater than unity. This is due to the fact that below this point the apex of the hyperbolic isogyre and the circle of reference cannot be tangent. In such cases it is necessary to choose a smaller circle of reference.

The value of E can readily be found by means of the graphical solution of the equation shown in Fig. 680. The line drawn from the center O through the intersection of whichever circle represents the angle of rotation φ, with that horizontal line which represents the constant sin O, will give the value of E.

417. Viola Method (1893).—Viola[1] determined the value of the axial angle in sections whose optic axes emerge beyond the field of view, by measuring the curvature of the hyperbola in any position of the stage.

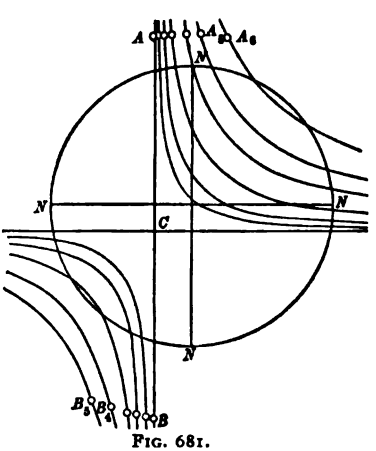


FIG. 681.

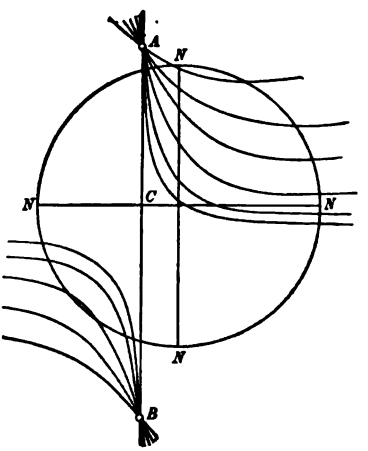


FIG. 682.

FIGS. 681–682.—Viola's method for determining the axial angle.

The accessories necessary are a Klein ocular, or simply two oculars used according to the Klein method,[2] and a thin glass plate engraved with con-

[1] C. Viola: Giornale di Miner, etc., IV (1893), pt. 3.*

Idem: *Ueber den Albit von Lakous* (*Insel Kreta*), T.M.P.M., XV (1896), 135–158, especially 150–156.

[2] Art. 392.

centric circles and graduated with divisions 5° apart. This engraved disk is placed in the focal plane of the upper ocular in such a position that its center does not correspond with that of the ocular (Fig. 681). If the stage is rotated, the successive positions assumed by the isogyres of a biaxial mineral are as shown in Fig. 681. If the stage is stationary and the nicols are rotated, they will appear as in Fig. 682. The latter positions are the ones to be used. If one uses a microscope whose nicols do not rotate, it is necessary to compute the positions which the isogyres would have assumed had they done so. To do this one must lay off, in the opposite direction from that in which the stage was rotated, the angle NCN_2, the amount of the rotation.

To determine the axial angle, the stage is rotated until the hyperbola passes through the center of the graduated disk. The angles γ_1 and γ_2 (Fig. 683), measured from the left and clock-wise to the points where the hyperbola cuts a circle of radius ρ, are read. The angle ω (N_2CN) through which the stage has been turned from the position of forming a cross to its position through the center of the auxiliary circle, is also determined. If R is the distance of the center of the circle from the center of the hyperbola, we have

$$\frac{R}{\rho}=\frac{\sin 2\gamma_1}{2\sin(\gamma+\psi)}=\frac{\sin 2\gamma_2}{2\sin(\gamma_2+\psi)}, \tag{1}$$

where

$$\tan\psi=\tan\gamma_1\tan\gamma_2\frac{\gamma_1+\gamma_2}{2}, \tag{2}$$

and

$$\frac{AC}{R}=\frac{\sin 2\psi}{\sin 2\omega}. \tag{3}$$

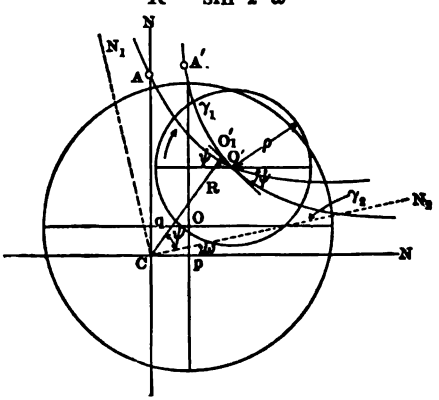


FIG. 683.—Viola's method for determining the axial angle.

The value of ρ is determined, in degrees, by means of a mineral of known optic angle, and such an engraved circle is chosen that its diameter corresponds with the value of $2E(=2\rho)$ of that mineral. γ_1 and γ_2 are determined by the position of the hyperbola across the auxiliary circle. $AC=E=$ one-half the apparent axial angle. ω is determined by the amount of rotation of the stage. ψ is determined from equation (2), and its value is substituted in

equation (1) to determine R (in degrees). The latter value is substituted in (3) to give E.

BECKE METHOD FOR DETERMINING GRAPHICALLY THE AXIAL ANGLE IN SECTIONS WHICH DO NOT SHOW THE POINTS OF EMERGENCE OF BOTH OPTIC AXES (1894)

418. Determination of the Point of Emergence of an Optic Axis.—Becke's[1] method for the determination of axial angles was developed from a method for determining the point of emergence of an optic axis. The measurement of the angle will be more readily understood if this is previously explained.

If one rotates the two nicols simultaneously and leaves the stage stationary, the only spot which will remain continually dark is the point of emergence of an optic axis.[2] If the stage is rotated instead of the nicols, the point of emergence of the optic axis will rotate in the same direction, following the path of the former but without movement of its own (Fig. 681). If, now, one can accurately locate the isogyre in two positions, their intersection will give the point of emergence of the optic axis.

To fix the position of the isogyre correctly in any position, it is sufficient to determine the distance from its nearest point to the center of the field of view, and the two points where it touches the periphery. Two positions of the isogyre will locate the optic axis, but a third is used as a check. The first position taken is one in which some crystallographic line lies parallel to a cross-hair. The second position is so chosen that the new isogyre crosses the position of the first approximately at right angles, and is determined by rotating the stage through 45°. The third is the position in which the isogyre lies parallel to one of the cross-hairs. In the process, the direction of rotation of the stage must be taken into consideration, otherwise confusion will result. The lines representing the three positions of the isogyre should intersect in a point, although as a matter of fact they usually form a small triangle, whose size is an indication of the accuracy of the measurements. The center of the triangle may be taken as the desired point.

An example will best illustrate the procedure. Since few microscopes are fitted with simultaneously rotating nicols, the appearances as seen with a rotating stage are given. Let Fig. 684 represent the interference figure in a section of plagioclase when it is placed with its twinning lamellæ parallel to the vertical cross-hair (zz') of the microscope, and let this be the first position to be measured. Let it be supposed that the vernier of the stage in this position reads 128°. The ocular is now turned until the vertical cross-hair

[1] F. Becke: *Bestimmung kalkreicher Plagioklase durch die Interferenzbilder von Zwillingen* T.M.P.M., XIV (1894–5), 415–442.

[2] Cf. Fig. 682, Art. 417.

cuts the center of the dark bar where it leaves the field at u. The Bertrand lens[1] is now removed and the stage rotated, clock-wise, until the twinning line (zz') coincides with the cross-hair in its new position at u. The stage

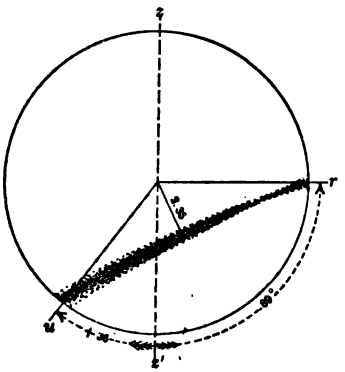


FIG. 684.

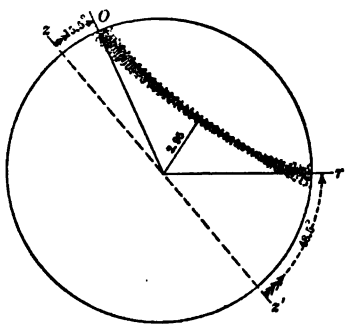


FIG. 685.

vernier reads, let us say, 154.5° ($=u$). Turn back the stage to zz' and set the vertical cross-hair of the ocular on r, the point where the bar leaves the field in the other direction. Upon rotating the stage, counter clock-wise,

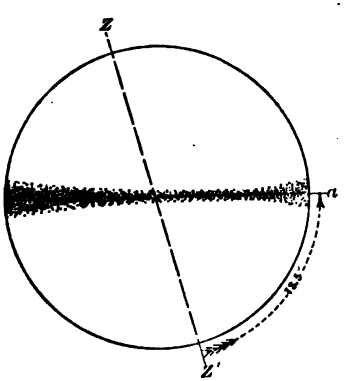


FIG. 686.

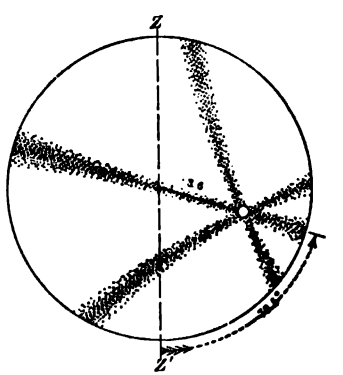


FIG. 687.

FIGS. 684 to 687.—(After Becke.)

it may read $r=42°(-)$. Turn back to 128° and set the ocular so that its scale extends from northwest to southeast and read the distance between the intersection of the cross-hairs and the bar, for example $d=3$.

On account of unavoidable eccentricity in centering, the observations are now repeated with the crystal rotated through 180°, whereby, in the above example, $zz'=128°+180°=308°$, the new reading of the stage vernier. The other values may be $u=353.5°$, $r=216°(-)$, $d=1.7$. Subtracting 180° from

[1] If a Klein ocular is used, the *insertion* of the Bertrand lens permits a much reduced image of the *mineral* to be seen.

each of the first three readings to reduce them to the first position, we have, upon tabulation,

	z	*u*	*r*	*d*
I reading.....	128°	154.5° (+)	42° (−)	3.0
II reading.....	128°	173.5°	36°	1.7
Averages.....	128°	164°	39°	2.35

Whereby the distances $u-z=36°$ (+), $r-z=89°$ (−).

A circle is now drawn with a diameter equal to that shown by the micrometer divisions of the eyepiece, for example, 13.0, and within it a small circle with a radius of 2.35. The line *zz'* is drawn, the points *u* and *r* are found, and a curve is sketched through the three points as shown in Fig. 684.

If the stage is now rotated approximately 45° (in the example 43°) counter clock-wise, the isogyre appears as is shown in Fig. 685. The position of the twinning line *zz'* is determined in its new position, and the above process is repeated, the following readings resulting after deducting 180° from the second set.

	z	*r*	*o*	*d*
I position.....	85°	41° (−)	99.5° (+)	3.3
II position	85°	32°	101.5°	2.6
Averages.....	85°	36.5°	100.5°	2.95

Whereby $r-z=48.5°$ (−), and $o-z=15.5°$ (+).

The third reading is taken by rotating the stage until the isogyre lies parallel to one of the cross-hairs, which in this particular case happened to be when it passed through the center of the field (Fig. 686). The readings, after reducing the second set by 180°, are,

	z	*a*	*d*
I position.....	110°	36° (−)	0
II position.....	110°	37°	0
Averages.....	110°	36.5°	0

Whereby $a-z=73.5°$ (−).

If the three bars are drawn in a single circle, and so placed that the line *zz'* of each coincides, they will appear as in Fig. 687, which is identically what would have been seen by using a microscope with simultaneously rotating nicols and stationary stage. The point of intersection lies 73.5° (−) from *zz'* and 3.6 micrometer divisions from the center, the latter corresponding to an apparent angle of 27°.

419. Becke's Rotating Drawing Stage (1895).—Becke[1] later simplified the process of construction by a device which permits a complete graphical solution. Instead of measuring the various positions of the isogyre, a specially constructed drawing-board is used (Fig. 688), and the image is drawn by means

[1] F. Becke: *Messung von Axenbildern mit dem Mikroskop.* T.M.P.M., XIV (1894-5), 563-656.

of a camera lucida (Z) so set that the reflected ray falls vertically upon the center of the drawing-table. The latter is made similar to the rotating stage of the microscope, and is attached by three centering screws, a, a_1, and a_2, to the support. It carries a vernier at one side, and may be rotated as desired. The microscope is placed between guide plates on the drawing-board so that it may always be returned to the same position. The drawing paper is attached to the rotating stage by means of spring object clips.

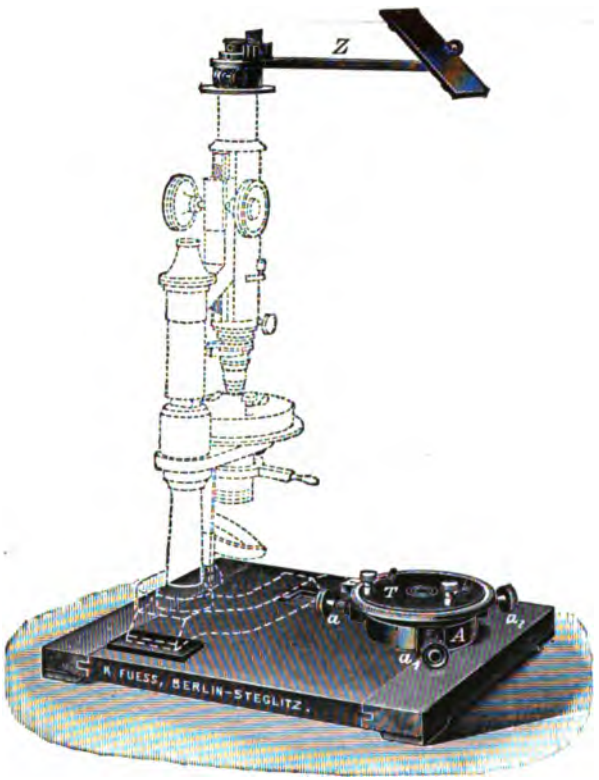


FIG. 688.—Becke drawing table. About 1/5 natural size. (Fuess.)

To use the instrument, the microscope is first centered on the thin section, the camera lucida is put in place, the ocular is removed, and the image of the center of the drawing-table is made to coincide with that of the center of the microscope stage. After obtaining a distinct image of the interference figure, the stage vernier is read, the drawing stage is set to the same reading, and the bar sketched in this position.[1] The microscope-stage and the drawing-stage are now both rotated through approximately 30° in the same direction, and again the black bar is drawn. The intersection of the two gives the posi-

[1] Becke suggests using black drawing paper and light colored crayons for sketching the isogyres.

tion of the optic axis, although for a check a third drawing is made with the stage rotated 30° in the opposite direction. The position of twinning or cleavage lines may be drawn by inserting the ocular.

420. Becke Method for Determining, by Means of the Curvature of the Isogyres, the Value of the Axial Angle in Sections which show the Point of Emergence of but a Single Optic Axis (1905).—Not only may the value of the optic angle be determined when both melatopes appear in the field of view, but this may be done when but a single one appears. Becke[1] has shown that the curvature of the bar depends upon the value of the axial angle. In the determination, use is made of the camera lucida and the Becke revolving drawing-board. The first step is to indicate the direction of the principal section of one of the nicols by laying, upon the drawing-stage, a visiting card with its edge parallel to the horizontal cross-hair. This is necessary since the interference figure is to be viewed by the Lasaulx method, consequently the ocular with its cross-hairs is to be removed. The ocular is removed and the microscope-stage is rotated until the isogyre lies parallel to the edge of the visiting card, as seen by the aid of the camera lucida. In this position the trace of the optic axes lies parallel to the principal section of one of the nicols, consequently both melatopes must lie somewhere along the dark bar. The card is now removed from the drawing-stage, and the latter is turned until its vernier shows the same reading (a) as does the microscope-stage. In this position the horizontal isogyre is sketched on the stage.

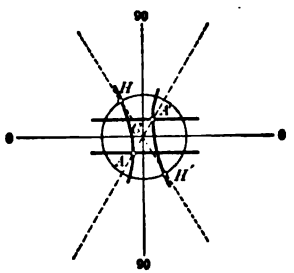


FIG. 689.—(After Becke.)

Both microscope- and drawing-stage are now rotated, clock-wise, through 45° (reading b), and the isogyre is sketched in its new position. The point in the drawing where the two lines cross is the point of emergence of one of the optic axes (A, Fig. 689).

The next step is a rotation through 180° of the microscope- or drawing-stage, one or the other, and the process of drawing the isogyre in two positions is repeated. In this manner a second point A' is obtained. Half the distance AA' determines C, the center of the field.

With C as a center, a circle is drawn with a radius $r = \frac{\beta \sin \rho}{K}$, in which β is the mean refractive index of the mineral, K the Mallard constant for the combination of lenses used, and ρ a value so chosen that the circle will not lie too near the edge of the field of view, for here the bars are too broad and the lenses themselves partially polarize the light.[2] With Fuess objective No. 7,

[1] F. Becke: *Messung des Winkels der optischen Achsen aus der Hyperbelkrümmung.* T.M.P.M., XXIV (1905), 35–44.

[2] Art. 357.

$\rho = 20°$ is a good value to use. The circle just drawn will cut the hyperbolæ in the points H and H'.

It is now necessary to draw the points A and H in stereographic projection, which is most readily done by the use of transparent paper and a Wulff net. The point of emergence of the optic axis is located by means of its azimuth and central distance. The former is determined by laying the projection drawing upon the stereographic net in such a position that the lines o–o and 90–90—drawn through C and perpendicular and parallel to the horizontal isogyre—correspond with the central meridian and the equator of the net. By placing a straight edge along AA' (Fig. 689) the angle which CA' makes with the vertical can be read from the graduations around the net. This gives the azimuth of the desired point. The central distance AA', measured with a millimeter scale, is proportional to $2D$ and, since $D = K \sin E = K\beta \sin V$, we have $V = \frac{D}{\beta K}$, where V is the central distance in degrees, K the Mallard constant for the instrument, and β the mean refractive index of the plate. As a control one may measure the distance between the horizontal isogyre and the center ($= 2a$), whereby $\sin \omega = \frac{a}{\beta K}$, in which ω is the angular distance between the axial plane and the center.

The point H is similarly transferred for azimuth. Its central distance (ρ) is known by the construction.

Having located the points A and H on the tracing paper, the drawing is rotated about the center over the Wulff net until the horizontal diameter is parallel to the horizontal isogyre through A_1 (Fig. 690). The great circle is then drawn through A_1 and the ends of the horizontal diameter. This is the trace of the plane of the optic axes, since it contains one of the points of emergence of the optic axes and lies parallel to the principal section of one of the nicols.

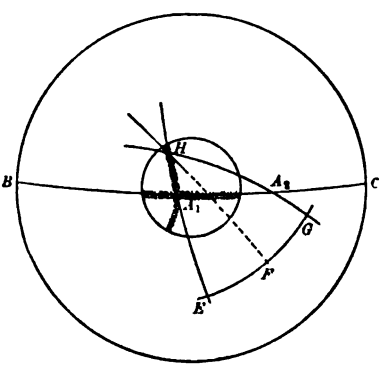


FIG. 690.—(After Becke.)

It is necessary, next, to determine the vibration direction of the ray at H.[1] According to the Biot-Fresnel[2] law, the extinction directions in any section

[1] See simplified method at the end of this Article.

[2] J. B. Biot: *Mémoire sur les lois générales de la double réfraction et de la polarisation dans les corps regulièrement cristallisées.* Mém. Acad. France, Année 1818. III (1820), 177–384, especially 228.

A Fresnel: *Mémoire sur la double réfraction.* Mém. Acad. France, VII (1827), 45–176.

Idem: *Ueber die doppelte Strahlenbrechung.* Translation of preceding. Pogg. Ann., XXIII (1831), 372–434, 494–56c, especially 542–545.

of a biaxial crystal are parallel to the traces, on that section, of the planes bisecting the angles between the two planes determined by the optic axes and the normal to the section. Since the extinction directions are shown by the directions of vibration of the nicols, it is only necessary to draw, through H, a line parallel to the trace of the polarizer. This line may readily be determined since its inclination to the horizontal line of the net is equal to the amount which the stage has been rotated to produce the isogyre through A_1H; in the present case 45°. A line, therefore, is drawn through H inclined 45° to the horizontal. The stereographic projection of the vibration plane through H, however, will not be this straight line, but a great circle tangent to it at H. It may readily be drawn by rotating the Wulff net until a great circle is tangent at H.[1] Two points, E and F, are now laid off, 90° from H, and a great circle is drawn through these points. Since this circle is the polar circle to H, it is laid off by rotating the net until H lies on the equator, and tracing the meridian through F. On this meridian a distance $FG=EF$ is laid off, and the great circle, drawn through GH, is the trace of the plane passing through the other optic axis. But it is already known that the plane BA_1C is the plane of the optic axes, consequently the unknown axis, lying in both the BA_1C and GH planes, must be at their intersection A_2. This point, therefore, is the point of emergence of the other optic axis. The axial angle, $A_1A_2=2V$, may be read directly from the stereographic net.

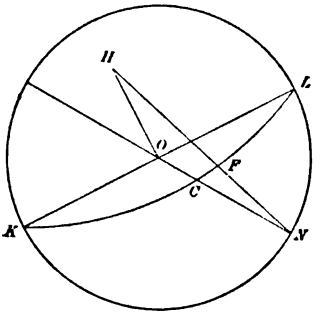


FIG. 691.—(After Becke.)

The error of observation, according to Becke, is about 1° in the value of $2V$, an error of small consequence for practical purposes. Observations should be repeated in a position at 180° from the first, to eliminate errors of eccentricity of the instrument, and the mean values of the four sets of readings should be transferred to the stereographic projection. The amount of rotation of the stage to obtain the best position for H depends upon the situation of the melatope upon the stage. The angle A_1HA_2 should be neither too acute nor too obtuse, and the angle HA_2B not too small. It has been found that if the melatope is too near the center of the field, the base A_1H will be too small. If the melatope falls too near the margin of the field, the length of the isogyre seen is too short to determine the position at which it is parallel to the vibration plane of the nicol. Here, also, the polarization effect of the edge of the lens acts as a disturbing factor. The most satisfactory position is when the horizontal isogyre lies at a distance of between one-half and one-

[1] The pole of this great circle (HF) must lie on a line through H, normal to the tangent, and 90° distant. The stereographic net is rotated until the equator passes through P. The required great circle is the meridian of the net now passing through H.

third of the radius of the field from the center, and the acute bisectrix lies in one half of the field and one of the optic axes in the other.

In a later paper, Becke[1] simplified the method of finding the vibration direction of the ray through H by a construction similar to that previously proposed by Wright.[2] The location of the point differs slightly from that obtained by the latter.[3]

The new method is much more quickly performed and, if repeated in the 180° position, there is less chance for error. The great circle KL (Fig. 691), polar to H, is drawn as before, as is also the vibration direction of the nicol ON. H is then connected by a straight line with N. Where it cuts the great circle KL is the desired point F.

421. Wright's Modification of the Becke Method for Determining the Axial Angle by Means of the Curvature of the Isogyres (1907).—The principle used by Becke for determining the value of the axial angle by means of the curvature of the isogyres, was also employed by Wright,[4] but instead of using a revolving drawing table, he used a double screw micrometer ocular (Fig. 386). This is an instrument in which, in place of the single movement of the usual screw micrometer oculars, there are two movements, at right angles to each other, whose readings determine the position of any point of the interference figure, and correspond to rectilinear coordinates in the orthographic projection or small circle coordinates in the stereographic. By means of the constant K of the microscope, which must have been determined previously for each of the movements, each reading is reduced to its angular value by the formula $\sin V = \frac{D}{K\beta}$, in which β is the mean refractive index of the mineral.

The readings are made as follows: The microscope stage is rotated until the dark isogyre is parallel to the horizontal cross-hair of the ocular; the horizontal cross-hair is moved by means of the vertical micrometer screw V until it coincides exactly with the center of the dark axial line (A_1C, Fig. 692), and the nicols (not the stage) are rotated about a suitable known angle, for example, 30° to 45°. The optic axis A_1 now corresponds with the intersection of the isogyre with the horizontal cross-hair (HA_1 with A_1C). The vertical cross-hair is next moved by means of the horizontal micrometer screw until it coincides with this intersection. The two readings are recorded, after

[1] F. Becke: *Zur Messung des Achsenwinkels aus der Hyperbelkrümmung.* T.M.P.M., XXVIII (1909), 290–293.

[2] Fred. Eugene Wright: *The measurement of the optic axial angles of minerals in the thin section.* Amer. Jour. Sci., XXIV (1907), 331–341.

Idem: *Das Doppel-Schrauben-Mikrometer-Okular.* T.M.P.M., XXVII (1908), 293–314.

[3] See Art. 421 *infra* and Fig. 691.

[4] *Op. cit.*

which the stage is rotated through an angle of 180°, and similar readings are taken for A_1 in its new position A'_1 to determine the exact center of the field. This point (O) lies half-way between CC' and $A_1A'_1$.

A_1, being thus fixed, its position can be plotted in stereographic projection by reducing the values to the true angles within the crystal by the formula $\frac{\sin i}{\sin r} = n$. Another point H of the isogyre is now determined by a single set of two readings, thus giving coordinates from the center. These are likewise reduced to their true angles and are plotted on the projection.

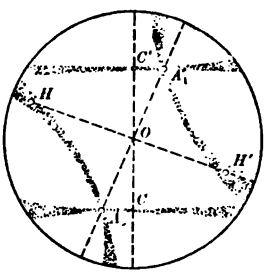


FIG. 692.

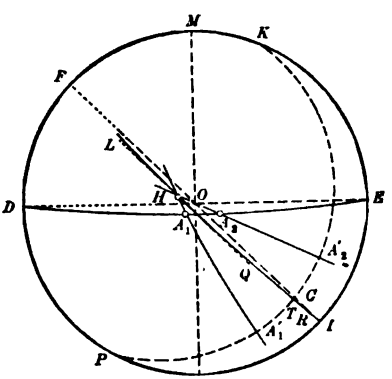


FIG. 693.

FIGS. 692 and 693.—(After Wright.)

Having located the points A_1 and H, the optic angle may be determined as in the Becke method or by the following, which differs from the former in the manner of determining the position of the vibration direction through H. According to Wright, this vibration direction is the great circle through H and C (Fig. 693). The latter is determined by the intersection of the great circle PK—polar circle to H—and the trace of the principal plane FOI of the lower nicol.

Having determined the point C, the distance A'_2C is laid off equal to A'_1C. The intersection of the great circle through H and A'_2 with the great circle DE gives the location of the second melatope A_2.

The difference in position of the point C, as located by Wright and by Becke, may be seen from Fig. 691 in which C is the location by Wright's method and F by Becke's. Wright[1] claims that the vibration for any dark spot of the isogyre must be parallel to the extinguishing plane of the upper nicol, regardless of the fact that two corresponding points darkened by vibrations at right angles to each other do not lie 90° apart, a condition deemed essential by Becke.[2] In practice, the points located by the two methods fall so close together that the accuracy of the two is about equal, both being approximations to the true position. The disturbing element of the rotation

[1] Fred. Eugene Wright: *The methods of petrographic-microscopic research.* Carnegie Publication No. 158, Washington, 1911, 160.

[2] F. Becke: *Op. cit.* T.M.P.M., XXVIII (1909), 290–293.

of the polarized light by the lenses and slide, and the indistinctness of the isogyres, produce greater errors than those caused by the location of the points.

Instead of the double-screw micrometer ocular, an ocular with a coordinate micrometer scale (Fig. 384) may be used. If the graduations are made to 0.1 mm., it will give results nearly as accurate as the former and is much less expensive. The graduations should cover the entire field given by the Bertrand lens, and should be calibrated in the same manner as the screw micrometer ocular. By the use of cross-section paper the isogyres may be plotted directly for any angle of rotation.

A simple method of calibration, independent of Mallard's formula, is that which makes use of a Zeiss apertometer (Fig. 214), by means of which it is only necessary to determine the number of divisions of the scale covered by the different angles.

422. Modifications of Becke's Method. Wright (1907).—Various modifications of the Becke method for determining the optic angle have been proposed.

In place of a revolving stage fixed to a board beneath the microscope, Wright[1] attached a small revolving stage directly to the stand.

423. Stark (1908).—Stark,[2] following a suggestion from Becke, did not use the revolving stage, but rotated the polarizer and a cap nicol above the camera lucida through equal angles. Later he used a microscope with simultaneously rotating nicols, thus expediting the determinations and eliminating the error of centering. He made his drawings by camera lucida, as in the Becke method, and likewise transferred them to stereographic projection. He claims the method requires but one-third the time necessary to make readings with the double screw micrometer ocular.

424. Tertsch (1910).—Tertsch[3] eliminated all errors produced by parallax and by lack of parallelism between microscope- and drawing-stage, and between microscope axis and reflected ray, by inserting a long focus lens in the tube. This projects a real image, enlarged and inverted, to the end of the tube, where it is received on tracing paper placed over an ocular made similar to a cap nicol, with a circle divided to 5° and a vernier reading to degrees. The paper is placed over the top and is held in place around the edges by a slip-over ring. It lies exactly in the plane of projection of the interference figure and therefore may be traced with a pencil. If outside light is shut off from the top of the microscope by means of a hood or cloth, as is done in a

[1] Fred. Eugene Wright: *The measurement of the optic axial angle of minerals in the thin section.* Amer. Jour. Sci., XXIV (1907) 331, and Fig. 7, page 332.

[2] Michael Stark: *Geologisch-petrographische Aufnahme der Euganeen.* T.M.P.M., XXVII (1908), 412–413.

[3] Hermann Tertsch: *Ein neues Zeichenokular.* T.M.P.M., XXIX (1910), 171–172.

photographic camera, the figure may be seen much more clearly. This apparatus has the advantage over the revolving table in its cheapness, in the use of white paper and fine pencil, in simplicity, and in rapidity of use. Its chief disadvantage is the reduction of light in transmission through the paper, especially noticeable with small minerals whose interference figures require small diaphragms. If a piece of ground glass, slightly oiled, were used instead of paper, the loss of light would not be so great.

In an earlier paper, Tertsch[1] described a method for determining, with the Becke drawing table, the axial angle in sections cut at right angles to a bisectrix. The method is hardly more accurate than that given by Michel-Lévy,[2] and is much more complicated.

[1] Hermann Tertsch: *Versuch einer Achsenwinkelmessung in einem Mittellinienschnitt* T.M.P.M., XXVII (1908), 589–594.

Review by St. Kreutz: Zeitschr. f. Kryst., XLIX (1910–11), 291–2.

[2] Art. 416.

CHAPTER XXXV

MEASUREMENT OF THE OPTIC AXIAL ANGLE BY MEANS OF A ROTATION APPARATUS

425. The Rotation Apparatus.—Various forms of rotation apparatus and the general method for the orientation of mineral sections have been described in an earlier chapter. It now remains to show the applicability of these instruments to the measurement of extinction and axial angles.[1]

As mentioned previously, modern rotation apparatus or universal stages are due almost entirely to the work of Klein and of von Fedorow. The work of the former was confined more especially to instruments adapted to the examination of single crystals, while that of the latter was to instruments for the examination of minerals in rock sections. Ordinarily it is necessary to examine a considerable number of differently orientated grains of the same mineral in order that all of its properties may be determined. While this is usually a simple enough procedure, it sometimes happens that but a single fragment of a mineral occurs, or it may be that one is unable to determine whether or not a particular grain in the slide is the same as some other. In such cases, if one can tilt the section to a different angle, the effect is that of having a differently orientated section. Further, in certain cases, as for example in the determination of the feldspars, one may desire to obtain the maximum extinction angle in a crystal. With a rotation apparatus it is possible, by a slight inclination of the section, to determine whether or not the angle is at its maximum. Ordinarily, when sufficient grains of the same mineral are present in the slide, the Fedorow methods will not be used, at least in full, but under certain conditions, they offer the only possible solution to the determination. Some of the methods are simple and quickly applied, while others are complicated and may require a great deal of time, perhaps hours, for a single mineral.

Observations with the universal stage are usually made by parallel and not by convergent light, which makes it possible to use lower power objectives and to cover a larger field. The rotation instrument is fixed on the stage of the microscope in such a position that the outer horizontal axis (*J*, Fig. 695) is parallel to the principal section of one of the nicols, ordinarily parallel to the left-to-right cross-hair. The inner disk being a glass plate, accurately divided into quadrants, it is easy to fix the universal stage in proper position;

[1] Besides the papers mentioned below see also L. Duparc et R. Sabot: *Les méthodes de Fedorow.* Arch. d. Sci. Phys. et Nat. Genève, XXXIV (1912), (Juillet), pp. 12.

it is only necessary to make these intersecting lines coincide with the cross-hairs of the microscope. If the microscope has a mechanical stage, the determination as to whether the central plate is truly at right angles to the axis of the microscope is made by moving the universal stage across the field in a direction at right angles to the axis, and noting whether the scratch in the glass remains sharply in focus.

Glass hemispheres are sometimes used to increase the angle of vision.[1] In size they are as much less than perfect hemispheres as the thickness of the glass stage and the object-glass, so that when attached above and below the section by a thin film of glycerine or cedar oil, their outer contours form a perfect sphere. Thus if the glass plate of the central stage is just 1 mm. in thickness and the object-glass of the preparation is the same, the latter is turned cover-glass downward, and the two hemispheres are each 1 mm. less in thickness than a perfect hemisphere. For the small apparatus (Fig. 405), in which no glass stage is used and the upper converging lens is placed directly upon the cover-glass of the preparation, the upper hemisphere is almost perfect while the lower one is cut down.

To set the segments properly, the lower one is first put into position by attaching it with a very thin film of glycerine or cedar oil,[2] the latter being rather more sticky. Upon looking through the microscope with a low-power objective, a bright, central circle of light is seen surrounded by a dark circle, the latter, with higher powers, lying beyond the field of view. The lower hemisphere should now be moved laterally until the bright ring lies concentric with the field of the microscope. The rock section is next placed upon the stage, and the mineral to be tested is centered. Careful note is taken of the exact part of the mineral, perhaps marked by a small inclusion, coinciding with the cross-hairs. The upper glass segment is now attached with glycerine or cedar oil. In general, a displacement of the mineral will be noticed. The segment is moved laterally until the original grain is centered, that is, until there is no displacement. If the stage is now rotated in altitude the mineral grain should remain exactly on the cross-hairs, for it lies at the exact center of the sphere, through which, also, all the rotation axes of the microscope and the universal stage pass.

For the rapid centering of the glass hemisphere, as is desirable in examining grains to determine their uniaxial or biaxial character, they may be set in

[1] E. von Fedorow: *Optische Mittheilungen. Noch ein Schritt in der Anwendung der Universalmethode zu optischen Studien.* Zeitschr. f. Kryst., XXV (1895–6), 353.

Idem: *Universalmethode und Feldspathstudien. I. Methodische Verfahren.* Zeitschr. f. Kryst., XXVI (1896), 229–231.

[2] If cedar oil is too thin, it may be rendered much less fluid by spreading it out in thin layers and exposing it for a long time to the influence of air and light. By this means it becomes of the consistency of castor-oil without increasing in dispersive power. The refractive index is also raised to 1.518–1.520. The index can be reduced to 1.510 if desired, by adding olive or castor-oil. (E. Abbe. Botan. Centralbl., X (1882), 224–225.)

carriers[1] which are attached to screws at the side of the inner stage. By this means they must necessarily be placed in proper position. It is better practice to attach the lower lens by glycerine or cedar oil, since it need not be removed upon placing a different mineral under the cross-hairs, and only use a carrier with the upper segment. For fine determinations, however, it is better to use the loose hemispheres, the field being clearer and larger on account of the liquid film used for their attachment.

The size of the field of view will depend upon the refractive index of the glass segments; the greater the index, the greater the angular view. Fedorow used, originally, glasses with an index of 1.7469, which is higher than that of most rock-forming minerals, consequently their values for $2E$ were less than for $2V$. Glass of such high index is very expensive, and the hemispheres usually provided with the instrument have indices of 1.5233. They are, however, just as good for the great majority of rock-forming minerals.

The object-glasses for mounting the preparations, as used by Fedorow,[2] are 2 cm. in diameter and circular instead of rectangular. For this special purpose they possess the advantage of permitting every portion of the slide to be examined, and yet do not interfere with the free rotation of the inner stage. To preserve such sections, they are kept in boxes into which are placed cardboard strips, 1 mm. in thickness (same thickness as the slides), cut as shown in Fig. 694. Between these strips are placed thin rectangular sheets to keep the sections apart, each fifth one being thicker than the others to aid in counting and to permit the writing of a number on its edge. With the Fuess theodolite microscope,[3] sections 28×48 mm. may be used.

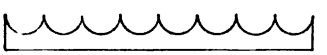


FIG. 694.—Septum in the von Fedorow slide boxes.

426. Locating one Optic Axis.—To determine the position of the point of emergence of an optic axis[4] in its relation to the normal to the section, the universal stage is set upon the stage of the microscope, and is placed in horizontal position with the axis J (Fig. 695) from left to right and the axis H exactly at right angles to it. The section, now in a random position with respect to the orientation of its vibration axes, is rotated about the axis H until, between crossed nicols, darkness ensues. Should there be no position in which it is possible to produce darkness by rotation about the H axis, the

[1] C. Leiss: *Vervollständigte neue Form des E. v. Fedorow'schen Universaltisches.* Neues Jahrb., 1897 (II), 93–94.

[2] E. von Fedorow: *Universalmethode und Feldspathstudien. I. Methodische Verfahren.* Zeitschr. f. Kryst., XXVI (1896), 227.

Idem: *Universalmethode und Feldspathstudien. III. Die Feldspäthe des Bogoslowsk'schen Bergreviers.* Zeitschr. f. Kryst., XXIX (1897–8). 617–618.

[3] Art. 184.

[4] E. von Fedorow: *Universal- (Theodolith-) Methode in der Mineralogie und Petrographie. II. Theil. Krystalloptische Untersuchungen.* Zeitschr. f. Kryst., XXII (1893–4), 232.

Idem: *Op. cit.* Zeitschr. f. Kryst., XXVI (1896), 242–243.

inner disk S, carrying the mineral section, is rotated through a not too small angle, and the operation is repeated. The section is now placed as nearly as possible in the position of darkness by rotation about H. It is evident that the trace of one of the principal sections of the optical ellipsoid now lies parallel to the vibration plane of one of the nicols. In general, however, this plane of the ellipsoid will not be parallel to the axis of the microscope, consequently upon rotating the mineral section about the axis J, the trace will become inclined to the cross-hairs. With the mineral set at some angle about J, the stage is tilted about H, and notice is taken whether darkness appears with greater or less inclination. One will soon find that for a rotation about J in one direction, darkness will appear at a greater angle, and for a rotation in the opposite direction at a lesser angle. There is thus determined the direction of rotation of the preparation necessary to bring one of the principal sections of its optical ellipsoid parallel to the principal section of one of the nicols. When in this position, the field will remain dark during the rotation about J. If it does not quite do so, a very slight rotation about H will correct the error.

FIG. 695.—Von Fedorow universal stage, large model. (Fuess.) See Fig. 407 for an improved form.

The stage of the microscope, or the disk T_1, may now be rotated, with successive settings about J, until the point is reached where the stage remains completely dark during this revolution also; the position of darkness being accurately determined by the fact that a unit retardation plate remains of uniform color during the rotation. Strictly speaking, such a point is never reached in biaxial minerals,[1] owing to dispersion, but the error is ordinarily so slight that it may be neglected. If it is great, monochromatic light may be used, or the section may be oriented simply by the position of maximum darkness.

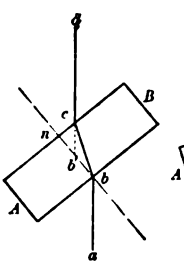


FIG. 696.

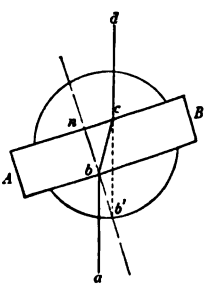


FIG. 697.

In the position of darkness one of the optic axes (bc, Fig. 696) has its apparent direction ($b'c$) parallel to the axis of the microscope. The position of the point of its emergence is located on the section by two readings of

[1] E. Kalkowsky: *Ueber die Polarisationsverhältnisse von senkrecht gegen eine optische Axe geschnittenen zweiaxigen krystallplatten.* Zeitschr. f. Kryst., IX (1884), 486–497.

the stage. The true angle of inclination of the optic axis with respect to the normal (*nb*) to the section may be calculated from the angles *nbc*, *nb'c*, and the mean refractive index of the mineral. This calculation may be performed by means of the formula $n \sin V = \sin E$, where n is the mean refractive index of the mineral, V (*nbc*) the true, and E (*nb'c*) the apparent angle which the axis makes in air with the normal. Several graphical

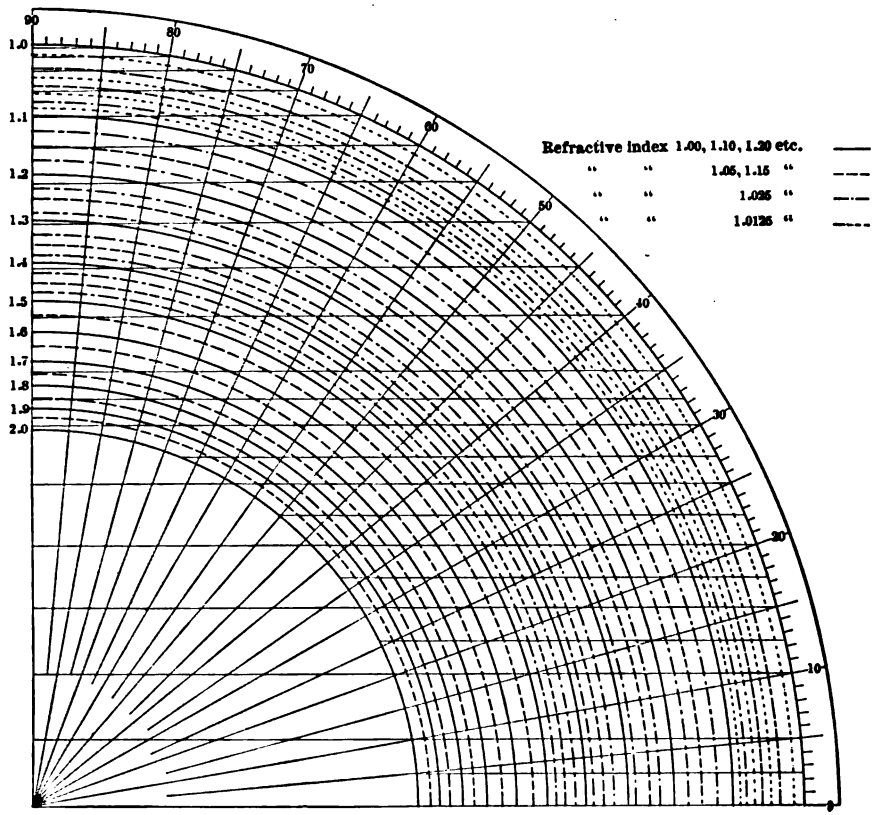


FIG. 698.—Graphical solution of $n \sin V = \sin E$. (After von Fedorow.)

solutions of this equation have been given,[1] the one shown in Fig. 698 being applicable to the determination of the true angle whether the apparent angle was measured in air, oil, glass (Fig. 697), or any other medium.

[1] E. von Fedorow: *Cit. supra*, Zeitschr. f. Kryst., XX (1893–4), 247–8. Gives a diagram for converting $2E$ in air to $2V$.

Idem: *Cit supra*, Zeitschr. f. Kryst., XXV (1895–6), 354–5. Gives a diagram for converting 2H in glass to $2V$.

Idem: *Cit. supra*, Zeitschr. f. Kryst., XXVI (1896), 246–247, and plate IV, Fig. 3. Gives the diagram referred to above. The same diagram is given by Fred. Eugene Wright: *The methods of petrographic-microscopic research*, Washington, 1911, plate 7.

The method of use is as follows: Suppose a mineral with a mean refractive index of 1.5 gave an apparent angle in air of 48° 10′. Find this value on the circumference of the circle (index of air = 1.0), follow the radius to its intersection with the circle of 1.5 refractive index, and then the horizontal line to the right (Fig. 699) to the degree marks at the circumference, in this case 30°.

If the second medium is glass, for example, with a refractive index greater than the mineral, the formula used is

$$\sin H = \frac{n}{m} \sin V,$$

where n is the refractive index of the mineral and m that of the second medium. If, for example, the glass has a refractive index of 1.75, the value of H, corresponding to $V = 30°$, in the same mineral ($n = 1.5$) as above, is 25° 25′.

Using the diagram (Fig. 698), the observed angle being 25° 25′, follow the radius from this value to its intersection with 1.75, follow the horizontal line to the left to its intersection with the 1.5 circle, and read the angle at the end of

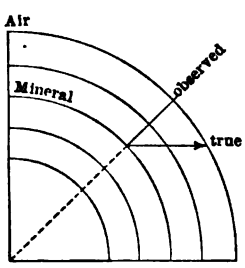


FIG. 699.

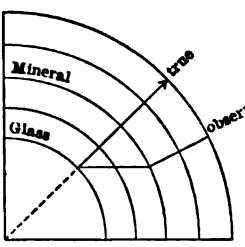


FIG. 700.

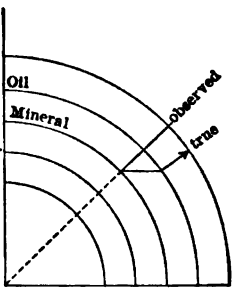


FIG. 701.

FIGS. 699 to 701.—Index sketches showing methods of using the preceding diagram. Fig. 699. Denser medium to air. Fig. 700. Rarer medium to denser. The true angle is greater than the observed. Fig. 701. Denser to rarer medium. The true angle is less than the observed.

the intersecting radius (Fig. 700). If the refractive index of the second medium is less than that of the mineral, follow the horizontal line to the right, instead of to the left, in the same manner as that used when this medium is air (Fig. 701). The general rule to be followed in every case is to follow the radius from the observed value to the curve representing the refractive index of the mineral. The horizontal line passing through this point is followed to its intersection with the curve of the second medium. The angle desired is obtained by following the radius through the latter point to the circumference of the circle. The diagram (Fig. 698) shows, likewise, the critical angle between any two substances, this being the point where a horizontal line is tangent to the circle of the refractive index of the denser medium. Thus the critical angle between water ($n = 1.335$) and air is found by the intersection of the horizontal tangent to the 1.335 curve and the curve of air (1.0).

Its value is 48° 30′. For crown glass ($n=1.608$) the critical angle with air is 38° 30′. As between quartz ($n=1.54$) and water ($n=1.33$) the angle is 60°.

Another method for determining the location of the principal vibration planes in a crystal and the position of the optic axes, was given by Klein[1] in 1895, and later by Evans.[2] The method was intended to be used for the determination of the optic axial angle in mineral sections immersed in a fluid having a refractive index as nearly as possible equal to that of β of the mineral. By parallel light and crossed nicols, the preparation is tilted until, as in the case previously described, the section remains dark during a rotation about the other axis. In this position the axis of rotation is the optic normal or one of the bisectrices of the optic axial angle. In the former case the section will remain dark except at the points where the optic axes emerge. Here, on account of internal conical refraction, there will be a very slight increase in light, but the intensity will remain the same during a complete rotation about the vertical axis M (axis of the microscope). The determination, however, cannot be made very accurately, and it is therefore better to rotate the nicols to the 45° position. The mineral will now appear uniformly light. Insert the gypsum plate and determine whether the vibrations along the axis of rotation are faster or slower than in the direction at right angles to it. If the axis of rotation is one of the bisectrices, it will be the direction of greatest or least ease of vibration, consequently, if the crystal be rotated about the horizontal axis of the stage, there will be no change in the sign (+ or −) of the mineral, although the birefringence will vary from $\gamma-\alpha$ to $\gamma-\beta$, or $\gamma-\alpha$ to $\beta-\alpha$. If, however, the axis of rotation is the optic normal ($\mathfrak{b}$), the optic sign will change four times during the rotation, depending upon whether the section which happens to be horizontal gives a birefringence of $\gamma-\beta$ or $\beta-\alpha$. That is, the position of greater ease, in the particular section which happens to lie at right angles to the axis of the microscope, will first be along the axis of rotation and then at right angles to it. The optic axes emerge at the points where the change from positive to negative character occurs, and here the retardation, except for dispersion, etc., is zero. Its exact position is given by the gypsum plate when the sensitive tint appears, or, better, by a combination wedge, such as the Evans double-quartz[3] or the Wright double-combination wedge,[4] when the black bars of the two halves coincide in position.

For thin sections, it is advantageous first to find, by convergent light, a section which gives the emergence of the acute bisectrix or to locate the

[1] C. Klein: *Ein Universaldrehapparat zur Untersuchung von Dünnschliffen in Flüssigkeiten.* Sitzb. Akad. Wiss. Berlin, 1895 (II), 1151–1159.

[2] John W. Evans: *Determination of the optic axial angle of biaxial crystals in parallel polarized light.* Mineralog. Mag., XIV (1905), 157–159.

[3] Art. 315.

[4] Art. 317.

melatopes by means of a Johannsen auxiliary lens and a low power objective. The glass hemispheres suggested by Fedorow cannot be used in this method since the light does not remain strictly parallel and this makes it difficult to locate the desired point. Immersion is not necessary with thin sections.

427. Determination of the Position of an Optic Axis by Means of the Optical Curves.—The point of emergence of an optic axis may be determined by plotting, in stereographic projection, the curves of zero extinction in certain zones,[1] and finding the point of their intersection. This method, called by von Fedorow[2] the method by optical curves, is best used with a microscope having simultaneously rotating nicols, although an ordinary polarizing microscope may be used. It is based upon the Biot-Fresnel[3] law which states that the trace, on the plane of the section, of the plane bisecting the angle between the two planes, each determined by an optic axis and the normal to the section, is the extinction angle in that section. To obtain the curves, the nicols are first crossed and set at some fixed angle in relation to the cross-hairs. The thin section is placed in a horizontal position and rotated to extinction about the M axis (axis of the microscope). This angle is read, reduced to the true angle by means of diagram Fig. 698, and plotted in stereographic projection. The inner stage T_1 is now turned, by successive 5° or 10° rotations, and at the same time there is determined, by rotation about the axis J, which remains parallel to its original position, the angle through which it is necessary to tilt the section to obtain darkness. The procedure may be reversed, and the stage tilted about J by successive 5° or 10° rotations and the angle on T_1 determined. These values are all plotted in stereographic projection, after being reduced to their true values; the mean value of the refractive index (β) being used instead of the varying indices in each different position with no appreciable error in minerals having weak or medium birefringence. The curve thus obtained must pass through the point of emergence of the optic axis of the crystal, and necessarily also through the center of the projection. It is called the curve of extinction and has a fixed position for a definite position of the nicols. If the relative position between the latter and the axis J is changed by rotating the nicols or the stage of the microscope, and the same method of procedure followed, a different curve, also passing through the point of emergence of the optic axis, is obtained. The intersection of two such curves serves to locate the desired point. As a check, it is better to determine three or more curves, for example with the nicols set at 0°, 22 1/2°, and 45°. Owing to slight inaccuracies in determination, the different curves ordinarily do not intersect in a point, but form a polygon, the center of which is taken as the true point

[1] Cf. Art. 351.

[2] E. von Fedorow: *Cit. supra*, Zeitschr. f. Kryst., XXVI (1896), 231–9.

[3] See Art. 351 *supra*. The reason for this extinction in inclined sections may appear more clearly from the demonstration in the next section.

of emergence. It is, of course, not necessary to determine the complete curves but only that part in the neighborhood of the optic axis. The curves may be named from the inclination of the principal sections of the nicols to the cross-hairs, 0°, 15°, 30°, 45°, etc., extinction curves. In Fig. 702 the 0° curve (nicols parallel to the cross-hairs) is drawn out in full, the 22 1/2° and 45° curves in part.

This method of optical curves can be used for the determination of the optic axial angle only when the points of emergence of both optic axes appear in the field of the microscope, in which case they may both be located by intersections. The values of the angles of inclination having already been reduced to their true values, the measured distance on the projection gives the value of $2V$.

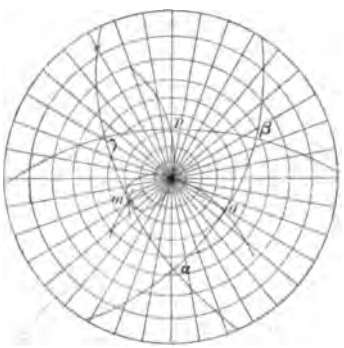


FIG. 702.—Determination of the optic axial angle by the method of optical curves.

428. Locating the Point of Emergence of the Second Optic Axis.—Not commonly will the points of emergence of both optic axes appear in the field at the same time, although one may be brought in, in the majority of cases, by tilting the stage, and its position may be determined by direct observation or as the point of intersection of several optical curves; the other, however, must be located by different means.

The simplest method[1] for determining the second optic axis is as follows: The section is placed in a horizontal position and is then rotated about the vertical axis (M) of the microscope by means of the disk T_1 (Fig. 695), until the known optic axis lies in the plane at right angles to the axis J (EOA', Fig. 703). When in this position let HOC represent the extinction angle. By the Biot-Fresnel law, the unknown axis OB must lie in the vertical plane OD, so placed that the angle $COD = HOC$. If the stage is now tilted about the axis J, the extinction direction OC must change for every different inclination, since by this rotation the angle HOB changes its value. When the axis OB lies in the vertical plane through the J axis (OJ plane) the extinction angle OC' must be 45° since $HOB' = 90° = 2\ HOC'$.

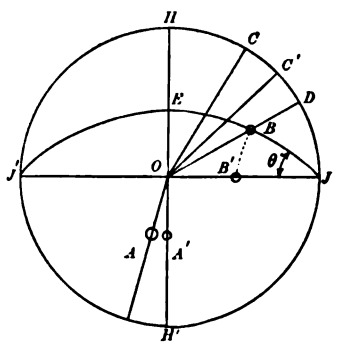


FIG. 703.—Locating the point of emergence of the second optic axis.

We have here, then, an indirect method by which we can determine the

[1] E. von Fedorow: *Cit. supra*, Zeitschr. f. Kryst., XXVI (1896), 234–5.

melatope B, for when the mineral section has been rotated about J to the point where the extinction angle is 45°, it lies in the OJ plane. The position of 45° extinction may be obtained readily by setting the crossed nicols at 45° and rotating the mineral about J to extinction.

It is now necessary to locate B in the stereographic projection from the two known positions. When the mineral section is horizontal we know that the first optic axis emerges at A', and the second somewhere along OD. When the mineral section is tilted about J through the angle θ', the second axis lies in OJ. If this plane ($J'OJ$) be now tilted through an angle θ' (corrected for refractive index by Fig. 698) in the opposite direction from that in which the stage was inclined, the mineral section will again lie in the horizontal plane, and we will have located two planes in which the unknown axis lies. The intersection gives the position of the axis. In the projection the line OD, at twice the extinction angle from H, is known. Draw $J'EJ$, the great circle of the plane inclined at an angle of θ' with the vertical, through the point E. The intersection of OD and EJ is the point of emergence of B.

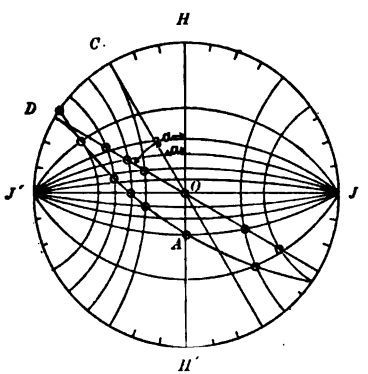


FIG. 704.

This method cannot be applied to such cases where the position of the second axis requires a steeply tilted stage. The effect of the elliptical polarization in such sections makes the exact position of extinction a matter of uncertainty, and a different method, unexpectedly exact, was used by von Fedorow.[1]

This second method is likewise based on the Biot-Fresnel law. Instead of setting the crossed nicols in some definite position with respect to J and determining the inclination required to produce darkness in the section, the preparation, after having its known optic axis placed on the HH' line, is inclined at various angles, and the crossed nicols are rotated to determine the extinction angles. For accuracy, the readings are repeated two or three times for each position of the nicols, which are then rotated through 90° four successive times, and the readings repeated in each position. The stage is tilted up or down, depending upon which side of the axis J the point of emergence of the optic axis falls. Best results are obtained by tilting the section rather steeply and using the same values on either side of the axis. For simplicity in plotting, such angles for tilting the section are chosen as have their true values, and not their measured, in even degrees.

Having obtained, in this manner, several readings, the results are plotted in stereographic projection. The method of plotting is similar to that pre-

[1] E. von Fedorow: *Cit. supra*, Zeitschr. f. Kryst., XXVI (1896), 235–6.

viously described. For example, let the observed extinction angle (that is the angle at which the nicols were set) be α. Draw through the center the line DO (Fig. 704) so that $DOH = 2\alpha$. DO, therefore, is the trace of the vertical plane containing the optic axis in its rotated position. Let A be the true (not observed) angle of inclination of the section about the axis J. Evidently when the mineral section is revolved back to the horizontal position, every point on OD must revolve $A°$ in planes perpendicular to J. Since these vertical planes are represented by the vertical small circles of the net, it is only necessary to lay off $A°$ (30° in the figure) in the proper direction along the vertical small circles, and connect these points by a great circle.

By thus constructing several great circles for different angles of extinction, each containing the desired optic axis, it is clear that the desired position is at their intersection. Ordinarily they do not intersect in quite the same point, but they fall so closely together that there is no difficulty in determining the mean.[1]

This method is very accurate if the position of the first axis has been correctly determined. If it has not been, the variation in the points of intersection of the second axis at once furnish a measure of the amount of inaccuracy and its direction, thus permitting a relocation of the first point and a new trial for the second. If the great circles finally intersect in a point, the two axes are accurate to $1/2°$.

Of all methods of locating the optic axes by means of the universal stage, that of optical curves is the most accurate, but, as von Fedorow[2] himself says, "The method is practically unavailable on account of the length of time required. Even the method of the direct determination of the symmetry planes, which with sufficient practice may be performed in not over two hours . . . . . causes too long an interruption in practical petrographic determinations."

An algebraic computation of the positions of the two optic axes as determined by the optical curves is given by Wallerant.[3]

429. Locating the Symmetry Planes and the Axes of the Optical Ellipsoid within the Crystal.—The methods previously given can be used with a universal apparatus having but two axes of rotation. The following method requires the use of three axes, and is more conveniently performed with four (Fig. 407). By it the positions of the symmetry planes are directly located, and from these the various optical properties.

[1] A slight modification of this method, for use when the intersecting angle is very acute, is given by Fred. E. Wright: *Measurement of the optic axial angle of minerals in the thin section.* Amer. Jour. Sci., XXIV (1907), 351–353, and in *Methods of petrographic-microscopic research*, Washington, 1911, 181–183.

[2] E. von Fedorow: *Cit. supra*, Zeitschr. f. Kryst., XXIX (1897–8), 606.

[3] Fr. Wallerant: *Sur la méthode de déterminations des axes optiques de M. E. v. Fedorow.* Bull. Soc. Min. France, XIX (1896), 356–363.

J. Beckenkamp: Review of above in Zeitschr. f. Kryst., XXIX (1897–8), 431–432.

In determining the symmetry planes by this method,[1] the universal stage is first placed in horizontal position with the J axis at right angles to the H axis, and the position of one of the principal sections of the optical ellipsoid is determined by the method given in Art. 426. In this position the section should remain uniformly dark during the rotation about the axis J. Care should be used in determining correctly this position of maximum darkness, since any error in the result will be from neglect in this respect. Having the angle of inclination about the horizontal axis H, and the angle of rotation about the axis M on the stage T_1, the points g, m, and p, representing lines in the symmetry planes at right angles to the axis of rotation, are to be plotted in stereographic projection, using, of course, the true and not the observed angles. Thus in Fig. 705, the symmetry plane $\alpha\beta$ is determined by the angle Mg, indicating the rotation about the axis H, and HMH_1, the rotation about M. In a similar manner the planes $\alpha\gamma$ and $\beta\gamma$ are determined by the angles Mm and HMH_2, and Mp and HMH_3. Having determined the points g, m, and p, the great circles representing the traces of the symmetry planes may be drawn in the projection through these points, thus locating by their intersections, the points α, β, and γ, which represent the points of emergence of the fastest, intermediate, and slowest rays of the crystal. As a check on the accuracy of the construction, it may be noted (1) that the lines connecting α and p, β and m, and γ and g should be straight and should pass through the center M of the projection, and (2) that the angular distances between α and p, β and m, γ and g, α and β, β and γ, and α and γ should each be 90°. The first condition gives a check upon the accuracy of the determinations; the second upon the accuracy of the value assumed for β in reducing the observed to the true angular values. By noting whether the distances between α and p, β and m, and γ and g are greater or less than 90°, it permits a correction to be applied to the assumed value of β, and a redrawing of the projection. The method thus serves for the rough determination of the value of the mean refractive index.

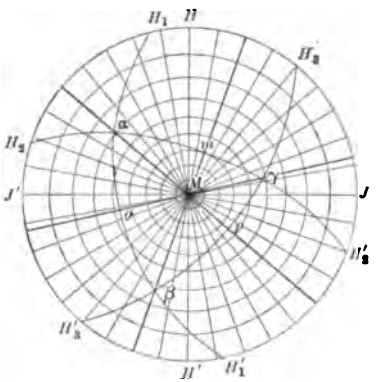


FIG. 705.—Method of locating the symmetry planes and the axes of the optical ellipsoid.

430. Determination of the Position of the Second Optic Axis when the First is Determinable by Optical Curves.—If one optic axis (A) can be determined by means of optical curves, the other one (B) is easily located after having found the symmetry planes, since $\alpha\gamma$ must be the plane of the optic

[1] E. von Fedorow: *Cit. supra*, Zeitschr. f. Kryst., XXVI (1896), 240–244.

axes, and either α or γ must be the acute bisectrix, depending upon the optical character of the mineral. In many cases where it could not otherwise be determined, the optical character can be found by means of the Johannsen auxiliary lens,[1] used with a low-power objective and a tilted stage. Knowing the optical character, $A\alpha$ or $A\gamma$ may be made equal to $B\alpha$ or $B\gamma$.

The time required for the determination of the optic axes by means of symmetry planes is about two hours.[2]

431. Approximate Determination of the Optic Axes when the Section lies nearly Parallel to the Plane of the Optic Axes.—To determine the position of the optic axes when the plane of the optic axes makes an angle of not over 25° with the plane of the section, the following method may be used.[3] Having located the position of β, it is brought into coincidence, by means of the axis H, with the axis of the microscope, thus bringing the plane of the optic axes into the horizontal plane. This horizontal plane will not be disturbed by rotating the stage T_1 about the vertical axis M. By the previous construction the symmetry planes $\alpha\alpha'$ and $\gamma\gamma'$ (Fig. 706) have been located, the third being the horizontal plane. Let the mineral section now be rotated about M on the T_1 stage until one of the optic axes (B) coincides with the axis HH', a position determined by trial. Let γ be the acute bisectrix, for example. The other optic axis will therefore occupy the position MA such that $BM\gamma = \gamma MA$, $M\gamma$ being a direction of extinction. The section is now rotated about the axis J through a definite angle θ (corrected for refractive index) so that the point B falls at E. The extinction angle for the new position (BMn) is now read; it should bisect the angle BMA', the latter being determined from the stereographic projection, since it must lie at the intersection of the vertical small circle through A and the great circle through $J'EJ$. If the two values do not agree, it indicates that the first optic axis did not coincide exactly with OB. If the angle is larger than it should be, it shows that the optic axis lies on the side OB'; if too small, on the side OB''. The section, therefore, should be rotated through small angles in the proper direction, and new sets of determinations made until the observed and constructed values agree. In this position the extinction angle should be determined carefully; twice its value is the value of $2V$.

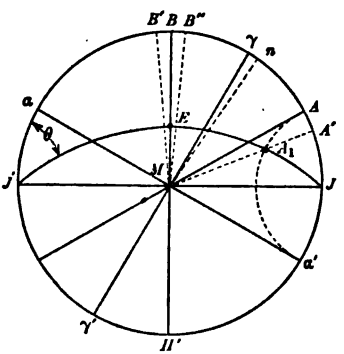


FIG. 706.

[1] Albert Johannsen: *An accessory lens for observing interference figures of small mineral grains.* Jour. Geol., XXI (1913), 96–98.

[2] E. von Fedorow: *Cit. supra*, Zeitschr. f. Kryst., XXIX (1897–8), 606.

[3] E. von Fedorow: *Cit. supra*, Zeitschr. f. Kyrst., XXVI (1896), 245–6.

This case is the most difficult of the methods for determining the optical properties, and it is therefore advisable to choose a different fragment, if this is possible.

432. Simplified Methods.—As mentioned above, the most accurate of the methods for determining optic axial angles by means of the universal stage, is by optical curves, but it is practically unavailable on account of the time required for its execution. Even the method by the direct determination of symmetry planes, which requires, with sufficient practice, not over two hours, is too slow for practical work. For this reason von Fedorow simplified further, as much as possible, the methods of determination, and used only the most accurate of the rapid methods. He gives the following:[1]

433. (a) Both Optic Axes appear in the Field of the Microscope at the most Satisfactory Angle, namely, Inclined between 15° and 55° (Corrected Values) with the Normal to the Section.—By means of the rotation axes J and M, bring, as nearly as possible, the more inclined optic axis to the vertical position, and find, from the stereographic projection by means of optical curves, its exact position. Determine whether the bisectrix of the axial angle is parallel to α or γ by means of the mica wedge. Greater accuracy may be obtained if the other symmetry planes likewise are brought into position at right angles to the axis J, and corrections applied to the projection.

434. (b) One Optic Axis makes an Angle of less than 20° with the Normal to the Section.—The extinction angle, in this case, will be very indistinct. Place the universal stage with the central disk horizontal, the M axis coinciding with the axis of the microscope, and the H axis at right angles to J and M. Turn the inner glass circle, and at the same time incline the section on the H axis, until the optic axis coincides as nearly as possible with the axis of the microscope. Now tilt to a considerable angle on the J axis, and at the same time rotate on the M axis to darkness. Since the plane of the optic axes in this position is at right angles to J, darkness will remain during rotation about it. In Fig. 707 if ab represents the plane of the optic axes, the axis J will coincide with the optic normal (b axis of the Fresnel ellipsoid or β of the indicatrix). The angle at which the axis H is inclined in the horizontal plane may now be read from the outer ring (T_1, Fig. 695). The stereographic net may be turned through a similar angle so that the principal

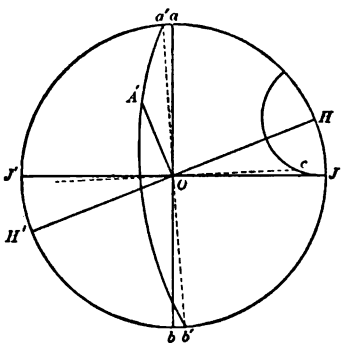


FIG. 707.

[1] E. von Fedorow: *Op. cit.*, Zeitschr. f. Kryst., XXIX (1897–8), 606–610.

diameter coincides with the direction of this axis (Fig. 707). Determine the inclination of H, and indicate this position on the stereographic projection as it would appear rotated to the horizontal plane. Every point on the sphere will describe a circle at right angles to the H axis and be projected as a vertical small circle. Thus in the figure a rotation of 22° is shown, the end of the J axis ($=\beta$) appearing at c, and the desired optic axis as OA' on a line at right angles to HH'. The plane of ab will appear as the circle $a'b'$ in the projection, every point in it lying 22° distant on the small circles. An arc may be drawn through the points so found or, more simply, the curve may be sketched by rotating the paper above a Wulff net. If the inner glass circle of the universal stage has been rotated, the orientation of the optic axes with respect to crystallographic directions may be obtained by simply rotating the entire net through the proper angle; for example, by transferring points by means of a transparent Fedorow net.

To determine the location of the other optic axis, rotate about J through some round number of degrees, and determine the extinction angle in this position. The extinction curves thus obtained will intersect $a'A'b'$ at the second optic axis.

Determine graphically the positions of γ and α. The first determination, on account of the indistinct extinction, is to be regarded as approximate, and is to be corrected by the redetermination of the symmetry planes. This, however, is a simple process, since their approximate positions are now known.

435. (c) One Optic Axis makes an Angle of between 20° and 55° with the Normal to the Section, the Other lies beyond 55°.—The first step in the determination of extinction curves is used for this determination. The first optic axis is rotated until it lies in the plane at right angles to the axis J. In this position the extinction angle is determined with the stage in horizontal position, as well as inclined to some round number of degrees (corrected angle). By this means are obtained a diameter and a great circle in the projection, and their intersection gives the location of the other optic axis. Now determine graphically γ, β, and α, and verify by symmetry planes.

436. (d) Both Optic Axes are Inclined more than 55° to the Normal to the Section.—In this case the inner glass stage is set at 0°, and the mineral section is rotated about M and H to the point of darkness. The section is now rotated about J to test whether the darkness remains. If it does not do so, it is rotated to a different position of darkness about M and H, until finally, after repeated trials, it remains dark also during the rotation about J. In this position the axis J coincides with one of the axes of the optical ellipsoid, and therefore a symmetry plane lies at right angles to this axis.

The amount of inclination of the H axis to the vertical cross-hair, that is, its rotation about M in the horizontal plane, is shown by rotating the stereographic net to an equal angle (Fig. 707). The pole of the ellipsoid axis may

now be located on the vertical small circle Jc, by laying off from J to c a distance equal to the amount of the vertical inclination about the H axis. The symmetry plane, corresponding to this pole, is located directly by laying off from the line ab, along vertical small circles, angles equal to Jc. This gives the great circle $a'b'$, which is the rotated position of ab.

The operation is now repeated for other angles of H, to locate a second pole and a second symmetry plane. Knowing two planes, the third may be constructed graphically.

The position of the symmetry planes must be verified now in the usual way, since the above operation gives simply the approximate positions. If one optic axis is not inclined over 70°, its position may best be determined by means of an optical curve. Its position is at the intersection of this curve with the plane of the optic axes. The other optic axis may easily be determined graphically.

If both optic axes are inclined over 70° they may be determined by the method given in Art. 431.

CHAPTER XXXVI

DETERMINATION OF OTHER PROPERTIES THAN 2V BY MEANS OF THE UNIVERSAL STAGE

437. Opaque Minerals.—The universal stage offers a ready means for changing the angles at which the incident light falls upon opaque minerals, thus aiding in their examination by reflecting the light from their surfaces.

438. Isotropic, Uniaxial, or Biaxial Character.—Isotropic crystals remain dark in every position between crossed nicols, consequently if an isotropic section is rotated in altitude, for example about the J axis of the universal stage, it will remain dark. This will also occur in uniaxial and biaxial crystals if a symmetry plane of the optical ellipsoid happens to lie at right angles to the J axis. A slight rotation about M, however, with J in an inclined position, will definitely show whether or not the crystal is isotropic.

Uniaxial crystals may be separated from biaxial crystals by tilting the mineral section until a position is reached in which, during a complete rotation about the M axis, the stage will remain uniformly dark. If the crystal is uniaxial, two symmetry planes will pass through this point, consequently the stage may be rotated about the J and the H axes, at right angles to each other, and darkness will remain. If the crystal is biaxial, but one plane of symmetry, the plane of the optic axes, will pass through this point.

439. Positive or Negative Character of an Anisotropic Mineral.[1]—A uniaxial crystal, placed between crossed nicols and with its principal axis parallel to the J axis of the stage, will show equal birefringence (ω) in every direction in the zone at right angles to the axis, except for such differences as may be caused in the thickness of the section by the rotation. When rotated about the axis at right angles to J, the birefringence will gradually change from o to $\omega-\epsilon$. To determine the optical character of the crystal, it is only necessary to determine whether the ease of vibration, in the direction of rotation about the J axis, is greater or less than in the direction at right angles to it.

A biaxial crystal will show, in general, increasing birefringence in both directions of rotation. Select the section showing the highest interference colors, that is, the section nearest the plane of the optic axes. Determine in it the slow ray (c). Tilt the section as much as possible, and rotate it about

[1] E. von Fedorow: *Ein einfaches Verfahren zur Bestimmung des absoluten optischen Zeichens eines unregelmässigen Mineralkörnchens in Dünnschliffen.* Zeitschr. f. Kryst., XXIV (1894–5), 603–605.

M until it comes to the position where the interference colors are lowest. Now turn the section back to the horizontal position and determine the angle between the axis of rotation and the axis c. If this angle is less than 45° the mineral is positive, if greater negative. The above method is only roughly approximate. For accurate determinations it is necessary to measure accurately the angle $2V$.

As an example, von Fedorow gave a determination made on a crystal of epidote. A section nearly parallel to the plane of the optic axes showed green of the third order. Inclined about 50°, and rotated about the *M* axis, a position was found in which the second order blue appeared. The inclination of *H* to the direction of the slow ray was 10° to 15°, consequently the mineral was negative.

440. Maximum Extinction Angle.—In the determination of feldspars, pyriboles,[1] and many other minerals, it is necessary to determine the maximum extinction angle. Ordinarily a search is made through the slide, and the maximum angle of all those found is considered the maximum angle of the mineral. This value may not always be correct, for in a schistose rock the crystals may lie more or less parallel, consequently the orientation may be such that the maximum angle cannot be obtained. In slides containing feldspars, there may be two kinds of plagioclase and, unless combined Carlsbad and albite twinning occurs or some other property aids in the determination, one determines simply the feldspar having the maximum extinction angle. With the universal stage it is a simple matter to rotate a section to various positions and note whether the angle increases or decreases. Thus the maximum angle may be readily obtained.

441. Mean Refractive Index of a Mineral.—A rough method for determining the value of β follows from the method of determining symmetry planes. It has already been given.[2]

442. Orientation of the Crystal Section with Reference to the Axes of the Optical Ellipsoid.—The inclination of the mineral section to the axes of the optical ellipsoid or to the optic axes, or the inclination of the optic axes with respect to the section, may be determined from the methods given above for locating the axes and symmetry planes of the optical ellipsoid.[3]

443. Determination of the Maximum Birefringence of an Unknown Mineral from that of One which is Known.—The method of determining the maximum birefringence of a mineral is best illustrated by an example.[4]

[1] Family of pyroxenes and amphiboles. See Albert Johannsen: *Petrographic terms for field use.* Jour. Geol., XIX (1911), 319.

[2] Art. 429.

[3] Art. 429.

[4] E. von Fedorow: *Op. cit.*, Zeitschr. f. Kryst., XXV (1895–6), 355–356.

See also W. Nikitin: *Beitrag zur Universalmethode. Zur Bestimmung der Doppelbrechung.* Zeitschr. f. Kryst., XXXIII (1900), 133–146.

It is required to determine, from a section of a quartz epidosite, the thickness of the slice and the maximum birefringence of the epidote from a grain selected at random.

A quartz grain is selected and rotated until its *c* axis lies parallel to the axis of the microscope. The angle of inclination of the section is found to be, say, 42°. This represents a true angle of 49 1/2° (Fig. 698) since the light, in this case, passes from quartz, with a refractive index of 1.544, to glass with an index of 1.74. The stage of the microscope is now turned until the *J* axis of the universal stage coincides in direction with the principal section of one of the nicols. In this position, no matter how much the crystal is rotated, it remains dark, since crystallographic *c* constantly remains in a plane perpendicular to the direction of the axis *J*.

The crystal is now rotated so that crystallographic *c* lies in the plane at right angles to the axis of the microscope. Since the inclination of this axis was 49 1/2°, the true angle must be 90°—49 1/2°=40 1/2°, which corresponds to a rotation of the stage of 35°. The quartz crystal now lies in the position giving the maximum birefringence ($\epsilon-\omega$). Measuring the amount of the retardation by means of a quartz or mica wedge, it is found to be, say, 310$\mu\mu$ which, since quartz has a maximum birefringence of 0.009, gives a thickness of section of 0.035 mm., as may be found from Fig. 453. But the thickness of section which produced this color was not the true thickness of the section but the thickness along the inclined line *Oa* (Fig. 708). The true thickness *Oc* equals $Oa \cdot \cos 40\ 1/2° = 0.035 \cdot \cos 40\ 1/2° = 0.026$ mm.

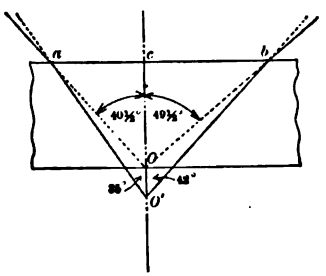


FIG. 708.

Instead of computing the values of the cosines for each mineral, they may be computed, once for all, and shown graphically, as in Fig. 709. In this diagram the true thickness is shown by the perpendicular through the intersection of the true angle with the curve representing the measured thickness.

Coming now to the second part of the problem. A grain of epidote is rotated to determine the positions of the optic axes, which may not appear very clearly on account of the strong dispersion of epidote. Having located each of the optic axes by means of two angles, they are corrected for their refractive indices, and are located on a stereographic projection net. This determines the plane of the optic axes, from which may be obtained the position of *b* and the amount of rotation, likewise corrected, necessary to revolve the stage in order to bring it parallel to the axis of the microscope, say 43° for the *J* axis and $x°$ for the *M*. Carrying out this rotation, the double refraction is determined—perhaps green of the third order—representing a wave difference of 127.5$\mu\mu$. From the determination on the quartz it was

found that $Oc=0.026$ mm. whereby $Oa=\frac{0.026}{\cos 43^\circ}=0.035$ mm. Again, from Fig. 453 we find that a section 0.035 mm. in thickness and having a retardation of $127.5\mu\mu$ has a maximum birefringence of approximately 0.037. This is the desired maximum birefringence of the epidote.

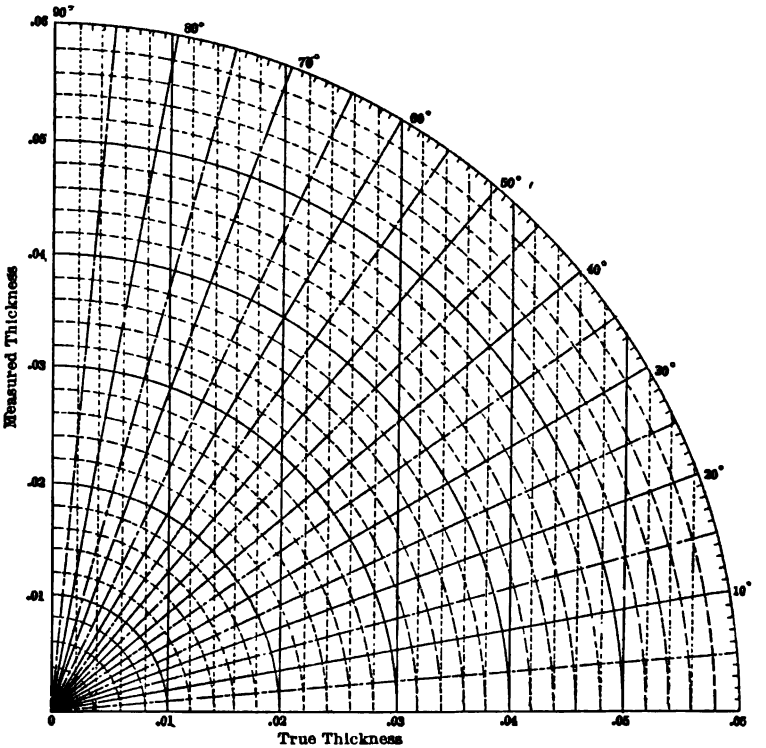


FIG. 709.—Graphical solution of the equation, $b=c\cos\alpha$, used in reducing the measured thickness of a section to its true thickness.

444. Graphical Representation of the Variation in the Double Refraction in Different Directions.—The variation in the double refraction in different directions in a crystal, as obtained by the rotating stage, may be shown in stereographic projection in a manner similar to that given by Schneiderhöhn[1] for the variation observed by means of a sliding diaphragm (Blendenschieber) above the ocular. By moving the slide in this ocular from the center across the field, parallel to one or the other of the nicol prisms, only a single inclined beam of light will pass through the section, the amount of retardation increasing as the distance traveled through the slice, consequently the inclination,

[1] H. Schneiderhöhn: *Die Beobachtung der interferenzfarben schiefer Strahlenbündel als diagnostisches Hilfsmittel bei mikroskopischen Mineraluntersuchungen.* Zeitschr. f. Kryst., L (1912), 231–241.

increases. While the variations are too slight to be detected with accuracy in ordinary cases in the small space permitted by the opening angle of the ocular, the diagrams given by Schneiderhöhn may be studied with advantage. Instead of showing lines of equal retardation, as was done by Michel-Lévy and others,[1] Schneiderhöhn indicates an increase or decrease by increasing or de-

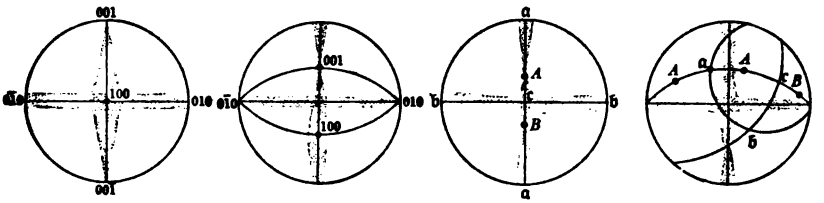


FIG. 710. FIG. 711. FIG. 712. FIG. 713.

FIGS. 710 to 713.—Graphical representation of the variation in the strength of the double refraction in different directions in a crystal.

creasing the width of a broad shaded line. Thus Fig. 710 represents the increasing and decreasing birefringence in zones along the vibration planes of the nicols of a section of a uniaxial mineral cut parallel to the optic axis. Fig. 711 shows an inclined uniaxial crystal, Fig. 712 a biaxial crystal cut perpendicular to the acute bisectrix, and Fig. 713 a random section of a biaxial crystal.

[1] Art. 289.

CHAPTER XXXVII

OPTICAL ANOMALIES

445. The Cause of Optical Anomalies.—So long ago as 1815, Brewster recognized the fact that certain crystals show optical properties which are not in harmony with their physical characters. Thus many crystals of leucite, analcite, garnet, and boracite, which are isometric and should therefore appear isotropic, show low interference colors between crossed nicols, and may even show a uniaxial or a biaxial interference figure in convergent light. In other cases the same minerals show bands which extinguish in different positions. This is well shown in leucite. Another anomaly in an isotropic substance is the double refraction seen in stained glass.

Again, crystals of the hexagonal or tetragonal systems may show biaxial interference figures. This is very common in quartz, eudialyte, and nephelite, the former, in many cases, showing an apparent axial angle of 18°, while eudialyte sometimes has one as great as 50°. Another anomaly is the separation of basal sections of tetragonal crystals into sectors of different illumination. This is well shown in apophyllite and vesuvianite[1] which, between crossed nicols, appear to be made up of a number of separate triangles joined at their apices.

Various theories have been advanced to account for these optical anomalies, and it is probable that not all cases are due to the same cause. That compression or tension is able to change the optical character of a mineral was already recognized by Brewster, and many experiments have been made by subsequent investigators on crystal sections and on colloids.

The effect of pressure on an isotropic substance may most easily be shown by inserting a perfectly circular disk of soft gelatine between two object glasses and placing it on the stage of a polariscope. If the ocular tube is lowered until it touches the upper glass, and a little pressure is applied, a perfect uniaxial cross will appear. The reason is not far to seek. The gelatine, when not under pressure, was isotropic, and the light passed through with equal ease in every direction. Upon the application of vertical pressure, the stress developed in this direction became greater than in the direction at right angles to it, which, in a circular disk, is radially equal. As a result, the indicatrix was changed from a perfect sphere to an ellipsoid of rotation. If the interference figure thus produced is examined by the aid of the gypsum plate,

[1] See Klocke, Neues Jahrb. 1881 (I), 204–205; Klein, Neues Jahrb., 1884 (I), 253–256. References in bibliography at end of chapter.

it will be found that it is negative, which follows from the fact that the increase in pressure has increased the ease of vibration in the same direction. The indicatrix, being the inverse of the ease of vibration figure, will consequently be oblate. If the pressure is applied around the periphery, as may be done by surrounding the gelatine disk by a brass strip and drawing it together by a cord, the ellipsoid will be prolate and the figure positive.

To show the effect of pressure on mineral sections, Bücking devised the instrument shown in Fig. 714. A brass plate *b* is clamped to the stage of the polariscope so that the opening *o* is in the center. A steel plate *d* is screwed to *b* on one side, and on the other is attached the sliding plate *e*. The latter may be forced against a crystal, placed over the opening *o*, by the screw *m*, and the amount of the pressure may be measured by the compression indicated on the frame *r*.

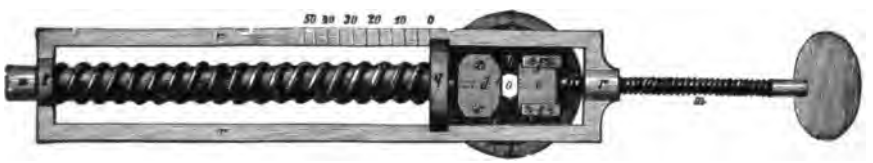


FIG. 714.—Bücking's instrument for showing the effects of pressure upon the interference figure of a mineral. (Fuess.)

If a cube of a uniaxial crystal is placed between the jaws of this instrument, so that the optic axis lies at right angles to the plate *b*, and pressure is applied, a gradual opening of the uniaxial cross is seen. The instrument should be so arranged on the stage that the compression comes at 45° to the principal sections of the nicols. By increasing the pressure, the hyperbolæ separate still farther, but not in proportion to the amount of pressure, for while a very slight pressure will make a uniaxial crystal biaxial, considerable pressure is necessary to increase the size of the optic angle.

If a cube of glass is placed in the instrument, the effect of the lateral pressure produces an interference figure resembling that of a uniaxial crystal cut at right angles to the direction of its optic axis.

A biaxial crystal, cut with its acute bisectrix vertical, when compressed will show an increase or a decrease in the optic axial angle, depending upon the direction of the pressure and the optical character of the mineral. In every case, pressure increases the ease of vibration in the direction in which it is applied.

Tension and compression, then, easily account for the anomalous biaxial character of certain uniaxial crystals, and the greater or less size of the optic axial angle of those that are biaxial. One may visualize the change produced by imagining compression or tension exerted upon the optical ellipsoids.

To account for the separation of certain minerals into differently illuminated fields, between crossed nicols, Reusch supposed that such minerals

contracted in certain directions during the process of crystallization and, upon solidification, retained the strain thus induced. That such is actually the case was shown by Klein and by Ben Saude who filled molds with gelatine and allowed them to dry for two to three days. It was found that slices, mounted in Canada balsam to prevent further drying, showed a separation into fields, as do certain isotropic minerals, and that these fields were dependent upon the outlines of the molds used.

Other optical anomalies, such as double refraction in leucite or boracite, may be explained by the fact that these minerals are dimorphous, that is, possess two forms. Above 433° leucite is truly isometric, while below this temperature it has weak double refraction. It therefore crystallized from the igneous magma in the isometric system, and retained its original form upon cooling.

Another cause for optical anomalies may be the intergrowth of lamellæ in slightly different optical orientation, as in prehnite, or in lamellæ of slightly different chemical composition, as in alum.

The abnormal interference colors spoken of in Art. 290, and caused by different retardations in rays of different wave lengths, are sometimes called anomalous interference colors.

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1898. Reinhard Brauns: *Ueber Polymorphie und die optischen Anomalien von chlor- und bromsaurem Natron.* Neues Jahrb., 1898 (I), 40–59.

1898. E. von Fedorow: *Ueber eine besondere Art der optischen Anomalien und der Sanduhrstructur.* Zeitschr. f. Kryst., XXX (1898), 68–70.

1898. F. Wallerant: *Théorie des anomalies optiques, de l'isomorphisme et du polymorphisme déduite des théories de MM. Mallard et Sohncke.* Bull. Soc. Min. France, XXI (1898), 188–256.

1899. E. S. Dana: *A Textbook of Mineralogy.* New York, 1899, 228–231.

1900. E. von Fedorow: *Constatirung der optischen Anomalien in Plagioklasen.* Zeitschr. f. Kryst., XXXI (1898–9), 579–582.

1901. O. Mügge: *Krystallographische Untersuchungen über die Umlagerungen und die Structur einiger mimetischer Krystalle.* Neues Jahrb., B. B., XIV (1901), 246–318.

1904. Rosenbusch-Wülfing: *Mikroskopische Physiographie*, I–1, Stuttgart, 4 Aufl., 1904, 356–359.

1905. P. Groth: *Physikalische Krystallographie.* Leipzig, 4 Aufl., 1905, 234–236.

CHAPTER XXXVIII

DETERMINATION OF SPECIFIC GRAVITY

446. Specific Gravity.—The specific gravity or density of a substance is the ratio of its weight in air to its weight in water at 4° C. (39.2° F.). In other words, it is the ratio of the weight of any fragment of a substance to the weight of an equal amount of water. The specific gravity of a mineral, provided it is pure and free from inclusions of solids, liquids, or gases, is a constant quantity. In isomorphous series, or in minerals whose chemical composition differs in different specimens, there is, however, a variation, and this serves as a means of separation.

The determination of specific gravity properly belongs to the province of mineralogy and not to petrography, for which reason the usual methods will be little more than mentioned here. For more detailed descriptions the student is referred to the standard works on mineralogy.[1]

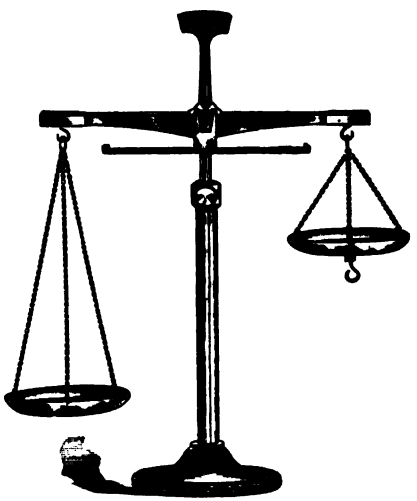

FIG. 715.—Specific gravity balance. (Central Scientific Co.)

447. Hydrostatic Balance.—The mineral, after examination under the microscope for impurities, is weighed in air (w) and then in water (w'). The difference between these two weights represents the weight of an equal amount of water ($w-w'$). Therefore the specific gravity (G) is represented by the equation

$$G=\frac{w}{w-w'}.$$

The usual form of hydrostatic balance is shown in Fig. 715. It differs from an ordinary balance only in having one pan suspended by a shorter wire, and in having beneath it a hook to which is attached a thin wire with or

[1] See also V. Goldschmidt: Verhandl. k. k. Geol. Reichsanst. Wien, 1886, 439.*
Idem: *Bestimmung des specifischen Gewichtes von Mineralien.* Ann. d. k. k. naturhist. Hofmuseum., I (1886), 127–134.

without another pan. For the determination of the weight in air the upper pan is used, the lower one being immersed in a beaker of water. For the determination of the weight in water, the mineral is transferred to the lower pan, if one is present, or is attached to the wire.[1]

A very convenient balance which gives at once the value of the specific gravity from the reading on a graduated arm, was designed by Rogers[2] (Fig. 716).

If the mineral fragment is too small to be thus determined, or if it is in a powdered state, its specific gravity may be obtained, as suggested by Penfield,[3] by placing it in a small glass tube, closed at one end, and having a platinum wire at the other by which to suspend it. The fragments are first weighed dry. They are then boiled in water to remove all air, are transferred to the

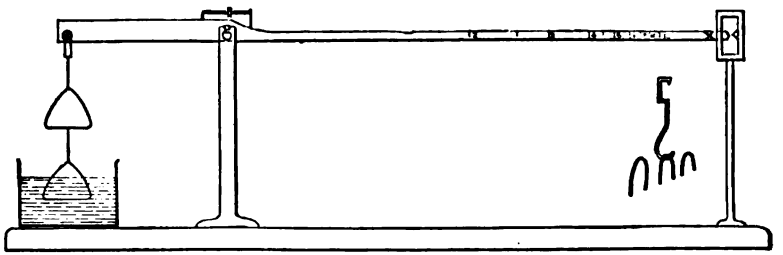

FIG. 716.—Roger's specific gravity balance.

tube, which is suspended from a balance, and are weighed immersed in water. The weight of the tube in water without the mineral is subtracted from the former weight to give the weight in water. The specific gravity is found by the same formula as before.

448. Jolly Balance.—In the Jolly[4] balance (Fig. 717) the specific gravity is determined by noting the amount of lengthening of a spring when the mineral is placed in the upper pan in air (w), and the amount when it is in the lower pan and immersed in water (w'). Here also

$$G = \frac{w}{w - w'}.$$

[1] See Axel Gadolin: *Eine einfache Methode zur Bestimmung des spezifischen Gewichtes der Mineralien.* Pogg. Ann., CVI (1859), 213–225.

G. Tschermak: *Ein einfaches Instrument zur Bestimmung der Dichte der Mineralien zugleich für annähernde Quantitätsbestimmung, bei chemischen Versuchen brauchbar.* Sitzb, Akad. Wiss. Wien, XLVII (1863), 294–301.

Franz Toula: *Hydrostatische Schnellwage.* T. M. P. M., XXVI (1907), 233–237.

[2] Austin F. Rogers: *A new specific gravity balance.* Science, XXXIV (1911), 58–60.

[3] S. L. Penfield: *Ueber einige Verbesserungen der Methoden zur Trennung von Mineralien mit hohem specifischen Gewicht.* Zeitschr. f. Kryst., XXVI (1896), 134–137.

[4] P. Jolly: *Eine Federwage zu exacten Wägungen.* Sitzb. Akad. Wiss. München, 1864 (I) 162–166.

An improved form of Jolly balance was designed by Linebarger,[1] and another by Kraus.[2]

449. Pycnometer for Determining the Specific Gravity of Powders.—For determining the specific gravity of small fragments or of powders, the mineral may first be weighed dry in air (w), then placed in a vessel—called a pycnometer[3] (Fig. 718)—previously weighed full of water (w'), the air excluded, the water brought to the same level as before, and the whole weighed (w'').

$$G = \frac{w}{w + w' - w''}.$$

FIG. 718.—Pycnometer (Central Scientific Co.)

450. Smeeth's Method for Mineral Powders (1888).—Smeeth[4] determined the specific gravity of mineral powders by first heating a small amount of vaseline in a watch crystal to remove bubbles. After cooling, the glass and vaseline were weighed in water (w') by suspending them by a fine wire. The watch crystal was now taken out, the water poured off, and any remaining drops carefully removed by means of filter paper. After heating the vaseline again, a weighed amount (w) of the powdered mineral was scattered over the surface to which it adhered. The whole was weighed in water, after cooling (w'').

$$G = \frac{w}{w - (w'' - w')}.$$

The result is entirely independent of the specific gravity of the vaseline.

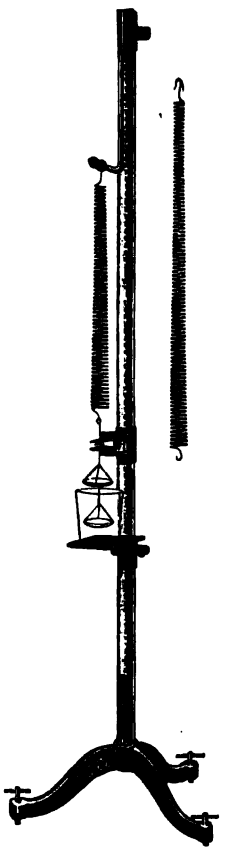

FIG. 717.—Jolly balance (Central Scientific Co.)

[1] C. E. Linebarger: *A new form of the spiral spring balance.* Physical Review, XI (1900), 110–111.

[2] Edward H. Kraus: *A new Jolly balance.* Amer. Jour. Sci., XXXI (1911), 561–563.

Idem: *Eine neue Jolly'sche Federwage zur Bestimmung des spezifischen Gewichts.* German translation of preceding. Centralbl. f. Min., etc., 1911, 366–368.

[3] James P. Joule and Lyon Playfair: *Researches on atomic volume and specific gravity.* Jour. Chem. Soc. London, I (1849), 123.

Earl of Berkeley: *On an accurate method of determining the densities of solids.* Mineralog. Mag., XI (1895), 64–68.

W. Leick: *Ueber specifische Gewichtsbestimmung.* Mittheil. naturwiss. Ver. Neu Vorpommern und Rügen. XXVII (1895).*

[4] W. F. Smeeth: *On a method of determining the specific gravity of substances in the form of powder.* (Communicated Feb. 14, 1888.) Proc. Roy. Dublin Soc., VI (1888–1890), 61–62.

451. Specific Gravity of Porous Substances.—In the determination of the specific gravity of porous substances, such as pumice, chalk, etc., two values should be obtained. First the specific gravity of the mineral with its included air spaces, and second the specific gravity of the material itself. The former value may be determined by coating the mineral with a thin, and negligible, coating of wax or varnish, and proceeding as above. The second value is determined on the mineral powder.

452. Specific Gravity of Substances Soluble in Water.—If the substance under examination is soluble in water,[1] its specific gravity with reference to some other fluid, such as absolute alcohol, should be determined. The resulting value should be multiplied by the specific gravity of the fluid used. Linck covered the mineral with a very thin coating of paraffine, prepared by dissolving a small amount in much ether.

453. Determination of Specific Gravity by Heavy Fluids.—Instead of determining the specific gravity of a mineral by weighing it in water, the determination may be made by immersing it in a liquid of known density and noting whether it sinks or floats. For such determinations a series of fluids of different specific gravities, or two fluids of widely different specific gravities, may be used.[2] In the latter case the mineral to be determined is first placed in the heavier solution. If it floats, the lighter fluid is slowly added, with constant and thorough stirring, until the mineral begins to show signs of sinking. The lighter fluid is now added more slowly until a point is reached at which the mineral remains stationary for a short time at any depth at which it may be placed. That the point of equal density is being approached may be seen by the fact that, after stirring, the movements of the mineral particles are more sluggish, while flakes stand on their edges, and laths on their ends. When the specific gravity of the mineral and the fluid are equal, it is only necessary to determine that of the latter.

Various fluids have been suggested for the determination of density, but these so-called heavy solutions have been used more frequently for the mechanical separation of the different components of a composite rock than for the determination of specific gravity.

The first use of a heavy solution for the determination of density is ascribed by Kalkowsky[3] to Scheibler who used, in 1861, sodium meta-

[1] J. Linck: *Beitrag zur Kenntniss der Sulfate von Tierraamarilla bei Copiapó in Chile.* Zeitschr. f. Kryst., XV (1888), 1–28, especially 9.

J. W. Retgers: *Die Bestimmung des spezifischen Gewichts von in Wasser löslichen Salzen.* Zeitschr. physikalische Chemie, III (1889), 289–315; IV (1889), 189–205; XI (1893), 328–344.

[2] See caution in regard to the use of heavy solutions, Art. 497.

[3] Review by Ernst Kalkowsky of A. Karpinskij: *Petrographische Notizen* (Iswestija des geol. Comites, III, No. 8, 263–280), St. Petersburg, 1884, in Neues Jahrb., 1886 (I), 263–264. In this work, written in the Russian language, Karpinskij gives a history of the discovery and use of heavy fluids for mechanical separation of rock constituents. In the review the original reference to Scheibler's work, if published, is not given.

tungstate with a specific gravity of 3.02. According to the same authority Marignac, in 1862, used a solution of sodium silico-tungstate ($4Na_2O \cdot SiO_2 \cdot 12WO_3 \cdot 7H_2O$) with a specific gravity of 3.05. In the same year Schaffgotsch[1] used an aqueous solution of acid mercuric nitrate. Into this solution the mineral was placed. If it floated, dilute nitric acid was added until the mineral slowly sank, whereupon a glass rod was dipped into the concentrated solution of acid mercuric nitrate and placed in the test glass enough times to cause the mineral to be suspended. The temperature of the solution was now raised to 17 1/2° C. by warming the beaker with the hand, and the specific gravity of the solution determined. On account of the acid character of the solution it acts upon many minerals, and was but little used. It cannot be diluted with water on account of the precipitation of a basic salt.

454. Sonstadt (or Thoulet) Solution (1874, 1877).—The so-called Thoulet solution, with a maximum specific gravity of 3.196, is an aqueous solution of potassium mercuric iodide. It was first described by Sonstadt,[2] in 1874, and his name should be given to it. He used it for the determination of the specific gravities of alkali salts, and prepared it by making a saturated solution, at room temperature, of iodide of potassium, into which as much mercuric iodide was stirred as would dissolve

Though the Sonstadt solution was used by Church[3] in 1877, it did not become generally known until Thoulet[4] published his experiments in 1878–9. It became still more widely known after Goldschmidt[5] published his careful investigations of the properties of the solution in 1881.

To prepare the solution, 80 c.c. of cold distilled water are taken, and in it 270 grm. of mercuric iodide (HgI_2) and 230 grm. of potassium iodide (KI) are dissolved by stirring. The solution is placed in a porcelain evaporating dish on a water-, not sand-bath, and is evaporated until a crystalline film forms on the surface, or until a crystal of tourmaline ($G=3.1$) or fluorite ($G=3.18$) floats. Upon cooling, the solution contracts, and the specific gravity rises to its maximum of 3.196. Needles of hydrous potassium mercuric iodide may crystallize out upon cooling, but if sufficient liquid has been prepared, the clear portion may be decanted. Should it be necessary to use all of the fluid, a *few drops* of water added will cause the crystals to be

[1] F. G. Schaffgotsch: *Ermittelung des Eigengewichts fester Körper durch Schweben.* Pogg. Ann., CXVI (1862), 279–289.

[2] E. Sonstadt: *Note on a new method of taking specific gravities, adapted for special cases.* Chem. News, XXIX (1874), 127–128.

[3] A. H. Church: *A test of specific gravity.* Mineralog. Mag., I (1877), 237–238.

[4] J. Thoulet: *Séparation des éléments nonferrugineux des roches fondée sur leur différence de poids spécifique.* Comptes Rendus, LXXXVI (1878), 454–456.

Idem: *Séparation méchanique des éléments minéralogiques des roches.* Bull. Soc. Min. France, II (1879), 17–24.

[5] V. Goldschmidt: *Ueber Verwendbarkeit einer Kaliumquecksilberjodlösung bei mineralogischen und petrographischen Untersuchung.* Neues Jahrb., B.B. I (1881), 179–238.

dissolved. So long as the proportions of HgI_2 and KI remain approximately as given above, the solution may be diluted with water in any amount to a minimum density of 1.0, and it may be restored by evaporation over a water-bath to its maximum value. A small excess of KI, according to van Werveke[1] is beneficial rather than harmful. If either salt is in excess, it crystallizes out, the mercuric iodide as a yellow hydrous double salt in needle-like crystals, or the potassium iodide in small cubes. If the HgI_2 is greatly in excess, the crystallization may take place suddenly upon evaporation, and the solution turn into a stiff, felty mass of fine needles. Should this take place, a KI solution, and not water alone, should be added. If the solution remains in contact with air for a long period, both salts may separate.

In spite of its high specific gravity, the solution may be filtered readily through filter paper. It should be transparent and of a yellowish-green color. After long use the solution may turn reddish-brown, due to the separation of iodine. It may be restored to its original condition by the addition, with stirring during evaporation, of a small quantity of pure mercury.

The density of Sonstadt's solution varies with the humidity of the air. It reaches its maximum of 3.196 in winter. In damp summer weather it may be as low as 3.17. If exposed to air, the specific gravity of the concentrated solution changes but slightly; when diluted, it rapidly takes up or gives off water and changes in value, the maximum changes taking place when the specific gravity is between 2.0 and 2.5.

Metals and organic substances such as dust or filter paper, act upon the solution. It is therefore necessary to remove carefully, with a magnet, all chips of iron derived from mortar or hammer which may have become mixed with the powder, and care should be taken not to use metallic forceps in removing minerals from the solution. It has the further disadvantage of being very poisonous, and of corroding the skin.

To determine the specific gravity of a substance whose density is between 3.196 and 1.0, about 25 to 40 c.c. of the solution are placed in a narrow beaker. The mineral is crushed in a steel mortar and passed through a series of fine mesh sieves. The coarsest material which appears homogeneous under the microscope should be used. If only the finest powder appears to be uniform, it should be separated from the dust by washing. The homogeneous material is now thrown into the heavy solution and water is added from a burette, drop by drop, with constant stirring, until the mineral remains suspended where placed. If too much water has been added, a little of the concentrated solution will restore the density. The addition of a single drop of water has a marked effect upon the concentrated solution. For example, if one uses as

[1] Leopold van Werveke: *Ueber Regeneration des Kaliumquecksilberlösung und über einen einfachen Apparat zur mechanischen Trennung mittelst dieser Lösung.* Neues Jahrb., 1883 (II), 86–87.

much as 59 c.c. of it, one drop of water changes the density from 3.196 to 3.194. It is advisable, therefore, to add a dilute solution of the preparation instead of pure water when working with that which is concentrated.

To restore a dilute solution to its maximum density, it should be evaporated over a water-bath. During the concentration it sometimes happens that crystallization begins. This may be prevented entirely, according to Laspeyres,[1] if the concentration is carried on over the water-bath only until a piece of glass floats. Final concentration, until a piece of tourmaline floats, may be carried on under an air pump, or in a desiccator in which some calcium chloride has been placed.

Owing to variation in the solution, it is not possible to determine the specific gravity of a mixture by measuring the amount of water which was added. The density should therefore be determined by one of the methods described below.

The initial cost of Sonstadt's solution is considerable, but since it may be used over and over and there is very little waste, it does not amount to a great deal in the end. At the present time the cost of potassium mercuric iodide crystals is about 70 cents per ounce, and the solution costs $1.65 per 100 grm., duty free.

The relation of the density of Sonstadt's solution to the refractive index has been given above.[2]

455. Klein Solution (1881).—The heavy fluid proposed by D. Klein[3] is an aqueous solution of cadmium borotungstate ($2Cd(OH_2)\cdot B_2O_3\cdot 9WO_3\cdot 16H_2O$). The process of preparation is quite complicated, and is given by Edwards[4] as follows:

The apparatus necessary are two large porcelain evaporating dishes 10 in. in diameter, two 6 in. in diameter, and two 3 in. in diameter, two beakers 10 in. deep and two 6 in. deep, and a glass funnel. The operation should be performed under a hood to carry off the fumes. The weights given below will make 160 grm. of cadmium borotungstate or about 50 c.c. of the solution.

[1] H. Laspeyres: *Vorrichtung zur Scheidung von Mineralien mittelst schwerer Lösung.* Zeitschr. f. Kryst., XXVII (1896), 45.

[2] Art. 215.

Besides the references given above see also Rapp: *Erfahrungen bei der Anwendung der Thoulet'schen Flüssigkeit.* Berichte Versam. oberrhein. geol. Vereines, Stuttgart XVI (1883), 11.*

[3] Daniel Klein: *Sur la séparation mécanique par voie humide des minéraux de densité. inférieure à* 3.6. Bull. Soc. Min. France, IV (1881), 149–155.

Idem: *Sur une solution de densité* 3.28 *propre à l'analyse immédiate des roches.* Comptes Rendus, XCIII (1881), 318–321.

Review by H. Rosenbusch of both preceding articles. Neues Jahrb., 1882 (II), 189–191.

[4] W. B. D. Edwards: *On the preparation of a cheap heavy liquid for the separation of minerals.* Geol. Mag., VIII (1891), 273–275.

Dissolve 450 grm. of crystallized sodium tungstate in as little boiling water as possible. When quite dissolved add 675 grm. of boric acid in small crystals, a little at a time and with constant stirring. This should be done in a large beaker. When entirely dissolved the solution should be poured into a large evaporating dish and put aside, covered up from dust, in a place where it will not be disturbed or shaken. In about twenty-four hours or longer, the liquid, which is of a light purple color, should be poured off quickly from the crystals into another evaporating dish. These crystals should be in the form of a hard solid deposit at the bottom of the dish. They should be washed with hot water three or four times, the washings being added to the mother liquor. The latter will now probably be in a thick pasty condition due to the formation of small crystals, which will be found to dissolve on heating the dish and its contents on a water-bath. At the same time about half the water can be driven off, care being taken not to go so far that a crust begins to form on the surface of the hot liquid. The solution is again set aside as before and left to cool and crystallize. The liquid is poured off into one of the smaller dishes and the crystals washed as before. This process of crystallization is gone through again until a piece of orthoclase will float on the liquid, the principal point being always to make the polyborates of soda crystallize out either as single large crystals or as a hard crystalline crust. It is impossible to separate and wash the crystals if they are very small. Owing to the high density of the liquid, in the later stages a longer time is necessary for the sodium borates to crystallize out than at first. A piece of glass or feldspar will be found to float when the liquid has been evaporated down to about 220 c.c.

The next process is to heat the sodium borotungstate on the water-bath to 100° C., pour into a large beaker, and add a boiling saturated solution of barium chloride. This should be done carefully, a little at a time, stirring the solution at the same time. The barium chloride solution should consist of 150 grm. of crystals in about 100 c.c. of distilled water. A dense white precipitate forms in pouring this solution into the sodium borotungstate, and this precipitate should be stirred for some minutes so as to thoroughly mix the two liquids. After a few minutes, hot water should be added and the precipitate stirred up thoroughly. In a short time the supernatant liquid can be siphoned off. This washing process should be repeated some ten or fifteen times. The white precipitate is next transferred to a large evaporating dish, and about 300 c.c. of dilute HCl added (1 : 10 H_2O). The mixture of precipitate and solution is now evaporated to dryness on a water-bath, about 40 c.c. of strong HCl being added toward the end. The dried mass is then treated with about 300 c.c. of hot distilled water, the former being thoroughly broken up into fine powder with a glass rod flattened at one end.

The green sediment of tungstic hydrate is filtered off and washed, the washings being added to the solution of barium borotungstate. The liquid is

evaporated down and allowed to stand. Yellow crystals are formed, and with a little care these can and should be obtained as single large crystals. The latter crystallize in two forms, one as modified tetragonal prisms with well-developed basal planes, and the other in flattened hexagon-like forms.

Nearly the whole of the barium borotungstate can be obtained, the mother-liquor being evaporated down a little more after each crop of crystals has been obtained. Toward the end of the process, transparent colorless platy crystals of barium borate may separate out as well. The barium borotungstate crystals should be dissolved in water and recrystallized once more. They should then be dissolved in 200 c.c. of distilled water, and a solution of $CdSO_4$ added from a burette or a pipette, care being taken to add it very slowly, drop by drop, as long as a precipitate falls. The precipitate of $BaSO_4$ is then filtered off, and the filtrate is evaporated in a porcelain dish in a water-bath till a piece of olivine floats on the surface.

This liquid will be found to have a specific gravity of 3.46 at 60° F., and it takes some hours before some of the salt crystallizes out and the specific gravity falls to 3.28. It is of a clear yellow color.

The quantity of cadmium borotungstate obtained by the above process is about 160 grm. or enough to make 50 c.c. of the solution. According to Edwards the cost (in 1891, with sodium tungstate at 1 s. for 450 grm.) was 2 s. 4d. for the materials and no account taken of the time spent in preparation. Cadmium borotungstate crystals are listed at the present time at $1.50 per ounce, or the solution at $8.00 a pound.

Klein's solution may be diluted to any amount with water, and again concentrated to its former density by evaporation on the water-bath. If the final crystals produced in the process of preparation are heated in a tube on a water-bath to 75° C. they will melt in their own water of crystallization, and a rather oily fluid with a specific gravity of 3.55 will be obtained. It is not, however, suitable for the separation of powders. The solution with a specific gravity of 3.36, such as is obtained by the concentration of a dilute solution by evaporation until a crystalline film forms over the surface, is not yet of oily consistency.

The solution is not very poisonous, nor does it act upon the skin nor upon filter paper, through which it readily passes. After repeated use, the solution becomes dark, but it may be cleared, according to van Werveke, by the addition of a few drops of peroxide of hydrogen. It possesses the disadvantages of being decomposed by metallic lead, zinc, and iron, and of being acted upon by carbonates, wherefore it is necessary, beforehand, to treat the mineral to be examined, with dilute acetic acid.

Mann[1] found the keeping qualities of this solution to be superior to Sonstadt's or Rohrbach's; a preparation in continual use for a number of years,

[1] Paul Mann: *Untersuchungen über die chemische Zusammensetzung einiger Augite aus Phonolithen und verwandten Gesteine.* Neues Jahrb., 1884 (II), 179–180.

and repeatedly diluted and condensed, showed not the slightest alteration from its original condition.

456. Rohrbach Solution, (1879, 1883).—According to Karpinskij,[1] a solution of barium-mercuric iodide was used for the determination of specific gravities by Suschin in 1879, and described by Karpinskij in 1880.[2] The publication, however, being in the Russian language, was seen by few investigators and it was not until 1883, when Rohrbach[3] rediscovered it, that the solution came into general use.

In preparing this solution it is necessary, on account of the hygroscopic character of the barium iodide and its rapid decomposition in solution, to work quickly until the double salt is formed. It is prepared as follows: 100 parts of barium iodide and 130 parts of mercuric iodide are rapidly weighed out and are shaken up together in a dry flask, after which 20 parts of distilled water are added and the whole is placed upon an oil-bath previously heated to 150°–200° C. The salts are more rapidly dissolved and the formation of the double salt promoted if the material in the flask is stirred by rapidly twirling in it a crutch-shaped glass rod, held between the fingers. When all is dissolved, the solution is boiled a short time longer, after which it is poured into a porcelain evaporating dish and is placed over a water-bath until an Untersulzbachthal epidote crystal ($G=3.4$) will just float. A small amount of a yellow double salt will separate out on cooling. In spite of this, however, on account of the contraction of the liquid, its specific gravity rises, so that, when cold, a topaz crystal ($G=3.55$) will float upon it. Since the solution acts upon filter paper and converts it into a parchment-like substance, it is not possible to filter off the clear liquid. It should be left undisturbed for a few days in a closed flask and then decanted. The solution is of a clear yellow color but it becomes darker upon heating. It boils at 145° C. and gives off steam and red mercuric iodide vapor. It has a high refractive index.[4]

Rohrbach's solution is not affected by carbonates, but it is hygroscopic and should be kept in closed vessels. It is also extremely poisonous. Its great disadvantage, however, is the difficulty of diluting it, for on mixing with water at ordinary temperatures, crystals of red mercuric iodide separate, and these will not dissolve again in the cold solution. The reduction in density must therefore be made by adding, very slowly, a dilute solution of the same preparation, the latter being made by adding water, drop by drop, with

[1] Review by Ernst Kalkowsky of A. Karpinskij. *Cit. supra.*

[2] Trudy St. Petersburgh Obschtsch. jestjestw., XI (1880), 146.*

[3] Carl Rohrbach: *Ueber eine neue Flüssigkeit von hohem specifischen Gewicht, hohem Brechungsexponenten und grosser Dispersion.* Wiedem. Ann., N. F. XX (1883), 169–174.

Idem: *Ueber die Verwendbarkeit einer Baryumquecksilberjodid-Lösung zu petrographischen Zwecken.* Neues Jahrb., 1883 (II), 186–188.

[4] Art. 217.

constant stirring, to a portion of the solution heated nearly to the boiling-point, or by adding carefully a thin stratum of water to a portion and leaving it, for twenty-four hours, to mix by diffusion.

The minerals whose specific gravities are to be determined must be absolutely dry. Upon removing them from the solution they must be washed, not with pure water, but with water to which a few drops of potassium iodide have been added.

Instead of making the complete separation of rock constituents by means of the Rohrbach solution, it is advisable to separate first the heavier from the lighter constituents by means of the Sonstadt, and use the Rohrbach only for those whose density is greater than 2.9. Since the solution is hygroscopic, the separation should be performed in closed vessels, such as the Thoulet or Harada tubes.

The cost of the components of this solution, at the present time, is about 35 cents an ounce.

457. Methylene Iodide (Brauns) (1886).—So long ago as 1873, Sonstadt[1] used ethyl iodide (C_2H_5I, with G = 1.93), prepared from commercial methylated spirits, therefore containing a few per cent. of methyl iodide, and having a density of about 2.0. For the determination of density he diluted it with bisulphide of carbon or chloroform, preferably the former since it is less volatile. To prevent the liquid from becoming discolored by the separation of iodine, he placed in it magnesium (or copper) wire or filings.

The first use made of methylene iodide (CH_2I_2) was by Brauns,[2] in 1886. This substance is a thin, light yellow fluid of high refractive index, boiling at 180° C. with partial decomposition, and freezing at 5° C. It can be diluted with neither water nor alcohol, but may be, in all proportions, with benzol. It is unaltered by exposure to the atmosphere and is slow to evaporate when concentrated, so that one can work for hours with no apparent change in the refractive index or specific gravity. When diluted it changes its refractive index and density rapidly by the evaporation of the benzol. It does not act upon the skin, metals, nor carbonates, but is decomposed by sulphur. The cost is rather more than the other heavy fluids already mentioned, being, at the present time, about $3.25 per 100 grm., duty free, as against $1.65 for the same amount of Sonstadt's.

Both refractive index and specific gravity change rapidly with change in temperature, as may be seen from the table in Article 218, and the following:

[1] *Op. cit.*

[2] R. Brauns: *Ueber die Verwendbarkeit des Methylenjodids bei petrographischen und optischen Untersuchungen.* Neues Jahrb., 1886 (II), 72–78.

C. Chelius: *Zur Benutzung des Methylenjodids.* Notizbl. Ver. f. Erdk. Darmstadt, 1890 (4), 16.*

TABLE SHOWING THE RELATION BETWEEN TEMPERATURE AND SPECIFIC GRAVITY OF METHYLENE IODIDE

Temp.	G	Temp.	G	Temp.	G	Temp.	G
5° C.	3.3485	11° C.	3.3353	17° C.	3.3221	23° C.	3.3089
6	3.3463	12	3.3331	18	3.3199	24	3.3067
7	3.3441	13	3.3309	19	3.3177	25	3.3045
8	3.3419	14	3.3287	20	3.3155	.....	.....
9	3.3397	15	3.3265	21	3.3133	33	3.2890
10	3.3375	16	3.3243	22	3.3111	74	3.1890

The chief advantage of this fluid, especially for mechanical separation of minerals, is its mobility, whereby even fine powders may be separated, a thing impossible with Sonstadt's or Klein's. The cleaning of the recovered powder is also very simple, a washing in benzol being usually all that is necessary. If a little of the methylene iodide should remain, it may be driven off by gentle heat. It possesses the further advantage of being usable for the separation of minerals soluble in water.

To concentrate a dilute solution it is only necessary to place it on a water-bath, or to expose it in shallow vessels to an air current, which will rapidly volatilize the benzol. Some of the methylene iodide will be lost at the same time, a disadvantage on account of the expense. Upon exposure to sunlight or heat the fluid may turn brown by the separation of iodine. It may be cleared by shaking it up with dilute potassium hydroxide (KOH), washing with clean water, drying by the addition of a piece of calcium chloride, and filtering. It has no effect upon filter paper and readily passes through. A simpler method of clearing[1] is to reduce the temperature to 5° C., whereupon it solidifies, leaving a small amount of a brown liquid, which may be poured off and cleared when convenient with potassium hydroxide. The amount necnecessary to so clear, however, will not be great. The crystallized portion, upon melting, will be perfectly clear and transparent. Another method of clearing is given by Schroeder van der Kolk,[2] who says that the iodine may be removed with copper.

458. Retgers' Heavy Fluids (1889).—A great number of experiments were made by Retgers[3] to obtain fluids having higher densities than any previously used. He found that after concentrating Sonstadt's solution on the water-bath until a surface film was produced, he could, by stirring, dissolve flakes of iodine in it, and thus obtain a black, opaque liquid. Upon cooling, a cer-

[1] R. Brauns: *Eine einfache Methode Methylenjodid zu klären.* Neues Jahrb., 1888 (I), 213–214.

[2] J. L. C. Schroeder van der Kolk: *Tabellen zur mikroskopischen Bestimmung der Mineralien.* Wiesbaden, 1900, 13.

[3] J. W. Retgers: *Ueber schwere Flüssigkeiten zur Trennung von Mineralien.* Neues Jahrb., 1889 (II), 185–192.

tain part of the iodine separated, but the decanted liquid itself had a specific gravity of 3.30–3.40.

Evaporating Rohrbach's solution on the water-bath to the formation of a crystalline surface film and saturating with iodine gives a nearly opaque fluid from which, on cooling, a portion crystallizes out. The decanted liquid has a density of 3.6 to 3.65 at 20° C. In one experiment a value of 3.70 was obtained.

Iodine and sulphur had been dissolved in methylene iodide so long ago as 1888 by Bertrand[1] to obtain a fluid of high refractive index. Retgers used iodine alone and obtained an opaque, black fluid, more mobile than the two solutions just mentioned. and of a refractive index of 3.543–3.549. It does not alter in air.

All of the above, however, are of practically no use in the determination of specific gravities since they are opaque, and the point of suspension of the mineral fragments cannot be seen. To a certain extent they may be used as separating fluids.[2] None can be filtered through paper.

A transparent fluid of high specific gravity was prepared by Retgers[3] by slightly warming methylene iodide (CH_2I_2) and dissolving in it as much iodoform (CHI_3) as it would take up. Although much was dissolved, a considerable amount recrystallized upon cooling. Generally some decomposition of the iodoform takes place and gives the solution a dirty brown color. It may be cleared, however, by shaking with potassium hydroxide, leaving a deep yellow, transparent fluid with a density of 3.456 at 24° C. The solution thus prepared will still dissolve iodine, and when saturated and cold has a density of 3.60–3.65. It is opaque.

Among other solutions prepared by Retgers[4] are a saturated solution of SnI_4 in $AsBr_3$, giving a density of 3.73 at 15°C., and a saturated solution of AsI_3 and SbI_3 in a mixture of $AsBr_3$ and CH_2I_2, giving a density of 3.70 at 20°. A solution of selenium in selenium bromide (SeBr) would probably have a density of 3.70. Lead tetra-chloride ($PbCl_4$) is a clear yellow fluid which solidifies at −15° C. and has a density of 3.18 at 0°. The analogous $PbBr_4$ probably has a density of 3.5 and is also transparent.

As a result of his experiments, Retgers came to the conclusion that it is probably hopeless to expect to find a fluid having a density greater than 4.0, mercury being an exception.

[1] Émile Bertrand: *Liquides d'indices supérieurs à* 1.8. Bull. Soc. Min. France, XI (1888), 31.

[2] See Chapter XXXIX, *infra*.

[3] *Op. cit.*

[4] J. W. Retgers: *Die Darstellung neuer schwerer Flüssigkeiten.* Zeitschr. f. physik. Chemie. XI (1893), 328–344.

Idem: *Ueber die mineralogische und chemische Zusammensetzung der Dünensande Hollands und über die Wichtigkeit von Fluss- und Meeressanduntersuchungen im Allgemeinen.* Neues Jahrb., 1895 (I), 16–74, especially 28–31.

459. Tabulation of the Properties of Heavy Fluids.—The properties, advantages, and disadvantages of the most important of the fluids described above are shown for convenience in tabular form.

Name	Maximum sp. gr.	Cost per 100 grm. duty-free	Miscible with	Composition	Mobility	Color	Preparation	Restoration after decomposition	Concentrated by	Poisonous	Decomposed by	Acts on filter paper	Disadvantages	Mineral cleaned by
Sonstadt (Thoulet) Solution	3.196	$1.65	Water in any amount	Aqueous sol. potassium mercuric iodide	Medium	Yellowish-green, transparent	Simple	Easily. Evaporation with addition of Hg	Evaporation on water-bath	Very	Organic substances, iron	Yes	Hygroscopic when diluted	Repeated boiling with water or dilute KI
Klein	3.55	.....	Water in any amount	Aqueous sol. cadmium borotungstate	Oily when concentrated	Pale golden yellow	Complicated	Easily by H_2O_2	Evaporation on water-bath	Not very	Carbonates metallic Pb, Zn, Fe	No	Oily when concentrated	Repeated boiling with water or dilute KI
Rohrbach	3.588	$1.65	Dilute solution of same. Difficult.	Aqueous sol. barium mercuric iodide	Medium	Clear yellow	Quite simple	.....	Evaporation on water-bath	Very	Water, metals	Yes	Hygroscopic. Density reduced in damp air	KI solution
Braun	3.3485	$3.25	Benzol, xylol	Methylene iodide	Very fluid	Light yellow	Commercial preparation	Shake with KOH or freeze	Evaporation on water-bath or exposure to air current	Not very	Sulphur	No	Density alters with temperature	Benzol
Retgers	3.456	.....	Benzol, xylol	Bromoform in methylene iodide	Medium	Deep yellow	Simple	.....	Evaporation on water-bath or exposure to air current	.....	Sulphur	No	.....	Benzol

460. Schroeder van der Kolk (1895).—Schroeder van der Kolk,[1] in 1895, used bromoform ($CHBr_3$) as a heavy solution for the separation of the constituent minerals of sands. It has a density of 2.88 and is useful for the separation of many minerals. It is much less expensive than methylene iodide, costing about 10 cents an ounce. Its melting-point is 2.5° C., its boiling-point 151°, and its refractive index by sodium light at 15°, 1.588. It is not decomposed by air or light. It should be used without dilution.

461. Muthmann (1898).—Muthmann[2] used symmetrical acetylene tetrabromide ($CHBr_2—Br_2HC$), with a specific gravity of 3.01, as a heavy solution. Its melting-point is below − 20° C. and its boiling-point 137° under a pressure

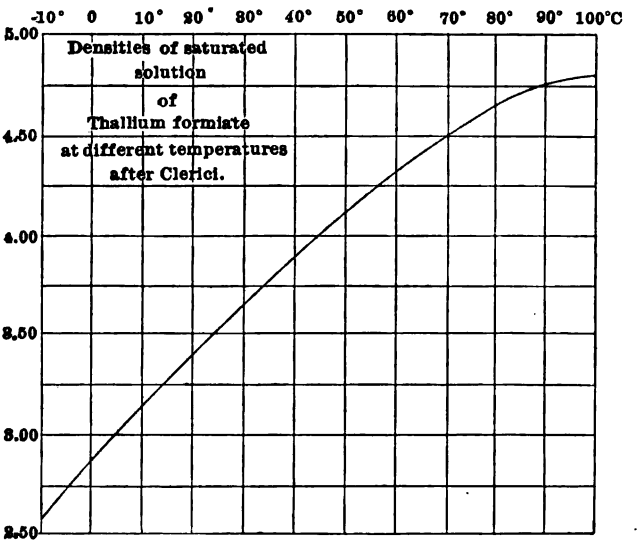

FIG. 719.—Diagram showing densities of a saturated solution of thallium formiate at different temperatures. (After Clerici.)

of 3.6 of mercury. It may be diluted with ether, benzol, or toluol in any proportions, and may be restored to its original density by evaporation. It is decomposed by neither air nor minerals, not even by ores. Since its density is almost exactly 3.0, it is useful as one of a series of density fluids. Minerals removed from this fluid may be cleaned by washing with ether. It costs less than 10 cents an ounce.

462. Clerici (1907).—Clerici[3] examined numerous thallium salts of the

[1] J. L. C. Schroeder van der Kolk: *Beitrag zur Kartirung der quartären Sande.* Neues Jahrb., 1895 (I), 272–276, especially 274.

[2] W. Muthmann: *Ueber eine zur Trennung von Mineralgemischen geeignete schwere Flüssigkeit.* Zeitschr. f. Kryst., XXX (1898), 73–74.

[3] Enrico Clerici: *Preparazione di liquidi per la separazione dei minerali.* Riend. Acad. Lincei. Roma (Ser. 5), XVI (1907), 1 Semsetre, 187–195.

organic acids and found the one best adapted as a heavy solution to be thallium formiate, aqueous solutions of which are colorless and free-flowing as water. The amount of the salt necessary for saturation depends upon the temperature, 5.0 grm. being soluble in 1 c.c. of water at 10° C. The density of the saturated solution varies with the amount of salt dissolved (Fig. 719), being 2.58 at −10° C., 2.86 at 0°, 3.14 at 10°, 3.40 at 20°, 3.64 at 30°, 3.87 at 40°, 4.11, at 50°, 4.32 at 60°, 4.50 at 70°, 4.67 at 80°, and 4.76 at 90°. At 95° thallium formiate melts and has a density great enough to float ilmenite. The solution is miscible with water.

To regain the thallium, the filtered liquid is concentrated on a water-bath, or H_2SO_4 is added and the thallium precipitated on a zinc plate.

463. Joly (1886).—Joly[1] determined the specific gravity of mineral fragments by placing them on a piece of previously weighed paraffine, carefully melting them in, and proceeding as by the method of Thoulet.

464. Hubbard (1887).—A method for approximately determining the specific gravity of a mineral was given by Hubbard.[2] He simultaneously placed the unknown mineral fragments and a number of other minerals of known density in a cadmium borotungstate solution ($G=3.3$). By noting the rapidity with which the different minerals sank, he determined their relative densities.

465. Streng (1887).—To determine the specific gravity of minerals heavier than the immersion fluid, Streng[3] made use of small glass cups constructed from glass tubing. They were made as thin as possible, from 5 to 6 mm. deep and 5 mm. in diameter at the top. To the bottom of each, three platinum wires were fused to serve as counterpoises to keep the vessel erect. Into the cup a fragment of the mineral to be tested was placed, and the cup was filled with the heavy solution, all bubbles being removed by means of a piece of platinum wire. The cup and mineral were now placed in the heavy solution which was diluted, with constant stirring, until they were suspended.

[1] J. Joly: *On a method of determining the specific gravity of small quantities of dense or porous bodies.* Phil. Mag. XXVI (1888), 29–33.

Idem: *On a method of determining the specific gravity of small quantities of dense or porous bodies.* Proc. Roy. Dublin Soc., V (1886), 41–47.

[2] L. L. Hubbard: *Beiträge zur Kenntnis der Noseanführenden Auswürflinge des Laacher Sees.* T. M. P. M., VIII (1887), 390.

[3] A. Streng: *Ueber die Bestimmung des specifischen Gewichts schwerer Mineralien.* Ber. oberhess. Gesell. Giessen, XXV (1887), 110–113.

If m = absolute weight of the cup,
s = sp. gr. of the cup,
m' = absolute weight of the mineral,
S = sp. gr. of mineral and float, therefore also of the heavy solution when they are suspended,
x = desired sp. gr. of the mineral,
we have,

$$\frac{m}{s}+\frac{m'}{x}=\frac{m+m'}{S}, \text{ and } x=\frac{S}{1-\frac{m}{m'}\left(1-\frac{S}{s}\right)}.$$

466. Retgers (1889).—Retgers[1] pointed out that in Streng's method errors were likely to occur through the incomplete mixing of the heavy solution during dilution, and through the probability that the solution within the cup remains heavier than that in the surrounding beaker. The absolute weight of the cup is also too great, as compared with the mineral, to give very accurate results, except with very large fragments. To remedy this, Retgers made glass clips by bending fine glass threads into the form of horseshoes, the spring of the glass keeping the ends together or pressed against the mineral fragment whose specific gravity is to be determined. A great number of these clips of various sizes and forms may quickly be constructed, their absolute weight and specific gravity being determined once for all.

In determining the density of a mineral by the use of these glass clips, it is essential that they be chosen no larger than necessary to give the desired buoyancy. The formula to be used is the same as in Streng's method.

467. Behr (1903).—Owing to the fragility of Retgers' glass floats, Behr[2] replaced them by glass tubes, such as are used in organic chemistry for the determination of melting-points. These tubes were shortened to 10 or 12 mm., and into one was inserted the crystal whose density was to be measured, its absolute weight having been determined previously. To prevent the crystal from slipping out, each end of the tube was slightly constricted by heating it. The specific gravity of the combination was now determined, and the true density computed by the formula given under Thoulet's method above.[3]

For the best results, the absolute weight of the tube in proportion to that of the crystal must not be too large; the best proportions being such that the combination has a specific gravity but little less than the original immersion fluid. During the process of dilution, the cylinder must be taken out repeatedly so that the concentration of the solution inside becomes the same as that outside.

[1] J. W. Retgers: *Die Bestimmung des specifischen Gewichtes in Wasser löslichen Salzen.* Zeitschr. f. phys. Chemie, IV (1889), 189–196.

[2] J. Behr: *Beiträge zu den Beziehungen zwischen eutropischen und isomorphen Substanzen.* Neues Jahrb., 1903 (I), 136–159, especially 143–146.

[3] Art. 464.

DETERMINATION OF THE SPECIFIC GRAVITY OF THE HEAVY SOLUTION

468. Goldschmidt's Method (1881).—The method of Goldschmidt[1] for the determination of the specific gravity of the heavy solution, after dilution to the point of suspension of the mineral, is as follows: First enough of the solution is poured into a glass flask to reach a mark indicating exactly 250 c.c. The exact point may best be determined by filling the flask slightly above the mark, which should extend entirely around the neck, bringing it to the level of the eye so that the scratch appears as a straight line, and removing enough of the fluid, with a pipette or piece of filter paper, to bring the lower curve of the meniscus tangent to this mark. The flask and contents are now weighed together, after which the fluid is returned to the beaker containing the mineral, and notice is taken that the mineral is again suspended. It is again poured to the 25 c.c. mark, and again flask and fluid are weighed. The same process is repeated a third time. These results are now averaged. Should the values differ widely, the process is repeated a fourth time to see if an error has been made. Note particularly that the mineral is suspended between each reading, and that the flask is filled exactly to the mark each time. Should the mineral and fluid not be of exactly the same specific gravity at any reading, owing to evaporation or other cause, the density is corrected by adding a few drops of water or of concentrated solution.

The weight of the flask alone, which was determined once for all, is now subtracted from the average reading, and the result is divided by 25, thus giving the weight of 1 c.c. of solution. Since 1 c.c. of water weighs 1 grm., the result expresses the specific gravity of the solution.

Goldschmidt gives several examples, among which is the following for Herkimer County, New York, quartz.

Flask and 25 c.c. fluid	77.991
	77.972
	77.901
	233.864
Average	77.955
Weight of flask	11.682
	25)66.273
	$2.651 = G$

The triple readings, repeated three times, gave averages of 77.959, 77.955, and 77.981, but the value for the density in each case was 2.651.

469. Sprengel Tube.—The method with the Sprengel tube (Fig. 720) is very similar. The heavy liquid is sucked up into the tube to a mark indicating

[1] V. Goldschmidt: *Ueber Verwendbarkeit einer Kaliumquecksilberjodidlösung bei mineralogischen und petrographischen Untersuchung.* Neues Jahrb., BB. I (1881) 197–199.

a known volume, the apparatus is hung from a balance by means of a platinum wire, and the tube and contained liquid are weighed (w). The weight of the tube alone (w'), and of the tube filled to the mark with distilled water (w''), are known, therefore

$$G=\frac{w-w'}{w''-w'}.$$

470. Sollas' Modification of the Sprengel tube (1886).—Sollas[1] proposed a modification of the Sprengel tube by connecting the two enlarged parts by a thin calibrated tube. The method otherwise is the same.

471. Westphal Balance (1883).—One of the simplest methods for the determination of the specific gravity of a liquid is by means of a balance originally manufactured by G. Westphal in Celle, and now generally known as the Westphal balance (Fig. 721). It was first described by Cohen.[2] The method of determining the specific gravity is as follows:

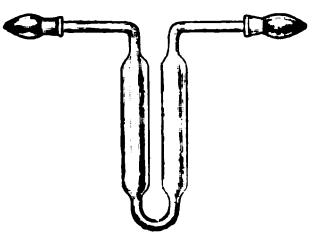

FIG. 720.—Sprengel tube. (Central Scientific Co.)

FIG. 721.—Westphal balance. (Central Scientific Co.)

The fluid whose density is to be determined is placed in a cylindrical vessel, and in it is placed a weighted sinker, generally made to serve as a thermometer for determining the temperature of the fluid as well. This sinker is attached to a balance so arranged, when it is in equilibrium, that the pointer, shown at the left, indicates zero. With any fluid heavier than water in the glass vessel, the sinker rises, and to reestablish equilibrium a certain number of riders must be hung upon the right arm of the balance. This arm is divided into ten parts. The density of the liquid is equal to the number of unit riders hung at the end of the arm, plus the number of tenths indicated by a unit rider hung at one of the ten divisions, plus the number

[1] W. J. Sollas: *On a separating apparatus for use with heavy fluids.* Proc. Roy. Dublin Soc., N. S., V (1886–7), 621–622.

[2] E. Cohen: *Ueber eine einfache Methode das specifische Gewicht einer Kaliumquecksilberiodidlösung zu bestimmen.* Neues Jahrb., 1883 (II), 87–89.

of one-hundredths indicated by a one-tenth unit rider, plus the number of one-thousandths indicated by a one-hundredth rider. If the volume of the sinker is exactly 1 c.c., the unit rider will be of 1 grm. weight, the second of 0.1 grm., and the third of 0.01 grm.

With this balance, determinations may be made very quickly, and the results are accurate to 2 in the third decimal place. The chief source of error is the adhesion of air bubbles to the sinker. They should be carefully removed by means of a glass rod or platinum wire. All of one set of determinations should be made at the same temperature.

472. Salomon's Apparatus (1891).—An apparatus, based on the principle that two fluids of different specific gravities placed in communicating tubes will stand at heights inversely proportional to their densities, was designed by Salomon.[1] The operation of determining the density of a fluid by this apparatus, however, is much more complicated than by the Westphal balance, and the results are less accurate.

473. Sollas Hydrostatic Float (1891).—Another method of determining the specific gravity of a fluid is by means of a hydrostatic float, such as that proposed by Sollas.[2] This consists of a thin glass tube, drawn out at one end into a capillary tube, and closed at the other. The closed end contains enough mercury to cause the instrument to float in a vertical position, the length of the capillary tube projecting from the immersing fluid serving as a measure of the specific gravity of the latter. The instrument may be calibrated by placing it in several fluids of known densities, marking the projection, and interpolating values. By employing several such hydrostatic floats with different ranges in values, results accurate to the third decimal place may be obtained rapidly.

474. Merwin's Method by Refractive Indices (1911).—A method for determining the density of Rohrbach's solution by means of its refractive index—the solution having a fixed density for a given refractive index—was given by Merwin.[3] The advantage of this method is that the amount of liquid required is not so great as in some of the other methods. It is necessary, however, to have a refractometer to measure the indices. The purity of the solution may be checked by bringing it to the density of clear quartz ($G=2.6495$) and determining its index of refraction for sodium light at 20° C. It should be 1.6208. The values found are given in the following table and are graphically represented in Fig. 722.

[1] W. Salomon: *Ein neuer Apparat zur Bestimmung des specifischen Gewichtes von Flüssigkeiten.* Neues Jahrb., 1891 (II), 214–220.

[2] W. J. Sollas: *Contributions to a knowledge of the granites of Leinster.* Read Nov. 30, 1889. Trans. Roy. Irish Acad., Dublin, XXIX (1887–1892), 427–514, especially 430–431.

[3] H. E. Merwin: *A method of determining the density of minerals by means of Rohrbach's solution having a standard refractive index.* Amer. Jour. Sci., XXXII (1911), 425–432.

Density at 20° C.	Refractive index
3.449	1.7686
3.396	1.7590
3.246	1.7312
3.180	1.7195
3.046	1.6944
2.980	1.6823
2.748	1.6391
2.649	1.6207
2.648	1.6205
2.367	1.5685
2.163	1.5320
1.067	1.5148

For accurate work, if the temperature differs more than 3° from 20°, a correction for density of −0.001 for each 2° below 20° or of +0.001 for each 2° above 20° should be made.

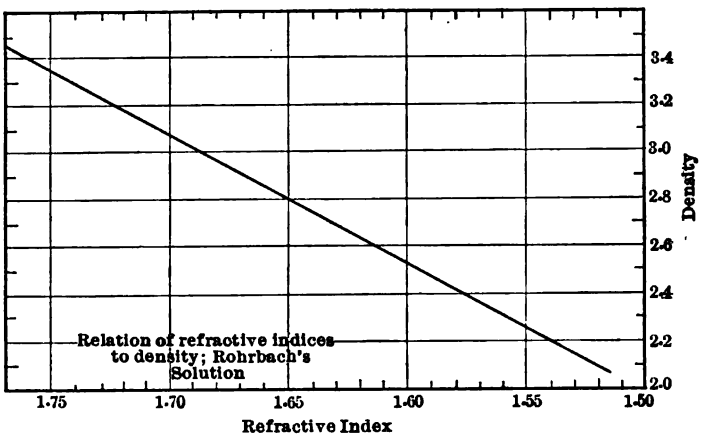

FIG. 722.—Diagram showing the relation between refractive indices and density in Rohrbach's solution.

475. Molten Substances as Specific Gravity Fluids.—Molten substances are rarely used for the determination of densities, although they may be so used. They are chiefly adapted to the separation of the mineral constituents of a rock and are described in full below.[1]

DETERMINATION OF THE SPECIFIC GRAVITY OF A MINERAL WHOSE DENSITY IS GREATER THAN THAT OF THE FLUID

476. Thoulet (1879).—If the specific gravity of the mineral whose density is to be determined is greater than that of the fluid in which it is to be immersed, it may be determined by a method devised by Thoulet.[2] In a

[1] Art. 484.

[2] J. Thoulet: *Sur un nouveau procédé pour prendre la densité de minéraux en fragments très-petits.* Bull. Soc. Min. France, II (1879), 189–191.

piece of wax, well smoothed and about the size of a grain of wheat, is enclosed, as a weight, a fragment of a mineral of such size that the specific gravity of the two combined is between 1 and 2. The weight of this sinker is represented by G and the weight of the mineral to be determined by g. The latter is lightly attached to the wax by pressure, and the specific gravity of the whole is determined by inserting it in Thoulet's solution diluted until the mineral and wax neither sink nor float. Let the density of the fluid at this stage, and, consequently, that of the combined substances, be represented by Δ. The mineral and wax are now removed from the solution and washed, the adhering mineral fragments carefully detached, and the density of the weighted wax determined by further dilution of the heavy solution. Let this value be D. The volume of the wax, therefore, is $V=\frac{G}{D}$; the volume of the mineral, $v=\frac{g}{d}$, d being the desired specific gravity. We have, therefore:

$$\Delta=\frac{G+g}{V+v}=\frac{G+g}{\frac{G}{D}+\frac{g}{d}}, \text{ whereby } d=\frac{g\Delta}{G+g-\Delta\frac{G}{D}}.$$

CHAPTER XXXIX

THE MECHANICAL SEPARATION OF ROCK CONSTITUENTS

477. Preliminary Examination.—It is necessary to separate a rock into its component minerals for such purposes as chemical analysis, the determination of specific gravity, or the refractive index of a single component. Preliminary to such separation by any one of the following methods, it is necessary to break up the rock fragment. This should be done, first roughly in an iron-, then in a diamond-mortar. The rock should be crushed, not ground; the aim being to separate all mineral particles from those ad-

FIG. 723.—Preparation microscope. About 1/2 natural size. (Fuess.)

FIG. 724.—Preparation microscope after Weinschenk. (Seibert.)

jacent, and to obtain as uniform a grain as possible. After crushing, the dust and clay-like particles are removed by stirring the pulverized mineral several times into a beaker of water, and decanting. The residue is dried, and sifted through a series of metal sieves of varying mesh. Ordinarily a series of five, varying from 1.0 to 0.2 mm. mesh, is sufficient. Each grade of powder is now examined under a low power microscope to determine whether each grain consists of but a single mineral. A very convenient form of microscope for this purpose is that proposed by Kalkowsky[1] and shown in Fig. 723. The focus of the lens *A* can be adjusted by means of the screw *T* for the

[1] C. Leiss: *Neues Lupenstativ mit Polarisation für mineralogische, geologische und palæontologische Zwecke.* Neues Jahrb., 1897 (I), 81–82.
Idem: *Die optischen Instrumente.* Leipzig, 1899, 257–258.

examination of the mineral grains spread evenly over the glass plate *O*, which is 8 1/2×10 cm. in size and set into a brass frame. The two hinges, *s* and s_1, permit the lens *A* to cover every part of this plate. Should it be desired to use polarized light, this is obtained by the glass-plate polarizer *P* and the cap nicol *A*.

Another form, proposed by Weinschenk,[1] is shown in Fig. 724. In this the glass plate may be moved transversely across the field, the box *S* serving to distribute the powder evenly over it.

Having examined the different powders, the largest grained, homogeneous one is taken, and the remaining coarser material is reduced to the same size. The larger the material, the more readily is it separated mechanically but the less readily chemically. Should the mineral be found to be full of inclusions, it will be necessary to reduce it to still smaller size, and it may be necessary to use bolting cloth sieves.

The microscopic examination of the powder will reveal the nature of the component minerals. Should it be intended to make the separation by heavy fluids, such minerals as would decompose it must first be removed. Most of the minerals being determinable microscopically, the density of the separating fluid for the various separations will also be known approximately, and much time may thus be saved in the process, especially if a series of fluids of varying densities is on hand.

478. Separation by Means of the Electromagnet.—Metallic iron, which may have been derived from the mortar in which the rock was crushed, may be separated from the pulverized rock by means of an ordinary magnet. The removal of iron is necessary before proceeding to a separation by means of a heavy fluid, since many of them are decomposed by metals.

By simply drawing the magnet through the powder repeatedly, all magnetic particles, such as metallic iron and magnetite, will become attached to it and may be removed by brushing. They are more easily removed if the magnet is moved beneath the paper upon which the powdered mineral lies.

A more convenient method was used by Cohen.[2] A sheet of smooth paper is dampened and stretched upon a rectangular frame so that when dry it is perfectly flat. To this frame are attached four legs of such a height that the hand may be used beneath it. The well-dried powder is spread upon this frame, and the magnet is drawn underneath repeatedly, from the center toward the edge, carrying with it the magnetic particles, which are swept off by means of a brush after removing the magnet. The

[1] E. Weinschenk: *Die gesteinsbildenden Mineralien.* Freiburg im Breisgau, 1901, 20. English translation by Clark, New York, 1912, 161.

[2] E. Cohen: *Zusammenstellung petrographischer Untersuchungsmethoden.* Stuttgart, 3 Aufl., 1896, 12–13.

finer the powder, the oftener is it necessary to repeat the operation, since some of the particles are held back by the adjacent grains.

Instead of a simple horseshoe magnet, a magnetized metallic graining comb, such as is used by painters, may be employed.

Many iron-bearing minerals, which in themselves are not magnetic, may be separated from the non-iron-bearing by a powerful electromagnet, as was first pointed out by Fouqué,[1] who used it to separate the ferro-magnesian minerals from the feldspars in the lavas of Santorin. The same process was used by Doelter,[2] who found that all minerals are not attracted with the same ease. In the following table the minerals are arranged in the order of decreasing susceptibility. Those last on the list contained inclusions.

1. Magnetite.
2. Hematite, ilmenite.
3. Chromite, siderite, almandite.
4. Hedenbergite, ankerite, limonite.
5. Iron-rich augite, pleonaste, arfvedsonite.
6. Hornblende, iron-poor augite, epidote, pyrope.
7. Tourmaline, bronzite, vesuvianite.
8. Staurolite, actinolite.
9. Olivine, pyrite, chalcopyrite.
10. Biotite, chlorite, rutile.
11. Hauynite, diopside, muscovite.
12. Nephelite, leucite, dolomite.

As may be seen from the table, the degree of attraction is not proportional to the iron content, for many iron-rich minerals, such as pyrite and biotite, are less attracted than others which are poorer in iron. Heating to redness sometimes increases the power of being attracted, although the amount of iron is thereby in nowise changed.

Depending upon the degree of attraction, the component minerals of a rock may sometimes be separated by increasing or decreasing the strength of the electromagnet. This may be done by changing the strength of the current, but more readily, as suggested by Rosenbusch,[3] by decreasing or increasing the distance between the poles of the magnet.

[1] F. Fouqué: *Nouveaux procédés d'analyse médiate des roches et leur application aux laves de la dernière éruption de Santorin.* Comptes Rendus, LXXV (1872), 1089–1091.

Idem: *Nouveau procédés d'analyse médiate des roches et leur application aux laves de la dernière éruption de Santorin.* Mem. Acad. France, XXII (1874), 11.

[2] C. Doelter: *Ueber die Einwirkung des Elektromagneten auf verschiedene Mineralien und seine Anwendung behufs mechanischen Trennung derselben.* Sitzb. Akad. Wiss. Wien, LXXXV (1882), pt. I, 47–71.

Idem: *Ueber die mechanische Trennung der Mineralien.* Ibidem, 442–449.

Idem: *Die Vulkane der Capverden*, 1882, 72.*

[3] H. Rosenbusch: Comment in review of Doelter, *Op. cit.*, Neues Jahrb., 1882 (II), Ref. 252.

An electromagnet for separating minerals is shown in Fig. 725. The current is supplied by a battery consisting of two strong accumulators, or one or two strong chromic acid Bunsen cells. To perform the separation, the two L-shaped soft-iron poles are separated an appropriate distance, clamped, and the current turned on. The pulverized mineral, scattered over a sheet of paper, is now brought beneath the magnet, which may pick up a certain proportion of the constituents. When the magnet carries as much as it will hold, the paper is removed, and a blank sheet is substituted. On turning off the current the mineral particles will drop to the paper. The process is repeated until no more minerals can be extracted from the powder. Any adhering mineral particles are now removed from the magnet

FIG. 725.—Electromagnet. (Fuess.)

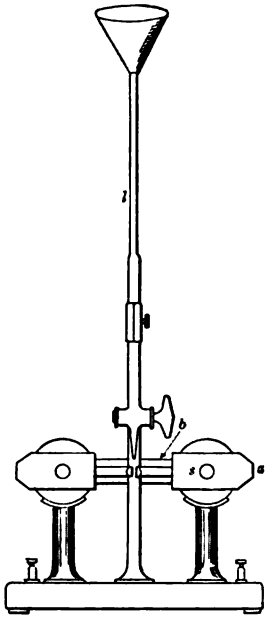

FIG. 726.—Mann's electromagnet.

with a brush or a trimmed feather, the poles are moved closer together, and another series of mineral fragments is extracted, and so on until all are removed.

The ordinary electromagnet is adapted to the separation of large quantities of iron-rich constituents from a rock powder. If only a small amount of such minerals be present, they may be better separated by a method first proposed by Pebal.[1] Instead of extracting the minerals from the dry

[1] L. Pébal: *Ueber die Anwendung von Elektromagneten zur mechanischen Scheidung von Mineralien.* Sitzb. Akad. Wiss. Wien, LXXXV, Abt. 1, (1882), 147–148.

Idem: *Notiz über mechanische Scheidung von Mineralien.* Ibidem, LXXXVI (1882), 192–194.

powder, they are removed from suspension in water. The powder is stirred in a beaker of water, after which a rod-shaped electromagnet is inserted and with it the stirring is continued until all of the iron-bearing minerals are attached.

An improvement on this method was given by Mann,[1] who placed his electromagnet horizontal and fastened the two poles so that their knife-like edges were vertical and separated 0.5 mm. Attached to this was a burette-like tube (*l*, Fig. 726), 50 cm. long and with an inner diameter of 6 mm. At the upper end was a funnel, immediately below which a constriction in the tube narrowed it to 4 mm. At the lower end of the tube, which ended a few millimeters above the knife edges of the poles, was a stop-cock.

The mineral powder was stirred up with water in a beaker, and was poured, with one motion, into the funnel. This imprisoned the air within the tube *l*, and when the stop-cock was slowly opened, the water flowed between the tube and the bubble, which practically filled the whole of it and prevented the clogging of the valve by the settling of a quantity of the powder. The water and the uniformly distributed powder passed in a slow stream over the magnet, which picked out the iron-bearing minerals. When enough were attached, the stop-cock was closed, the electric current shut off, and the powder washed into a beaker. The process was then repeated until all of the powder had passed over.

479. Separation by Means of Water.—The method of separating minerals by means of water is not much used in petrographic work, although it is applicable in certain cases. It is largely used in the separation of the constituents of soils, but that is a different problem, and the student is referred to the publications of the Bureau of Soils.[2] It may be used with advantage in the separation of heavy residual minerals from the decomposition products of the rocks, thus determining their original igneous or sedimentary character,[3] or even in the separation of minerals from crushed unaltered rocks.[4] The method is simply the washing of the material in a

[1] Paul Mann: *Untersuchungen über die chemische Zusammensetzung einiger Augite aus Phonolithen und verwandten Gesteine.* Neues Jahrb., 1884 (II), 181–185.

[2] See also F. Steinriede: *Anleitung zur mineralogischen Bodenanalyse.* Leipzig, 1889.*

F. Wahnschaffes: *Anleitung zur wissenschaftlichen Bodenuntersuchung,* Berlin, 1887, 2 Aufl., 1903.*

A. Nowacki: *Kurze Anleitung zur einfachen Bodenuntersuchung.* Berlin, 1885.*

Konrad Keilhack: *Lehrbuch der praktischen Geologie.* Stuttgart, 1896, 373–389.

[3] Orville A. Derby: *On the separation and study of the heavy accessories of rocks.* Proc. Rochester Acad. Sci., I (1891), 198–206.

E. W. Dafert and O. A. Derby: *On the separation of minerals of high specific gravity.* Ibidem, II (1893), 122–132.

[4] Hans Thürach: *Ueber das Vorkommen mikroskopischer Zirkone und Titanmineralien in den Gesteinen.* Verhandl. phys. med. Gesell. Würzburg, N. F. XVIII (1884).*

J. J. H. Teall: *On the occurrence of rutile needles in clays.* Mineralog. Mag., VII (1887). 201–204.

prospector's pan, a process so familiar that it hardly needs explanation. The material is washed in water, running if possible, to remove all of the light material. The pan is given a rotary motion, the water is poured off, the upper layers of sand, etc., removed by the hand, more water is added, another swing, further removal of material, and so on, until a final rotary swing in a very little water sends the small remaining amount of lighter material beyond the heavy. A final sorting may be made by hand when the minerals are dry. Calcareous material, such as shale, limestone, or dolomite, may first be treated with dilute hydrochloric acid, and the residual insoluble material separated as above.

480. Separation by Means of Heavy Fluids.—One of the simplest and most useful of the methods for separating minerals is that by which they are sorted according to their specific gravities by means of heavy fluids. The process simply consists in placing the powdered rock in a vessel containing a fluid of high specific gravity, and gradually diluting the latter until the mineral particles of a certain density sink. These, with some of the liquid, are withdrawn from the bottom of the vessel, a little of the concentrated solution is added to bring the mineral into suspension, and the specific gravity of the fluid determined. The remaining minerals are separated in the same manner by the further dilution of the heavy fluid, and their specific gravities are likewise determined.

Preceding a separation by a heavy fluid there should always be a microscopical examination, as mentioned in Art. 477. This permits the removal of such minerals as iron, calcite, etc., which act upon the particular fluid used.[1] Knowing the minerals contained in the rock, the possibility of making a specific gravity separation may be determined. An idea may also be obtained as to the amount of dilution necessary to bring down certain known minerals, thus establishing the limits between which the separation must take place. This may sometimes be of importance since in the powder there is no way of distinguishing which mineral is falling and, if there should be present two which have nearly equal densities, it is quite possible that the proper point of dilution might be passed.

481. Indicators.—Knowing most of the minerals, the separation limits may be marked by certain indicators placed in the fluid. One may have to separate, for example, from an olivine gabbro, olivine (3.36), augite (3.36), hornblende (3.12), labradorite (2.70), and magnetite (5.10). The magnetite should first be removed by means of a magnet. The proper amount of dilution of the heavy fluid may readily be found by using as indicators, apatite (3.18) and aragonite (2.92). The indicators are placed in the heavy solution with the powder to be separated. When the apatite sinks all of the olivine and augite must have gone down; when the aragonite

[1] See Art. 462.

sinks there remains, on the surface of the fluid, only the labradorite. The vessel containing the heavy fluid must be thoroughly and continuously shaken during the dilution.

Goldschmidt[1] gives the following table of indicators:

No.	Indicator	Locality	Sp. gr.	Difference
1	Sulphur	Girgenti	2.070	.....
2	Hyalite	Waltsch	2.160	0.090
3	Opal	Scheiba	2.212	0.052
4	Natrolite	Brevig	2.246	0.034
5	Pitchstone	Meissen	2.284	0.038
6	Obsidian	Lipari Islands	2.362	0.078
7	Perlite	Hungary	2.397	0.035
8	Leucite	Vesuvius	2.465	0.068
9	Adularia	St. Gotthard	2.570	0.105
10	Nephelite	Brevig	2.617	0.047
11	Quartz	Middleville, N. Y	2.650	0.033
12	Labradorite	Labrador	2.689	0.039
13	Calcite	Rabenstein	2.715	0.026
14	Dolomite	Muhrwinkel	2.733	0.018
15	Dolomite	Rauris	2.868	0.135
16	Prehnite	Kilpatrick	2.916	0.048
17	Aragonite	Bilin	2.933	0.017
18	Actinolite	Zillerthal	3.020	0.087
19	Andalusite	Bodenmais	3.125	0.105
20	Apatite	Ehrenfriedersdorf	3.18	0.055

As to whether natural or artificial indicators are most desirable, Goldschmidt[2] concluded that while the former are easier to recognize by their colors, the latter are homogenous and may be obtained of almost any specific gravity. He suggested that they might be artificially colored or made of different forms, so that successive indicators could be easily recognized in the solution. Later[3] he prepared sets of indicators, partly natural and partly artificial. One such set comprised indicators of 34 different densities as follows:

No.	Sp. gr.	No.	Sp. gr.	No.	Sp. gr.	No.	Sp. gr.	No.	Sp. gr.	No.	Sp. gr.
1	2.060	7	2.311	13	2.531	19	2.699	25	2.962	31	3.147
2	2.148	8	2.363	14	2.552	20	2.720	26	2.981	32	3.189
3	2.164	9	2.404	15	2.570	21	2.740	27	3.013	33	3.224
4	2.209	10	2.448	16	2.612	22	2.762	28	3.044	34	3.295
5	2.252	11	2.476	17	2.646	23	2.883	29	3.058	.....	.....
6	2.298	12	2.492	18	2.661	24	2.936	30	3.091	.....	.....

[1] V. Goldschmidt: Verh. k. k. geol. Reichsanst., Wien, 1883, 68.*

Idem: *Ueber Verwendbarkeit einer Kaliumquecksilberjodidlösung bei mineralogischen und petrographischen Untersuchungen.* Neues Jahrb., B.B., I (1881), 215.

[2] V. Goldschmidt: Verh. k. k. geol. Reichsanst., Wien, 1883, 68.*

[3] Idem: *Ueber Indikatoren zur mechanischen Gesteinsanalyse und spezifischen Gewichts-Bestimmung.* Centralbl. f. Min., etc., 1913, 39–44

These indicators may be obtained from P. Stoë in Heidelberg, F. Krantz in Bonn, or R. Fuess in Berlin. The cost is about 25 cents per indicator, sets being made of various sizes.

Another set of indicators, given by Johnsen and Mügge,[1] consists of cubes of various kinds of glass, 4 mm. along edges. To make them visible in the solution they were ground on five sides and blackened with graphite. On the sixth side the value of the density was scratched with a diamond.

Still another set was proposed by Linck.[2] It consists of 24 cubes of glass with 6-mm. edges and ranging in specific gravity from 2.24 to 3.55, with a mean difference between two successive indicators of 0.057. Each cube is marked with a number corresponding to values given in a table. Linck suggests that a heavy solution containing indicators be stirred, not with a glass rod, but with one of hard rubber, as less likely to injure indicators or beaker.

482. TABLE OF SPECIFIC GRAVITIES

The following table of specific gravities may assist in choosing proper division points.

Cassiterite	6.65	Titanite	3.48
Hematite	5.20	Ægirite-augite	3.46
Magnetite	5.10	Arfvedsonite	3.45
Monazite	5.10	Hypersthene	3.45
Pyrite	5.05	Wöhlerite	3.43
Pseudobrookite	4.98	Barkevikite	3.43
Ilmenite	4.65	Titan-augite	3.41
Zircon	4.53	Vesuvianite	3.41
Xenotime	4.52	Diaspore	3.40
Chromite	4.45	Piedmontite	3.40
Rutile	4.25	Pistacite (Green Epidote)	3.39
Fayalite	4.14	Augite	3.36
Picotite	4.08	Olivine	3.36
Perofskite	4.03	Astrophyllite	3.35
Melanite	3.95	Prismatine	3.34
Corundum	3.95	Jadeite	3.34
Siderite	3.94	Zoisite	3.31
Brookite	3.94	Rosenbuschite	3.31
Anatase	3.88	Diopside	3.30
Orthite (Allanite)	3.85	Bronzite	3.29
Pleonaste	3.82	Dumortierite	3.29
Ænigmatite	3.81	Axinite	3.28
Pyrope	3.75	Diallage	3.27
Staurolite	3.72	Cornerupine	3.27
Grünerite	3.71	Forsterite	3.26
Periclase	3.65	Sillimanite	3.24
Disthene (Cyanite)	3.62	Johnstrupite	3.20
Spinel	3.60	Monticellite	3.20
Grossular	3.56	Fluorite	3.19
Topaz	3.55	Thuringite	3.17
Hedenbergite	3.54	Apatite	3.16
Lavenite	3.53	Andalusite	3.15
Ægirite	3.53	Anthophyllite	3.15
Uwarowite	3.51	Humite	3.15
Sapphirine	3.49	Chondrodite	3.15
Rinkite	3.48	Clinohumite	3.15

[1] A. Johnsen und O. Mügge: *Verbesserungen am Harada'schen Trennungsapparat.* Centralbl. f. Min., etc., 1905, 152–153.

[2] G. Linck: *Indikatoren zur Bestimmung des spezifischen Gewichts von Flüssigkeiten.* Centralbl. f. Min., etc., 1912, 508–509.

A set of 24 indicators costs 20 Marks. For sale by F. Krantz, Bonn.

Enstatite	3.15	Clinochlore	2.71
Crossite	3.15	Bastite	2.70
Spodumen	3.15	Labradorite	2.70
Tourmaline	3.12	Alunite	2.70
Hornblende	3.12	Andesine	2.67
Magnesite	3.10	Quartz	2.65
Glaucophane	3.10	Cordierite	2.63
Lawsonite	3.09	Albite	2.62
Lazulite	3.06	Kaolin	2.62
Eucolite	3.05	Mizzonite	2.61
Actinolite	3.02	Anorthoclase	2.59
Melinophane	3.01	Nephelite	2.58
Biotite	3.00	Dipyr	2.57
Gehlenite	3.00	Serpentine	2.57
Tremolite	3.00	Orthoclase	2.55
Melilite	3.00	Microcline	2.55
Mosandrite	2.98	Leucite	2.47
Datolite	2.95	Brucite	2.39
Anhydrite	2.95	Hauynite	2.38
Carpholite	2.94	Sodalite	2.38
Aragonite	2.94	Hydrargillite	2.38
Eudialyte	2.92	Thomsonite	2.35
Dolomite	2.90	Gypsum	2.32
Delessite	2.89	Tridymite	2.30
Wollastonite	2.88	Laumontite	2.30
Prehnite	2.87	Glauconite	2.30
Muscovite	2.85	Scolecite	2.28
Lepidolite	2.85	Hydronephelite	2.26
Paragonite	2.84	Epistilbite	2.25
Phlogopite	2.81	Natrolite	2.21
Pectolite	2.81	Opal	2.21
Anorthite	2.76	Heulandite	2.20
Talc	2.75	Phillipsite	2.20
Meionite	2.73	Hyalite	2.17
Bytownite	2.73	Analcite	2.16
Pennine (Chlorite)	2.73	Stilbite	2.16
Calcite	2.71	Hydromagnesite	2.16

483. Heavy Solutions.—In the chapter[1] dealing with the determination of the specific gravities of minerals, there were given full descriptions of various heavy solutions. Any of the fluids there mentioned may be used for the mechanical separation of minerals. They are, in fact, more often used in petrographic work for that purpose than they are for the determination of specific gravities.

484. Heavy Melts.—While the heavy liquids previously mentioned suffice for ordinary separations, it occasionally happens that fluids of still greater densities are needed. With the exception of mercury, there appears to be no liquid or solution with a density greater than 4.0, consequently recourse must be had to melts of salts having low fusing points.

The first heavy melt proposed was that of Bréon[2] who used a molten mixture of lead chloride ($PbCl_2$) and zinc chloride ($ZnCl_2$). The first fuses at about 500° C. and has a density of 5.0, the second at 262° C. and has a

[1] Chapter XXXVIII, *supra*.

[2] R. Bréon: *Séparation des minéraux microscopiques lourds.* Bull. Soc. Min. France., III (1880), 46–56.

density of 2.4. By the use of various mixtures of the two salts, a range of from 2.4 to 5.0 is obtained. They are prepared by placing the proper amounts of the two substances in a test-tube on a sand-bath at about 400° C., and are fused to a homogeneous mixture. The rock powder, which must not be too fine grained, is introduced in small installments with continuous stirring with a platinum rod. After the heavy minerals have sunk, the tube is allowed to cool and the salts to solidify. The mass is broken apart in the middle, and the separated mineral recovered by dissolving the salt in hot water to which some acetic or nitric acid has been added.

In 1889, Retgers[1] proposed as a heavy melt, silver nitrate ($AgNO_3$) which melts at 198° C. to a clear transparent water-like fluid with a density of 4.1. The salt is fused in a small beaker placed on wire gauze over an open flame. Its density may be reduced by the addition of KNO_3 ($G=2.092$) or $NaNO_3$ ($G=2.244$).

A concentrated solution of $AgNO_3$ will dissolve a considerable amount of silver iodide (AgI), and soon a separation to a yellow oil-like and a white watery fluid takes place. The former, which may also be obtained by melting together the dry salts, has a density of about 5.0 and a melting-point of only 70° C., which makes it possible to work over a water-bath. In composition it is an anhydrous double salt of about the composition $2AgNO_3+3AgI$. If the AgI is in excess, the fluid is thick and useless, wherefore it is necessary to use as much $AgNO_3$ as possible in the combination. After separating two minerals, the lighter floating one is removed with a glass spoon or by rapidly pouring off the upper part of the liquid. The adhering melt is removed from both the light and heavy mineral grains by boiling in water.

In 1893 Retgers[2] proposed the double salt, thallium silver nitrate ($AgTlN_2O_6$). Although $AgNO_3$ has a melting-point of 224° C. and $TlNO_3$ one of 205°, the combination melts at 75°. In the above proportions it has a density of 4.5.[3] With $AgNO_3$ to $TlNO_3$ in the proportion of 3 to 4, it melts below 100° and has a density of 4.678+. In the proportion 2:4 it melts below 150° and has a density of 4.8+. In the proportion 1:4 it melts below 200° and has a density of 4.85+. Thallium nitrate alone has a density of 4.94+ at the melting point.

[1] J. W. Retgers: *Ueber schwere Flüssigkeiten zur Trennung von Mineralien.* Neues Jahrb., 1889 (II), 185–192.

[2] Idem: *Thalliumsilbernitrat als schwere Schmelze zu Mineraltrennung.* Neues Jahrb., 1893 (I), 90–94.

Idem: *Beiträge zur Kenntniss des Isomorphismus.* Zeitschr. f. physik. Chem., V (1890), 451–452, footnote.

[3] S. L. Penfield: *On some devices for the separation of minerals of high specific gravity.* Amer. Jour. Sci., L (1895), 446–448.

Idem: *Ueber Verbesserungen der Methoden zur Trennung von Mineralien mit hohem specifischen Gewicht.* Zeitschr. f. Kryst., XXVI (1896), 134–137. (Same paper as preceding.)

The double salt forms a fluid, thin and colorless as water, and is well suited to the separation of minerals except sulphides which it attacks. It may be diluted with water in any proportion.

Retgers,[1] in 1896, made many experiments on heavy melts to obtain one which would not attack sulphides and yet be of high specific gravity. With acetates he obtained rather unsatisfactory results, the specific gravities in general being too low, seldom exceeding 4.0. The most satisfactory one was thallium-nitrate-acetate ($TlAc+TlNO_3$) which melts at 65° (TlAc, 110° C; $TlNO_3$, 205°), has a density of approximately 4.5, and does not act upon sulphides. It is, however, readily decomposed at high temperatures. Below 100° it is stable.

Among the nitrates several appear to be serviceable. Mercuro-nitrate ($HgNO_3+aq$) has a density of 4.3 and a melting-point of 70° C. It is clear, mobile, miscible in all proportions with water, and cheap. It is decomposed during heating but this is of little consequence if one works rapidly. Thallium mercuri-nitrate ($TlHg(NO_3)_4$) has a density of 5.0 and a melting-point of 110° C. It is a mobile fluid, miscible in all proportions with water, does not act on sulphides, but is rather cloudy and therefore not so useful as thallium mercuro-nitrate ($TlHgN_2O_6$). This has a density of 5.3 and a melting-point of 76° C. It is a transparent mobile fluid, miscible in all proportions with water, and does not act on the sulphides of the metals. It is not decomposed below 100° C. It may be prepared by adding crystals of mercuro-nitrate ($HgNO_3+aq$) to an equal quantity of molten thallium nitrate. It forms a double salt with a melting-point much lower than that of either component. The fusion can be performed in a test-tube over an open flame, but care must be used not to fill the tube too full since the material boils up considerably, due, probably, to the escape of the water from the mercuro-nitrate. Should the temperature be raised too high, brown vapor passes off but this ceases when the heat is reduced. On cooling, the pale yellow fluid becomes cloudy and viscous at 85°, and soldifies at 74° . To make separations with this fluid, one should work over a water-bath.

SEPARATING APPARATUS

485. Thoulet (1879).—The first apparatus used for the separation of minerals by means of heavy solutions was that proposed by Thoulet[2] (Fig. 727). This consists of a graduated tube, closed at the lower end by a pair

[1] J. W. Retgers: *Versuche zur Darstellung neuer schwerer Flüssigkeiten zur Mineraltrennung. I. Die Acetate der Schwermetalle als schwere Schmelzen.* Neues Jahrb., 1896 (1), 212–221.

Idem: *Versuche usw. II. Die Nitrate und Doppelnitrate der Schwermetalle als schwere Schmelzen.* Neues Jahrb., 1896 (II), 183–195.

[2] J. Thoulet: *Séparation méchanique des divers éléments mineralogiques des roches.* Bull. Soc. Min. France, II (1879), 17–24.

of large-way stop-cocks *C* and *D*. About 60 c.c. of the heavy solution are placed in the tube, and from 1 to 2 grm. of the pulverized rock, previously boiled with water to free it from air, are slowly added from above. The upper end of the tube is closed by a stopper *E* through which passes a tube *F* connected with an air pump. The stop-cock *C* is carefully opened, and at the same time a continuous current of air is admitted through *B*. This serves to keep the fluid and pulverized mineral in commotion and intimately mixes them, and also prevents the formation of dry lumps. No fluid should enter *B* until the mixing is complete. The cock *C* is now closed and all minerals heavier than the solution fall to the bottom of the tube *A*. When all have settled, the valve *C* is opened and the powder falls into the space between *C* and *D*. The valve *C* is now closed, the end (*H*) of the tube is placed in a beaker of water, and the mineral allowed to fall into it by opening the cock *D*. The space between *C* and *D* may be rinsed by repeatedly sucking the water as high as necessary into the tube *B* and allowing it to fall back into the beaker.

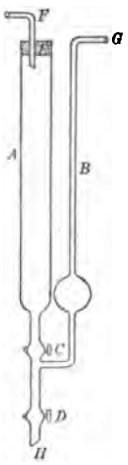

FIG. 727.—Thoulet apparatus.

The dilute solution in the beaker is now poured off, the mineral is repeatedly washed, and finally dried. On account of the cost of Sonstadt's solution, the dilute solution which was poured off, as well as all of the wash water, is concentrated over a water-bath, and used again and again.

Water is now carefully added, drop by drop, and air is blown through the solution in *A* until minerals of a less density fall. The process is repeated as before and so on, again and again, until all of the powder has come down in installments of different specific gravities.

The Thoulet apparatus is complicated and easily broken. The proper control of the valve *C*, to prevent the mineral from falling below the entrance of *B* before it is thoroughly separated from other particles, is difficult. There is also a likelihood of choking the narrow tube above *C*, and of particles adhering just above the constriction and coming down later with minerals of less density.

486. Goldschmidt (1881).—To overcome the difficulties attending the use of the Thoulet apparatus, and especially to make possible a stirring together of powder and fluid, Goldschmidt[1] preferred using, especially for preliminary separations, small, slender, lipped-breakers with a capacity of from 40 to 50 c.c. By their use a much larger quantity of material may be worked and the density at any time may be tested readily. Further, no special apparatus is required, and there is no difficulty in cleaning up.

[1] V. Goldschmidt: *Ueber Verwendbarkeit einer Kaliumquecksilberlösung bei mineralogischen und petrographischen Untersuchungen.* Neues Jahrb., B.B., I (1881), 214–261.

Goldschmidt's method consists in placing the pulverized mineral in a beaker with approximately 30 c.c. of the solution, stirring vigorously with a glass rod, allowing it to settle, and pouring off some of the fluid with the floating minerals. If the quantity is large, most of the floating minerals may be removed with a glass or platinum spoon. Upon pouring off the solution, there will remain, hanging to the sides of the vessel, a partial ring of the lighter minerals, broken on the side where the liquid was poured out. This clear passageway may be widened somewhat by wiping with the glass rod, and along it the heavy minerals may be poured or rinsed into another beaker. Finally the adhering lighter minerals may be washed into a beaker, dried, and further separated. To obtain perfect separation the process should be repeated with each portion of different density thus obtained.

487. Harada (1881).—One of the simplest and most convenient of separating instruments, and one after which many others are patterned, is that designed by T. Harada[1] (Fig. 728). It consists of a long, pear-shaped glass vessel, closed at the upper end by a tight fitting stopper, and at the lower by a stop-cock whose passage-way is of the exact size of the inner diameter of the tube, thus leaving no shoulder. The mixing of powder and fluid is carried out by shaking, after which the heavy portion is allowed to settle, the lower end of the tube is placed in the bottom of the vessel *b*, the cock is carefully opened, and the mineral particles are permitted to pass through. A portion of the fluid, only enough to cover the opening of the tube, will also escape, but most of it will be held in the vessel by the air pressure. After all of the mineral has passed out, the stop-cock is closed and a thin layer of water is poured over the solution in the vessel *b*. Upon raising the tube slightly, the water will rush into the narrow extension while the heavy solution will fall and carry with it any remaining particles.

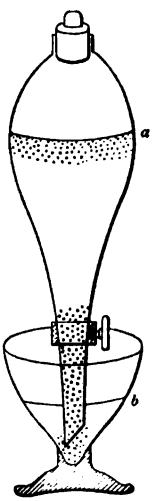

FIG. 728.—Harada tube.

The separating operation is repeated with the addition of a small amount of water to permit minerals of lesser density to fall. If perfectly pure material is desired, it is necessary to perform the separation a number of times, since the heavier particles mechanically carry down with them some of the lighter and the lighter hold up part of the heavier.

Johnsen and Mügge[2] suggest that the receiving cup be made cylindrical and very little larger than the lower portion of the tube. If indicators[3]

[1] K. Oebbeke: *Beiträge zur Petrographie der Philippinen und der Palau-Inseln.* Neues Jahrb., B.B., I (1881), 457.

[2] A. Johnsen und O. Mügge: *Verbesserungen am Harada'schen Trennungsapparat.* Centralbl. f. Min., etc., 1905, 152–153.

[3] Art. 480.

are used, they will pass with the mineral grains freely through the stop-cock below, and the density of the mineral may at once be determined from the figures inscribed on the accompanying glass cube. Johnsen and Mügge also suggest adding the water necessary for dilution as well as the indicators at the lower end of the tube, inverting it for this purpose. The upper stopper need not be removed except for the insertion of the mineral powder and for cleaning purposes.

488. Oebbeke (1881).—Oebbeke[1] designed a simplified form of the Thoulet tube which possesses all of the good qualities of the latter, but is more readily cleaned and is much easier to handle. Through a glass tube (Fig. 729), extending through the stopper nearly to the bottom of the vessel, a current of air is blown which thoroughly mixes the fluid and the powder. The space between the two stop-cocks is cleaned by means of a stream of water forced through the tube *b*, which is bent and drawn out to a fine end.

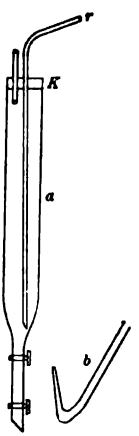

FIG. 729.—Oebbeke apparatus.

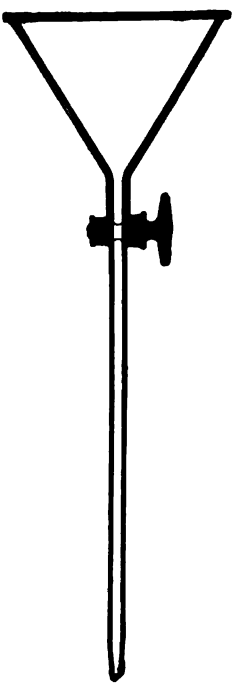

FIG. 730.—Separating funnel. 1/4 natural size. (Fuess.)

489. Van Werveke (1883).—A very simple separating funnel was suggested by Van Werveke[2] in 1883. It consists of a funnel (Fig. 730) with a stop-cock having an opening of the exact size of the bore of the tube to prevent the lodging of any mineral particles. If such a funnel is not at hand, the tube of an ordinary funnel may be cut to a convenient length and a rubbler tube

[1] K. Oebbeke: *Op. cit*, p. 456.

[2] L. van Werveke: *Ueber Regeneration der Kaliumquecksilberjodid Lösung und über einen einfachen Apparat zur Trennung mittelst dieser Lösung.* Neues Jahrb., 1883 (II), 86–87.

and pinch-cock attached. The mineral powder and heavy fluid are mixed in the funnel by means of a glass rod, or the upper rim of the funnel is ground, covered by a ground glass plate, and the apparatus, with the contained fluid, is shaken thoroughly.

490. Brögger (1884).—To avoid the necessity of performing the operation of separation a number of times to obtain pure material, as is necessary with the Harada tube, Brögger[1] devised the repeating separator shown in Figs. 731–733. This instrument differs from the Harada tube in having a second large stop-cock above the first. The heavy solution and powdered

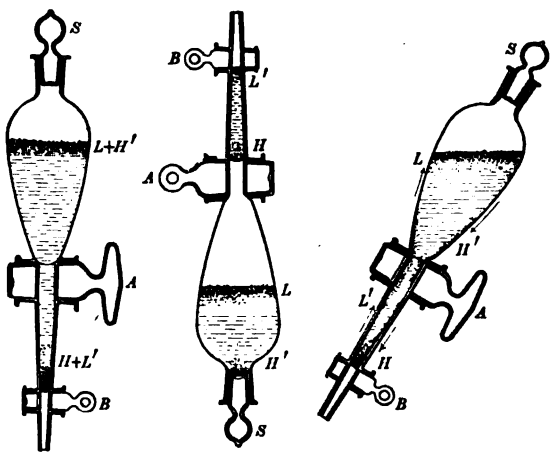

FIG. 731. FIG. 732. FIG. 733.

FIGS. 731 to 733.—Brögger's method for separating minerals.

rock are placed in the tube with the valve *A* open. It is thoroughly shaken and the minerals are allowed to settle, as shown in Fig. 731. The grains now will be in the same incomplete state of separation as when the Harada tube is used. Above the lower valve will lie the greater part of the heavy mineral (*H*) mingled with some of the lighter (*L'*) while floating on top will be a little of the heavy (*H'*) supported by the light (*L*). The valve *A* is now closed, the instrument is inverted, again thorougly shaken, and set away in this position for an hour (Fig. 732). Between the stopper *S* and the valve *A* and between the two valves, a further separation of the constituents has taken place as shown in the figure. The instrument is now carefully turned to the position shown in Fig. 733, and the valve *A* is slowly opened, whereby the separated portions of the light and the heavy minerals unite by the movement shown by the arrows. After placing the end of the apparatus

[1] W. C. Brögger: *Om en ny konstruktion af et isolations-apparat for petrografiske undersögelser.* Geol. Fören. i Stockholm Förh., VII (1884), 417–427.

E. Cohen: Review in Neues Jahrb., 1885 (1), 395–396.

in a vessel whose cup is narrow at the bottom, the lower valve may be opened and the heavy mineral removed as from the Harada tube. After the valve *B* is closed the process is repeated until no further separation takes place. For the remaining minerals of less density the solution is diluted and the process repeated.

491. Smeeth (1888).—The apparatus constructed by Smeeth[1] possesses so many good qualities that it seems strange that it has not been more commonly used. It contains no stop-cocks to leak or get clogged, and is

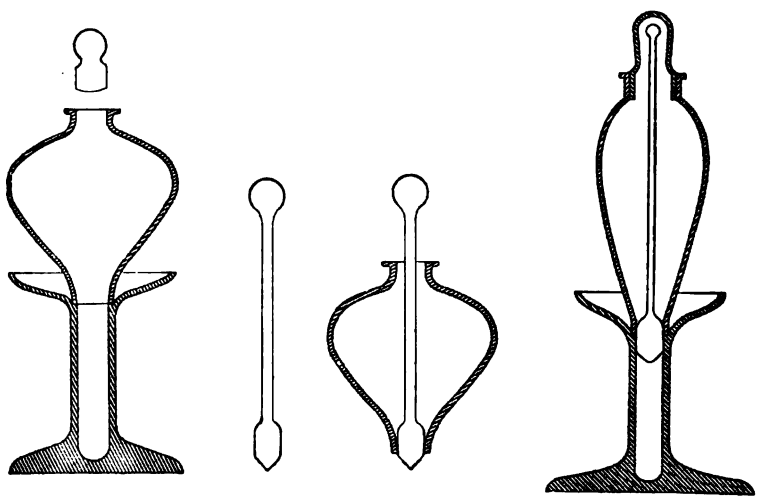

FIG. 734. FIG. 735. FIG. 736. FIG. 737.

FIGS. 734 to 736.—Smeeth separating apparatus. FIG. 737.—Diller's modification of the Smeeth apparatus.

easily cleaned. It consists, essentially, of a turnip-shaped separating vessel, closed above and below by ground glass stoppers (Fig. 734). The lower stopper (Fig. 735) is in the form of a glass rod with enlarged end, and the whole apparatus fits tightly in a flat, candle-stick-shaped foot.

The separation is begun with the inner rod removed and the apparatus set up as shown in Fig. 734. On closing the upper stopper, the whole instrument may be vigorously shaken. After the heavy mineral settles, the stopper (Fig. 735) is inserted, and the whole upper part (Fig. 736) is removed. By using several cup-bases, the minerals of different densities may be separated without interruption. Afterward the densities of the fluids contained in the cups may be determined, without removal, by the Westphal balance.

[1] W. F. Smeeth: *An apparatus for separating the mineral constituents of rocks.* Proc. Roy. Soc. Dublin, VI (1888), 58–60.

492. Diller (1896).—The Smeeth apparatus was improved by Diller[1] who gave it a pear- rather than a turnip-shape (Fig. 737). It thus offers less chance for the lodgment of particles on the sides. By making the upper stopper hollow and shortening the rod-stopper, the apparatus may be shaken with the latter in place and a second partial separation made. By tilting the instrument in a manner similar to that described for the Brögger tube, and then removing the rod, repeated separations may be made very easily.

493. Laspeyres (1896).—A simple and cheap instrument, proposed by Laspeyres,[2] is shown in Fig. 738. It may be made of any size, the most convenient being one which requires but 12 c.c. of fluid and may be used with one hand. It consists of two pear-shaped bodies connected at their small ends by the stop-cock *C*, whose opening is equal to that of the inner diameter of the tube at that point. At each end are large, carefully ground stoppers. By placing the thumb and two fingers in the depressed ends of the stop-cock and the stoppers, a firm grip may be obtained upon the instrument, thus permitting vigorous shaking with no danger of losing the enclosed liquid. To use the instrument, the cock *C* is opened and the double containers filled as full as possible with the powdered rock and separating liquid. The end is closed and the vessel is thoroughly shaken, producing the first partial separation. After permitting the minerals to settle, the valve is closed and the shaking repeated. When the valve is again opened, the minerals of the second separation unite with those of the first, as in previously described methods

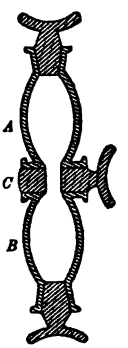

FIG. 738.—Laspeyres separating apparatus.

A somewhat similar instrument is described by Hauenschild.[3]

494. Wülfing (1890).—Another repeating separator, based somewhat on Brögger's principle but of quite different form, is that proposed by Wülfing.[4] Instead of being pear- or turnip-shaped this one is link-like (Fig. 739). At the upper and lower ends are valves by which the connection between the two parts may be shut off. When open, the bore is the same as the tube adjacent, so that there is no chance for the collection of mineral particles at this point. The capacity of the instrument is about 40 c.c. It is filled, through two glass-stoppered openings, about three-fourths full of the heavy fluids, the connecting valves being opened to bring it to the same level on either side. The powder is then added to both sides, the connecting valves are closed, and the first partial separation takes place as in the Brögger

[1] J. S. Diller: *The Smeeth separating apparatus.* Science, N. S., III (1896), 857–858.

[2] H. Laspeyres: *Vorrichtung zur Scheidung von Mineralien mittelst schwerer Lösungen.* Zeitschr. f. Kryst., XXVII (1896), 44–45.

[3] Alb. Hauenschild: Zeitschr. f. Baumaterialienkunde, 1898.*

[4] E. A. Wülfing: *Beitrag zur Kenntniss des Kryokonit.* Neues Jahrb., B.B., VII (1890), 164–165.

instrument. Upon opening the lower valve and inclining the apparatus, the heavy material, with the small amount of light material which is carried down with it, will sink into the lower tube. If, in the inclined position, the upper valve is opened quickly, the difference in pressure will carry every particle of the heavy material down. The inclination of the apparatus is now increased until the liquid in the lower tube reaches the upper valve, when the lower valve is closed. By slightly shaking, the light material can be washed over into the now partially emptied tube. Should some still remain in the first tube, the instrument need only be once more inclined, the lower tube again filled to the upper valve, and the light powder shaken over. With both stop-cocks closed, the instrument may be shaken again, after which the separation may be repeated, again and again, in the same manner as before, until all of the material is separated according to densities. The heavy material may now be removed through one of the glass-stoppered openings, more fluid added, diluted to the proper density, and the separation continued as with other instruments.

FIG. 739.—Wülfing's separator. (Fuess.)

495. Luedecke (1911).—An apparatus similar to that of Smeeth was made by Leudecke and, after his death, described by Dreibrodt.[1] It differs from

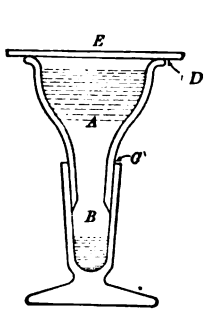

FIG. 740.

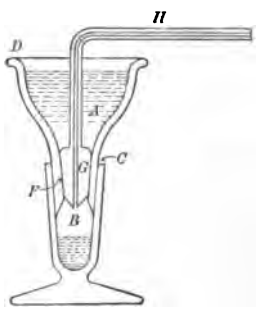

FIG. 741.

FIGS. 740 and 741.—Leudecke apparatus.

the Smeeth apparatus chiefly in the form of the upper vessel, which is here covered with a ground glass plate (*E*, Fig. 740), and in the stopper *G* (Fig. 741), which is provided with a capillary tube *H* to take up the displaced solution when it is inserted in it.

496. Separation Apparatus for Heavy Melts.—To simplify separation by

[1] O. Dreibrodt: *Trennungsapparat nach Prof. Dr. O. Luedecke.* Centralbl. f. Min., etc., 1911, 425–426.

means of melts, Penfield[1] proposed the apparatus shown in Fig. 744. In its first form (Fig. 742) it consisted of a glass tube with tapering end, into which a glass rod was ground with fine emery to serve as a stop-cock. The apparatus was placed inside a test-tube and was so adjusted that it reached to within a few millimeters of the bottom. Test-tube and contained separating

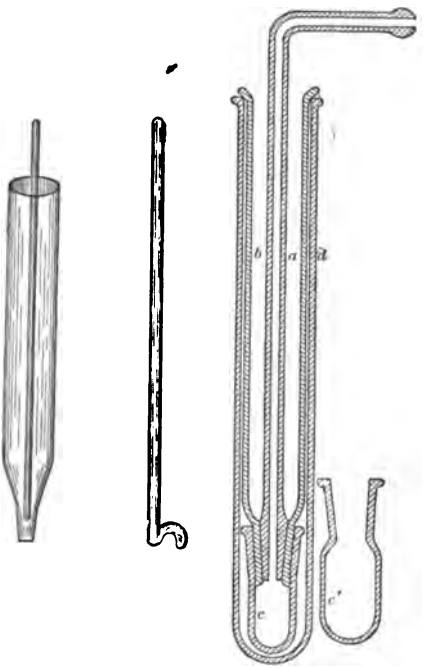

FIG. 742. FIG. 743. FIG. 744.

FIGS. 742 to 744.—Penfield separating apparatus for heavy melts.

apparatus were placed in a beaker of hot water and were heated until the double nitrate of silver and thallium, which was used in the separation, were fused. A glass rod, bent as shown in Fig. 743, was used as a stirring rod. When the separation had taken place, the inner rod was raised and the heavy mineral and a certain amount of the melt were allowed to escape into the test-tube.

In its improved form[2] the instrument is shown in Fig. 744. The glass

[1] S. L. Penfield and D. A. Kreider: *On the separation of minerals of high specific gravity by means of the fused double nitrate of silver and thallium.* Amer. Jour. Sci., XLVIII (1894), 143–144.

[2] S. L. Penfield : *On some devices for the separation of minerals of high specific gravity.* Amer. Jour. Sci., L (1895), 446–448.

Idem: *Ueber einige Verbesserungen der Methoden zur Trennung von Mineralien mit hohem specifischen Gewicht.* Zeitschr. f. Kryst., XXVI (1896), 134–137. (Translation of preceding.)

tube *b* is about 20 cm. long and has an internal diameter of 2.2. cm. At its lower end, with well-ground joints, it fits the cap *c* and the hollow stopper *a*. The whole apparatus is placed within a test-tube *d*, and is heated in a beaker of water.

In making a separation by means of a heavy melt, Penfield recommended first melting the double salt in a dish on the water-bath until it becomes perfectly clear. It is then diluted with water until it is of approximately the specific gravity of the mineral to be separated, and is poured into the warm separating apparatus after removing the stopper *a*. The tube *b* is filled about half full, after which the mineral powder is added and thoroughly mixed by blowing a stream of air into the liquid through a small glass tube. Successive portions of water are added until the desired separation takes place. To obtain the heavy material in *c*, the previously warmed stopper *a* is inserted, the cap *c* removed, and the contents washed out with hot water. The cap *c*, or one similar, may be replaced, more water added, and another separation made. For large quantities of material the larger cap *c'* may be used, though it is not so convenient as the smaller one.

497. Causes Likely to Produce Errors in Separating Minerals or in Determining Specific Gravities by Means of Heavy Fluids.—The causes which operate to produce wrong determinations of the specific gravity, or incomplete separation, by means of heavy solutions, are: (a) The variations in the specific gravity of grains of the same mineral. This may be caused by inclusions of bubbles which decrease the apparent density, or by inclusions of other minerals which increase or decrease it, or by partial alteration or weathering. (b) The impossibility of completely breaking apart the different mineral grains by crushing, wherefore certain grains may be composed of more than one mineral. This is especially the case with minerals intimately intergrown, as microperthite, etc. (c) The practical identity of the specific gravities of certain minerals. (d) The likelihood of the heavy minerals carrying down the lighter, and *vice versa*. (e) The consolidation of the grains into little lumps.[1]

A preliminary examination under the microscope will act as a check on *a* and *b*, while repeated separation will eliminate *d*, the purity of the material being determined microscopically. In the third case, *c*, the separation may be made with great care when the densities are not quite the same, the microscope in this case also showing when the separation is complete. If the densities are exactly equal, they cannot, of course, be separated by this method. To prevent the mineral grains from gathering together into little dry lumps (*e*) which are afterward almost impossible to separate, one should

[1] If the material is being separated for chemical analysis, one must be certain that no chemical change has taken place. See W. F. Hillebrand: *A danger to be guarded against in making mineral separations by means of heavy solutions.* Amer. Jour. Sci., XXXV (1913), 439–440.

first pour over the powder only just enough of the heavy solution to cover it, and then stir with a glass rod until all of the grains are thoroughly wet, after which more fluid may be added. If the mineral was powdered some time previously, and it has been exposed to the air, it is usually advisable to boil it or place it under an air pump before performing the separation.

498. Separation of Thin Flakes and Fine Needles.—Micaceous minerals may be readily separated from a rock powder by means of water. If the powdered rock is placed in a beaker, and a fine stream of water is conducted to the bottom by means of a glass or rubber tube, the flakes will be carried upward by the eddies and thus be washed over the edge of the beaker into a larger one surrounding it.

Rosenbusch[1] separated mica from a mica trachyte by letting the powder glide many times down a slightly inclined sheet of rather rough writing paper. The mica particles adhered to the paper while the feldspar slipped down.

Linck[2] allowed small portions of rock powder to fall from a considerable height into a glass funnel upon the inner surface of which a slight coating of moisture had been placed by breathing upon it. The mica flakes adhered to the funnel while the rounded grains fell through. By tapping the funnel lightly upon a sheet of paper on a table, the mica dropped down. The process was repeated several times to obtain pure material.

499. Separation by Hand.—In spite of the most careful separation or repeated separations by heavy solutions or electromagnet, it often will be found, upon examination under the microscope, that the constituents thus obtained are not perfectly homogeneous. It may be that the specific gravity of two minerals is so nearly the same that they come down together, or that occasional grains in an otherwise homogeneous powder are composed of two minerals grown together. Perhaps it is desired to isolate certain grains before making a separation by heavy solutions, or to pick certain constituents from a sand. In all such cases the only method is to perform the operation by hand. The mineral grains are strewn on the stage of a preparation microscope (Figs. 723–724), and are picked out by means of the moistened end of a pointed piece of wood or the end of a heavily waxed thread.[3] Zirkel[4] suggested picking out the grains by means of a dissecting needle upon whose point a trace of Canada balsam had been placed. As soon as the drop of balsam is covered everywhere with mineral grains, the

[1] H. Rosenbusch: *Glimmertrachyt von Montecatini in Toscana.* Neues Jahrb., 1880 (11), 207.

G. Linck: Abhandl. zur geolog. Spez.-Karte von Elsass-Lothringen. III, Strassburg, 1884, 41–42.*

[3] Rosenbusch-Wülfing: *Mikroskopische Physiographie*, Stuttgart, 4 Aufl., I_1, 1904, 434–435.

[4] F. Zirkel: *Lehrbuch der Petrographie.* Leipzig, 2 Aufl., 1893, I, 107.

needle is held for a moment in a vial of benzol. This dissolves the balsam and permits the mineral fragments to accumulate in the bottom of the flask. Instead of Canada balsam, the needle may be dipped in glycerine and the minerals transferred to water.

500. Separation by Chemical Means.—Ordinarily it is not necessary to make chemical separations of mineral constituents. At most it may be necessary to remove carbonates, which is readily done by means of dilute hydrochloric acid. Separation methods will readily suggest themselves to a chemist, to others the directions which could be given within the limits of this book would be useless.

CHAPTER XL

MICROCHEMICAL REACTIONS

501. General Mitrochemical Reactions.—For petrographical purposes general microchemical reactions for the determination of the elements are used but little. The subject is a study in itself, and the student is referred especially to Boricky's paper and Behrens' English translation of his own "Manual of Microchemical Analysis," published in 1894. For convenience, a bibliography is given at the end of this chapter.

Chemical Reactions on Rock Slices

502. Apparatus.—Certain special materials and apparatus are required in microchemical researches, those necessary for the examination of reactions on thin rock sections are:

Microscope.—Special chemical microscopes have been designed, some of them inverted. A cheap microscope is all that is necessary, provided it is fitted with nicol prisms. No higher magnification than 200 diameters is required. If the objective used is corrected for cover-glasses, it will add to the clearness of the image if an ordinary cover-glass is stuck to the lower lens with cedar oil or glycerine. This serves to counteract the effect of the removal of the cover-glass from the rock section, and also acts as a protecting screen for both the objective and its casing against the reagents used. Another method is to work under a kind of table made of a large cover-glass resting on pieces of cork attached at the corners. For most observations, objectives with a clear working distance of 3 cm. should be used.

Canada Balsam.—For immediate use, it is sometimes desirable to have Canada balsam dissolved in some medium which will evaporate quickly, without heat, leaving the balsam hard. It may be prepared by heating the balsam in a shallow dish until a sample, cooled in water, is of sufficient hardness. It should then be broken up and dissolved in bisulphide of carbon or ether to the consistency of cream. The first has an unpleasant odor, and the latter, if dried in a damp atmosphere, may become turbid, owing to the absorption of moisture; a mishap not likely to happen in our steam-heated laboratories, however. If the balsam thus prepared is too hard, more or less soft balsam may be added to it. Fifteen minutes should be sufficient for the material to harden.

Capillary Tubes and Pipettes.—For transferring reagents to slides or removing worked out liquids, capillary tubes and pipettes made of glass are useful. Being readily made, they may be thrown away after using, thus doing away with cleaning.

Glass Dropper.—A glass rod drawn out rather thin and with a slightly enlarged end is useful in taking large drops from a reagent bottle.

Platinum Dropper.—A most convenient dropper is made of a platinum wire, 0.5 mm. in diameter and bent into a small hook or loop at the end. If quickly withdrawn from a reagent bottle, a large drop will be carried away; if slowly, a small one, provided the wire is clean. Thicker straight wires may be used instead, and have the advantage of being cleaned more readily.

Burner.—A small Bunsen burner giving a flame 5 to 10 mm. in length is useful.

Water-bath.—A small water-bath, about the size of a cigar box, is very convenient. One may be improvised from a large evaporating dish covered with a glass plate over which a small pasteboard box is inverted. The slide may be laid directly upon the glass plate, upon a piece of pasteboard within the box, or upon the box itself, depending upon the degree of heat desired.

503. **Preparing the Slide.**—The chemical determination of certain properties of minerals is often necessary, and special processes and methods must be used. Ordinarily these examinations must be made on rock sections which are also to be used for the general determinations of the rock, and it is usually desirable that the section be spoiled as little as possible by the operation.

To make a chemical test on a mineral it is necessary, of course, that the cover-glass be removed from that portion of the slide. This can be done roughly by cutting on the cover-glass, with a marking diamond, or more neatly, with a slide marker (Figs. 760–761), a circle around the mineral, placing the slide for a moment, cover-glass downward, on a heated plate (Fig. 755) or a water-bath, and lifting the small circular section by means of a needle. The balsam underlying the opening is removed by placing a few drops of alcohol upon it by means of a camel-hair brush, allowing it to act for a short time, removing the white gum with a rolled-up piece of filter paper, applying more alcohol, and so on until the mineral is uncovered.

If the mineral grain to be treated is very small and a smaller opening is desired, the old cover-glass must first be removed by placing the slide with the cover-glass downward on the hot plate until the balsam is softened, then sliding, not lifting, it from the preparation. If a few drops of turpentine are placed on the upper side of the thin section, its evaporation will assist in keeping the balsam film between rock slice and mounting slip hard. Care

must be used not to heat the slide too much, otherwise it is likely to go to pieces on the mount. A few drops of well-cooked balsam are now placed over the section, allowed to spread, and then to cool. In a new cover-glass, a hole is drilled, or bitten by acid according to a method to be explained, in such a position that when it lies over the mineral section the cover-glass will cover approximately all of the rock slice. This glass is now placed over the rock slice and shoved about, under the microscope, until the hole lies over the mineral to be examined, when slide and cover are carefully removed and heated until the balsam softens enough to begin to push through the hole. After cooling, the balsam is removed from above the mineral by the method previously described.

Holes may be made in cover-glasses by means of a diamond drill. Since such is usually not at hand in the laboratory, a different process is necessary. The cover-glass may be dipped in melted wax[1] and, after cooling, by means of a needle point, a circle of the desired size (1/4 to 3/4 mm.) may be scratched upon it. A few drops of hydrofluoric acid, renewed as often as necessary, will soon bite through the glass, leaving a cone-shaped hole. If the small opening is not quite large or round enough, it may be enlarged by means of a needle. The wax may be removed from the remainder of the slide by means of hot water. A number of such cover-glasses, with holes in various parts, may be prepared beforehand and kept in stock. When placed over a slide the smaller end of the funnel-shaped hole should lie against the rock slice, the larger side up. The remainder of the slide, being protected by the glass and the Canada balsam, will not be acted upon by the reagents used. If hydrofluoric acid is the reagent, a piece of thin, perforated platinum foil should be substituted for the glass, its proper position on the slide being readily determined, under the microscope, by first centering the mineral under the cross-hairs and then sliding the platinum foil into place.

Another method[2] for protecting the remainder of the slice is to cover it, after removing the cover-glass, with rather a thick coating of balsam dissolved in ether. In a few hours the balsam will be hard, and a hole may be scratched through it directly over the mineral to be examined.

504. Microchemical Filtrations.—The most satisfactory device for fitering small quantities of liquid is that of Streng.[3] The slide, with the liquid to be filtered upon it, is placed on a small box turned upside down and

[1] A. Streng: *Ueber eine Methode zur Isolirung der Mineralien eines Dunnschliffs behufs ihrer mikroskopisch-chemischen Untersuchung.* Ber. oberhess. Gesell. Giessen, XXII (1883), 260–262.

Idem: *Ueber einige mikroskopisch-chemische Reaktionen.* Neues Jahrb., 1885 (I), 26.

Idem: *Erwiderung.* Neues Jahrb., 1885 (I), 174–175.

[2] Arthur Wichmann: *Ueber eine Methode zur Isolirung von Mineralien behufs ihrer mikrochemischen Untersuchung.* Zeitschr. f. wiss. Mikroskop., I (1884), 417–419.

A. Streng: *Erwiderung.* Neues Jahrb., 1885 (I), 174–175.

[3] A. Streng: *Anleitung zur Bestimmung der Mineralien*, 65.*

slightly inclined. With a width of 5 cm. the box should be about 10 mm. high on one side and 12 mm. on the other. A piece of filter paper, cut in the form of a letter Y, with a width of from 1 to 2 mm. and a length of 10 to 25 mm., is so placed that the reentrant angle of the forked end is in contact with the liquid to be filtered while the lower end touches a clean slide placed near the box. To retain it in place, the upper arms may first be slightly moistened. The liquid will now run through, perfectly clear and transparent, while the solids remain above. For the filtration to be successful there must be enough fluid so that it is not all retained in the filter, at least 0.01 c.c. being necessary, if it is not to be diluted, with paper of the size mentioned. The solid portion may be washed by the gradual addition of water, the lower end of the filter strip being placed on a folded piece of filter paper to absorb the wash water.

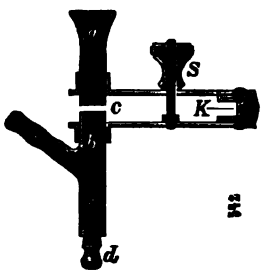

FIG. 745.—Haushofer filter. (Fuess.)

For larger quantities of liquid the Haushofer[1] filter is the best. It consists of two thick glass tubes (*a* and *b*, Fig. 745) with an inner diameter of about 4 mm. The abutting ends *c* are smoothly ground and are kept in contact by the screw *S*. Between the two ends *c* is placed a double piece of filter paper with a diameter a couple of millimeters greater than that of the outside of the tube, and clamped in place by the screw *S*. A rubber tube is attached to *e*, and the liquid to be filtered is poured into the funnel-shaped end of the upper tube. By sucking through the rubber tube, the liquid is rapidly filtered into *b*. The filtrate is removed by opening the stopper *d*, the precipitate remaining in a compact ring 4 mm. in diameter on the filter paper.

505. Gelatinizing and Staining Minerals.—A general gelatinization test may be made by entirely removing the cover-glass from the slide and cleaning off the Canada balsam by means of alcohol or ether. Usually it will be found sufficient if only a part of the slide is uncovered, which may be done by cutting across with a diamond, heating, and sliding the cover-glass from one side. If the cover-glass is entirely removed, the reagent may be confined to a portion of the slide by surrounding it with a ring of hardened balsam.

A few drops of hydrochloric acid are now spread in a thin film over the part of the slide to be tested. Only enough acid is used to make a surface etching, otherwise the resulting gelatine will spread over the surrounding unattacked minerals and cause confusion. It is better to make several successive trials than to cause too vigorous action at once. After allowing the acid to act for a short time, perhaps with gentle heating, it is washed off,

[1] K. Haushofer: *Beitrage zur mikroskopisch-chemischen Analyse.* Sitzb. Akad. Wiss. München, XV (1885), 224–226.

care being taken not to remove the gelatine coating. It may be well to use a few drops of dilute ammonia to neutralize the acid.

The section will now be found to have a thin film of gelatine over such minerals as gelatinize with acid. To make it more visible it is necessary to stain it. This is done by covering the slide with an aqueous solution of some dye and allowing it to act for about 15 minutes. Sometimes slight heating will aid the staining, as will also a trace of ammonia. The slide is washed to remove the stain from such minerals as were not attacked by the acid, and it is examined under the microscope. Gelatinized minerals will have taken the stain, as will also cracks in other minerals. If it is found that the action was not continued long enough, it is repeated, the new acid destroying the dye.

The coloring matter generally used is fuchsine, first recommended by Behrens[1] and afterward by Haushofer.[2] While it has great staining power, it fades in the light and is not permanent in the presence of Canada balsam. Malachite green surpasses fuchsine in staining power, and is permanent. Methylene blue is nearly equal in staining power but is likely to form films on rough surfaces.

If the solvent is to be examined, the acid is allowed to act for a longer time and is then removed with a capillary tube, placed on an object glass, and tested for various chemical reactions.[3]

SPECIAL REACTIONS, CHIEFLY ON THIN SECTIONS

506. Hauynite, Noselite, Sodalite, Melilite, and Zeolites.—Minerals of the sodalite-noselite-hauynite group may be treated with acid and the solvent removed by means of a capillary tube and placed upon a clean object glass. Sodalite will be found to dissolve without gelatinization in HNO_3, and cubes of NaCl form upon drying the solution.[4] If a few drops of a dilute, slightly acid solution of lead acetate are placed upon a thin section of sodalite upon which, previously, a few drops of dilute Cl-free HNO_3 or acetic acid have been placed, thin, flat needles of strongly refracting lead chloride form over it.[5] Noselite and hauynite gelatinize in thin sections with HCl. Upon

[1] H. Behrens: *Mikroskopische Untersuchungen über die Opale.* Sitzb. Akad. Wiss. Wien, LXIV, I Abth. (1871), 521.

[2] K. Haushofer: *Mikroskopische Reactionen. Eine Anleitung zur Erkennung verschiedener Elemente unter dem Mikroskop als Supplement der qualitativen Analyse.* München, 1885.*

[3] H. Behrens: *A manual of microchemical analysis.* London, 1894. See also other papers mentioned in the preceding pages and in the General Bibliography at the end of the chapter.

[4] G. A. Sauer: *Untersuchungen über phonolithische Gesteine der Kanarischen Inseln.* Zeitschr. f. d. gesammten Naturw., XIII (1876), 322.*

[5] Franz F. Graeff: *Mineralogisch-petrographische Untersuchung von Eläolithsyeniten von Serra de Tinguá, Provinz Rio de Janeiro, Brasilien.* Neues Jahrb., 1887 (II), 230.

G. Freda: *Sulle masse trachitiche rinvenute nei recenti trafori delle colline di Napoli.* Rendiconti della Acad. di. Napoli, III (1889), (2), 39.*

drying the solution derived from the former, much NaCl (cubes) and a little $CaSO_4$ (needles) separates; from hauynite much $CaSO_4$ separates.[1] Sodalite and noselite may be separated by placing over them a drop of dilute acetic acid (one part acid to three or four of water) to which a little $BaCl_2$ solution has been added. To prevent complete drying, the section and a watch crystal containing some of this fluid are set away under a bell jar. It will be found, according to Osann,[2] that the sodalite will remain clear, although it will be etched, while the noselite will be covered with an opaque film of $BaSO_4$.

That colorless members of the hauynite groups may be colored blue by heating, was shown by Vogelsang.[3] The same result was obtained by Knop[4] by heating the uncovered thin section in a closed vessel, in the bottom of which was placed a pinch of flowers of sulphur.

Analcite gelatinizes with HNO_3. It differs from sodalite in that no cubical crystals of sodium chloride form from the evaporated solution.

Noselite, hauynite, and analcite will take stain readily since they gelatinize easily.[5]

Melilite gelatinzes readily with HCl. If a drop of H_2SO_4 be added to the hydrochloric acid solution, crystals of gypsum are formed on the slide.[6]

507. Nephelite, Cancrinite, and Hydronephelite.—If a thin section containing nephelite and cancrinite is heated, no changes appear in the former but the latter becomes cloudy, probably due to the driving off of the CO_2.[7] Nephelite gelatinizes readily with HCl and takes stain. From the solution, cubes of NaCl are formed. Cancrinite gelatinizes after it is acted upon by warm HCl. There is a slight evolution of CO_2 which may readily be observed under a cover-glass as described under carbonates (Art. 510).

[1] G. A. Sauer. *Op. cit.*

[2] A. Osann: *Ueber ein Mineral der Nosean-Hauyn-Gruppe im Eläolithsyenit von Montreal.* Neues Jahrb., 1892 (I), 224.

[3] H. Vogelsang: *Ueber die natürlichen Ultramarineverbindungen.* Versl. en Meded. Akad. Weten. Amsterdam, VII (1873), 161–199.

[4] A. Knop: *Ueber eine mikrochemische Reaction auf die Glieder der Hauynfamilie.* Neues Jahrb., 1875, 74–76.

[5] J. Lemberg: *Zur mikrochemischen Untersuchung einiger Minerale.* Zeitschr. d. deutsch. geol. Gesell., XLII (1890), 738–740.

H. Dressel: *Mittheilungen vom Laacher See.* Neues Jahrb., 1870, 565.

G. vom Rath: *Mineralogisch-geognostische Fragmente aus Italien.* Zeitschr. d. deutsch. geol. Gesell., 1866, 547.

[6] Alfred Stelzner: *Ueber Melilith und Melilithbasalte.* Neues Jahrb., B. B., II (1883), 382.

[7] A. E. Törnebohm: *Om den s. k. Fonolitenfraan Elfdalen, dess klyftort och förekomstsätt.* Geol. Fören. i Stockholm Förh., VI (1883), 383–405.

E. Cohen: Review of above. Neues Jahrb., 1883 (II), 370–371.

A. Streng: *Ueber die mikroskopische Unterscheidung von Nephelin und Apatit.* T. M. P. M., 1876, 168–169.

Idem: *Ueber einige mikroskopisch-chemische Reaktionen.* Neues Jahrb., 1885 (I), 29–33.

Hydronephelite is soluble in HCl, and, upon evaporation, gelatine is formed.

In making the gelatinization test, care should be taken not to mistake the mineral from which the gelatine was derived.

508. Olivine Family.—Olivine gelatinizes slowly with cold and rapidly with hot HCl or H_2SO_4. The iron rich members are more readily acted upon than are the iron poor.

509. Apatite.—Apatite is easily soluble in HCl or HNO_3. If ammonium molybdate is added to the solution, a yellow precipitate, consisting of isometric crystals, is formed.[1] If dilute H_2SO_4 is added to the nitric acid solution, gypsum crystals develop upon evaporation.

510. Carbonates.—Upon the addition of acids to carbonates, an effervescence arises from the escape of the carbon dioxide. This breaking up occurs in some carbonates upon the addition of acetic acid, in other with cold hydrochloric acid, and in still others only with hot. If the amount of carbonate is small, the escape of the gas may not be noticed. In such cases a drop of water may be placed on the section and over it a cover-glass. If a drop of acid is brought to the edge of the latter, it will gradually diffuse through the water. The cover-glass will prevent the escape of the gas and, if the latter is in small amount, will confine it immediately above the mineral from which it is being evolved, thus permitting its study under the microscope. The solvent may be removed with a capillary tube and studied, if desired.

Separation of Calcite, Dolomite, and Magnesite.—Calcite is acted upon by acetic or hydrochloric acid even when they are cold. Dolomite and magnesite require hot hydrochloric acid.

Calcite and dolomite may be separated by Lemberg's[2] method which depends upon the fact that aluminium hydroxide is quickly and completely precipitated from solutions of aluminium salts by calcite and very slowly by dolomite. Further, if the precipitation takes place in the presence of coloring matter, it generally combines with it to form an insoluble coating. To 60 parts of water are added 4 parts of dry aluminium chloride and 6 parts of logwood (hæmatoxylon campechianum). The ingredients are boiled together for twenty-five minutes with stirring, the evaporated water being constantly replaced. When cold, the deep violet solution is filtered. If a few drops of this solution are placed on a thin section of calcite, are allowed to stand from five to ten minutes, and are then carefully washed off with water, the section will be colored violet. A section of dolomite with the

[1] A. Streng: *Op. cit.*, 168.

A. Stelzner: *Ueber Melilith und Melilithbasalte.* Neues Jahrb., B. B., II (1883), 382.

[2] J. Lemberg: *Zur mikroskopischen Untersuchung von Calcit, Dolomit und Predazzit.* Zeitschr. d. deutsch. geol. Gesell., XC (1888), 357–359.

same treatment remains unchanged, and only after twenty minutes' treatment does it show faint stains in spots.

An earlier method of Lemberg[1] consisted in treating both minerals with ferric chloride. A solution of one part crystallized hydrochloric-acid-free ferric chloride ($Fe_2Cl_6+12H_2O$) in ten parts of water is used. If any basic salt separates the solution is filtered. When this solution is placed on calcite, the latter, within a minute, precipitates the iron as a hydroxide. If the slide or mineral grains are washed, this precipitate appears as a brown coating which becomes black, by changing to FeS, if a solution of ammonium sulphide ($(NH_4)_2S$) is poured over it. Dolomite treated with ferric chloride for the same length of time shows no change to the eye although pouring ammonium sulphide over it changes it to pale green by incident light while it remains colorless by transmitted. Brucite acts like dolomite.

Sections treated by the method just described do not show a permanent coloration, since the FeS readily oxidizes. It may, however, be made permanent as follows: Immediately after the ammonium sulphide has converted the FeOH to FeS, it is washed off the slide and a concentrated solution of potassium ferricyanide is quickly poured over it and allowed to remain about half a minute. It is then renewed and allowed to remain eight minutes. The resulting Prussian blue is permanent. If time is allowed to elapse before the addition of the potassium ferricyanide, oxidation will set in and spoil the reaction.

Linck[2] prepared a solution of 20 c.c. of ammonium phosphate in 30 c.c. of dilute acetic acid. If this preparation is allowed to remain on a slide of pure calcite, complete solution will take place, while slides of dolomite or magnesite are but slightly altered on the surface, being immediately protected from further action of the acid by a coating of magnesium ammonium phosphate. The film forms with as little as 10 to 15 per cent. of $MgCO_3$. The solution should be allowed to act for twenty-four hours.

Another reaction depending upon the precipitation of iron hydroxide or copper carbonate, was given first by Lemberg[3] and later by Hinden.[4] If 1 grm. of powdered calcite is thoroughly shaken up with 5 c.c. of a 10 per cent. solution of iron chloride, a violent effervescence takes place, and the solution becomes dark reddish brown. After two or three minutes the solution in the test-tube becomes thick and jelly-like and of a rust-brown color, due to the separation of FeOH. If 5 c.c. of a 5 per cent. solution of potassium thiocyanate (KCNS) be now added to the solution, no further change takes

[1] J. Lemberg: *Zur mikrochemischen Untersuchung von Calcit, Dolomit und Predazzit.* Zeitschr. d. deutsch. geol. Gesell, XXXIX (1887), 489–492.

[2] G. Linck: *Geognostisch-petrographische Beschreibung des Grauwackengebiets von Weiler bei Weissenburg.* Abh. zur geol. Spezialkarte von Elsass-Lothringen., III (1884), 17.*

[3] J. Lemberg: *Op. cit.*, Zeitschr. d. deutsch. geol. Gesell., XXXIX (1887), 489–492.

[4] Fritz Hinden: *Neue Reaktionen zur Underscheidung von Calcit und Dolomit.* Verhandl. d. Naturforsch. Gesell. in Basel, XV (1903), Hft. 2.

place, since all of the iron was previously precipitated (1 grm. of calcite will precipitate the iron from 14 c.c. of a 10 per cent. ferric chloride solution.)

If, in the same manner, ferric chloride is added to dolomite powder, no change takes place unless the solution is heated. If 5 c.c. of the potassium thiocyanate solution are added to the solution, not previously heated, the well-known deep-red iron reaction color appears. This test may be used quantitatively. To 1 grm. of the rock powder, in a flask, there is added 5 c.c. of a 5 per cent. potassium thiocyanate solution and then enough ferric chloride from a burette to give a permanent blood-red color, the ferric chloride being added a little at a time with vigorous shaking. By experiment it was found that 1 c.c. of the ferric chloride solution represented 8 per cent. $CaCO_3$ in the mineral examined.

The Hinden test may be made directly on a hand specimen or thin section, the ferric chloride giving, after one or two minutes, a dark red-brown color to calcite while dolomite shows no change. Magnesium rich calcite shows a more or less pale brown color, depending upon the amount of the calcium carbonate present.

A similar reaction takes place by boiling 1 grm. of calcium carbonate or dolomite with 5 c.c. of a 10 per cent. solution of $CuSO_4$. The former gives the blue color of basic copper carbonate while the latter shows no change. Ammonia added to the filtered or decanted solution derived from the calcite shows no change, while that from the dolomite becomes dark brown.

If any hydroxide of iron was present in the slide itself, this acts as a disturbing cause, especially if through the addition of $(NH_4)_2S$ it is changed to FeS. To overcome this, and likewise to make permanent mounts, Lemberg transformed the FeS into Turnbull's blue ($Fe_3[Fe(CN)_6]_2$) by treating it with potassium ferricyanide ($K_3Fe(CN)_6$).

Both Lemberg's and Link's methods are practically useless for rocks in which the carbonate is very finely distributed, the stain not holding with short action and sinking into cracks with longer action. Heger,[1] therefore, proposed a method which consists in treating the section with dilute HCl (2–3 c.c. of $\frac{n}{10}$) to which a few drops of potassium ferricyanide has been added. The reaction should be watched under the microscope. If calcite is present the reaction is great enough to cause effervescence and the acid should be washed off after a few seconds. The calcite will be found colored a deep blue *if it is not entirely free from iron as an impurity*. The slides should then be washed gently in water. With dolomite or other carbonates the reaction is much slower.

Separation of Calcite from Hydromagnesite and Brucite.—If grains of calcite, hydromagnesite, and brucite are heated until the latter two lose their

[1] W. Heeger: *Ueber die mikrochemische Untersuchung fein verteilter Carbonate im Gesteinsschliff.* Centralbl. f. Min., etc., 1913, 44–51.

water, and are then immersed in a silver nitrate solution, the calcite will remain unaltered but the others will be colored brown or black on account of the precipitation of silver oxide.[1]

Separation of Calcite and Aragonite.—Calcite and aragonite may be separated by Meigen's[2] method. This has been modified by Panebianco[3] who gives the following: If pulverized calcite is boiled for one minute with a dilute cobalt nitrate solution, it becomes blue, and, after four minutes' boiling, lavender-blue. Aragonite, similarly treated, immediately becomes lilac and, with continued boiling, violet. The reaction for aragonite is very sensitive. In a mixture of one part of aragonite and nineteen parts of calcite, the color characteristic of the former will be produced. Magnesite acts like calcite while witherite and strontianite act like aragonite. Kreutz[4] found that siderite, rhodochrosite, and smithsonite give no reaction with cobalt nitrate.

511. Separating Quartz from Feldspar.—In fine-grained rocks it is sometimes difficult to separate quartz from feldspar, especially when the grains are scattered through a great mass of another mineral, for example, quartz and orthoclase in amphibolite. In such cases, according to Harada,[5] the slide may be treated for a moment with hydrofluoric acid whereby the quartz, though acted upon, remains clear, while the surface of the feldspar is altered to aluminium fluosilicate and becomes cloudy.

Becke[6] carried the process a step farther in that he colored the etched minerals. He proceeded as follows: Upon the clean surface of the slide to be stained, a large drop of hydrofluoric acid was allowed to act from 1/4 to 1 minute. With a piece of filter paper, the drop was absorbed without disturbing the slide, which was then placed on the water-bath and the remaining moisture rapidly evaporated. A drop of some aniline dye was now placed on it and allowed to remain from five to ten minutes, after which it was removed with a pipette and the slide washed by carefully dropping

[1] J. Lemberg: *Ueber die Kontactbildungen bei Predazzo.* Zeitschr. d. deutsch. geol. Gesell., XXIV (1872), 225–229.

[2] W. Meigen: *Eine einfache Reaktion zur Unterscheidung von Aragonit und Kalkspath.* Centralbl. f. Min., etc., 1901, 577–578.

Idem: *Die Unterscheidung von Kalkspath und Aragonit auf chemischem Wege.* Ber. Versam. oberrhein. geol. Vereins; XXXV (1902), 31–33.*

[3] G. Panebianco: Rivista di min. crist. Ital., XXVIII (1902), 5.*

[4] Stef. Kreutz: *Ueber die Reaktion von Meigen.* T. M. P. M., XXVIII (1909), 487–488.

[5] T. Harada: *Das Luganer Eruptivgebiet.* Neues Jahrb., B. B., II (1883), 14, footnote.

[6] F. Becke: *Unterscheidung von Quarz und Feldspath in Dünnschliffen mittelst Farbung.* T. M. P. M., X (1889), 90.

Idem: *Unterscheidung von Quarz und Feldspathen mittelst Färbung.* Ibidem, XII (1891), 257.

See also R. Sokol: *Ueber die Methoden einzelne Bestandtheile einer fein körnigen Grundmasse im Dünnschliffe zu unterscheiden.* Centralbl. f. Min., etc., 1911, 276–279.

water upon it. The water was removed by washing with absolute alcohol, after which the slide was laid for a moment in benzol, then wet with a few drops of oil of lavender, and finally covered with a cover-glass over Canada balsam dissolved in ether. By stopping the etching at the proper time it is possible, by this method, to separate the different members of the feldspar group, the calcium feldspars being acted on most vigorously, the soda less so, and the potassium least.

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Idem: *The elements of a new method of chemico-microscopic analysis of rocks and minerals.* Translation of preceding. 19 Ann. Report Geol. and Nat. Hist. Survey of Minnesota for 1890. Minneapolis, 1892, 1–77.

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1883. A. Streng: *Ueber eine neue mikrochemische Reaction auf Natrium.* Ber. oberhess. Gesell., Giessen, XXII (1883), 258–260.

1883. Idem: *Ueber eine Methode zur Isolirung der Mineralien eines Dünnschliffe behufs ihrer mikroskopisch-chemischen Untersuchung.* Ibidem, 260–262.

1884. K. Haushofer: *Mikroskopische Reactionen.* Sitzb. Akad. Wiss. München, XIV (1884), 590–604.

1885. Idem: *Mikroskopische Reactionen. Eine Anleitung zur Erkennung verschiedener Elemente unter dem Mikroskop, als Supplement der qualitativen Analyse.* München, 1885.*

1885. Idem: *Beiträge zur mikroskopisch-chemischen Analyse.* Sitzb. Akad. Wiss. München, XV (1885), 206–226.

1885. Idem: *Erkennung verschiedener Elemente und Verbindungen unter dem Mikroskop.* Braunschweig, 1885.*

1885. H. Behrens: *Sur l'analyse microchimique des minéraux.* Ann. École Polyt. Delft. Leiden, I (1885), 176–212.*

1885. A. Streng: *Mikroskopisch-chemische Bestimmung von Kobalt und Nickel.* Ber. oberhess. Gesell., Giessen, XXIV (1885), 58–59.

1885. Idem: *Ueber eine neue mikroskopisch-chemische Reaction auf Natrium.* Ibidem, 56–58.

1885. Idem: *Ueber einige mikroskopisch-chemische Reactionen.* Ibidem, 54–55.

1885. Idem: *Ueber einige mikroskopisch-chemische Reactionen.* Neues Jahrb., 1885 (I), 21–42.

1886. Idem: Same title, Ibidem, 1886 (I), 49–61.

(1888) Idem: Same title, Ibidem, 1888 (II), 142–150.

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CHAPTER XLI

PREPARATION OF THIN SECTIONS OF ROCKS

512. Early History.[1]—Almost as soon as the microscope was known, attempts were made to study the internal structure of minerals and rocks. The first attempts were made directly upon the minerals themselves or upon chips, as, for example, when Robert Boyle[2] in 1663, examined the inclusions in a diamond to see if he could find anything peculiar in it. Later the rocks were pulverized before microscopical examination, as by an unknown writer in 1774,[3] and subsequently by Dolomieu,[4] de Bellevue,[5] and others. Cordier[6] improved upon the method somewhat by suggesting the preliminary separation or concentration of like minerals by washing or sliming in water.

Shortly after the discovery of polarized light, minerals were studied by its aid, as by Sir David Brewster in 1816 and later, but while plane-parallel plates were used by him and by Biot, no systematic attempts were made to use such for the general study of the different minerals. Polarized light was made more available for the microscope by Nicol's invention, in 1828, of the polarizing prism named after him, and he prepared thin sections of minerals.[7] The first thin sections of fossil woods were thus prepared by him. Later, Witham,[8] in his studies on fossil woods, made use of the same method, rough grinding on a grindstone, rough polishing on a lead plate with coarse emery, and finally on a copper plate with fine emery. Sorby[9] expressed the opinion that Witham did not prepare his own sections but purchased them from Nicol, and also had some one else write his book for him.

[1] See also F. Zirkel: *Die Einführung des Mikroskops in das mineralogisch-geologische Studium.* Decanato Programme, Leipzig, 1881.

[2] Robert Boyle: *Experiments and considerations upon colours with observations on a diamond that shines in the dark.* 1663.*

[3] I. D. in Rozier's *Observations sur la physique*, IV (1774), 225.*

[4] D. Dolomieu: Jour. d. Physique, XLIV, 198.*

[5] Fleuriau de Bellevue: *Mémoire sur les cristaux microscopiques des laves.* Jour. d. Physique, LI (1800), 442.*

[6] P. Cordier: *Sur les substances minérales dites en masse qui entrent dans la composition des roches volcaniques de tous les âges.* Ann. Chim. et Phys., III (1816), 285.

[7] H. C. Sorby: *Preparation of transparent sections of rocks and minerals.* Northern Microsc., II (1882) 101–106, 133–140.

[8] Henry Witham: *Observations on fossil vegetables, accompanied by representations of their internal structure as seen through the microscope.* Edinburgh and London, 1831, 48 pp.*
Review of above in Neues Jahrb., 1833, 456–457.

[9] H. C. Sorby: *Op. cit.*

This, however, was the first published account of the process. Slides of silicified wood were placed upon the market by Andrew Pritchard of London and were evidently quite extensively prepared by him. A more detailed account of the grinding of sections than that by Witham was given by Professer Unger[1] of Grätz, who first sliced his material on a stone-cutter's saw and then ground it down by hand with emery on plates of bell metal or cast iron. His final polishing was done by means of a circular motion on damp cloth, tightly stretched, and covered with tripoli powder. He said the thinness of section necessary for study must be such that fine print can be read through them. He attached his chip to the support with a cement composed of 2 parts gum mastic in grains, 4 parts of white wax, and 1 part of yellow rosin, a cement which he claimed to be better than Canada balsam, water glass, shellac, or any other cementing material, since the chip would not separate from the mount except by heat. The cover-glass was attached by means of Canada balsam.

The first real use made of thin sections was by Sorby[2] in 1850. He speaks of the preparation of sections not much more than 1/1000 of an inch (0.025 mm.) in thickness. If his sections were actually of this thinness they compared very favorably with modern sections, especially since the rocks he described were such, namely calcareous grits, which ordinarily do not permit very thin grinding.

In this, his first paper on the use of thin sections, Sorby gave no description of the methods used, and not much use was made of it by other investigators even in England, since two years later Andrews,[3] in a paper before the British Association, described determinations made on rock splinters by reflected and polarized light with no mention of any knowledge of thin sections.

Sorby[4] continued his method, however, and made continual use of rock sections so that, with right, he may be called the Father of Modern Petrographic Methods.

In the meantime the making of thin sections had been taken up in Germany by Oschatz,[5] probably without knowledge of Sorby's work. In the reports of the meetings of the German Geological Society he is spoken

[1] Professor Unger: *Ueber die Untersuchung fossiler Stämmeholzartiger Gewächse.* Neues Jahrb., 1842, 149–171, especially 153–159.

[2] Henry Clifton Sorby: *On the microscopical structure of the calcareous grit of the Yorkshire coast.* Q. J. G. S., VII (1851), 1–6.

See also Idem: *Op. cit.*, Northern Microsc., II (1882), 101–106.

[3] T. Andrews: *On the microscopic structure of certain basaltic and metamorphic rocks and the occurrence of metallic iron in them.* Rept. British Asso. Adv. Sci., Belfast, 1852. Transact. of the Sections, 34–35.

[4] Henry Clifton Sorby: *On the microscopical structure of crystals indicating the origin of minerals and rocks.* Q. J. G. S., XIV (1858), 453–500, especially 469.

[5] Dr. Oschatz: Reports of meetings of the society. Zeitschr. d. deutsch. geol. Gesell., III (1851), 383; IV (1852), 13; VI (1854), 261–263; VII (1855), 5, 298; VIII (1856), 308.

of, in a number of notices, as exhibiting sections. No methods of preparation are given, although it is mentioned that most of the slides were mounted in Canada balsam. The statement is made that sections as thin as 1/100 of a line (0.0226 mm.) were prepared. Collections of 73 rock sections were offered for sale at 35 Thalers 22 1/2 Sgr. ($25.38) and separate slides from one Thaler (71 cents) to 6 Sgr. (14 cents). Oschatz was the first man to attempt to grind minerals soluble in water, he having prepared a section of carnallite by grinding it under an ethereal oil. According to Zirkel,[1] a collection, prepared by Oschatz, is in the University of Leipzig. The sections are mounted on glass slips 34×21 mm. and are "extraordinarily thin and well ground" but only about 10 to 15 sq. mm. in area.

Slides made by Oschatz were used by many subsequent investigators, and more and more were thin sections used, although not in a systematic way until Zirkel's[2] *Mikroskopischen Gesteinsstudien* appeared in 1863. Zirkel had himself prepared a great number of thin sections in the laboratory of the Geologische Reichsanstalt in Vienna, and had made a systematic study of the material, giving directions for the preparation of thin sections based upon his own experience. The paper was of great importance, as was also a paper by him in 1866[3] in which he speaks of his slides as being 1/2 to 3/4 sq. in. in size. Directions for the preparation of thin sections were given by Vogelsang[4] in 1867; directions which have been copied, more or less word for word, by numerous text-books up to the present time, showing how little change there has been in the method of preparation.

513. Section-cutting Machines.—In preparing thin sections of rocks the writer has found it advantageous first to cut, with a diamond saw, a slice as thin as possible in order that the subsequent work of grinding to the necessary thinness may be reduced to a minimum Certain workers, Forbes,[5] Ady,[6] and others, maintain that it is more economical to use thin chips, doing away with all cutting, simply reducing the chip to proper thinness by grinding. Where this grinding is done by hand, as universally seems to be the custom, the value of the time lost certainly is much more than the maximum of 2/5 of a cent per section (two cuts, each 1 sq. in.) for diamond dust used. With

[1] F. Zirkel: *Op. cit.*, 14, footnote.

[2] Idem: *Mikroskopische Gesteinsstudien.* Sitzb. Akad. Wiss. Wien, XLVII (1863), 226–270.

[3] Idem: *Ueber die mikroskopische Zusammensetzung und Struktur der diessjährigen Laven von Nea-Kammeni bei Santorin.* Neues Jahrb., 1866, 769–787. With plate of thin sections.

[4] H. Vogelsang: *Philosophie der Geologie und mikroskopische Gesteinsstudien.* Bonn, 1867, 225–228.

[5] David Forbes: *On the preparation of rock sections for microscopic examination.* Monthly Microsc. Jour., I (1869), 240–242.

[6] John Ernest Ady: *Observations on the preparation of mineral and rock sections for the microscope.* Mineralog. Mag., VI (1885), 127–133.

the mechanical grinder suggested below, it may be possible to equal this low cost, but certainly the time required to examine the section during the process would be worth as much as the bort.

There is not a great variety in cutting machines, three types having been made, two of which are still in use. One of the earliest instruments was made by Rumpf.[1] It was based on the saws used by stone cutters and consisted of a foot-power machine with a horizontal, hack-saw-like arrangement which was drawn across the rock. The blade, however, had no teeth, was made of soft tin plate stretched taut, and was fed with emery and water, the specimen being held against the blade by means of a weight. Slices as thin as 1/2 mm. and with parallel faces could be cut from a homogeneous rock. The speed with which a specimen was cut depended upon the kind of rock, 100 sq. cm. of limestone being sawn in from three-fourths to three hours, the same amount of granite in three to five hours, and porfido rosso antico in twelve hours.

Another type of saw, and one still in use for cutting large slabs, is also based on a stone cutter's instrument. It consists of an endless wire of soft iron or brass, 0.5 to 0.7 mm. in diameter, running over two wheels. The lower wheel is attached to the motive power, the upper to a weight which serves to keep the wire taut. The specimen is placed on the sawing table and so arranged that a weight will draw it forward mechanically as required. The cutting material, emery or carborundum, should be fed automatically to the saw, as should also enough water to keep it moist. Such a machine may be set working and left, with only occasional inspection, until the cut is completed. The speed, in a granite slab 1 in. thick, is approximately 1 1/2 in. an hour. Slices cut with such a machine do not have perfectly plane faces but show the striations made by the saw and require considerable after grinding. It is not safe to attempt slices too thin, for the cuts are likely to run together. A preliminary kerf with a diamond saw is a great aid in starting the cut. When large blocks of soft or medium hard rocks are to be cut, the instrument may be used to advantage, but it is questionable if there is any gain over the diamond saw with small specimens or with hard rocks, not only on account of its wastefulness of material, for it cuts a wide path for itself, but on account of the time required.

The third type of saw has a disk-shaped blade revolving on a spindle. It may be either vertical or horizontal and be fed with carborundum or have its edge set with diamond chips or diamond dust.

While diamond dust was long used in the commercial cutting of certain stones, its use in the laboratory for cutting rock sections appears to have been described first by Lehmann,[2] who used saws made of tin with notched edges

[1] J. Rumpf: *Eine Cabinets-Steinschneide-Maschine.* T. M. P. M., IV (1882), 409–414.

[2] J. Lehmann: *Einige auf das Durchschneiden von Gesteinsstücken und die Herstellung von Mineral- und Gesteinsdünnschliffen bezügliche Erfahrungen.* Verh. naturhist. Ver. preuss. Rheinl., Bonn., XXXVII (1880), Sitzb. 228–231.

set with diamond splinters. About the same time Cohen constructed, at the University of Strassburg, a rather simple machine for such section cutting but does not appear to have described it. A modified form was made and described, in 1882, by Steinmann,[1] also of the University of Strassburg. This instrument (Fig. 746) is very similar to modern instruments and, with the exception of more delicate adjustments on some of them for accurate orientation of crystals, serves as a model for modern makers. The instrument is worked by foot power and is about the size of a large sewing machine. The top (M) is made of wood, covered by a sheet of zinc (T) which slopes to the left-hand rear corner so that all water spilled upon it will drain, through the tube a, to a pan set on the floor. P is a cast-iron plate to which are attached the tracks SS and S_1, carrying the guiding apparatus parallel to the cutting saws s and s_1. S_1 may be displaced or removed by the screws μ and μ_1. The carrier at the left may be shoved forward by the plate B, and the specimen inclined at any angle by means of the plates C and D. D has, at the back, a slit for the entrance of the saw. Attached to F is a horizontal plate with a V-shaped cut-out, likewise for the entrance of the saw when the plate FF is inclined in azimuth by means of the screws δ and ϵ. At the right, the sledge B_1 carries the bar J_1 to which the cylinder N_1 is attached. In the latter is the T-shaped bar C_1D_1. The bar D_1 is hollow and carries the plate F_1, which may be shoved forward by means of the screw γ and clamped by ζ. To the axle A are attached the two saws s and s_1 which are turned by means of the pulley R. The bottle G contains the petroleum which is used as a lubricant and is conducted to the saws by means of the lead tubes r and r_1, which may readily be bent to proper positions. The tin shields H–H, whose front halves H_1–H_1 may be thrown back, serve as mud guards, and are provided with windows f and f_1 through which the process of cutting may be observed.

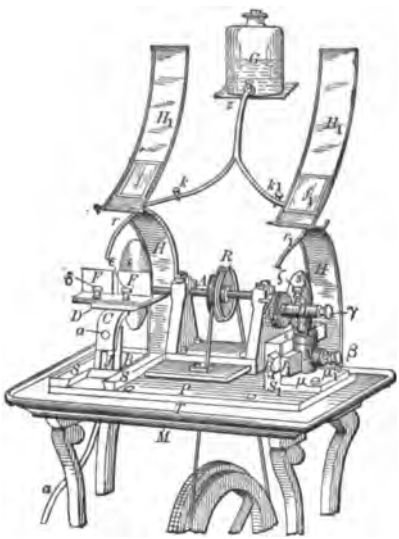

FIG. 746.—Steinmann's section cutting machine.

The saws of tin plate (tinned iron) used by Steinmann were from 10 to 22 cm. in diameter. The central holes were made a trifle smaller than the counter shaft upon which they were to be placed, and were enlarged by means of a

[1] Gustav Steinmann: *Eine verbesserte Steinschneidemaschine.* Neues Jahrb., 1882 (II), 46–54.

rat-tail file until they fitted exactly. After being clamped to the shaft, they were rotated against a hard substance, such as the flat edge of a smoothly ground triangular file, until no more shavings were removed, and they were, consequently, perfectly true and had edges at right angles to the sides. The edge was now nicked and charged with diamond dust.[1]

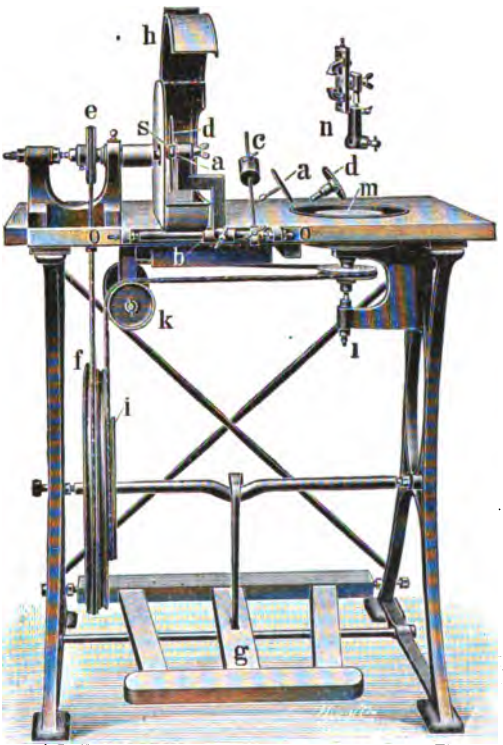

FIG. 747.—Section cutting and grinding apparatus. (Dr. Steeg and Reuter.)

The rock to be cut is attached to the plate by a prepared cement made of 1 part beeswax and 1 part yellow rosin. For small sections the s_1 saw is used, the specimen being fastened to F_1. If the chip has no flat surface, it is set in wax and further supported by burnt matches stuck into wax-filled holes in the plate F_1. These small wooden supports are cut by the saw as it passes through the rock. Should the plate S_1 be not quite vertical, it may be adjusted by placing thin sheets of paper or card-board under one side or the other of its base. To test whether the saw is perfectly vertical, saw a thick piece of a soft rock, such as a homogeneous limestone, fasten the newly sawed face of the piece removed to the plate, and make a new cut. The

[1] See Art. 514, *infra*.

difference in the thickness of top and bottom will represent the departure from parallelism between saw and holder plate. To test whether the plate F_1 is parallel to the saw in azimuth, measure the front and back of the soft rock just cut. The thickness should be the same. If not, correct it, and then cut, with a file, a scratch across C_1 and N_1 to mark this position. For mineralogical work it is advantageous if the top of the cylinder N_1 is marked in 2° divisions. The instrument was made complete, by Carl Benz in Mannheim, for 275 Marks.

Made somewhat on the same principle, although with but a single saw, are the cutting machine first described by Groth[1] in 1885, and the one

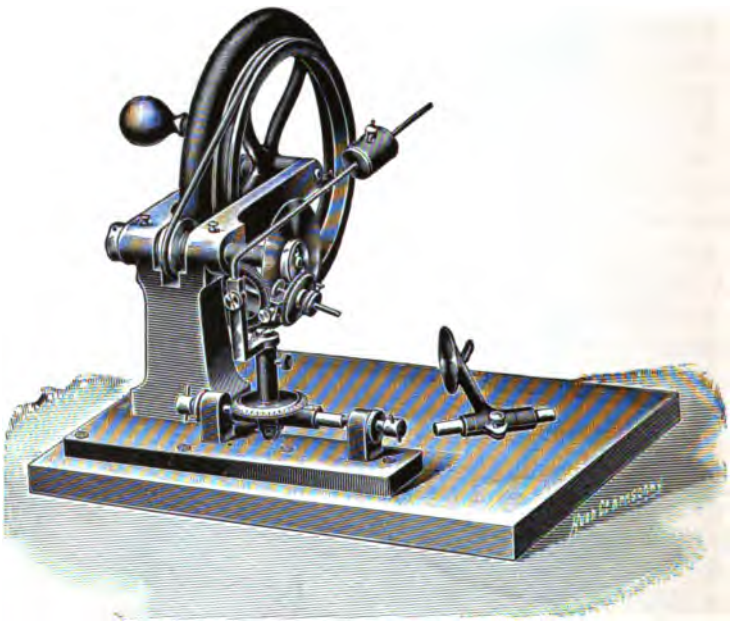

FIG. 748.—Hand section cutting machine. (Dr. Steeg and Reuter.)

shown in Fig. 747. Both machines are made for combined cutting and grinding. In the one shown in the illustration, the specimen to be cut is cemented to the plate *a*, which may be moved along the rod *b* for a considerable distance. At the end of this rod are two screws (*O*) for fine adjustment, permitting the cutting of a section of any thickness. The weight *c* may be removed up or down the rod to which it is attached, thus changing the pressure against the saw *d*, which rotates from the top downward. For large specimens the clamp *n* is used instead of the disks *a* and *d*. Should the saws become eccentric they may be trued up with a turning chisel, using the object carrier as a tool rest.

[1] P. Groth: *Physikalische Krystallographie*. Leipzig, 2 Aufl., 1885, 670.

Another instrument described by Groth[1] was a small hand apparatus, similar to the one shown in Fig. 748 except that the power was transmitted by geared wheels instead of a belt. The orienting device shown at the side is extremely useful in cutting sections along certain definite directions. A similar device is described by Fuess.[2]

The cutting machine shown in Fig. 749 has one decided advantage over those previously described in its extremely rigid holder (K–S) for the specimen. This carrier is mounted on a rod and may be moved very accurately to any required distance, by means of the screw-head s, in a direction at right angles to the plane of the saw. The latter has a diameter of 6 1/2 to 7 in. (16–18 cm.) and should be rotated at a speed of about 400 revolutions per minute, the power being applied either by foot or motor. The special recommendation for this instrument is its compactness, the freedom of the saw from vibration, and the rigidity of the specimen carrier. If the saw becomes eccentric, as it is likely to do when carborundum and not diamond dust is used with it, it may be trued up by swinging out the carrier K and using it as a rest for a turning tool. In a sawing machine described by Rauff[3] a turning tool could be clamped in the specimen carrier and advanced by means of a cranked screw similar to that in a machine lathe.

FIG. 749.—Section cutting machine. (Fuess.)

The cutting apparatus described by Grayson[4] differs from those previously described in a number of particulars. The ordinary machines run

[1] P. Groth: *Op. cit.*, 668.

[2] R. Fuess: *Ueber eine Orientirungsvorrichtung zum Schneiden und Schleifen von Mineralien nach bestimten Richtungen.* Zeitschr. f. Instrum., Oct. 4, 1899, 4 pp. separate.

[3] H. Rauff: *Ueber eine verbesserte Steinschneidemaschine, sowie über einen von M. Wolz in Bonn construirten damit verbundenen Schleif-Apparat zur Herstellung genau orientirter Krystalplatten.* Neues Jahrb., 1888 (II), 230–246.

Idem: Same title in Verhandl. d. naturhist. Vereins. d. preuss. Rheinlande u. Westfalens. 1886, 130–139.*

[4] H. J. Grayson: *Modern improvements in rock-section cutting apparatus.* Proc. Roy. Soc. Victoria, XXIII, Pt. I (1910), 65–81.

best at about 500 revolution per minute with disks 8 in. or less in diameter, while Grayson's, with a disk 10 in. in diameter, is speeded to 1000 revolutions. Instead of being vertical, the cutting disk is horizontal, and is clamped between two collars. With his instrument he claims to be able to slice, grind, and mount a granite section an inch in diameter, in not more than ten minutes. The cost of charging a 10-in. disk with diamond dust is one shilling, and with it he is able to cut, without recharging, about 95 sq. in. of average rock, making the cost about 1/4 cent per square inch.

514. Diamond Saws.—Diamond saws are of three kinds, those set with diamond chips, those charged with diamond dust directly upon the smooth edge of a disk, and those charged in notches. The first kind is made by setting the chips in the edge of a tin disk much in the manner of the setting of writing diamonds, namely by gouging out a hole, inserting the chip, and burnishing down the burr. The saws are expensive, costing, at the present price of bort, about $9.50 for a 7-in. saw, and $13.50 for one 10 in. in diameter. For the purpose of cutting rock slices they are not nearly so satisfactory as disks charged with diamond dust. The makers claim that with proper usage a 7-in. saw will cut about 470 sq. in. of rock of average hardness, making the cost about 2 cents per square inch. As ordinarily used they cut decidedly less. Aside from the expense, the fact that if slightly eccentric they cannot be turned true, is a serious objection to them.

The second class of saws are those charged directly upon the edge of a smooth disk of ordinary tin plate, about 0.50 to 0.75 mm. in thickness. The disk should be from 6 to 10 in. in diameter; a 7-in. saw being quite satisfactory for ordinary work. The smaller the saw, the more rigid it is, consequently the truer it will run. The large saws are chiefly of use in cutting slabs from large blocks. The depth of cut which can be made will depend upon the size of the counter-shaft and the nut and bearing plates holding the saw. Usually the cut will be about one-third the diameter of the disk. By reversing the specimen, a piece somewhat less than two-thirds the diameter of the saw may be cut.

A saw should run perfectly true and with no eccentricity. The central hole should be a trifle smaller than the counter-shaft, and the disk should be fitted as described by Steinmann.[1] If there is any eccentricity, it should be removed by turning off the edge by an ordinary metal-turning chisel, using the rock holder as a tool rest, or even clamping the chisel in, if the holder is suitable for so doing. The edge of the saw should be exactly at right angles to the sides, otherwise it will cut in at an angle and soon bind.

To charge the disk the following method is recommended by Leiss.[2] In a fragment of quartz or flint, a cut, 5 to 10 mm. in depth, is made, and in it is placed a very small quantity of diamond dust moistened with petroleum.

[1] Art. 513.

[2] C. Leiss: *Die optischen Instrumente, etc.* Leipzig, 1899, 274.

The saw kerf is slipped over the edge of the tin disk and is pressed hard against it. The belt or pulley which turns the spindle is now rotated by short forward and backward motions until the entire disk has passed through the quartz. By this means the diamond dust is forced into the rim and the adjacent sides of the tin.

Another method of charging the saw is to apply a thin paste of diamond dust and olive oil to the edge of the rapidly rotating disk by means of a very small, spatula-shaped piece of wood, and at the same time hold, with the other hand, a smooth piece of quartz or agate, moistened with petroleum, against the running edge. The whole process of charging requires but a few moments and but a very small quantity of diamond dust is used.

The objection made to smooth-edge saws has usually been that when rocks which are composed of minerals of different hardness are cut, the diamond cutting-edge is quickly lost. Grayson[1] seems to have found them satisfactory for general use. For perfectly homogeneous material, such as agate, the saw is excellent and will cut from 25 to 35 sq. in. at a cost of about one-fourth of a cent per square inch. When the saw becomes dull it may be recharged in the same manner as before. If it becomes eccentric it is simply necessary to turn it true, and recharge.

The third type of saw is the tin disk with nicked edge. Such a saw was first used for petrographical sections by Steinmann[2] in 1882. He used disks of from 10 to 22 cm. in diameter, and with a knife hacked the edges in a tangential rather than a radial direction, thus making notches, rip-saw-like, close together all around the rim. The saw is so placed on the counter-shaft that the notches point downward in front, thus forcing the diamond dust deeper as the saw rotates from the top forward. The saw is charged by mixing diamond dust with kerosene or other oil to form a thick paste, and placing it on the edge of the saw at intervals of one-sixth to one-eighth the circumference. As little as possible should be placed on the sides for there the material is practically wasted. The pulley should now be rotated by hand while a smooth piece of quartz is pressed against the edge of the disk to force the diamond dust into the soft iron. More dust is now placed at intervals around the edge and the rotation repeated, and thus the process is continued until the entire disk has been charged. If any of the paste adheres to the side of the saw it may be recovered by washing it into a beaker by means of oil from a small wash bottle; the diamond dust soon settling to the bottom of the beaker. When a saw is dull it may be recharged as before. If eccentric it may be turned circular, rehacked, and recharged.

Recent use seems to indicate that radial nicks are more efficient than tangential. After truing up a saw, hack its edge, by means of an old, rather heavy, thin-bladed case knife, with a force sufficient to make incisions about

[1] H. J. Grayson: *Op. cit.*, p. 76.

[2] Gustav Steinmann: *Op. cit.*

a millimeter in depth. Rotate the disk by hand until the entire edge is covered with nicks about a millimeter apart. Even distribution in the incisions is not at all essential although it makes a better looking saw. The edge may be charged with diamond dust in the manner described above, or it may be rubbed into the edge with the finger, or the disk may be removed from the counter-shaft and the edge placed in a little heap of the oil and diamond dust placed on a flat iron plate and the upper edge gently hammered while rotating the disk. To be certain that it is completely charged, it should be thus rotated two or three times through the powder. After charging the edge, a smooth piece of quartz or a chilled steel roller is held against the edge, very gently at first, to force the chips deeper into the iron.

The diamond used, if possible, should be purchased as coarse bort and not in powdered form. It should be crushed in a steel diamond mortar and sifted during the process so that the material obtained is of quite uniform size. Grayson[1] made sieves out of inch-long sections of 3/4-in. glass tubing, to one end of which, after grinding level, a piece of very fine bolting silk was cemented. One-forth of a karat (1/20 grm.) of dust is sufficient to charge a half dozen 6-in. saws, and costs about 35 to 40 cents, making the cost per saw, including tin, less than ten cents.

Saws should run in perfectly true planes. They may be tested for adjustment as described in Art. 513. Upon this, and upon lack of eccentricity, much of the efficiency depends. They should not be used as long as they will cut, but should be recharged whenever they begin to be dull. One can make his own disks by cutting, with a tin shears, an approximately circular disk from a piece of perfectly flat tin, and then, placing it on the spindle, truing it up with a turning tool. It is well to cut and charge a number of saws at the same time, so that they may be on hand when wanted.

A saw should run from above downward, and during the process of cutting it should be kept constantly wet with a lubricant of some kind. Kerosene, sweet oil, water, sodium carbonate and water, and soap emulsion have all been recommended. The first is probably best for compact rocks and the last for those which are soft or porous. Sweet oil is likely to become sticky and is hard to remove from the specimen. Some sort of reservoir placed above the cutting bench, and with a dropping tube conducting to the upper edge of the saw, is most convenient. It should be so adjusted that two or three drops fall per second, the right amount depending somewhat upon the character of the rock, but is easy to determine since it must be great enough to keep the saw wet and not enough to spatter a great deal. A good suggestion, made by Grayson, is to attach two bits of sponge beneath the drip and so arranged that one piece touches either side.

The size of the saw to be used depends upon the nature of the work and the size of the specimen. The smaller the saw, the truer will it run. If a

[1] H. J. Grayson: *Op. cit.*, p. 75.

large saw is used, it is well to make a preliminary cut, 5 to 10 mm. deep, with a small one, and start the larger saw in the kerf. All through the operation, but especially at the beginning, the specimen should be but lightly pressed against the saw. Only a certain amount of material can be removed by each diamond chip, and pressure does not make this any greater. The number of rotations depends upon the size of the saw. Usually 400 to 600 revolutions are considered sufficient for a 7-in. saw, and less for those that are larger, although Grayson recommends a speed of 1000 revolutions for a 10-in. saw. The higher the speed, the more true must the saw run.

The rapidity with which a saw will cut a section depends entirely upon the rock. With a saw in fair condition, a square inch of chalcedony should be cut in about three minutes, granite in two, and syenite in one.

The life of a 7-in. notched saw is from 50 to 60 sq. in. of average rock. Very soft rocks should not be cut with the diamond saw, but with a circular disk fed with carborundum. Rocks which consist of minerals of very different degrees of hardness should be moved forward very slowly.

515. Sawing a Rock Slice.—The method of sawing a rock slice has not altered very much since the first one was cut by Trautz, as described by Steinmann.[1] If one has brought chips as well as hand specimens from the field; the former may be taken, ground to a plane surface on one side with carborundum upon a lap, and cemented to the receiving plate (*a*, Fig. 747, *K*, Fig. 749). If a number of sections are to be made, three or four chips from rocks of approximately the same degree of hardness may be cemented to the plate at the same time. If one grinds his own sections, he will soon learn to bring from the field chips which are thin, nearly plane on one side, free from cracks, and about 1 1/2 in. in diameter.

Different workers prefer different cements. One of the most common is Canada balsam, which, however, requires previous boiling.[2] Steinmann's cement of wax and rosin is very good when properly prepared. If too soft the section will glide in it, if too hard the cement will fracture. An excellent cement is common shellac, or half shellac and half Canada balsam. This should be used with a heated plate and a heated specimen. Like chips mounted in balsam alone, they may be removed by heat or by placing them for several hours in alcohol. If no chip was brought from the field or if they are too large or irregular to be readily ground to a flat surface on one side, or if it is desired to make a section transversely across a schistose rock, it will be necessary to make a preliminary saw cut by clamping the specimen in the holder provided and sawing off the desired amount. Usually such pieces will be found to have surfaces flat enough to be cemented directly to the holder plate, although it may be a wise precaution to give them a few rubs on the lap to assure a flat surface. If this face is

[1] Gustav Steinmann: *Op. cit.*

[2] See Art. 519.

to be the side attached to the permanent mount, it must, of course, be carefully ground.

After cementing the chips to the disk, the holder is screwed up until the saw is at the desired distance from the plate. The thinness to which a rock may safely be sawed depends upon its character. Compact homogeneous rocks may be made much thinner than those which are porous, 0.5 to 1.0 mm. being an average cut. If a piece of tin of the proper thickness is laid against the face of the holder plate adjacent to the mineral chips, the holder may be pushed forward until the two rest against the saw. Leaving the tin in position when the saw is started, the slide will be of the proper thickness. Instead of a tin strip the writer recommends a screw with a flat end and extending through the holder plate near the rear edge. This may be made to project the proper distance and serve as a guide. Care must be used to let the screw touch the side of the saw and not let the diamond-charged edge slice off the end.

Toward the end of a cut, the pressure against the saw should be decreased, although the speed of rotation should remain the same. The hand should be held against the fragment being cut off, otherwise its weight may tear away a piece of the specimen.

If, at any time during the cutting, the rock tears loose from the mount, refasten it immediately by heating. Upon starting up the saw, begin anew from the opposite side and meet the former cut, but do not proceed in the old kerf.

Sometimes a saw will bind. This is due to a deflection from the true plane of the saw, and may be caused by having pressed forward too rapidly during the process, by a lack of lubricant, or by a false start. Remove the saw and make a fresh cut to meet the old. If there are a few very hard minerals, such as garnets, scattered through the rock, the pressure which forces the saw forward should be decreased, otherwise upon meeting a face of the hard mineral at a very small angle, the saw will be forced aside and bind.

Having cut the rock, the slice may be removed from the iron supporting plate by heating. Instead of mounting the chips directly on the iron supporting plate, it is sometimes desirable to mount them upon a glass support which later serves as a holder for grinding. This is usually desirable when the rocks are too brittle to support themselves in thin slices. In such cases, after the preliminary cut, the flat side is ground with particular care to a flat surface as described below, and mounted on 1 1/4-in. squares of plate glass, or even on ordinary object-glasses. The lower sides of these glass slips are now fastened to the iron supporting plate with beeswax, or a beeswax and rosin cement, the former being the easier to remove, and the cutting is done as before. With whatever cement the glass slip is attached to the plate, it should have a lower melting-point than the Canada balsam with which the slice is mounted to the object-glass, so that it will loosen first.

516. Grinding a Section.—Before making a saw cut it is sometimes necessary, as was mentioned above, to grind upon the chip one perfectly plane surface. This may be done by hand on a metal or glass plate covered with wet emery or carborundum, or it may be done upon a rotating lap. The old method of grinding by hand is still employed by some men in preference to using a machine, but it requires more time and certainly need not be employed until the final stage of grinding the second face is reached.

For grinding it is necessary to have several grades of emery or carborundum. The writer uses No. 120 carborundum for coarse grinding, follows with No. 180, and concludes with FFFF emery. For the very last grinding it is better to use old emery rather than that which is fresh and unused. Extreme care must be taken not to mix the coarse with the fine, a single grain of coarse carborundum on the final plate may cut a slide to pieces in an instant.

For hand grinding, three plates of metal or glass should be used. Ady[1] recommends plates of soft metal such as pewter, zinc, copper, or lap-metal, each about 12 in. square and 1/4 in. thick, and says that glass plates are "not to be tolerated" since they rapidly wear down irregularly. He says further that the plates should be raised slightly in the center so that the slide, from the natural greater pressure on the edges, will not be thicker in the center. By heating a metal plate in the center, by means of the Bunsen burner, it will buckle by expansion and give enough curvature. It should then be set in a wooden frame on a base of plaster of Paris. Zirkel[2] and Rosenbusch[3] recommend a plate of cast iron for the rough grinding, and one of ground glass for the fine. The writer has used pieces of plate glass for coarse and fine, and has had no difficulty with their grinding away unevenly, but when used by students they do so rapidly. A little care is necessary and all parts of the plate should be used. Nor is there any need of a plate with a raised center when one has had a little experience.

Very little grinding is necessary before moun ing if the chip has been sawed and is to be mounted on the iron support of the cutting machine, or if it is to be mounted on a piece of thick glass for the first grinding and is to be attached to the final object slip by the second sawed face. If the final grinding is done on the heavy glass and the slice is transferred by floating to the new glass, the first face, of course, must be ground with fine emery.

Upon one plate is placed a pinch of No. 120 carborundum which is wet, and kept wet, with considerable water; more water or more carborundum being added as either seems to be needed. Now, with a circular motion,

[1] John Ernest Ady: *Observations on the preparation of mineral and rock sections for the microscope.* Mineralog. Mag., VI (1885), 125–133.

[2] F. Zirkel: *Lehrbuch der Petrographie.* I, Leipzig, 1893, 21.

[3] Rosenbusch-Wülfing: *Mikroskopische Physiographie*, I–1, Stuttgart, 1904, 108.

first in one quarter of the plate and then in another, and with occasional sweeps over its entire surface, the slide is rubbed until it has become perfectly flat. If a chip and not a sawed face was used to begin with, grind until there appears a flat surface an inch in diameter without depressions. Wash well in water, using an old tooth brush to remove all grains of the coarse emery, and begin grinding in the same way on the second plate upon which is placed water and a pinch of No. 180 carborundum. Grind until no scratches remain, and be sure, by applying equal pressure over the entire surface while grinding, that the face is perfectly flat.

Again wash the slice carefully with water, and transfer it to the third plate upon which is placed some water and a little FFFF emery. Here grind till no traces of marks from the second emery remain. It is not necessary to polish the surface, the process being more than likely to spoil the flatness of the face.

Wash again, and mount the face just prepared in Canada balsam upon the final object-slip, and at the same time, while still warm, slip off the heavy glass base, if such was used. Press down firmly until the balsam is cool, and then repeat the process of grinding upon the other face. The most delicate part of the operation is the final grinding after the slide has reached almost its proper thinness. No amount of written instructions can teach the proper time to change from the second grade of carborundum to the emery, nor when new emery may no longer be safely added. This must be learned by experience. Do not expect to make a section at the first, or second, or third, trial. After five or ten you may succeed and, thereafter, the knack of keeping the slide absolutely level on the grinding plate having been obtained, the work is easy.

When a slice of a dark rock is thin enough to permit fine print to be read through it, it should be covered with a drop of water and the interference colors examined under the microscope to determine its thickness. Any old microscope, abandoned in the laboratory as out of date, or a preparation microscope (Figs. 723 and 724) may be used. Rocks which contain considerable quartz, such as granite, will be transparent long before they are of sufficient thinness. Print may first be read through the quartz, then through the orthoclase, then through the albite, and finally, perhaps, through the ferromagnesian minerals. After grinding a number of sections the proper degree of thinness will be learned by noting how the slide is acting in regard to breaking away at the edges. A good section should be between 0.025 and 0.040 mm. in thickness so that quartz will show an interference color no higher than yellow. Be very careful not to grind too long on the No. 180 carborundum before transferring to the emery, for you may have a square inch of rock one moment and the next, with but a single sweep across the plate, it has disappeared. It is better, toward the end, to place the slide often under the microscope and examine the interference colors.

If the microscopic examination shows that the section is thicker at one side than at the other, correct this by exerting greater pressure on the former. Sometimes, when a slice begins to break at one side, a drop of balsam, placed on the object-glass alongside the slice and allowed to harden, may protect it. Smith[1] recommends grinding the chip first in the form of a circular disk. Having no irregular edge, it will not so readily break away. In making the mount be sure that no bubbles lie between the slice and the glass, for above each bubble the rock is sure to break away.

Usually old emery is fine enough for the last grinding, but Ady[2] and other writers recommend the use of hone stones. The writer has rarely found this necessary.

If one is very finical and objects to having the object-slip scratched at the four corners, he may use as a protection, four bits of cover-glass or zinc cemented on and afterward removed, as was suggested by Forbes.[3] Bornemann[4] says three bits of glass are better than four.

Instead of grinding the section by moving the chip on a stationary plate, some form of grinding machine with a revolving lap may be used. The procedure is essentially the same as that described for hand grinding except that, since the lap revolves, it is not necessary to move the slide about so much. The section is first ground with coarse carborundum and water. At the proper time this is carefully removed from both lap and rock chip, finer carborundum is substituted, and the grinding renewed; this also is carefully removed at the proper time, and fine emery powder substituted. Instead of cleaning the carborundum from the lap each time, it is more economical in material and time to use three adjacent laps, a method also advisable on account of the likelihood of getting a grain of coarse carborundum mixed with the fine in the first method, with disastrous results.

In using a lap one has a certain latitude in rapidity of grinding due to the difference in velocity at the center and periphery of the wheel. In no case should the upper end of the spindle project through the surface of the lap, for by so doing it makes useless a large portion of the most useful grinding surface.

During the process of grinding, the chip, whether mounted or unmounted, should be so held between the thumb and the first three fingers, or by three fingers alone, that the finger nails are not ground down to the quick. For inspection, the slide may be removed from the quickly revolving wheel by slipping it toward the edge and passing the thumb beneath it. Toward the close of the grinding process this should be done quickly in order to avoid

[1] John Smith: *A method of making and mounting transparent rock sections for microscopic slides.* Jour. Postal Microsc. Soc., II (1883), 28–33.

[2] *Op. cit.*

[3] David Forbes: *Op. cit.*

[4] See Art. 517.

too much cutting away, although there is less danger of this than might be supposed, since the pressure being removed, the cutting action of the emery is much less. If desired, the final rubbing down may be done by hand on a glass plate. As in hand grinding, experience is necessary, and numerous slides will be spoiled before dexterity is obtained. This does not mean, however, that the first half dozen slides, which must necessarily be spoiled, should be carelessly ground. Each section, even from the beginning, must be treated as though no more of the rock material were to be obtained.[1]

Surrounding each lap there should be a guard to catch the water and abrasive thrown off. It is not necessary constantly to take fresh material, but that which has already been used may be taken up. It pays, occasionally, to clean out the box, wash the material free from dust, dry and sift to remove rock-fragments, and use again until too dull to cut. It will be found that while carborundum cuts much more rapidly than emery, it also becomes dull much more quickly, probably due to its brittleness and its consequent more rapid reduction to powder.

517. Various Grinding Machines.—Originally thin sections were made entirely by hand, or with preliminary grinding on an ordinary grindstone until the rock slices were approximately 1 mm. in thickness, after which they were completed by hand. Vogelsang[2] used a small emery wheel, constructed for the purpose, but suggested, if one had no stone at his disposal, that chips be turned over to a knife grinder for rough grinding.

One of the earliest machines made especially for section grinding was that described by Sellers.[3]

Another machine, and one of a pattern which has not been copied subsequently, was that made by J. G and L. G. Bornemann.[4] It followed the method of hand grinding more closely than other machines in that the grinding plate was stationary and a contrivance above moved the mineral chips. The grinding plate consisted of an ordinary iron griddle, 10 or 11 in. in di-

[1] Every student in petrography should be able to prepare his own thin sections. While under ordinary circumstances he may send his chips away to be ground, he may, some day, be called upon to prepare his own for immediate use.

Thin sections are prepared, from material sent in, by the following firms:

America,

W. Harold Tomlinson, Swarthmore, Penn.

Germany,

Voigt & Hochgesang, Untere Maschstrasse 26, Göttingen.

F. Krantz, Bonn.

[2] H. Vogelsang: *Philosophie der Geologie.* Bonn, 1867, 225–226.

[3] C. Sellers: *Beschreibung einer Maschine zur Herstellung dünner Schliffe von harten Substanzen für mikroskopische Zwecke.* Zeitschr. f. gesammten Naturwiss., N. F., II (1870), 417–419.

[4] J. G. and L. G. Bornemann jun.: *Ueber eine Schleifmaschine zur Herstellung mikroskopischer Gesteinsdünnschliffe.* Zeitschr. d. deutsch. geol. Gesell., XXV (1873), 367–373.

ameter. It was placed on a table and above it was erected a support for a vertical spindle, to the center of which was attached a horizontal pulley and to the lower part a horizontal wooden cross. The four arms of this cross were pierced with holes into which bent wire drags were placed. The chips, usually to the number of six or eight, and of approximately equal hardness, were mounted on small glass plates by means of pure beeswax, this material having been chosen in preference to Canada balsam on account of its ready fusibility. If it was necessary to grind rocks of unequal hardness at the same time, those which were harder were placed near the periphery of the plate where the speed of the drag was greater. To the center of the upper sides of the glasses carrying the chips, posts 0.5 to 1.0 cm. in height were cemented with sealing-wax, and over them were placed short upright sections of close fitting lead pipe. The latter served as weights, and to them the drag wires were attached. As the chips became thinner, the lead weights were exchanged for others not so heavy. Usually it was found advisable to allow the arms of the cross to drag the chips which were heavily loaded and push those which were not. The grinding was done by means of emery and water, the change from coarse to fine being made by simply changing the griddle. If a specimen was to be polished, the iron plate was replaced by one of glass covered with calf- or buckskin upon which tripoli powder and water were placed. Upon the completion of the first face, the chips were reversed and ground on the other side until the slices were as thin as "strong paper." They were then mounted on object-slips whose corners were protected by three fragments of cover-glasses, not four as suggested by Forbes, and the grinding was completed with fine emery, the chip being frequently removed and examined.

The ordinary type of machine used at the present time has a horizontal lap, without rock holders, and is fed with carborundum or emery and water. In form it is similar to all of the earlier instruments[1] and no particular improvement has been made. The table upon which the instrument works should be of a height so that it does not tire one's back or arm when working,

[1] J. Lehmann: *Einige auf das Durchschneiden von Gesteinsstücken und die Herstellung von Mineral- und Gesteinsdünnschliffen bezügliche Enfahrungen.* Verh. naturhist. Ver. preuss. Rheinl., Bonn. XXXVII (1880), Sitzb., 228–231.

H. C. Beasley: *On the preparation of rocks for microscopic examination.* Trans. Liverpool Geol. Asso., III (1883), 141–147.

P. Groth: *Physikalische Krystallographie.* Leipzig, 1885, 667–674.

H. Rosenbusch: *Mikroskopische Physiographie.* Stuttgart, 1885, 6–14.

John Ernest Ady: *Op. cit.*, 1885.

K. J. V. Steenstrup: *En formentlig Forbedring ved de saedvanlige Slibemaskiner.* Geol. Fören. i Stockholm Förh., X (1888), 114–115.

C. H. Caffyn: *A rock-grinding machine for amateurs.* Knowledge, XXXIV (1911), 30–31. Describes a "home made" cutting and grinding machine made from an old sewing-machine stand.

and it should have a top of reasonable size. Williams[1] used a table 3 1/2 ft. square and 2 ft. 9 in. in height. If one prefers to stand while working, the table should be about 40 in. high.

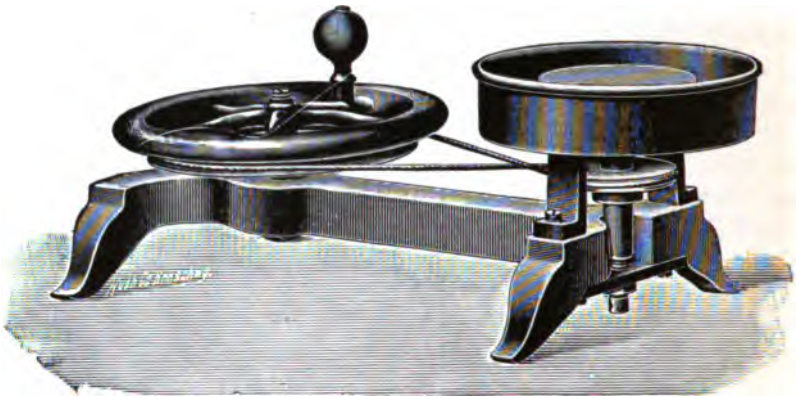

FIG. 750.—Hand section grinding machine. (Dr. Steeg and Reuter.)

Among modern grinding machines are those shown in Figs. 750 to 753. The first is a small apparatus with a horizontal plate, cast iron for coarse grinding and glass for fine, revolving on ball bearings. An instrument almost exactly like this is described by Leiss.[2]

FIG. 751.—Foot power lapidary's lathe. 1/15 natural size. (Fuess.)

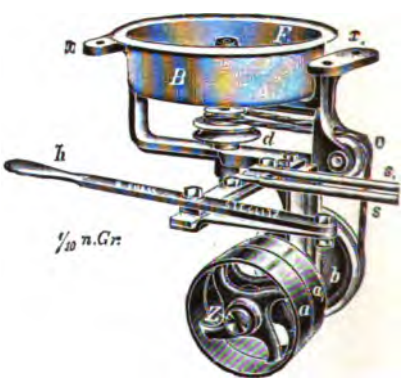

FIG. 752.—Motor lap. 1/10 natural size. (Fuess.)

[1] George H. Williams: *A new machine for cutting and grinding thin sections of rocks and minerals.* Amer. Jour. Sci., XLV (1893), 102–104.

[2] C. Leiss: *Die optischen Instrumente.* Leipzig, 1899, 275–276.

A foot-power lapidary's lathe is shown in Fig. 751. It consists of an iron stand with a rectangular wooden top into which is set an enameled iron grinding basin (*B*). The horizontal drive wheel (*R*) is set in ball bearings and is rotated by double treadles (t and t_1). Grinding disks of iron, 12 to 13 cm. (4 1/2 to 5 in.) in diameter, and a disk to which a piece of plate glass for fine grinding may be attached, are provided with the instrument.

Similar to the enameled basin of Fig. 751 is that shown in Fig. 752, but the machine is motor driven, the belt wheel (a_1) being attached to the spindle which operates the lap and carries the loose throw-off wheel (a). The machine is to be attached to the lower side of the work bench by means

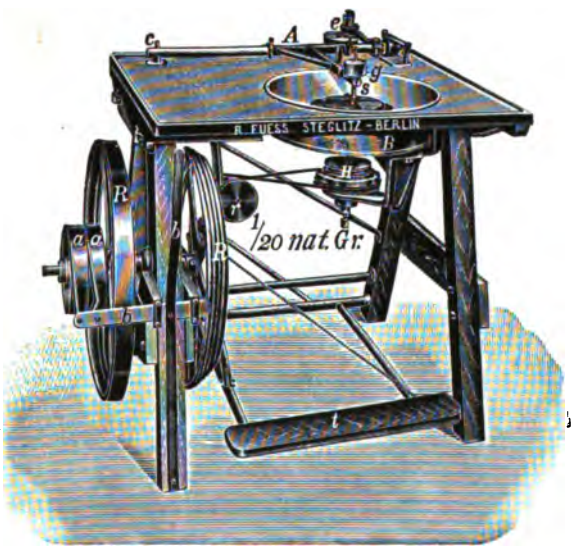

FIG. 753.—Large automatic grinding machine. 1/20 natural size. (Fuess.)

of four screws in the plate xx_1, the basin itself slipping from above into a hole of proper size. Two sizes are manufactured, one with disks of the same diameter as those in the preceding machine, and one with disks of 25 cm. (10 in.). The latter is much better adapted for practical work than the former. If possible a bench should be arranged for four of these laps side by side, to avoid the necessity of continually cleaning up in changing from one grade of abrasive to another. One should be used exclusively for coarse, one for medium, and one for fine grinding. The fourth could be used for polishing rock faces.

In Fig. 753 is shown a large grinding machine, too large for ordinary laboratory purposes, but provided with an attachment which might well be adapted to a smaller lap. The instrument appears to be intended only

for grinding and polishing large slabs of rock, the plate *s* for the attachment of the specimens being 25 cm. in diameter and the basin *B*, 35 cm. As may be seen from the illustration, the machine may be foot or power driven, the motion being transmitted to the horizontal pulley *H* which rotates the grinding lap. At the same time, the spindle *e* is set in motion, and a belt to *g* rotates the disk *s*. There is also imparted to the disk a forward-and-back motion by the eccentric at *e*, moving the arm hinged at *c*. The position of the plate can be altered somewhat by the slide *A*.

With such an attachment to two laps like those in Fig. 752 and a plate (*s*) approximately 4 in. in diameter, to which four to six chips could be attached at the same time, and used on a 10-in. grinding disk, the rough and intermediate grinding could be done mechanically, especially if an automatic feed for carborundum and water were provided. The whole upper part, *cAgs*, could be made to lift off and slip over the pin (*c*) and eccentric (*e*) of the second lap, so that sections could be advanced without removal from the carrier. There might also be arranged a stop, set on screws beneath the arm *A*, which would prevent too thin grinding if the machine were left unattended for a time, absolutely preventing the spoiling of material by over-grinding. With fairly thin chips, no sawing would be necessary, time being no great consideration with the machine.

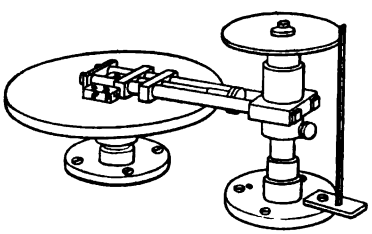

FIG. 754.—Grayson's lap.

The grinding lap described by Grayson[1] is of bronze, 10 in. in diameter, and provided on the lower side with a threaded boss by which it is screwed to the spindle of the machine, thus allowing the whole surface of the lap to be utilized. Tray-like shields or mud guards of galvanized iron, 5 in. deep, and with the upper edges rounded and brass bound, are provided, although not shown in the illustration (Fig. 754). The space around each spindle is raised and capped so as to exclude dust and grit, which otherwise would soon ruin the bearings. Somewhat to the right and behind the lap is a pillar supporting a horizontal clamping device, and arranged so that it may be swung by hand across the lap. The lower part of the supporting rod is threaded so that it may be raised or lowered during use. The machine is set on a table 3 ft. 2 in. high, and is driven by an electric motor at a speed of 980 revolutions per minute.

518. Orienting Devices.—For petrographic work orienting devices, by which sections may be cut at any desired angle, are very seldom used.

[1] H. J. Grayson: *Op. cit.*, 71–74.
[2] H. Rauff: *Op. cit.*

They are of great use in crystallographic work and reference should be made to the works mentioned below for detailed descriptions. One of the earliest instruments was that described by Rauff. In this the orienting device consisted of motions in two directions controlled by screws in the manner of a lathe. Another form is that found in the cutting machine described by Steinmann[1] and shown in Fig. 746, and another is that by Grayson.[2] More accurate is the device described by Fuess,[3] and still more so that by Tutton.[4] Differing in principle is Wülfing's[5] instrument, which is a multiple-screw device to be placed on the lap, instead of a goniometer specimen-holding-clamp as are the others.

519. Mounting the Section.—One of the operations upon which the eventual success of a section largely depends is that of mounting. Owing to poor cementation, a slide, evenly ground, may suddenly break loose from the object-glass, or it may break away over a bubble. Sometimes it may slide apart in undercooked balsam, or float apart when one attempts to attach the cover-glass.

As has been mentioned, various cements are used to attach the chip to the plate, or the rock to the glass. If the chip is attached directly to the iron holder-plate for preliminary sawing, a cement of half beeswax and half rosin is strong enough. If the chips are first ground to one flat surface and are then mounted with Canada balsam on the object-glass, these plates may be attached to the holder-plate by pure beeswax. This is one of the easiest cements to remove, since a slight heating will loosen the slide without melting the Canada balsam by which the rock is attached to the object slip.

Ady[6] proposed a cement, for preliminary mounting, made by heating Venice turpentine on a sand or water-bath and adding enough orange shellac to produce, on cooling, a thoroughly hard yet tough solid. During the process it should be tested, from time to time, by removing a small portion and cooling it. To attach a rock slice it is only necessary to melt a portion of the cement on the object-glass, and press the chip firmly down upon it. During the process of grinding this cement does not take up

[1] Gustav Steinmann: *Op. cit.*

[2] H. J. Grayson: *Op. cit.*, 71.

[3] R. Fuess: *Ueber eine Orientirungsvorrichtung zum Schneiden und Schleifen von Mineralien nach bestimmten Richtungen.* Zeitschr. f. Instrum., IX (1889), 349–352.

Also in Neues Jahrb., 1889 (II), 181–185.

[4] A. E. H. Tutton: *An instrument for grinding section-plates and prisms of crystals of artificial preparations accurately in the desired direction.* Proc. Roy. Soc., London, LX (1894), 108–110.*

Idem: *Crystallography and practical crystal measurement.* London, 1911, 681–691.

[5] E. A. Wülfing: *Ueber einen Apparat zur Herstellung von Krystallschliffen in orientirter Lage.* Zeitschr. f. Kryst., XVII (1889–90), 445–459.

[6] John Ernest Ady: *Observations on the preparation of mineral and rock sections for the microscope.* Mineralog. Mag., VI (1885), 127–133.

as much emery as does Canada balsam. To remove the chip from the glass after the preliminary grinding, it should be soaked in methylated spirits for a few hours. It should not be forced but should be allowed to float off of its own accord. It should then be transferred to clean spirits for an hour or two and washed gently with a camel-hair or sable brush. This cement possesses the advantages of hardening quickly and of requiring but little cooking.

Zirkel[1] recommended 16 parts by weight of thick Canada balsam and 50 parts of shellac. The shellac should be dissolved in the Canada balsam by heating on the water-bath for one or two hours. As soon as it is cool enough, but before hardening, it should be rolled between the hands into sticks 20 to 30 cm. in length and 1 cm. in diameter.

Another cement suitable for either preliminary or final mounting is a mixture of equal parts of gum damar (dissolved in pure benzol) and Canada balsam.[2]

Pure Canada balsam[3] alone may be used. The handiest method is to evaporate the balsam in a porcelain dish until a test piece is hard on cooling. The material is now taken up in balls on the end of glass rods and left to cool. To use the mass it is rubbed on a hot object-glass until the proper amount has come off. Pure paper-filtered Canada balsam (Therebinthina Canadensis) dissolved in xylol is preferred by the writer. The material is kept in a wide-mouth bottle, through the cork of which a glass dropping rod is inserted. Care must be taken to keep the inside of the bottle neck free from balsam or the cork will stick. For the final mounting the best grade of balsam should be used.

Grayson[4] says the tenacity and range of hardness of the balsam may be extended if a small quantity, not more than 1 to 3 per cent., of some clear and colorless organic oil, such as poppy, castor, clove, or linseed, is added to it in the right proportion. The amount must be learned by experience.

The method of preparing thin sections recommended by the writer is as follows: In the bottom of each compartment of a box 1 in. deep and divided by partitions into 2-in. squares, is placed a number card and on it the corresponding chip to be sliced, from 10 to 20 being the number best handled at the same time. The first chip is taken and is ground down to a flat surface on one side by the method described above, holding the chip in the hand and using coarse, medium, and fine emery. If the fragments are very thick or hard, they are sliced on the diamond saw. The pieces are then

[1] F. Zirkel: *Op. cit.*, 23.

[2] John Ernest Ady: *Op. cit.*

[3] [G. Marpmann]: *Die modernen Einschlusmittel.* Zeitschr. f. angew. Mikrosk., I (1895), 8–11, 36–46. Gives methods for determining the kind and purity of various embedding materials.

[4] H. J. Grayson: *Op. cit.*, p. 76.

returned to the proper compartments of the box and other chips are ground until each has one flat surface.

It was formerly customary to mount the chip first on a piece of thick glass and afterward transfer it to the final mount. This is now rarely done, the chip being usually mounted directly upon the object-glass.

Before being used, object- and cover-glasses should be made absolutely clean. For this purpose a cleaning solution[1] may be prepared by dissolving 2 oz. of bichromate of potash in 25 oz. of water and slowly adding 3 oz. of sulphuric acid. The solution should be left under a hood until it is cold and the fumes cease. A considerable number of object- or cover-glasses are now placed in a wide-mouth bottle and covered with the solution. The bottle should be gently tilted a number of times to cause the fluid to enter between the glasses and separate them, after which it should be left for three or four hours. The solution may now be poured back into the stock bottle, to be used over and over again, while the bottle of glasses should be repeatedly filled and emptied with clean water. The cover- or object-glasses may be left in the bottle covered with water and taken out as required with a pair of forceps and dried with a linen rag, or they may be placed upright to dry on lintless blotting paper. To support them on edge, use may be made of a piece of wood with vertical saw kerfs on the sides, or two glasses may be so placed that they will mutually support each other.

FIG. 755.—Cementing oven. (Fuess.)

The object-glasses, having been cleaned, the proper number is placed on an object-glass heater (Fig. 755) whose temperature is kept between 100° and 150° C., depending upon the kind of balsam mixture used. Grayson's heating arrangement is a piece of asbestos or a metal plate, over which is placed a sheet of white blotting paper. The plate is then laid in a well-filled sand-bath, supported by a tripod, and the heat of a Bunsen flame so regulated that it will not discolor or char the paper.

Canada balsam is next placed upon the slips. The writer prefers balsam dissolved in xylol, using only a drop or two—only enough to squeeze out a trifle on all sides of the chip when it is placed upon it. Too much balsam is "messy" and is likely to spread over oven, table, hands, clothes, and maybe hair as well.

One may determine when the balsam has been properly cooked by taking off a bit with a thin glass rod or burnt match, letting it cool, and testing it on the finger nail. If it does not stick it is ready to receive the chip. Another

[1] C. E. Hanaman: Note in Amer. Nat., XII (1878), 573–574.
C. Seiler: *Cleaning of slides and thin covers.* Amer. Jour. Microsc., V (1880), 50.

method[1] is to touch a bit, while hot, to the finger, and draw it out into a fine thread. It should just be beginning to get brittle. The chip is now taken up in a pair of forceps, heated for a moment with the flat side up in the flame to drive off the moisture, and placed on the balsam. By first placing one edge in the balsam and letting the other go down gradually, most of the bubbles will be avoided. By pressing down on the chip and moving it about a trifle, all others will be forced out. How successful one has been may be seen from the under side. If bubbles appear, the slide should be reheated and the bubbles removed, otherwise in the final grinding, the slice is likely to break away over them.

The object-glass and chip should now be removed from the hot plate, and the chip pressed down a few moments, but not moved, until the balsam cools. After this the glass slip should be marked on the *back*, by means of a diamond, with its proper number to avoid all chance of confusion, and returned to the box. If numbered on the face there is danger of obliterating the marks by grinding.

After having treated all of the slides in this manner, they are ready for the second face. Chips which are rather thin and of material which is not too hard may be ground down without cutting. If one has difficulty in holding the thin slips without grinding the finger nails, they may be attached to bits of plate glass by means of beeswax which later may be removed by very slight heat. If the chips are to be sawed, five or six are cemented by wax and rosin to the holder-plate, and the saw is passed as close to the object-glass as possible. The slide is now ground down to proper thinness, being covered with a drop of water and tested under the microscope from time to time. When the grinding is finished, the emery-filled balsam is scraped away from the sides of the chip, and it is covered with another drop of old or cooked balsam, which will require but little heating, and a warmed cover-glass is laid over the whole. When thin fresh balsam is used it is necessary to heat the slide until the balsam boils. This softens the underlying balsam, and the xylol of the later addition creeps under the chip and dissolves it, so that, upon pressing down the cover-glass, the slice is likely to break apart and be squeezed out at the sides. If only enough balsam to hold the cover-glass is used and the slide is set away for three or four days in a drying oven maintained at a temperature of 45° C., the drying will proceed without danger of losing the section.

Instead of placing the cover-glass upon the slide immediately after completion, Ady[2] covered the finished rock-slice with a drop of Canada balsam and put it aside for ten or twelve hours in a dust-proof box. Another drop of balsam was now added and a slightly warmed cover-glass placed above it.

[1] H. C. Sorby: *Preparation of transparent sections of rocks and minerals.* Northern Microsc., II (1882), 134.

[2] *Op. cit.*

This method, however, makes too thick a layer of balsam above the slice and causes difficulty when high power objectives are used. Bornemann[1] avoided bubbles under the cover-glass by placing a drop of turpentine under it and upon the completed rock-slice. Upon placing a drop of thin Canada balsam adjacent to one side of the cover, the balsam rapidly flowed under it, mixing with the turpentine. It was put aside to harden naturally or aided by gentle heat.

Another method is to boil the balsam upon the cover-glass and, when of the proper consistency, to invert it over the finished rock-slice, previously warmed. There is no danger of the slide separating by this method. Both the object- and cover-glass should be in close contact with the rock-slice with only a thin, but even, balsam film between. No instrument,[2] such as is often used in biologic work for holding the section in the balsam until it cools, is necessary, although a pair of tweezers (Fig. 756)[3] which close when released, large enough to hold a cover-glass transversely, is convenient. The cover-glass should be inserted 1/16 in. from the tips of the tweezers, which should then be placed on the object-glass to hold it steady and be released when the cover-glass is in the proper position. The drop of Canada balsam being convex will be touched by the cover-glass first at the center, and as it is pressed down the balsam will be squeezed out on all sides. By so doing, there is less danger of the section floating away than if the cover-glass is placed down with one edge first.

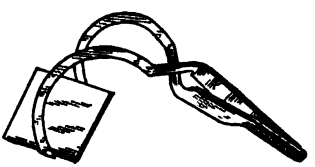

FIG. 756.—Henniges' tweezer for holding cover-glasses.

After having thus mounted and covered the rock-slice, the excess of balsam around the edge is removed with a heated knife blade or putty knife. The slide is then placed for a short time in alcohol, is brushed with it by the aid of a medium soft brush, such as an old tooth brush, and is washed in water and dried. It must not be left too long in the alcohol, or the balsam beneath the edge of the cover-glass will be dissolved out, giving a projecting edge which affords a good hold for eventually springing it off by the object clips.

The size of the object-glasses used is a matter of personal taste and convenience. English slides are usually 1×3 in. While they afford a large space at either end for labels, they are too long for convenience, the end projecting over the stage being likely to be struck with the hand, throwing the mineral under examination out of the field. The slides used at the

[1] *Op. cit.*, p. 371.

[2] L. Henniges: *Ueber einen Hilfsapparat beim Einlegen von Gesteinsdünnschliffen in Kanadabalsam.* Centralbl. f. Min., etc., 1911, 158–160.

[3] Dr. Seiffert: *Eine neue Pincette zum Halten der Deckgläschen.* Zeitschr. f. angew. Mikrosk., I (1895–6), 84.

University of Heidelberg are 30×30 mm., a size too small to label with anything more than the number. The most convenient size seems to be about 28×48 mm. (Fig. 759). They are small enough to be out of the way on the stage and large enough to label. Those used by the U. S. Geological Survey are 27×47 mm. For the Fedorow stage circular slides, such as are shown in Fig. 406, are necessary. For the new Fuess microscope with Fedorow stage the 28×48 mm. slides may be used.

It may sometimes be necessary to remount an old rock slice on account of the cracking or yellowing of the balsam, or the stripping off of the cover-glass. If the cover-glass is still in place and unbroken, a few drops of turpentine[1] may be placed upon it and the whole set on the heated plate until the lower balsam film melts. The evaporation of the turpentine will keep the upper film cool, consequently cover-glass and rock-slice may be slipped sidewise off the object-glass without breaking. The object-glass should be cleaned, or a new one taken, and a few drops of fresh balsam placed upon it and cooked. In the meantime the cover-glass should be turned upside down and the old yellowed balsam carefully scraped away from around the sides of the rock-slice. When the balsam on the object-glass is sufficiently cooked, the cover-glass with the attached section should be gently heated, though not enough to melt the balsam, pressed down upon the object-glass, and the whole removed to cool. If the cover-glass is also defective, the same process is repeated, inverting the slide and evaporating turpentine on the bottom to remove the cover. If the cover-glass alone has come off, scrape away the old balsam around the slice, cook a few drops of balsam to the proper state in a small watch crystal and pour it over the gently warmed old slide and cover immediately with a warmed cover-glass.

Sorby[2] replaced broken object-glasses by removing the cover-glass, scraping the balsam away from about the rock-slice, and covering it with plaster of Paris. When the plaster was hard the whole was heated and the plaster with the embedded slice was pushed off. It was remounted in the usual way. This method may be used to good advantage with broken slides, the pieces being fitted together and held with a piece of gummed paper on the back of the object-glass before removing the cover, thus keeping the rock fragments in proper position.

If both object-slip and cover-glass are broken, the remnants of the original rock-slice may be removed by placing a liberal amount of fresh balsam dissolved in xylol upon the pieces, and heating gently. The new balsam will work its way beneath the slice which will soon float upon its surface. It may be transferred to a fresh object-glass, upon which balsam

[1] E. von Fedorow: *Universalmethode und Feldspathstudien, III.* Zeitschr. f. Kryst., XXIX (1897–9), 617.

[2] H. C. Sorby: *Preparation of transparent sections of rocks and minerals.* Northern Microsc., II (1882), 137.

cooked to the proper stage has been placed, by slightly tilting the old mount and letting the rock-slice float to its new position.

SPECIAL METHODS FOR PREPARING SECTIONS OF UNUSUAL MATERIAL

520. Friable Material.—Soft or friable material, such as decomposed rock, clay, or chalk cannot be ground in the ordinary way but must be given a different treatment, depending upon the nature of the material. Forbes[1] soaked soft or porous rocks in turpentine, then in soft Canada balsam, and afterward heated them until quite hard. A similar method was used by Sorby.[2] Another method is to boil the material in Canada balsam until it will absorb no more and put it aside to harden.[3] Pfaff[4] prepared chalk and soft limestone by inverting the section, when ready for the final grinding, and rubbing down upon it with a very soft cork and the finest emery flour. The section should be completely surrounded by a ring of Canada balsam, and if this breaks away it should be replaced, otherwise a few rubs with the cork may break the edges of the rock slice. Wichmann[5] shaved flat, with a knife, one side of soft, fine-grained material and then rubbed it to a perfect plane upon a dry plate of glass. The flat side was then placed on an object-glass in Canada balsam which had been cooked and allowed to cool to a rather viscous state. On complete cooling the other side was shaved down with a knife as much as possible, cleaned from dust, and a cover-glass placed over it in Canada balsam dissolved in chloroform.

Bosscha[6] prepared sections of a friable meteorite by saturating it with melted copal gum. He first ground one side flat, placed it, with the ground side up, on a plate heated to about 125° C. and upon it laid pieces of copal gum, which melted and entered the pores. After cooling, the excess of gum was scraped off and the section cleaned by means of a rag dipped in ether.

Steenstrup[7] was able to preserve and show in thin sections, by a double procedure, the original arrangement of the grains in clays. A piece of perfectly dry clay was ground flat on one side upon fine sand or upon a glass plate without water, and was then fixed to a cover-glass by Canada balsam or a mixture of Canada balsam and shellac; the cement being allowed to

[1] David Forbes: *On the preparation of rock sections for microscopic examination.* Mon. Microsc. Jour., I (1869), 240–242.

[2] H. C. Sorby: *Op. cit.*, 136.

[3] F. Zirkel: *Lehrbuch der Petrographie*, Leipzig, 2 Aufl., 1893, 26.

[4] F. Pfaff: *Einiges über Kalksteine und Dolomite.* Sitzb. Akad. Wiss., München, XII (1882), 562–563.

[5] Arthur Wichmann: *Ein Beitrag zur Petrographie des Viti-Archipels.* T. M. P. M., V (1883), 33, footnote.

[6] J. Bosscha Jun.: *Ueber den Meteorit von Karang-Modjo oder Magetan auf Java.* Neues. Jahrb., B.B, V (1887), 126–144, in particular 127–129.

[7] K. J. V. Steenstrup: *Tyndprøver af Ler.* Geol. Fören. i Stockholm Förh., XII (1890), 647–648.

cool somewhat and become viscous before pressing down the clay, so that it would not enter too far into its pores. After standing about twenty-four hours to permit the balsam to harden without heat, the clay was broken off from the object-glass, leaving but a thin film upon it. The fresh face on the fragment of material was now cemented, without grinding, in the same manner as before, to another cover-glass, the Canada balsam allowed to harden, and the material again broken off. If the clay film was too thick, it was thinned by gently spraying it with water from a wash bottle. After drying, Canada balsam and a cover-glass were placed over it, and the cement allowed to harden without boiling. The reason for the double process was that the mineral particles were displaced by the grinding, in the first flat face, while in the second they retained their proper positions.

521. Vesicular Rocks.—Since the cavities of pumice and other vesicular rocks are closed except where they are fractured at the surface, boiling such rocks in balsam is useless. Johnston-Lavis[1] ground such rocks smooth on one side, blew or washed the dust out of the cavities, and placed them on a hot plate to dry. When well warmed, a stick of hard balsam was rubbed over the surface and an abundance of the cement left upon in. At the end of a minute or two more balsam was added, if the first had sunk in. The rock was now removed from the hot plate and allowed to cool in a horizontal position, after which it was ground down on a slab of sandstone, slightly inclined, over which a stream of water slowly flowed. The rock was ground until all broken septa were brought flush with the surface. It was then washed, heated, and more balsam added. The excess of balsam was removed by grinding and the specimen again washed, after which it was polished by being rubbed on an inclined hone upon which were placed a few drops of a solution made by dissolving 1 pint of yellow soap in 2 pints methylated spirits and then adding 3 pints of water. During the process of polishing, a small quantity of water, preferably soapy, constantly dripped upon the upper end of the hone. If the balsam began to "rool" and caused hitching, a few drops of the soap solution were added. The specimen was polished until the surface was brilliant. It was then put in a warm, dust-free place to dry, after which it was cemented to a slide by hard balsam. The opposite side was now ground down almost to transparency on a well-watered grindstone, then polished on the soapy hone. Too much soap causes a softening and saponification of the balsam, causing it to become opaque; too little causes it to stick to the stone and thus carries the section away with it. The slide was finally washed and dried. When completely dry the surface was brushed with equal parts of turpentine and benzol or chloroform until the network began to appear raised. The slide

[1] H. J. Johnston-Lavis: *On the preparation of sections of pumice-stone and other vesicular rocks.* Jour. Roy. Microsc. Soc., 1886, 22–24.

was drained, but not dried, and balsam dissolved in benzol or chloroform added, and the cover-glass placed on top.

522. Coal.—Harris[1] placed coal for a considerable time in turpentine, then in dilute Canada balsam till saturated. Upon evaporation by gentle heat, the balsam gradually hardened and the coal was ground down as any hard rock.

523. Clays and Soft Powders.—Materials, such as soft powders, which do not need to be mounted in such a way as to show the original texture of the rock, may be mixed to a paste with some other material and a section prepared of the united mass after it hardens. Pearcey[2] made sections of some of the Challenger material by uniting it with gum copal. He placed 1/2 lb. of the best gum in a strong glass quart jar having an air-tight ground-glass stopper, and added to it 20 oz. of ether (B. P. sp. gr. 0.735). After standing for at least two days, with frequent shaking or stirring, the gum was dissolved, and the resulting clear, thin, transparent liquid was ready for use. The substance from which a section was to be made was first well dried, then placed in a porcelain crucible and twice its amount of the gum copal and ether poured over it, care being taken to cap the stock bottle immediately. It was now placed on a moderately hot plate, since the ether is very inflammable, and allowed to simmer until it had partly evaporated, when greater heat was applied. If the material was a fine sand or ooze it was kept well stirred; if a soft, porous, or decomposed rock, it was only necessary to turn it several times.

If the proportions were right, after nearly all the ether had evaporated, the substance was of a stringy nature when stirred. If it was found that too little cement remained to hold the grains together, more was added; if too much, more of the substance as well as a little pure ether, and the boiling repeated. The mass was of a reddish-brown color when done, and a small portion, rapidly cooled by pressing it against some cold surface, hardened immediately. The crucible was removed and the material, while yet warm, was scraped out with a knife, pressed with the fingers into an oblong mass, and molded into little cylinders about 3/4×3/4 in. by pressing it into molds formed of strips of tin tied with wire and set on a piece of glass. The mass was cooled in water, the mold removed, and the material was ready to cut like any other rock section. If the sides began to crumble before the section was thin enough, a little cement, made of one part of beeswax to four of resin, was dropped, while hot, with a pipette around the edges to form a support.

[1] C. L. Lord and W. H. Harris: *Cutting sections of coal.* Science Gossip, 1882, 136–137.

[2] F. G. Pearcey: *Preparing thin sections of friable and decomposed rocks, sands, clays, oozes, and other granulated substances.* Proc. Roy. Soc. Edinburgh, VIII (1884–5), 295–300.

524. Sand and Other Loose Grains.—Sand and other loose grains may be examined by the method given by Thoulet,[1] who mixed them with about ten times their volume of zinc oxide, and then added enough potassium silicate (water glass) solution to make a thick paste. This paste was pressed into sections of glass tubing, several millimeters in length and with parallel ends. These ends were covered with paper and the material allowed to dry for several days. Sections can be cut from such a mass in the same manner as from the natural rock.

Mann[2] mixed the grains to be examined with a paste of zinc oxide and phosphoric acid and molded the mass into balls. When dry they were sectioned as usual.

Retgers[3] found methods of embedding grains in cements, which hardened later, to be impracticable for sands, on account of the breaking out, during the process of grinding, of hard minerals such as zircon, spinel, and corundum. He crushed the grains in an agate mortar to fragments, but not to powder, and immersed them in a fluid of high refractive index. For permanent mounts he fixed such grains in Canada balsam.

525. Hydrous Minerals.—Hydrous minerals cannot be mounted in the ordinary way since they will lose their water of crystallization by the heat during the process of preparation. They should, therefore, be placed in the balsam only after it is properly cooked, when it is not too hot and beginning to be viscous. The cover-glass should be laid on with rather thick balsam and set aside without heating to harden. Another method is to mount them in Canada balsam dissolved in ether, and then place them to harden, for several days, in a dust-proof box.

526. Minerals Soluble in Water.—Minerals soluble in water should be ground with emery and alcohol, turpentine, xylol, etc.

527. The Preparation of Polished Faces on Rocks.—To polish a rock which has been ground to as flat a face as possible with fine emery, it is held upon a leather-, felt-, or "beaver" cloth-covered lap impregnated with tin oxide (putty powder), chromic oxide, aluminium oxide, or iron oxide (rouge), and kept well wet with water. A final polish may be given by rouge on a dry chamois-covered lap. French chalk, rotten stone, or tripoli do not give as good results as the oxides mentioned above.

528. Rims.—While in biologic work it is customary to surround cover-

[1] J. Thoulet: *Note sur un nouveau procédé d'étude au microscope de minéraux en grains très fins.* Bull. Soc. Min. France, II (1879), 188.

[2] P. Mann: *Untersuchungen über die chemische Zusammensetzung einiger Augite aus Phonolithen und verwandten Gesteinen.* Neues Jahrb., 1884 (II), 187.

[3] J. W. Retgers: *Ueber die mineralogische und chemische Zusammensetzung der Dünensande Hollands und über die Wichtigkeit von Fluss- und Meeressanduntersuchungen im Allgemeinen.* Neues Jahrb., 1895 (I), 16–74, especially 32.

glasses with a rim of cement of some kind, this is rarely done with rock sections, although it would be of considerable advantage. It keeps the air from the balsam, thus preventing it from turning yellow, and acts as a guard to prevent springing off the cover-glass with the object-clips.

The rim is usually put on with a brush, the slide being placed on a turn table. This necessitates the use of circular cover-glasses, which in themselves are advantageous, being less likely to come off. If the cement is rather thick, the slide may be put on a turn table and a broad band[1] put on at the junction of cover- and object-glass. A knife blade may now be held, first on one side and then on the other, so that the cement is heaped up in a thick ring. Should there be a tendency for the cement to run, the slides may be put away with the cover-glasses downward.[2]

Various cements have been used, zinc white[3] and asphaltum varnish being the most common. The disadvantage of zinc white is that it is too brittle and soon breaks away. It may be improved[4] by draining off the oil from the usual paint and mixing the latter with Canada balsam very much thinned with chloroform. The mixture should be of the consistency of cream and flow freely from the brush. If it does not do so, add a little turpentine. The rim may be colored as desired with ordinary artist's oil paint, and then varnished.

Another good cement is dammar varnish, although it is rather brittle unless turpentine is added. It may be prepared by mixing gum dammar, benzine, and turpentine in equal parts, and setting away in a warm, not hot, place until dissolved. The clear liquid should now be poured off and allowed to evaporate until of the required consistency. Another method of preparation[5] is as follows: To 4 drams of crushed Indian dammar add 8 liquid drams of pure benzole, and allow the resin to dissolve at the ordinary temperature. After a day or two an insoluble residue will be found at the bottom of the vessel. Carefully decant the clear liquid, and add to it 1 1/3 drams of spirits of turpentine. Dammar cement may also be used as a mounting medium in the place of Canada balsam.[6] James[7] says a limpid solution of dammar may be obtained by adding enough benzol to make a solution which is readily filtered through paper. If too thin for immediate use, evaporate

[1] C. E. Hanaman: *Notes on microscopical technology.* Amer. Mo. Microsc. Jour., II (1881), 142–144.

[2] Frank L. James: *Microscopy.* National Druggist, V (1884), 216.

[3] C. E. Hanaman: Note in Amer. Mo. Microsc. Jour., V (1884), 220.

M. A. Booth: Note in Ibidem, VI (1885), 39.

[4] J. Ford: *Dr. Hunt's American cement for ringing slides.* Jour. Post. Microsc. Soc., I (1882), 193.

[5] C. J. M.: *The preparation of dammar varnish for microscopic purposes.* Science Gossip, 1882, 257.

[6] Wilhelm Pfitzner: *Die Epidermis der Amphibien.* Morphol. Jahrb., VI (1880), footnote 479.

[7] Frank L. James: *Microscopy.* National Druggist, VII (1885), 245.

to the proper consistency. If the dammar rims prove too brittle, a small amount of pure rubber dissolved in naphtha may be added. If a colored ring is desired one may flow on a ring of ordinary water color before the varnish, or the color may be mixed with the latter.

Less brittle than dammar are rims of copal varnish. The finest varnish that can be purchased should be used, Berry's hard finish being excellent, and enough dragon's blood may be added[1] to give it a red color without destroying the transparency. It should be left exposed to the air until it becomes rather thick, and may then be run around the edge of the cover-glass in the same manner as that just described. Another way is to spin the slide on the turn table and cut through the varnish, with a knife, a ring inside and outside the edge of the cover-glass, leaving a strip of the proper width. After drying for a week the superfluous varnish may be scraped off.

Another cement[2] is composed of 2 parts wax and 7 to 9 parts of colophony. The latter is added piece by piece to the melted wax and the resultant filtered. This cement is solid at ordinary temperatures but readily melts on being placed in a basin of hot water. It is insoluble in water, glycerine, or caustic potash, and, since it hardens quickly, the slide may be finished at once.

Venice turpentine[3] is another substance which may be used for ringing slides. It may be prepared by dissolving true Venice turpentine in enough alcohol so that the solution may be readily filtered. It is then placed on a sand-bath and evaporated until a small quantity dropped into cold water will be hard and break with a vitreous fracture. Parker[4] suggests using square cover-glasses. A piece of No. 10 to 12 copper wire, bent into a right angle and having the short arm just the length of the side of the cover-glass, is heated and dipped into the prepared turpentine, some of which adheres. The wire is now placed flat along the edge of the cover and the turpentine will be evenly distributed along the entire side. It becomes hard immediately and is of a pleasing green tinge from the copper.

A brown ring can be made by using a shellac cement,[5] made by adding enough litharge to a thin shellac varnish, to thicken it. It should be applied in at least two coats, the second added after the first is completely dry. This cement dries quickly and becomes dark brown by exposure to the air.

[1] W.: *Finishing slides.* Amer. Mon. Microsc. Jour., I (1880), 123–124.

[2] Dr. Krönig: *Einschlusskitt für mikroskopische Präparate.* Arch. f. Mikrosk. Anat., XXVII (1886), 657–658.

[3] Julius Vosseler: *Venetianisches Terpentin als Einschlussmittel für Dauerpräparate.* Zeitschr. f. wiss. Mikrosk., VI (1889), 292–298.

[4] C. B. Parker: *A new cement.* Amer. Mon. Microsc. Jour., II (1881), 229–230.

[5] Hamilton Smith: *New cement and new mounting medium.* Amer. Mon. Microsc. Jour., VI (1885), 182.

For the preparation of a shellac mounting medium see Romyn Hitchcock: *The preparation of shellac cement.* Ibidem, 1884, 131–132.

CHAPTER XLII

PETROGRAPHIC COLLECTIONS

Field Work

529. Working Tools.—The working tools of a field petrologist are a geological hammer, a hand lens, and a collecting-bag. The hammer should be made of the best cast steel, properly tempered. It should not be so hard that it will chip off at the corners, nor yet so soft that the edges will round over. The form depends upon the use to which it is to be put, and is usually a matter of personal preference. For a general petrographic hammer one weighing, without handle, between a pound and a pound and a quarter is best. It should have at least one rectangular face, the other end being shaped as a pick or wedge. If the latter, the sharp edge may run parallel to the direction of the handle or at right angles to it, the writer preferring the former for a heavy hammer and the latter for one which is light. For a trimming hammer, one with both ends rectangular and weighing about 6 oz. is very convenient. Another useful hammer is one weighing between four and six pounds, and having two rectangular faces, each about $1 \times 2\ 1/2$ in. This is especially useful in breaking off spalls from a large block. If one is traveling afoot and but a single hammer can be carried, it should be of medium weight, perhaps three-fourths of a pound, and may have two rectangular faces, or one rectangular and one wedge-shaped with its edge at right angles to the handle. In all hammers the center of gravity of the head should fall at the point of intersection of the handle. The opening through it should be larger above than below so that, after the insertion of a wooden or metal wedge, there will be no danger of the head working off. Very few hammers appear to be so made. The handle should be of hickory and about 14 in. in length. It should be trimmed down near the head so that its spring will absorb all shock and not transmit it to the hand.

The most convenient method of carrying the hammer is on the belt. A case may be made of a circular piece of leather (Fig. 758) about 7 in. in diameter, provided on the front below the center with a horizontal loop for the hammer handle and on the back with two that are vertical for the belt. The position of these loops should be such that when the hammer is placed in its loop the upper part of the leather disk will fold over the head and prevent it from slipping out, the curvature of the belt around the body preventing the flap from opening. Another form is shown in Fig. 757.[1]

[1] Ferdinand von Richthofen: *Führer für Forschungsreisende.* Berlin, 1886, 14–15.

For short excursions the handle may be slipped through the strap, if such is provided, at the back of the trousers, where the hammer will be concealed, entirely out of the way, and in no danger of being lost.

The method of carrying the hammer with its handle slipped through two straps on the front of the collecting bag is not a good one since it is always necessary to unstrap the bag to get at it.

It will be found a great convenience to have the end of the handle notched for 4 in. at 1-in. intervals, to serve as a measuring stick for hand specimens. The beauty of a collection depends largely upon the uniform size of the specimens. A thong passed through a hole in the handle and around the wrist

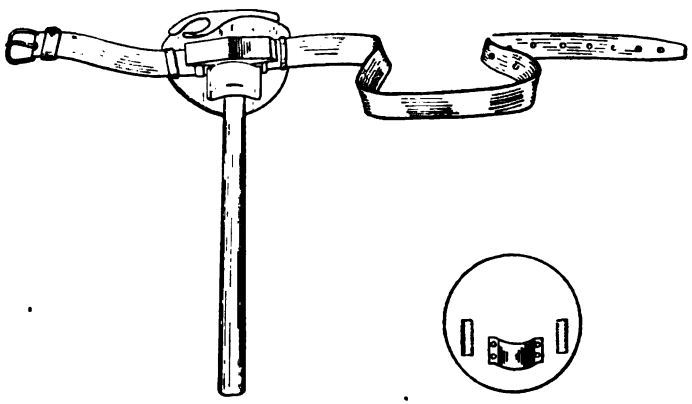

FIG. 757. FIG. 758.

FIGS. 757 and 758.—Hammer shields.

relieves the hand from cramping if one carries the hammer for a long time while walking. The hole should be far enough up so that the handle will rest in the hand when the thong is about the wrist. Of course when the hammer is used the thong should be slipped from the wrist.

Hand lenses have been described above.[1]

The collecting-bag may be in the form of a pouch to carry at the side, a knapsack, or a rucksack. If one is working afoot the former, when loaded with specimens, soon tires one's shoulder. A knapsack may be so arranged with straps and buckles that it may be readily converted into a bag to carry at the side—if one objects to walking through town with a bag on his back. In size it may be about 11 in. by 12 in. by 3 in. It should be divided into several compartments, perhaps including one for map and note-book, although many men prefer to have the latter of such size that it will fit the pocket. The rucksack has its advocates though the writer finds that the load hanging so low on the back is tiresome. Whatever kind of bag is chosen, it should be made of light and strong material, such as canvas; leather being altogether too heavy.

[1] Art. 99.

A U. S. army canvas haversack, fitted with straps as a knapsack, is most convenient.

530. **Hand Specimens.**—If conditions permit, hand specimens should be trimmed to uniform size and, unless for special purposes, should show fresh faces on all sides and no marks of the hammer. They should be about 3 by 4 in. in size and an inch or an inch and a half thick. The corners should be rectangular and not rounded. If one is doing reconnaissance work and his baggage is limited, the specimens may be made 3 by 2 by 3/4 in. or even 1 1/2 by 2 by 1/2 in. It will be found much more difficult to dress a small, neat specimen than a large one. Each hand specimen should be accompanied by a number of fresh chips from the same piece. These are to be used for thin sections, and for possible chemical analysis. For thin sections, pieces about 1 1/2 in. in diameter, free from cracks, and having one nearly flat face should be chosen. In wrapping they should be separated from the hand specimens by several sheets of paper. They may well be placed in separate envelopes and carried in a separate compartment of the collecting bag, for when wrapped with the hand specimen they make awkward packages which are likely to break open.

Immediately upon collecting the specimen it should be labeled, preferably by attaching a gummed label and writing the number upon it with ink. The labels should be small, 3/8-in. circular or oval being sufficiently large. They should be well gummed, better than ordinarily, so that if one is firmly pressed down into the irregularities of a rock, after all dust has been blown off the latter, and it is held down for a few moments, there will be little danger of its coming off. It is advisable to place a locality label within the wrapper around the specimen. This should be folded across the middle to prevent the obliteration of the writing. Some geologists recommend writing the locality upon the wrapper. This is no easier in the field than to prepare a label, and it necessitates the preparation of a label in the office as well. The reason for accompanying the specimen with a label is that if the notebook, containing the localities corresponding to the numbers, is lost, the specimen will permit the reconstruction of the notes to a certain extent. The locality should be so written that its position may be determined without reference to any points except such as are shown on the map. That is, no label such as "1/4 mi. W. of camp" should be used. A label such as "300 ft. above preceding in bed of creek" is permissible. It is advisable, every evening, to mark the exact locality of each specimen by a number on a map kept for that purpose. One should also insert on the label information such as the relative position of a specimen in a dike, sheet, flow, or laccolith, *e.g.*, "near top," "2 ft. from contact," etc. The labels should be so written that there will be no ambiguity in regard to the relationship to other rocks.

Specimens should be collected from rock in place and not from loose

blocks, no matter how large they may be, unless there is no question as to their source, as, for example, in a quarry, talus from a cliff, etc. This, of course, does not apply to material collected from glacial bowlders or terrace deposits.

Certain rocks are almost impossible to dress to proper size, and one must do the best he can. Thus granite, where it occurs in rounded bosses, offers no chance for breaking off a spall. One must take what he can get or resort to blasting. The length of time necessary to trim a neat specimen of a rock from which one can get a good spall, should not be over two or three minutes for such rocks as granite, granite-porphyry, or limestone. A specimen of gabbro, pyroxenite, or other tough rock may take considerably longer.

Where any variation in the type of rock occurs, specimens should be collected from the unusual as well as of the usual phase. This caution is hardly necessary; it would better be written, collect the usual as well as the unusual. It not infrequently happens that upon returning from the field one finds that the usual occurrences have been overlooked.

531. Wrappers and Labels.—A single leaf of an ordinary newspaper, folded in half, makes a good wrapper. It should be so folded that the number of thicknesses on either side is as nearly as possible equal. The final fold should be tucked under in such a way that there is no danger of the wrapper coming undone. Much more convenient are specimen envelopes made of heavy manilla paper. They should be at least 8 by 10 in. in size so that when an ordinary 3 by 4 by 1 1/2 in. hand specimen is placed in one corner, the envelope may be folded over first on one side and then on the other so that there will be three and five thicknesses of paper as a protection against rubbing. These bags also are convenient for wrapping specimens of tuff, clay, and so on.

Chips for thin sections should be sealed in small, strong manilla envelopes, and the number written in ink, outside. If the envelopes are about 2 1/2 by 3 1/2 in. in size, they may be doubled up to serve as a protection to the chip when mailing. There is usually little danger of rubbing through the wrappers, and it is unnecessary to use gummed labels upon chips.

532. Packing Specimens for Shipment.—Hand specimens should be packed in strong boxes, not too large, 10 in. by 12 in. by 14 in., inside measurement, being a good size. The wood need not be unnecessarily thick; a box with 1 1/8-in. ends and 5/8-in. sides, wired at the ends, is as strong as one made entirely of 7/8-in. stuff and not wired. The wire should be fairly heavy and should be held in place by staples or be given a twist around the heads of two or three nails on each side of the box. The safest way to pack hand specimens is to place a layer on edge in the bottom of the box, crowding as much as possible, and then fill the interstices completely with newspaper wads. A second and a third layer may then be packed, a box of the size mentioned

above holding three layers of about three rows each, the rows being rather irregular on account of the lenticular form of the specimens.

If a box is not quite full the remaining space should be crowded with excelsior, hay, or paper, but not with sawdust or other fine material. The tighter the box is packed the better it will stand shipment.

OFFICE WORK

533. Accession Catalogue.—Whether a collection of rocks should be listed in an accession catalogue or not depends upon the purpose of the collection. For the ordinary working collection of material from one restricted district this is not necessary, but if the field embraces a large territory, or if the collection is that of an institution, such a catalogue is necessary. A very good form, following the plan of one devised by Professor Weller, is that used at the University of Chicago. The pages are 8 1/2 by 11 in. in size, the entries extending across two opposite pages so that it makes an available length of 17 in. The columns are headed as shown below.

No.	Corrected name	Name under which received	Orig. No.	Date rec'd	Source	Locality	Remarks
0							
1							
2							
3							
4							

Two lines are given to each specimen, every alternate line being ruled heavier than the other. The numbers on each left-hand leaf run twice from 0 to 9, permitting twenty entries to a page. By beginning each page with 0, it is only necessary to fill in the number twice to a page instead of three times. A book of 200 leaves, giving space for 4000 specimens, makes a convenient volume. It should be substantially bound in canvas, ledger style, in preference to leather.

534. Permanent Labels for Hand Specimens.—After unpacking specimens, they should at once be given permanent numbers. These are best made by painting the number in white on a dark field in one corner. The field should be rectangular, about 6 by 16 mm. in size, and may be black, blue, dark green, or any other dark color. Different colors may be chosen for different collections, such as petrographic, mineralogic, or economic. If desired, a black or red number may be painted on a light field. Enamel paint seems best adapted for the field color. It should be rather thick and

be flowed on from a bristle brush made stiff and stubby by clamping the bristles close to the ends with a piece of tin. If the brush is just right the field may be made almost perfectly rectangular with one stroke. After drying, the painted area should be smooth and glossy, the paint having been laid on thick enough to fill all irregularities in the rock. If the paint is too thin it will run and spoil the appearance of the label. If it does not form a smooth coat, a second should be applied after the first is dry. After a week or more of drying and when the paint is hard, the numbers may be written on with white paint by means of a medium steel pen. They should be neither too coarse nor too fine, and as neat as possible. After these too are dry, a touch of dammar varnish will form a protecting coat. For neatness and uniformity, the numbers should always be placed in the same corner, and as nearly as possible in the same relative position in all specimens.

535. Labels for Thin Sections.—Upon the object-glass of each thin section, a number, corresponding to the number on the hand specimen from which it was taken, should be scratched with a writing diamond. Since this number is not easily read, a paper label should be pasted over it, the scratched number being for safety in case the paper label springs off.

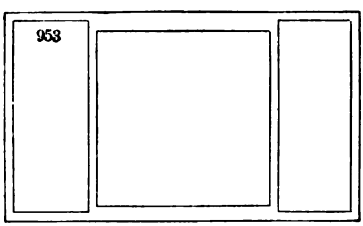

FIG. 759.—Labeled thin section.

A convenient way of numbering slides so that the figures may be easily read when the sections are placed in boxes, is shown in Fig. 759. With slides so numbered there is no excuse for misplacing them in the boxes after use.

Northrup[1] says a label written on the glass with Higgin's water-proof india ink is permanent so far as ordinary treatment is concerned. Before writing the label, the slide must be made free from grease by breathing upon it and rubbing with a dry cloth. Parts of the label may be removed, if desired, by scratching with a knife, or the whole by rubbing with a damp cloth.

Besides the number of the specimen, the name of the rock and the locality where it was collected may be written on the label. For collections to be used by students, however, there should be nothing more than the accession number.

Bryan[2] suggests that instead of one thin-paper label at one end, two made of slips of thick card be used. They should be attached to the object-glass at either side of the cover. Slides thus protected may be placed one against another, making a cabinet unnecessary.

[1] Zae Northrup: *A new method for labeling microscopic slides.* Science, XXXVIII (1913), 126–127.

[2] G. H. Bryan: *How to label microscopic slides.* Science Gossip, 1882, 64.

536. Marking Thin Sections.—It is sometimes desirable to mark a slide so that a certain noteworthy portion may be readily found on a future occasion. One of the most convenient instruments for this purpose is the object marker[1] shown in Fig. 760. The mineral, whose position is to be marked, is placed in the center of the field under the cross-hairs, after which the objective is removed and the object marker substituted. The diamond *D* is controlled by the screw *b* and the spring *F*, and may be placed out of center as far as the slider *a* will permit. *S* is a weak spiral spring by means of which the inner cylinder *C* is pressed downward in the casing *H*. It is kept from falling out by the screw *c* which works in a slot. Upon depressing the tube, the diamond

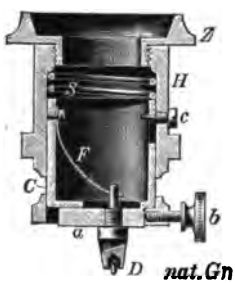

FIG. 760.—Section marker. Natural size. (Fuess).

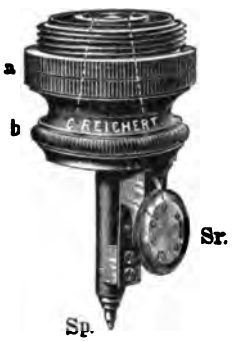

FIG. 761.—Section marker. (Reichert.)

touches the cover-glass with greater or less pressure depending upon the amount of the depression. If, now, the stage of the microscope be rotated, the diamond will scratch a circle upon the glass, its size depending upon the amount of the displacement. In the form shown in Fig. 761 the sizes of the circles are shown by the graduations on the screw *Sr*.

Instead of a permanent scratch, one may desire to place upon the cover-glass a mark to indicate temporarily a certain portion, as for example for micro-photography. For this purpose there may be used a holder similar to the above but provided at the lower end with a metal ring which, when inked by a stamping pad and depressed until it touches the cover-glass, leaves a circular mark. A spring prevents any injury to the slide. Cones with different-sized rings, interchangeable with the first, are furnished with

[1] C. Leiss: *Die optischen Instrumente, etc.* Leipzig, 1899, 248–249.

See also P. Schiefferdecker: *Ueber einen Apparat zum Markiren von Theilen mikroskopischen Objecte.* Zeitschr. f. wiss. Mikrosk., III (1886), 461–464.

R. Fuess: *Apparat zur dauernden Kennzeichnung bemerkenswerther stellen in mikroskopischen Objecten oder Präparaten.* Neues Jahrb., 1895 (I), 280–281.

the device.[1] If the circular ring were of rubber there would be less danger of breaking the slide and a better ring would be impressed upon the glass.

With a mechanical stage provided with guide strips, any desired mineral may be found on a subsequent occasion if both vernier readings are noted and the slide is inserted in the same position as it was before. It is necessary, however, to use the same microscope for the determination. The position of any point may likewise be determined with the Hirschwald stage as modified by Johannsen.[2]

537. Cases for Thin Sections.—The manner of storing sections depends upon the size of the collection. If the number of specimens is few they may be kept in boxes such as are shown in Fig. 762. The septa shown in Fig. 763

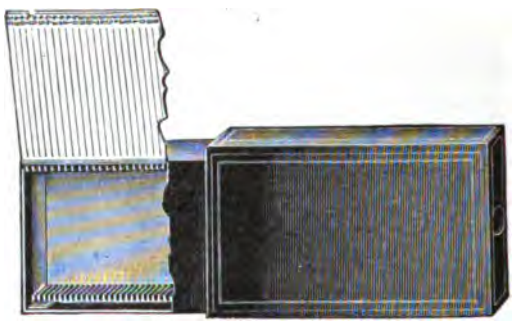

FIG. 762.—Box for thin sections. (Dr. Steeg and Reuter.)

are of compressed paper and are of sufficient length so that slides a few millimeters shorter or longer than normal may be inserted without difficulty. Being of paper they are much thinner than would be necessary were they saw kerfs in wood strips, consequently many more sections may be placed in a box of a given size.

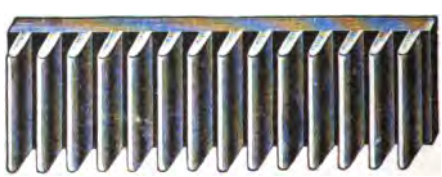

FIG. 763.—Septa in section box. (Dr. Steeg and Reuter.)

For a larger collection of sections a neat cabinet may be made by fastening a number of boxes like Fig. 762 tightly together in a frame like a sectional book-case, using small brass ring fasteners as drawer pulls.

[1] Manufactured by Klönne und Müller, Berlin. Originally described by P. Francotte, Bull. Soc. Belge de Micr., XI (1882), 48.* Reviewed in Jour. Roy. Microsc. Soc., V (1885): 325.

[2] See Art. 109, *supra*.

If it is desired to keep the sections flat, cases such as shown in Fig. 764 may be used. Such cases take up more space, are more expensive per unit, and the sections are more easily disarranged than in those previously described, but they permit the entire label to be read without moving the slide. If sections are labeled as suggested above this is hardly necessary, and only in such regions, as in the southwestern states, where the summer heat is so great that the balsam softens and the section slides away, is it necessary to keep sections flat.

A third type of case, intermediate between that for vertical and that for flat-lying sections, is one in which the sections are slipped into inclined grooves.

FIG. 764.—Slide cabinet. (Bausch and Lomb.)

There seems to be no particular advantage in this method so far as cheapness or saving of space is concerned.

Merrill[1] made cheap cases for storing thin sections by folding manilla wrapping paper into pleats. The slides were placed on end between the folds, which acted as springs, the whole being placed in proper-sized boxes.

538. Card Catalogue.—The working petrologist should make a card catalogue of all specimens collected. This method of keeping a record possesses several advantages over any other method. Cards for the same area or of the same type of rock may be studied together, and the cards may be arranged or rearranged in any manner most convenient for the time being. This is of special importance in cataloguing transition types which may have to be transferred from one group to another. The method also is elastic, and the descriptions may be extended at will. The form of card is a matter of personal preference. All that is necessary is that all possible information be written upon it. The aim should be to write the descriptions so that another

[1] George P. Merrill: *A cheap form of box for microscopic slides.* Science, XX (1892), 298–299.

person, from the description alone, may obtain a mental picture of the rock. The form given below for systematic collections is also very good for field collections.

The following description of the card catalogue of the University of Chicago collection, which is a modification of that used by the U. S. Geological Survey[1] for its reference collection, may serve as a model for other institutions.

The rock specimens are arranged in the order of accession, and the thin sections, which bear corresponding numbers, are numerically arranged in a case such as was suggested above, that is, in a series of boxes similar to Fig. 762 arranged in tightly fitting cases holding 2000 slides each, and occupying a space of 12 by 26 by 7 in.

The regular descriptive cards are 4 by 6 in. in size, and are similar to that shown below. They are arranged in numerical order in the files, the number at the left on the card being the accession number.

826 *Leucite basanite.*	Lava of 1760, Vesuvius, Italy.
Megascopic	Medium gray. Very many stout augite prisms, dark green in color. Groundmass, dark gray, aphanitic.
Microscopic	*Texture.*—Porphyritic, nearly sempatic.
	Phenocrysts.—About 45 per cent. Megaphyric. Short, stout prisms and regular basal sections.
	Groundmass.—Holocrystalline, hypautomorphic.
	Constituents.
	Phenocrysts.—Augite 90 per cent., leucite 5 per cent., olivine, 5 per cent.
	Groundmass.—Leucite 40 per cent., augite 25 per cent., plagioclase 20 per cent., magnetite 8 per cent., olivine 5 per cent. biotite 2 per cent.
	Accessory.—Apatite.
	Secondary.—Chlorite.
	Noteworthy.—Leucite with inclusions. Poikilitic augite.
	Remarks. —Section rather thick. Specimen taken from near surface.
	Occurrence.—Lava flow of 1760.
	Literature. —Rosenbusch: *Mikroskopische Physiographie*, II, 1907, 1376–1379.
	Zirkel: *Lehrbuch*, III, 1894, 13.

The form of the card differs for granular and for porphyritic rocks in that the constituents of the former are arranged under the headings: essential, minor accessories, occasional accessories, and secondary. Rocks which have been analyzed have their analyses written on the margin at the left.

To make the collection available for all purposes, several series of index cards are provided. The first is a classification index. This is divided into

[1] Published with the permission of the Director, U. S. Geological Survey.

families and subfamilies, and under each division is a card giving the numbers and localities of all rocks of this kind in the collection. This classification is temporary and can readily be changed. In the U. S. Geological Survey collection, the descriptive cards are themselves arranged according to rock terms. For the use of students it has seemed better to arrange these cards numerically, similar rocks being readily found from the cross reference classification index.

The advantage of keeping the descriptive cards in numerical order is that one is not tied down to any system of classification. The petrographical index may be arranged to suit any system, or several indices may be made for several different systems. If a rearrangement is desired, it is only necessary to revise comparatively few cards and not the complete collection.

As a specimen of the classification, the subdivisions of the granite family used in the temporary arrangement at the University of Chicago are given in part below.

100 Normal alkali-lime series.
110 Predominating feldspar orthoclase.
111 Granite-rhyolite family.
111.1 Plutonic.
111.11 Leucocratic.
111.111 Alaskite.
111.112 etc.
111.12 Normal.
111.121 Biotite granite = granitite.
111.122 Two mica granite.
111.123 Amphibole granite.
111.1231 Hornblende granite.
111.1232 etc.
111.124 Pyroxene granite.
111.1241 Augite granite.
111.1242 Diopside granite.
111.1243 etc.
111.125 Topaz granite.
111.126 Garnet granite.
111.127 etc.
111.13 Melanocratic.
111.131 Melano-granite.
111.2 Hypabyssal.
111.21 Leucocratic.
111.211 Alaskite porphyry.
111.22 Normal.
111.221 Granite porphyry.
111.3 Effusive.
111.31 Leucocratic.
111.311 Tordrillite.
111.32 Normal.
111.321 Rhyolite.
111.322 Rhyolite porphyry.
Quartz porphyry.
Orthoclase porphyry, etc.
111.4 Differentiation rocks.
111.41 Leucocratic.
111.411 Aplite.
111.412 Pegmatite.
111.413 etc.
111.42 Melanocratic.
111.421 etc., etc.

Under these subdivisions are arranged cards such as the following:

GRANITE. Biotite granite (Granitite).	
7	Gross Bieberau, Odenwald, Germany.
11	Mühltal, Eberstadt, Odenwald. (Hornblende bearing).
21	Heidelberg, Baden.
22	Burkersdorf, Erzgebirge, Sachsen (Porphyritic).
36	Eibenstock, Erzgebirge, Sachsen.
38	Königshain, Schlesien.
63	Grasstein, Tirol.
69	Frauenthal, Bohemia, Austria.
84	Koritnicza, Hungary.
101	Baveno, Lago-Maggiore, Italy (Red).
103	Bovey Tracey, Devonshire, England.
104	High Downs, Cornwall, England.
106	Dalbeattie, Scotland.
112	Ross of Mull, Scotland (Porphyritic).
115	Nystad, Finland (Rapakiwi).
117	Wyborg, Finland (Rapakiwi).

Besides this index there are: an index of noteworthy minerals; an index of other noteworthy features, such as textures, etc.; a geographical index. The following is an example of the first:

APATITE.	
809	Medium size in granite. Fine.
816	Usual small laths, showing parting, in granite.
848	Irregular grains and laths in granite.
859	In syenite.
864	In syenite.
867	Large and small grains, in syenite.
869	Small laths in syenite.
873	Cross parting well shown, in quartz monzonite.
899	Small, in diorite.
937	Showing inclusions and corrosion, in andesite.
954	Showing many inclusions and corrosion, in andesite. Etc. etc.

The geographical index, including cross reference cards, is subdivided, at the present time, as follows: (Further subdivisions may be added as needed.)

Ascension Islands.
Argentina.
Austria-Hungary.
    Austria.
        Bohemia.
            Erzgebirge.
            Mittelgebirge.
        Bukowina.
        Dalmatia.
        Carinthia (Kärnten).
        Carniola (Krain).
        Coastland.
        Erzgebirge, see Bohemia, Erzgebirge.

Galicia.
Lower Austria.
Moravia (Mähren).
Salzburg.
Silesia (Schlesien).
Styria (Steiermark).
Tyrol and Vorarlberg.
Upper Austria.
Bosnia.
Bolivia.
Brazil.
Hungary.
Croatia and Slavonia.
Hungary (Ungarn).
Transylvania (Siebenburgen).
Belgium.
Canada.
British Columbia.
Ontaria.
Quebec.
Chili.
Egypt.
England.
France.
Miscellaneous.
Vosges (Vogesen), see Germany, Alsace-Lorraine.
Germany.
Alsace-Lorraine (Elsass-Lothringen).
Baden.
Odenwald, see Hessen, Odenwald.
Bavaria (Bayern).
Fichtelgebirge.
Pfalz.
Rhön Gebiet, see Thuringen States, Rhön Gebiet.
Spessart Gebiet.
Brunswick.
Harz Gebiet.
Erzgebirge, see Austria-Hungary, Austria, Bohemia, Erzgebirge.
Eifel, see Prussia, Rhine, Eifel.
Fichtelgebirge, see Bavaria, Fichtelgebirge.
Harz Gebiet, see Brunswick, Harz Gebiet,
Hessen.
Odenwald.
Odenwald, see Hessen, Odenwald.
Pfalz, see Bavaria, Pfalz.
Prussia.
Brandenburg.
East Prussia.
Eifel, see Prussia, Rhine, Eifel.
Hanover.
Hessen-Nassau.
Rhön Gebiet, see Thuringen States, Rhön Gebiet.
Spessart Gebiet, see Bavaria, Spessart Gebiet.
Pomerania (Pommern).
Posen.
Rhine (Rheinland, Rhenish Prussia).
Eifel.
Siebengebirge.
Saxony.
Harz, see Germany, Brunswick, Harz.
Schleswig-Holstein.
Silesia (Schlesien).
Westphalia.
West Prussia.

Rhenish Bavaria, see Bavaria, Pfalz.
Rhön Gebeit, see Thuringen States, Rhön Gebiet.
Saxony.
Erzgebirge, see Austria-Hungary, Austria, Bohemia, Erzgebirge.
Schwarzwald, see Baden.
Siebengebirge, see Prussia, Rhine.
Spessart Gebiet, see Bavaria, Spessart Gebiet.
Thuringen States.
Rhön Gebiet.
Vogesen, see Alsace-Lorraine.
Würtemberg.
Italy.
Ireland.
Norway.
Mexico.
Peru.
Portugal.
Russia.
Finland.
Great Russia, Archangel.
Northern Caucasia.
Trans-Caucasia.
Ural Mountains.
Scotland.
Island of Skye.
Spain.
Sweden.
Switzerland.
United States.
Arizona.
Arkansas.
California, etc., etc., etc.
Venezuela.
Wales.

APPENDIX

Letter		Name	Corresponding letter in English
A	α	Alpha	A
B	β	Beta	B
Γ	γ	Gamma	G
Δ	δ	Delta	D
E	ϵ	Epsilon	Short E
Z	ζ	Zeta	Z
H	η	Eta	Long E
Θ	θ ϑ	Theta	Th
I	ι	Iota	I
K	κ	Kappa	K
Λ	λ	Lambda	L
M	μ	Mu	M
N	ν	Nu	N
Ξ	ξ	Xi	X
O	o	Omicron	Short O
Π	π	Pi	P
P	ρ	Rho	R
Σ	σ ς	Sigma	S
T	τ	Tau	T
Υ	υ	Upsilon	U
Φ	φ ϕ	Phi	F
X	χ	Chi	Ch
Ψ	ψ ϐ	Psi	Ps
Ω	ω	Omega	Long O

USEFUL FORMULÆ

Trigonometric

(1) $\sin A = \frac{a}{c}$ (Fig. 765)

(2) $\cos A = \frac{b}{c}$

(3) $\tan A = \frac{a}{b}$

(4) $\cot A = \frac{b}{a}$

(5) $\sec A = \frac{c}{b}$

(6) $\csc A = \frac{c}{a}$

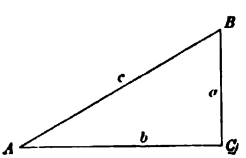

Fig. 765.

Each of the six principal functions may be expressed in terms of the other five as follows:

sin	.....	$\sqrt{1-\cos^2}$ (7)	$\frac{\tan}{\sqrt{1+\tan^2}}$ (8)	$\frac{1}{\sqrt{1+\cot^2}}$ (9)	$\frac{\sqrt{\sec^2-1}}{\sec}$ (10)	$\frac{1}{\csc}$ (11)
cos	$\sqrt{1-\sin^2}$ (12)	.....	$\frac{1}{\sqrt{1+\tan^2}}$ (13)	$\frac{\cot}{\sqrt{1+\cot^2}}$ (14)	$\frac{1}{\sec}$ (15)	$\frac{\sqrt{\csc^2-1}}{\csc}$ (16)
tan	$\frac{\sin}{\sqrt{1-\sin^2}}$ (17)	$\frac{\sqrt{1-\cos^2}}{\cos}$ (18)	.....	$\frac{1}{\cot}$ (19)	$\sqrt{\sec^2-1}$ (20)	$\frac{1}{\sqrt{\csc^2-1}}$ (21)
cot	$\frac{\sqrt{1-\sin^2}}{\sin}$ (22)	$\frac{\cos}{\sqrt{1-\cos^2}}$ (23)	$\frac{1}{\tan}$ (24)	.....	$\frac{1}{\sqrt{\sec^2-1}}$ (25)	$\sqrt{\csc^2-1}$ (26)
sec	$\frac{1}{\sqrt{1-\sin^2}}$ (27)	$\frac{1}{\cos}$ (28)	$\sqrt{1+\tan^2}$ (29)	$\frac{\sqrt{1+\cot^2}}{\cot}$ (30)	.....	$\frac{\csc}{\sqrt{\csc^2-1}}$ (31)
csc	$\frac{1}{\sin}$ (32)	$\frac{1}{\sqrt{1-\cos^2}}$ (33)	$\frac{\sqrt{1+\tan^2}}{\tan}$ (34)	$\sqrt{1+\cot^2}$ (35)	$\frac{\sec}{\sqrt{\sec^2-1}}$ (36)	.....

(37) $\sin^2 A+\cos^2 A=1$

(38) $\tan A=\frac{\sin A}{\cos A}$

(39) $\cot A=\frac{\cos A}{\sin A}$

(40) $\sin(-A)=-\sin A$

(41) $\cos(-A)=\cos A$

(42) $\tan(-A)=-\tan A$

(43) $\cot(-A)=-\cot A$

(44) $\sec(-A)=\sec A$

(45) $\csc(-A)=-\csc A$

(46) $\sin(90°+A)=\cos A$

(47) $\cos(90°+A)=-\sin A$

(48) $\tan(90°+A)=-\cot A$

(49) $\cot(90°+A)=-\tan A$

(50) $\sec(90°+A)=-\csc A$

(51) $\csc(90°+A)=\sec A$

(52) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

(53) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

(54) $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

(55) $\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha}$

(56) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

(57) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

(58) $\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

(59) $\cot(\alpha-\beta)=\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$

(60) $\sin\alpha+\sin\beta=2\sin 1/2(\alpha+\beta)\cos 1/2(\alpha-\beta)$

(61) $\cos\alpha+\cos\beta=2\cos 1/2(\alpha+\beta)\cos 1/2(\alpha-\beta)$

(62) $\sin\alpha-\sin\beta=2\cos 1/2(\alpha+\beta)\sin 1/2(\alpha-\beta)$

(63) $\cos\alpha-\cos\beta=-2\sin 1/2(\alpha+\beta)\cos 1/2(\alpha-\beta)$

(64) $\frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\frac{\tan 1/2(\alpha+\beta)}{\tan 1/2(\alpha-\beta)}$

(65) $\sin 2\alpha=2\sin\alpha\cos\alpha$

(65) $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$

(67) $\cos 2\alpha = 1 - 2 \sin^2 \alpha$

(68) $\cos 2\alpha = 2 \cos^2 \alpha - 1$

(69) $\tan 2\alpha = \dfrac{2 \tan \alpha}{1 - \tan^2 \alpha}$

(70) $\cot 2\alpha = \dfrac{\cot^2 \alpha - 1}{2 \cot \alpha}$

(71) $\sin 1/2\alpha = \sqrt{\dfrac{1 - \cos \alpha}{2}}$

(72) $\cos 1/2\alpha = \sqrt{\dfrac{1 + \cos \alpha}{2}}$

(73) $\tan 1/2\alpha = \sqrt{\dfrac{1 - \cos \alpha}{1 + \cos \alpha}}$

(74) $\tan 1/2\alpha = \dfrac{\sin \alpha}{1 + \cos \alpha}$

(75) $\tan 1/2\alpha = \dfrac{1 - \cos \alpha}{\sin \alpha}$

(76) $\cot 1/2\alpha = \dfrac{1 + \cos \alpha}{\sin \alpha}$

(77) $\cot 1/2\alpha = \dfrac{\sin \alpha}{1 - \cos \alpha}$

(78) USEFUL VALUES OF NATURAL TRIGONOMETRIC FUNCTIONS

Angle	Sin	Cos	Tan	Cot	Sec	Csc
0°	0	1	0	∞	1	∞
30°	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	$\sqrt{3}$	$\frac{2}{3}\sqrt{3}$	2
45°	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	2	$\frac{2}{3}\sqrt{3}$
90°	1	0	∞	0	∞	1
180°	0	−1	0	∞	−1	∞
270°	−1	0	∞	0	∞	−1
360°	0	1	0	∞	1	∞

CARTESIAN GEOMETRIC

Rectilinear equation to a right line.

$y = m'x + b$ (Fig. 766).

(79) $DP = m'OD + OF$

$$\left(\text{in which } m' = \frac{\sin \theta}{\sin (\omega - \theta)}\right).$$

Rectangular equation to a right line.

$\omega = 90°$ (Fig. 767).

(80) $y = mx + b$, where $m = \tan \theta$.

Polar equation to a right line.

(81) $\rho \cos (\varphi - \alpha) = p$, where p = perpendicular to OC (Fig. 767).

Equation to the circle, the origin being at the center.

(82) $$x^2+y^2=R^2 \text{ (Fig. 768).}$$

Axial equation to the ellipse.

(83) $$b^2x^2+a^2y^2=a^2b^2 \text{ (Fig. 769)}$$

$$\text{or } \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \text{ where } CD=x,\ PD=y,\ CA=a,\ CB=b.$$

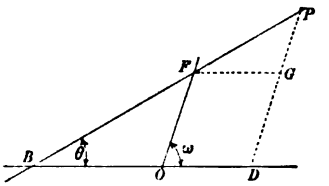

FIG. 766.

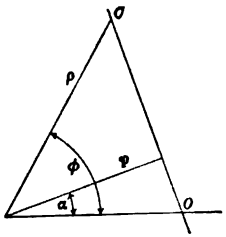

FIG. 767.

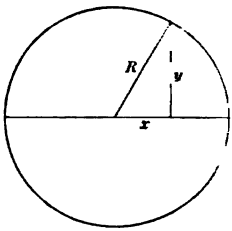

FIG. 768.

Equation of the hyperbola referred to its axes.

(84) $$b^2x^2-a^2y^2=a^2b^2$$

where $CD=x$, $DP=y$, $CA=a$, $CF=c$, $c^2=a^2+b^2$ (Fig. 770).

Equation to the equilateral hyperbola.

$a=b$ in equation (103) and

(85) $$x^2-y^2=a^2 \text{ (Fig. 770).}$$

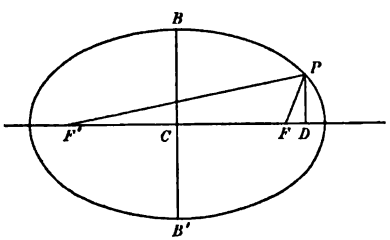

FIG. 769.

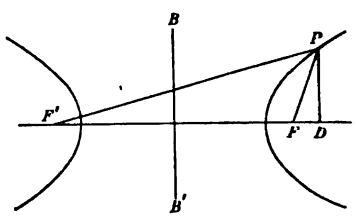

FIG. 770.

Equation of the tangent to the ellipse.

(86) $$\frac{xx'}{a^2}+\frac{yy'}{b^2}=1.$$

Equation of the tangent to the circle, origin at the center

(87) $$xx'+yy'=R^2$$

Equation of the tangent to the hyperbola.

(88) $$\frac{xx'}{a^2}-\frac{yy'}{b^2}=1.$$

Equation of the normal to the ellipse.

(89) $$\frac{a^2x}{x'}-\frac{b^2y}{y'}=a^2-b^2=c^2.$$

Equation of the normal to the circle.

(90) $$\frac{x}{x'}-\frac{y}{y'}=0.$$

Equation of the normal to the hyperbola.

(91) $$\frac{a^2x}{x'}+\frac{b^2y}{y'}=a^2+b^2=c^2.$$

CONVERSION TABLES FOR WEIGHTS AND MEASURES

Linear Measure

1 millimeter = 0.0394 inch.
10 mm. = 1 centimeter = 0.3937 inch.
10 cm. = 1 decimeter = 3.937 inches.
10 dcm. = 1 meter (m) = 39.37 inches.

1 inch = 25.399 mm.
1 foot = 0.30479+ m.
1 yard = 0.91439+ m.

Paris line = 2.2558 mm. = 0.089 in.
12 Paris lines = 1 Paris inch = 27.07 mm.
12 Paris inches = 1 Paris foot = 0.3248 m.
6 Paris feet = 1 toise = 1.9490 m.

English duodecimal line = 2.1166 mm.
English inch = 25.3997 mm.

Prussian line = 2.1802 mm.
Prussian foot = 0.31385 m.

Vienna line = 2.1952 mm.
Vienna inch = 26.3419 mm.

Measures of Capacity

	Cubic meas.	Dry measure	U. S. liquid measure
1 liter (1) =	1000 c.c. =	0.908 quarts = 61.022 cu. in.	1.0567 quarts. 33.8 ounces.

Weights

	Amount of water at maximum density to which equal	Avoirdupois weight
1 milligram (mg.)	= 1 cubic millimeter	= 0.0154 grain.
1000 mg. = 1 gram (grm.)	= 1 cubic centimeter (c.c.)	= 15.432 grains.
1000 grm. = 1 kilogram (kg.)	= 1000 c.c. = 1 liter	= 2.2046 pounds.

USEFUL RECIPES [1]

Acid-proof cement for glass cells, etc.

(a) Resin 24 parts, red ochre 4 parts, calcined plaster of Paris 2 parts, linseed oil 1 part. Unite by stirring together when melted.*

(b) Shellac in alcohol.

(c) Asphaltum in turpentine.

Water-proof cement.

(a) Shellac 4 parts, borax 1 part. Boil in a little water until dissolved. To use heat till pasty.*

(b) Dissolve as much gutta-percha as possible in 10 parts carbon bisulphide and 1 part turpentine.*

(c) Melt shellac, and mold into sticks. Warm the articles to be cemented sufficiently to melt the shellac when applied.

(d) For cementing glass, repairing troughs, etc Dissolve 5 to 10 parts gelatine in 100 parts water; add 10 per cent. saturated bichromate of potassium solution; mix thoroughly and keep in a dark place. After using the cement the articles are exposed to sunlight, by the action of which the medium is rendered insoluble in water. (M. I. Cross: Knowledge, XXVI (1903), 285–286.)

Cement for mending rock specimens.

(a) Dissolve shellac in alcohol. Apply to both parts and bind together till dry.

(b) Equal parts red and white lead mixed with boiled linseed oil to a proper consistency. Color is objectionable.

(c) White cement. Plaster of Paris in a saturated solution of alum.*

(d) White cement. Melt together resin 8 parts and wax 1 part, then stir in plaster of Paris 4 parts. Heat pieces to be mended.*

(e) Plaster of Paris in a solution of gum arabic to which a few drops of oil of cloves have been added. Color suitably with a small amount of lamp black, umber, or ochre.

(f) Gray cement. Litharge 20 parts, dry lime 1 part. Make into putty with linseed oil. Sets in a few hours.*

Cement for attaching rock chips to holder plate.

Besides those described in the text, the following may be used:

Black resin 4 parts, beeswax 1 part. Melt and add 1 part whiting previously heated red hot and still warm. The proportions may be varied within a wide range.

[1] Recipes starred have *not* been tested by the writer. They are given for convenience without recommendation.

Cement for attaching leather, felt, etc., to metal.

(a) Gelatine dissolved in acetic acid.

(b) Common glue 56 parts by weight. Add 3 1/2 parts gum arabic. Stir to an even paste with water over fire. Remove and add slowly 3 1/2 parts nitric acid.*

Cement for attaching glass laps to metal holders.

(a) Shellac 1 lb. dissolved in methylated spirits 1 pint.

(b) Fine litharge 2 parts, white lead 1 part. Make into a paste with 3 pints boiled linseed oil and 1 of copal varnish. Add more litharge and lead if required.*

Glue for attaching labels to metal or glass.

(a) Add a little calcium chloride to the glue. It will prevent cracking.*

(b) Break yellow glue into small pieces, soak in cold water for a few hours, then pour off the water. Place the softened glue in a wide-mouthed bottle and add enough glacial acetic acid to cover. The glue will dissolve more readily if placed on the water-bath.

(c) Dextrine mucilage. Dissolve 2 oz. dextrine in 1 oz. acetic acid diluted with 5 oz. water. When dissolved add 1 oz. alcohol. (Microsc. Bull., II (1885), 46.)

(d) Dissolve 120 grm. gum arabic in 1/4 liter of water and 30 grm. of gum tragacanth in a similar quantity. After a few hours shake the tragacanth solution until it froths and then add the gum arabic solution. Strain through linen and add 150 grm. glycerine, previously mixed with 2 1/2 grm. oil of thyme.* (Zeitschr. f. angew. Mikrosk II (1896), 151.)

Ink for writing on glass.

(a) Water glass (sodium silicate) 1 to 2 parts, fluid Chinese white 1 part.* (Zeitschr. f. angew. Mikrosk., I (1895), 183.)

(b) With the rubber stopper of a hydrofluoric acid bottle touch the slide or bottle where the label is desired. A frosted surface will result upon which the label may be written with a lead pencil. Such marks will withstand steam and ordinary handling, and may be removed with a rubber eraser when desired. This is especially useful for labels on beakers, flasks, etc., used in analysis. (Science, XXXVII (1913), 561–562.)

Simple formula for mixing any grade of alcohol required.

Let P represent the grade per cent. of the alcohol on hand, P' the grade per cent. required, v the number of volumes of water to be added to one volume of P to make alcohol P', and x the number of volumes of P desired to change to P'. Then $\frac{Px}{x+vx}=P'$, and $v=\frac{P-P'}{P'}$. (Ohio Naturalist, VI (1906), 352–353.)

NATURAL SINES AND COSINES

A	Sin	Cos	
0°	0.000000	1.0000	90°
10′	0.002909	1.0000	50′
20′	0.005818	1.0000	40′
30′	0.008727	1.0000	30′
40′	0.011635	0.9999	20′
50′	0.014544	0.9999	10′
1°	0.017452	0.9998	89°
10′	0.02036	0.9998	50′
20′	0.02327	0.9997	40′
30′	0.02618	0.9997	30′
40′	0.02908	0.9996	20′
50′	0.03199	0.9995	10′
2°	0.03490	0.9994	88°
10′	0.03781	0.9993	50′
20′	0.04071	0.9992	40′
30′	0.04362	0.9990	30′
40′	0.04653	0.9989	20′
50′	0.04943	0.9988	10′
3°	0.05234	0.9986	87°
10′	0.05524	0.9985	50′
20′	0.05814	0.9983	40′
30′	0.06105	0.9981	30′
40′	0.06395	0.9980	20′
50′	0.06685	0.9978	10′
4°	0.06976	0.9976	86°
10′	0.07266	0.9974	50′
20′	0.07556	0.9971	40′
30′	0.07846	0.9969	30′
40′	0.08136	0.9967	20′
50′	0.08426	0.9964	10′
5°	0.08716	0.9962	85°
10′	0.09005	0.9959	50′
20′	0.09295	0.9957	40′
30′	0.09585	0.9954	30′
40′	0.09874	0.9951	20′
50′	0.10164	0.9948	10′
6°	0.10453	0.9945	84°
10′	0.10742	0.9942	50′
20′	0.11031	0.9939	40′
30′	0.11320	0.9936	30′
40′	0.11609	0.9932	20′
50′	0.11898	0.9929	10′
7°	0.12187	0.9925	83°
10′	0.12476	0.9922	50′
20′	0.12764	0.9918	40′
30′	0.13053	0.9914	30′
	Cos	Sin	A

A	Sin	Cos	
30′	0.1305	0.9914	30′
40′	0.1334	0.9911	20′
50′	0.1363	0.9907	10′
8°	0.1392	0.9903	82°
10′	0.1421	0.9899	50′
20′	0.1449	0.9894	40′
30′	0.1478	0.9890	30′
40′	0.1507	0.9886	20′
50′	0.1536	0.9881	10′
9°	0.1564	0.9877	81°
10′	0.1593	0.9872	50′
20′	0.1622	0.9868	40′
30′	0.1650	0.9863	30′
40′	0.1679	0.9858	20′
50′	0.1708	0.9853	10′
10°	0.1736	0.9848	80°
10′	0.1765	0.9843	50′
20′	0.1794	0.9838	40′
30′	0.1822	0.9833	30′
40′	0.1851	0.9827	20′
50′	0.1880	0.9822	10′
11°	0.1908	0.9816	79°
10′	0.1937	0.9811	50′
20′	0.1965	0.9805	40′
30′	0.1994	0.9799	30′
40′	0.2022	0.9793	20′
50′	0.2051	0.9787	10′
12°	0.2079	0.9781	78°
10′	0.2108	0.9775	50′
20′	0.2136	0.9769	40′
30′	0.2164	0.9763	30′
40′	0.2193	0.9757	20′
50′	0.2221	0.9750	10′
13°	0.2250	0.9744	77°
10′	0.2278	0.9737	50′
20′	0.2306	0.9730	40′
30′	0.2334	0.9724	30′
40′	0.2363	0.9717	20′
50′	0.2391	0.9710	10′
14°	0.2419	0.9703	76°
10′	0.2447	0.9696	50′
20′	0.2476	0.9689	40′
30′	0.2504	0.9681	30′
40′	0.2532	0.9674	20′
50′	0.2560	0.9667	10′
15°	0.2588	0.9659	75°
	Cos	Sin	A

A	Sin	Cos	
15°	0.2588	0.9659	75°
10′	0.2616	0.9652	50′
20′	0.2644	0.9644	40′
30′	0.2672	0.9636	30′
40′	0.2700	0.9628	20′
50′	0.2728	0.9621	10′
16°	0.2756	0.9613	74°
10′	0.2784	0.9605	50′
20′	0.2812	0.9596	40′
30′	0.2840	0.9588	30′
40′	0.2868	0.9580	20′
50′	0.2896	0.9572	10′
17°	0.2924	0.9563	73°
10′	0.2952	0.9555	50′
20′	0.2979	0.9546	40′
30′	0.3007	0.9537	30′
40′	0.3035	0.9528	20′
50′	0.3062	0.9520	10′
18°	0.3090	0.9511	72°
10′	0.3118	0.9502	50′
20′	0.3145	0.9492	40′
30′	0.3173	0.9483	30′
40′	0.3201	0.9474	20′
50′	0.3228	0.9465	10′
19°	0.3256	0.9455	71°
10′	0.3283	0.9446	50′
20′	0.3311	0.9436	40′
30′	0.3338	0.9426	30′
40′	0.3365	0.9417	20′
50′	0.3393	0.9407	10′
20°	0.3420	0.9397	70°
10′	0.3448	0.9387	50′
20′	0.3475	0.9377	40′
30′	0.3502	0.9367	30′
40′	0.3529	0.9356	20′
50′	0.3557	0.9346	10′
21°	0.3584	0.9336	69°
10′	0.3611	0.9325	50′
20′	0.3638	0.9315	40′
30′	0.3665	0.9304	30′
40′	0.3692	0.9293	20′
50′	0.3719	0.9283	10′
22°	0.3746	0.9272	68°
10′	0.3773	0.9261	50′
20′	0.3800	0.9250	40′
30′	0.3827	0.9239	30′
	Cos	Sin	A

NATURAL SINES AND COSINES.—*Continued*

A	Sin	Cos		A	Sin	Cos		A	Sin	Cos	
30′	0.3827	0.9239	30′	30°	0.5000	0.8660	60°	30′	0.6088	0.7934	30′
40′	0.3854	0.9228	20′					40′	0.6111	0.7916	20′
50′	0.3881	0.9216	10′	10′	0.5025	0.8646	50′	50′	0.6134	0.7898	10′
				20′	0.5050	0.8631	40′				
23°	0.3907	0.9205	67°	30′	0.5075	0.8616	30′	38°	0.6157	0.7880	52°
				40′	0.5100	0.8601	20′				
10′	0.3934	0.9194	50′	50′	0.5125	0.8587	10′	10′	0.6180	0.7862	50′
20′	0.3961	0.9182	40′					20′	0.6202	0.7844	40′
30′	0.3987	0.9171	30′	31°	0.5150	0.8572	59°	30′	0.6225	0.7826	30′
40′	0.4014	0.9159	20′					40′	0.6248	0.7808	20′
50′	0.4041	0.9147	10′	10′	0.5175	0.8557	50′	50′	0.6271	0.7790	10′
				20′	0.5200	0.8542	40′				
24°	0.4067	0.9135	66°	30′	0.5225	0.8526	30′	39°	0.6293	0.7771	51°
				40′	0.5250	0.8511	20′				
10′	0.4094	0.9124	50′	50′	0.5275	0.8496	10′	10′	0.6316	0.7753	50′
20′	0.4120	0.9112	40′					20′	0.6338	0.7735	40′
30′	0.4147	0.9100	30′	32°	0.5299	0.8480	58°	30′	0.6361	0.7716	30′
40′	0.4173	0.9088	20′					40′	0.6383	0.7698	20′
50′	0.4200	0.9075	10′	10′	0.5324	0.8465	50′	50′	0.6406	0.7679	10′
				20′	0.5348	0.8450	40′				
25°	0.4226	0.9063	65°	30′	0.5373	0.8434	30′	40°	0.6428	0.7660	50°
				40′	0.5398	0.8418	20′				
10′	0.4253	0.9051	50′	50′	0.5422	0.8403	10′	10′	0.6450	0.7642	50′
20′	0.4279	0.9038	40′					20′	0.6472	0.7623	40′
30′	0.4305	0.9026	30′	33°	0.5446	0.8387	57°	30′	0.6494	0.7604	30′
40′	0.4331	0.9013	20′					40′	0.6517	0.7585	20′
50′	0.4358	0.9001	10′	10′	0.5471	0.8371	50′	50′	0.6539	0.7566	10′
				20′	0.5495	0.8355	40′				
26°	0.4384	0.8988	64°	30′	0.5519	0.8339	30′	41°	0.6561	0.7547	49°
				40′	0.5544	0.8323	20′				
10′	0.4410	0.8975	50′	50′	0.5568	0.8307	10′	10′	0.6583	0.7528	50′
20′	0.4436	0.8962	40′					20′	0.6604	0.7509	40′
30′	0.4462	0.8949	30′	34°	0.5592	0.8290	56°	30′	0.6626	0.7490	30′
40′	0.4488	0.8936	20′					40′	0.6648	0.7470	20′
50′	0.4514	0.8923	10′	10′	0.5616	0.8274	50′	50′	0.6670	0.7451	10′
				20′	0.5640	0.8258	40′				
27°	0.4540	0.8910	63°	30′	0.5664	0.8241	30′	42°	0.6691	0.7431	48°
				40′	0.5688	0.8225	20′				
10′	0.4566	0.8897	50′	50′	0.5712	0.8208	10′	10′	0.6713	0.7412	50′
20′	0.4592	0.8884	40′					20′	0.6734	0.7392	40′
30′	0.4617	0.8870	30′	35°	0.5736	0.8192	55°	30′	0.6756	0.7373	30′
40′	0.4643	0.8857	20′					40′	0.6777	0.7353	20′
50′	0.4669	0.8843	10′	10′	0.5760	0.8175	50′	50′	0.6799	0.7333	10′
				20′	0.5783	0.8158	40′				
28°	0.4695	0.8829	62°	30′	0.5807	0.8141	30′	43°	0.6820	0.7314	47°
				40′	0.5831	0.8124	20′				
10′	0.4720	0.8816	50′	50′	0.5854	0.8107	10′	10′	0.6841	0.7294	50′
20′	0.4746	0.8802	40′					20′	0.6862	0.7274	40′
30′	0.4772	0.8788	30′	36°	0.5878	0.8090	54°	30′	0.6884	0.7254	30′
40′	0.4797	0.8774	20′					40′	0.6905	0.7234	20′
50′	0.4823	0.8760	10′	10′	0.5901	0.8073	50′	50′	0.6926	0.7214	10′
				20′	0.5925	0.8056	40′				
29°	0.4848	0.8746	61°	30′	0.5948	0.8039	30′	44°	0.6947	0.7193	46°
				40′	0.5972	0.8021	20′				
10′	0.4874	0.8732	50′	50′	0.5995	0.8004	10′	10′	0.6967	0.7173	50′
20′	0.4899	0.8718	40′					20′	0.6988	0.7153	40′
30′	0.4924	0.8704	30′	37°	0.6018	0.7986	53°	30′	0.7009	0.7133	30′
40′	0.4950	0.8689	20′					40′	0.7030	0.7112	20′
50′	0.4975	0.8675	10′	10′	0.6041	0.7969	50′	50′	0.7050	0.7092	10′
				20′	0.6065	0.7951	40′				
30°	0.5000	0.8660	60°	30′	0.6088	0.7934	30′	45°	0.7071	0.7071	45°
	Cos	Sin	A		Cos	Sin	A		Cos	Sin	A

NATURAL TANGENTS AND COTANGENTS

A	Tan	Cot		A	Tan	Cot		A	Tan	Cot	
0°	0.000000	∞	90°	30′	0.1317	7.5958	30′	15°	0.2679	3.7321	75°
				40′	0.1346	7.4287	20′				
10′	0.002909	343.7737	50′	50′	0.1370	7.2687	10′	10′	0.2711	3.6891	50′
20′	0.005818	171.8854	40′					20′	0.2742	3.6470	40′
30′	0.008727	114.5887	30′	8°	0.1405	7.1154	82°	30′	0.2773	3.6059	30′
40′	0.011636	85.9398	20′					40′	0.2805	3.5656	20′
50′	0.014545	68.7501	10′	10′	0.1435	6.9682	50′	50′	0.2836	3.5261	10′
				20′	0.1465	6.8269	40′				
1°	0.017455	57.2900	89°	30′	0.1495	6.6912	30′	16°	0.2867	3.4874	74°
				40′	0.1524	6.5606	20′				
10′	0.02036	49.1039	50′	50′	0.1554	6.4348	10′	10′	0.2899	3.4495	50′
20′	0.02328	42.9641	40′					20′	0.2931	3.4124	40′
30′	0.02619	38.1885	30′	9°	0.1584	6.3138	81°	30′	0.2962	3.3759	30′
40′	0.02910	34.3678	20′					40′	0.2994	3.3402	20′
50′	0.03201	31.2416	10′	10′	0.1614	6.1970	50′	50′	0.3026	3.3052	10′
				20′	0.1644	6.0844	40′				
2°	0.03492	28.6363	88°	30′	0.1673	5.9758	30′	17°	0.3057	3.2709	73°
				40′	0.1703	5.8708	20′				
10′	0.03783	26.4316	50′	50′	0.1733	5.7694	10′	10′	0.3089	3.2371	50′
20′	0.04075	24.5418	40′					20′	0.3121	3.2041	40′
30′	0.04366	22.9038	30′	10°	0.1763	5.6713	80°	30′	0.3153	3.1716	30′
40′	0.04658	21.4704	20′					40′	0.3185	3.1397	20′
50′	0.04949	20.2056	10′	10′	0.1793	5.5764	50′	50′	0.3217	3.1084	10′
				20′	0.1823	5.4845	40′				
3°	0.05241	19.0811	87°	30′	0.1853	5.3955	30′	18°	0.3249	3.0777	72°
				40′	0.1883	5.3093	20′				
10′	0.05533	18.0750	50′	50′	0.1914	5.2257	10′	10′	0.3281	3.0475	50′
20′	0.05824	17.1693	40′					20′	0.3314	3.0178	40′
30′	0.06116	16.3499	30′	11°	0.1944	5.1446	79°	30′	0.3346	2.9887	30′
40′	0.06408	15.6048	20′					40′	0.3378	2.9600	20′
50′	0.06700	14.9244	10′	10′	0.1974	5.0658	50′	50′	0.3411	2.9319	10′
				20′	0.2004	4.9894	40′				
4°	0.06993	14.3007	86°	30′	0.2035	4.9152	30′	19°	0.3443	2.9042	71°
				40′	0.2065	4.8430	20′				
10′	0.07285	13.7267	50′	50′	0.2095	4.7729	10′	10′	0.3476	2.8770	50′
20′	0.07578	13.1969	40′					20′	0.3508	2.8502	40′
30′	0.07870	12.7062	30′	12°	0.2126	4.7046	78°	30′	0.3541	2.8239	30′
40′	0.08163	12.2505	20′					40′	0.3574	2.7980	20′
50′	0.08456	11.8262	10′	10′	0.2156	4.6382	50′	50′	0.3607	2.7725	10′
				20′	0.2186	4.5736	40′				
5°	0.08749	11.4301	85°	30′	0.2217	4.5107	30′	20°	0.3640	2.7475	70°
				40′	0.2247	4.4494	20′				
10′	0.09042	11.0594	50′	50′	0.2278	4.3897	10′	10′	0.3673	2.7228	50′
20′	0.09335	10.7119	40′					20′	0.3706	2.6985	40′
30′	0.09629	10.3854	30′	13°	0.2309	4.3315	77°	30′	0.3739	2.6746	30′
40′	0.09923	10.0780	20′					40′	0.3772	2.6511	20′
50′	0.10216	9.7882	10′	10′	0.2339	4.2747	50′	50′	0.3805	2.6279	10′
				20′	0.2370	4.2193	40′				
6°	0.10510	9.5144	84°	30′	0.2401	4.1653	30′	21°	0.3839	2.6051	69°
				40′	0.2432	4.1126	20′				
10′	0.10805	9.2553	50′	50′	0.2462	4.0611	10′	10′	0.3872	2.5826	50′
20′	0.11099	9.0098	40′					20′	0.3906	2.5605	40′
30′	0.11394	8.7769	30′	14°	0.2493	4.0108	76°	30′	0.3939	2.5386	30′
40′	0.11688	8.5555	20′					40′	0.3973	2.5172	20′
50′	0.11983	8.3450	10′	10′	0.2524	2.9617	50′	50′	0.4006	2.4960	10′
				20′	0.2555	3.9136	40′				
7°	0.12278	8.1443	83°	30′	0.2586	3.8667	30′	22°	0.4040	2.4751	68°
				40′	0.2617	3.8208	20′				
10′	0.12574	7.9530	50′	50′	0.2648	3.7760	10′	10′	0.4074	2.4545	50′
20′	0.12869	7.7704	40′					20′	0.4108	2.4342	40′
30′	0.13165	7.5958	30′	15°	0.2679	3.7321	75°	30′	0.4142	2.4142	30′
	Cot	Tan	A		Cot	Tan	A		Cot	Tan	A

NATURAL TANGENTS AND COTANGENTS.—*Continued*

A	Tan	Cot	
30′	0.4142	2.4142	30′
40′	0.4176	2.3945	20′
50′	0.4210	2.3750	10′
23°	0.4245	2.3559	67°
10′	0.4279	2.3369	50′
20′	0.4314	2.3183	40′
30′	0.4348	2.2998	30′
40′	0.4383	2.2817	20′
50′	0.4417	2.2637	10′
24°	0.4452	2.2460	66°
10′	0.4487	2.2286	50′
20′	0.4522	2.2113	40′
30′	0.4557	2.1943	30′
40′	0.4592	2.1775	20′
50′	0.4628	2.1609	10′
25°	0.4663	2.1445	65°
10′	0.4699	2.1283	50′
20′	0.4734	2.1123	40′
30′	0.4770	2.0965	30′
40′	0.4806	2.0809	20′
50′	0.4841	2.0655	10′
26°	0.4877	2.0503	64°
10′	0.4913	2.0353	50′
20′	0.4950	2.0204	40′
30′	0.4986	2.0057	30′
40′	0.5022	1.9912	20′
50′	0.5059	1.9768	10′
27°	0.5095	1.9626	63°
10′	0.5132	1.9486	50′
20′	0.5169	1.9347	40′
30′	0.5206	1.9210	30′
40′	0.5243	1.9074	20′
50′	0.5280	1.8940	10′
28°	0.5317	1.8807	62°
10′	0.5354	1.8676	50′
20′	0.5392	1.8546	40′
30′	0.5430	1.8418	30′
40′	0.5467	1.8291	20′
50′	0.5505	1.8165	10′
29°	0.5543	1.8040	61°
10′	0.5581	1.7917	50′
20′	0.5619	1.7796	40′
30′	0.5658	1.7675	30′
40′	0.5696	1.7556	20′
50′	0.5735	1.7437	10′
30°	0.5774	1.7321	60°
	Cot	Tan	A

A	Tan	Cot	
30°	0.5774	1.7321	60°
10′	0.5812	1.7205	50′
20′	0.5851	1.7090	40′
30′	0.5890	1.6977	30′
40′	0.5930	1.6864	20′
50′	0.5969	1.6753	10′
31°	0.6009	1.6643	59°
10′	0.6048	1.6534	50′
20′	0.6088	1.6426	40′
30′	0.6128	1.6319	30′
40′	0.6168	1.6212	20′
50′	0.6208	1.6107	10′
32°	0.6249	1.6003	58°
10′	0.6289	1.5900	50′
20′	0.6330	1.5798	40′
30′	0.6371	1.5697	30′
40′	0.6412	1.5597	20′
50′	0.6453	1.5497	10′
33°	0.6494	1.5399	57°
10′	0.6536	1.5301	50′
20′	0.6577	1.5204	40′
30′	0.6619	1.5108	30′
40′	0.6661	1.5013	20′
50′	0.6703	1.4919	10′
34°	0.6745	1.4826	56°
10′	0.6787	1.4733	50′
20′	0.6830	1.4641	40′
30′	0.6873	1.4550	30′
40′	0.6916	1.4460	20′
50′	0.6959	1.4370	10′
35°	0.7002	1.4281	55°
10′	0.7046	1.4193	50′
20′	0.7089	1.4106	40′
30′	0.7133	1.4019	30′
40′	0.7177	1.3934	20′
50′	0.7221	1.3848	10′
36°	0.7265	1.3764	54°
10′	0.7310	1.3680	50′
20′	0.7355	1.3597	40′
30′	0.7400	1.3514	30′
40′	0.7445	1.3432	20′
50′	0.7490	1.3351	10′
37°	0.7536	1.3270	53°
10′	0.7581	1.3190	50′
20′	0.7627	1.3111	40′
30′	0.7673	1.3032	30′
	Cot	Tan	A

A	Tan	Cot	
30′	0.7673	1.3032	30′
40′	0.7720	1.2954	20′
50′	0.7766	1.2876	10′
38°	0.7813	1.2799	52°
10′	0.7860	1.2723	50′
20′	0.7907	1.2647	40′
30′	0.7954	1.2572	30′
40′	0.8002	1.2497	20′
50′	0.8050	1.2423	10′
39°	0.8098	1.2349	51°
10′	0.8146	1.2276	50′
20′	0.8195	1.2203	40′
30′	0.8243	1.2131	30′
40′	0.8292	1.2059	20′
50′	0.8342	1.1988	10′
40°	0.8391	1.1918	50°
10′	0.8441	1.1847	50′
20′	0.8491	1.1778	40′
30′	0.8541	1.1708	30′
40′	0.8591	1.1640	20′
50′	0.8642	1.1571	10′
41°	0.8693	1.1504	49°
10′	0.8744	1.1436	50′
20′	0.8796	1.1369	40′
30′	0.8847	1.1303	30′
40′	0.8899	1.1237	20′
50′	0.8952	1.1171	10′
42°	0.9004	1.1106	48°
10′	0.9057	1.1041	50′
20′	0.9110	1.0977	40′
30′	0.9163	1.0913	30′
40′	0.9217	1.0850	20′
50′	0.9271	1.0786	10′
43°	0.9325	1.0724	47°
10′	0.9380	1.0661	50′
20′	0.9435	1.0599	40′
30′	0.9490	1.0538	30′
40′	0.9545	1.0477	20′
50′	0.9601	1.0416	10′
44°	0.9657	1.0355	46°
10′	0.9713	1.0295	50′
20′	0.9770	1.0235	40′
30′	0.9827	1.0176	30′
40′	0.9884	1.0117	20′
50′	0.9942	1.0058	10′
45°	1.0000	1.0000	45°
	Cot	Tan	A

INDEX

Names in the General Bibliographies have not been indexed

$\mathfrak{a}$, 91
α, Crystallographic, 1
α, Optic, 91
Abbe, E, 131, 132, 133, 180, 187
Abbe drawing apparatus, 297
Abbe test plate, 187
Aberration, 129
Abnormal birefringence, 359
Absorption axes, 322
Absorption coefficient, 326
Absorption of light, 320
Accession catalogue, 609
Accessories, Slot for, 148
Accessories, Testing, 231
Achromatic lenses, 130
Active substances, 108
Acute bisectrix, 105
Adams, W. G., 305
Adjustment, Coarse, 149
Adjustment, Fine, 149
Ady, J. E., 574, 585, 593, 594
Ahrens, C. D., 171
Ahrens' prism (1884), 171
Ahrens' prism (1886), 171
Airy, G. B., 363
Alcohol, Formula for mixing, 625
Allochromatic colors, 309
Amann, J., 379
Amann birefractometer, 379
Ambronn, H., 253, 254, 266
Ambronn's method for determining refractive indices, 253, 254
Amici, M., 450
Amici-Bertrand lens, 450
Amorphous substances, 1, 110
Amplitude, 33, 35, 49
Analyzer and polarizer, 176
Anastigmatic, 130
Andrews, T., 573
Angle, Extinction, 102
Angle, Incidence, 51
Angle, Measurement of, 293
Angle, Optic, 102
Angle, Polarization, 58
Angle of reflection, 51
Angle of refraction, 59
Angular aperture, 131
Angular velocity, 33
Angular velocity, Equation of, 33
Anisometric system, 2
Anisotropic media, 48, 61, 89, 110
Anlauffarben, 286
Anomalies, Optical, 508
Anomalous birefringence, 359
Anomalous dispersion, 442
Anorthic system, 3
Anterior focal plane, 139
Antipodal points, 6
Anthony, J., 152, 299
Apatite, Chemical reactions on, 565
Aperture of lenses, 131
Aperture table, 132, 190
Apertometer, 132
Aplanatic lenses, 130
Apochromatic lenses, 131
Apparent optic axial angle, 102
Arago, F. J., 108, 337, 362, 387
Aragonite, Separation from calcite, 568
Arago's law, 60
Areas, Measurement of, 290
Arons, Leo, 311
Arschinow, Wladimir, 307
Astigmatism of lenses, 130
Asymmetric system, 3
Attachable mechanical stage, 144
Attractive minerals, 457
Augitic system, 2
Automatic section grinding machine, 591
Auxiliary circle, 32
Axes of ease of vibration, 91
Axes of elasticity, 91
Axes of optical ellipsoid, 61
Axes of optical ellipsoid, Locating position in crystal by means of a rotation apparatus, 497
Axes of vibration, 61
Axes of vibration, biaxial, 91
Axial angle, See also Optic axial angle

Axial angle, 102
Axial angle diagram, 471, 491
Axial angle, Equation for true, 103
Axial angle, Relation between true and apparent, 104, 466
Axial angle scale, 469, 471
Axial plane, Crossed dispersion of, 444
Axis of isotropy, 89
Axis of a lens, 114
Axis of no double refraction, 64
Axis of single wave velocity, 100
Axis, Optic, 64

$\mathfrak{b}$, 91
β, Crystallographic, 1
β, Optic, 92
Babinet, M., 322, 387
Babinet compensator, 373
Baker's lamp, 224
Balance, Hydrostatic, 515
Balance, Jolly, 516
Balance, Roger's, 516
Balance, Westphal, 533
Barium mercuric iodide solution, 524
Bartholinus, Erasmus, 62
Bausch & Lomb, Instruments manufactured by, 137, 144, 147, 154, 184, 212, 227, 298, 613
Bausch, Edward, 187
Beasley, H. C., 589
Beck, Instruments manufactured by, 217, 221
Becke, F., 271, 275, 278, 374, 425, 426, 429, 453, 463, 464, 468, 476, 478, 480, 483, 484, 568
Becke-Exner mikrorefractometer, 275
Becke-Klein magnifier, 453
Becke line, 277
Becke rotating drawing stage, 478
Becke method for determining axial angles graphically, 476
Becke method for determining feldspars, 278
Becke method for determining refractive indices, 271
Becke method for determining $2E$ by means of the curvatures of the isogyres, 480, 485
Becke's graphical solution of $\sin E = n \sin V$, 468
Becker, G. F., 284
Beckenkamp, J., 172, 429, 497
Beckmann, E., 317
Becquerel, Edmond, 241
Becquerel, H., 322
Becquerel et Cahors method for determining refractive indices, 241
Beer, August, 429
Behr, J., 531
Behr's method for determining specific gravity, 531
Behrens, H., 563
Behrens, Wm., 260, 284
Bell, Louis, 313
Bellevue, F. de, 572
Bennett, A., 29
Berek, Max, 397
Berkeley, Earl of, 517
Bernhard, Wilh., 299
Bertin, A., 242
Bertin's method for determining refractive indices, 242
Bertrand, Emile, 169, 251, 301, 394, 449, 450, 451, 527
Bertrand immersion fluid, 251
Bertrand lens, 178, 450
Bertrand lens, Centering, 230
Bertrand method for observing interference figures, 449, 451
Bertrand ocular, 394
Bertrand ocular, Testing, 230
Bertrand prism, 169
Biaxial crystals, 91, 93
Biconcave lenses, 114
Biconvex lenses, 114
Binormals, 93, 100
Biot, J. B., 70, 108, 347, 351, 363, 365, 366, 386, 406, 457, 461, 481
Biot-Klein plate, see Biot quartz plate
Biot quartz plate, 386
Biot sensitive violet, 386
Biradials, 99
Birefringence, 61
Birefringence, Abnormal, 359
Birefringence, Calculation of, 351
Birefringence, Determination of, 369
Birefringence, Determination of by means of rotation apparatus, 504
Birefringence, Lines of equal, 355
Birefringence, Table of maximum, 373
Bisectrices, Dispersion of, 412
Bisectrices, Dispersion of in monoclinic crystals, 445
Bisectrix, 105
Blackburn, W., 131

Blackham, G. E., 180
Body tube, 145
Boeke, H. E., 5
Booth, M. A., 603
Borders in minerals, Colored, 249, 258
Borders in minerals, Dark, 257
Bornemann, J. G. and L. G., 588
Bosscha, J., 599
Boyle, Robert, 572
Brace, D. B., 388
Brace's half-shade polarizer, 388
Brandao, See Souza-Brandao
Brauns, R., 251, 525, 526
Brauns' use of methylene iodide, 251, 525
Bravais, A., 363, 374, 393
Bravais twinned mica plate, 387, 393
Bravíf, H. L., 285
Bréon, R., 545
Brewster, Sir David, 241, 284, 363, 448, 457, 460
Brewster lens, 136
Brewster's method for determining refractive indices, 241
Brögger, W. C., 301, 551
Brögger's microgoniometer, 301
Brögger's separation apparatus, 551
Brucite, Chemical reactions on, 567
Brücke, E., 311
Bryan, G. H., 610
Bücking, H., 403
Bücking's apparatus for showing effects of pressure, 509
Bullock's filar micrometer, 289
Butte granite, Mechanical analysis, 292
Bx_a, 105
Bx_o, 105

ſ, 91
Cadmium borotungstate solution, 521
Caffyn, C. H., 589
Cahours, Auguste, 241
Calcite, Chemical reactions on, 565
Calcite, Double refraction in, 62
Calcite, Separation from aragonite, 568
Calderon, L., 395, 449
Calderon plate, 395
Calkins, F. C., 284
Camera lucida, 296
Campbell, William, 286
Canada balsam, 559
Canada balsam, Index of refraction of, 283
Cancrinite, Chemical reactions on, 564
Cap diaphragm, 152
Cap nicol, 176
Carbonates, Chemical reactions on, 565
Card catalogue, 613
Cardinal points of lenses, 119
Cartesian geometric formulæ, 621
Catalogue, Card, 613
Catalogue of specimens, 609
Cathrein, A., 325
Cedar oil, To thicken, 488
Cement, Acid proof, 624
Cement for glass to metal, 625
Cement for leather, 625
Cement for specimens, 624
Cement, Water proof, 624
Cementing oven, 595
Centering objectives, 148
Central Scientific Co., Instruments made by, 107, 225, 240, 471, 515, 517, 533
Cesàro, G., 379, 401, 415
Cesàro wedge, 379
Chalk, Preparing sections of, 599
Chamberlain, C. J., 224
Changing oculars, 228
Chaulnes, Duc de, 238
Chaulnes' method for determining index of refraction, 238
Chauvenet, Professor, 14
Chelius, C., 525
Chemical reactions on rock slices, 559
Chemical separations, 558
Chromatic aberration, 130
Chromoscope, 311
Chrustschoff, K. von, 376
Chrustschoff twin compensator, 376
Church, A. H., 519
Circle of confusion, 130
Circle of reference, 32
Circles appear as true circles in stereographic projection, 8
Circular polarization, 107
Clay, R. S., 129
Clay, Thin sections of. 599, 601
Cleavage, 235
Cleaving, Method of, 236
Clerici, Enrico, 270, 529
Clerici's heavy fluid, 529
Clerici's method for determining refractive indices, 270
Clinorhombic system, 2
Clinorhomboidal system, 3
Clutch for objectives, 227

Coal, Thin sections of, 601
Coarse adjustment, 149
Coddington lens, 136
Coefficient of absorption, 326
Cohen, E., 323, 450, 533, 538, 551, 564
Collection bags, 606
Collections of petrographic material, 605
Collimation, Line of, 210
Color determination, 310
Color of light, 49, 110
Color of minerals, 309
Color of thin plates, 328
Color scale according to Kraft, 332
Color scale according to Newton, 330
Color scale according to Quincke, 331
Colored borders of minerals, Cause of, 249, 258
Compensating oculars, 194
Compensation, 366
Compound microscope, 138
Concave lenses, 114
Condensing system, 154
Confusion circle, 130
Conical refraction, Exterior, 102
Conical refraction, External, 102
Conical refraction, Interior, 100
Conical refraction, Internal, 100
Conjugate foci, Equation for, 117
Conjugate foci of convex lense., 116
Conoscope, 413
Converging lenses, 114
Convergence of lens, Equation of, 122
Convergent light, Observations by, 413
Conversion tables, weights and measures, 623
Convex lenses, 114
Cordier, P., 572
Corpuscular theory of light, 29
Correction collar, 186
Cosine table, 626
Cotangent table, 628
Cover-glass, Correction for, 186
Cover-glass, Compensation for in objectives, 186
Cover-glasses, Effect of, 185
Craig, Thomas, 9
Crew, Henry, 30
Crisp, Frank, 139, 188, 296
Critical angle, 56, 110
Cross, W., 292
Crossed axial plane dispersion, 444
Crossed nicols, Examination between, 336
Crossed dispersion, 447
Cross-hairs, Adjusting, 229
Cross-hairs, Focussing in ocular, 197
Cross-hairs, Replacing, 197
Crystal, 1
Crystal form, Determining, 233
Crystal system, Determining by rotation apparatus, 503
Crystallographic axes, 1
Crystallographic axes, Relation of to the optical ellipsoid, 390
Cubic system, 1
Curved ruler, 14
Curves of equal velocity, 429
Cylinder diaphragms, 152
Czapski, S., 187, 208, 288, 297, 413, 453, 467
Czapski ocular, 453

Dafert, E. W., 541
Dale, T. N., 285
Dallinger, W. H., 183
Daly, R. A., 402
Dana, J. D., 1, 2
Dark borders, Cause of, 257
Decomposed rock, Sections of, 599
Definition, Depth of, 180
Definition of objectives, 180
Delesse, A., 290
Deleuil, 166
De Lorenzo, See Lorenzo
Demonstration oculars, 196
Density to refractive index, Relation of, 285
Depth of definition, 180
Depth of focus, 180
Derby, Orville A., 541
Des Cloizeaux, A., 1, 2, 446
DeSouza-Brandao, See Souza-Brandao
Detmers, H. J., 188
Diamond saws, 580
Diaphragms, 151
Diatom test plates, 191
Dichroism, 320
Dichroscope, 325
Dichroscope ocular, 326
Dick, Allan, 177, 215
Dick microscope, 215
Diedrichs, K., 316
Diller, J. S., 553
Diller separating apparatus, 553
Dilute colors, 309
Dimetric system, 1

Dippel, L., 191
Direct vernier, 145
Direction of vibration in uniaxial crystals, 73
Dispersed white light for monochromatic illumination, 317
Dispersion, Anomalous, 442
Dispersion croisée, 447
Dispersion, Crossed, 447
Dispersion, Crossed axial plane, 444
Dispersion, Effect of temperature upon, 448
Dispersion, Horizontal, 446
Dispersion, Inclined, 445
Dispersion inclinée, 446
Dispersion in monoclinic crystals, 445
Dispersion in orthorhombic crystals, 443
Dispersion in triclinic crystals, 448
Dispersion, Influence upon extinction angles, 412
Dispersion, Normal, 442
Dispersion of bisectrices, 412, 445
Dispersion of light, 54
Dispersion of light in crystals, 442
Dispersion of optic axes, 443
Dispersion, Selective, 442
Dispersion tournnate, 447
Dispersion, Unsymmetrical, 448
Distance of most distinct vision, 133
Displacement in a circle, Equation of, 32
Distinct vision, Distance of, 133
Distinct cleavage, 236
Diverging lenses, 114
Doelter, C., 539
Dolomieu, D., 572
Dolomite, Chemical reactions on, 565
Double concave lenses, 114.
Double lenses, 114
Double refracting goniometer, 293
Double refraction, 61
Double refraction, Graphical representation of variation in different directions in a crystal, 506
Double refraction in calcite, 62
Double refraction in calcite, Apparatus for demonstrating, 69
Double refraction in uniaxial crystals (Huygens' construction), 82
Doublets, 136
Doubly oblique system, 3
Dove, H. W., 457
Dowdy, S. E., 227
Draw tube, 145
Drawing apparatus, 296
Drawing-board, Tilting, 298
Drei-und-einaxige system, 1
Dreibrodt, O., 554
Drescher, W. A. E., 289
Dressel, H., 564
Drude, Paul, 442
Dry objectives, 183
Du Bois, H. E. J. G., 316
Duboscq, Jules, 387
Duc de Chaulnes, See Chaulnes
Dufrénoy, 1
Duparc, L., 326, 331, 356, 374, 407, 487
Durand, W. F., 114

ϵ, 64, 89, 111
E (extraordinary ray), 64, 89
E (half the optic axial angle), 102, 112
Ease of vibration axes, 91
Ease of vibration curve, 79
Ease of vibration spheroid, 92
Ease of vibration surface, Equation of, 80
Ebner, V. von, 301
Edinger, L., 196
Edwards, W. B. D., 521
Ehlers, J., 322, 326
Ein-und-einaxige system, 2
Ein-und-eingliedrige system, 3
Elasticity axes, 91
Elasticity curve, 79
Elasticity ellipsoid, 91
Electromagnet, 538
Electromagnetic theory of light, 30
Elliptical polarization, 107, 312
Elongation, optical character of, 361
Emission theory of light, 29
Enlargement, Measurement of, 287
Entrance pupil of microscope, 138
Equal extinction curves, 410
Equal velocity curves, 429
Ether, 31
Evans, John W., 383, 384, 429, 493
Evans' double quartz wedge, 384
Evans' simple quartz wedge, 383
Ewell, M. D., 185
Exit pupil of microscope, 138
Exner, Sigm, 275
Exterior conical refraction, 102
External conical refraction, 102
Extinction angles, 339
Extinction angles, Calculating, 403

Extinction angles, Calculation in random thin sections, 399
Extinction angles, Graphical methods for determining, 406
Extinction angles, Influence of dispersion upon, 412
Extinction angle, Maximum, Determination by means of a rotation apparatus, 504
Extinction angle, Measuring, 392
Extinction angle of a face, 392
Extinction angle of a mineral, 392
Extinction angles on 110 cleavage plates of pyroxenes, 402
Extinction, Curves of equal, 410
Extinction diagram, 410
Extinction, Inclined, 62, 390, 392
Extinction, Measuring, 392
Extinction, Parallel, 62, 390
Extinction positions, 390
Extinction, Symmetrical, 391
Extraordinary ray, 64, 89
Eye lens, 138
Eyepiece, 138, 193
Eye shade, 226
Eye to be used in work, 225

Fedorow, E. von, 4, 14, 16, 18, 25, 173, 303, 324, 378, 379, 469, 488, 489, 491, 494, 495, 496, 497, 498, 499, 500, 503, 504, 598
Fedorow comparator, 379
Fedorow method for determining low interference colors, 378
Fedorow mica comparator, 366
Fedorow prism, 173
Fedorow slide boxes, 489
Fedorow stage, 303
Fedorow stereographic net, 16
Fedorow three point compass, 25
Feldspar, Separation from quartz, 568
Ferro, A. A., 403
Field, Flatness of, 181
Field lens, 138
Field of view, 140
Field of view, Measuring, 287
Filtrations, Microchemical, 581
Fine adjustment, 149
Finishing slides, 602
First order red plate, 365
Fischer, H., 309, 310
Fitzgerald, G. F., 31, 74
Flatness of field, 181
Fletcher, L., 80, 92, 97, 98, 99, 100
Fletcher's indicatrix, uniaxial, 80, 111
Fletcher's indicatrix, biaxial, 92
Flink, G., 301
Fluids for ray filters, 314, 315, 316
Focal distance of lens, 120
Focal length, 140
Focal length, Equation for, 125, 127
Focal planes of lenses, 119, 120
Focal point of a lens, 115, 119
Focus, Depth of, 180
Focus of combined lenses, 118
Focussing, 227
Foot of microscope, 142
Forbes, David, 574, 587, 599
Ford, J., 603
Formation of image in compound microscope, 138
Foucault, Leon, 165
Foucault prism, 165
Fouque, F., 439, 539
Francotte, P., 612
Frankenheim, M. L., 3
Freda, G., 563
Fresnel, Augustin, 30, 74, 92, 337, 343, 406, 481
Fresnel's curve of elasticities, 79
Fresnel's ellipsoid, 80
Friable material, Sections of, 599
Fripp, H. F., 154
Fuess, R., 143, 377, 579, 593, 611
Fuess, Instruments made by, 14, 25, 26, 63, 143, 144, 146, 153, 173, 203, 204, 206, 207, 220, 224, 236, 240, 275, 289, 290, 291, 294, 301, 302, 303, 304, 305, 306, 307, 317, 318, 325 326, 367, 374, 379, 384, 385, 453, 469, 471, 479, 490, 509, 537, 540, 550, 554, 562, 590, 591, 595, 611
Fuess microscopes, 202, 203, 205, 207, 218
Fuessner, K., 158, 166, 168, 175
Fuessner prisms, 168

γ, Crystallographic, 1
γ, Optic, 92
Gadolin, Axel, 516
Gauss, K. F., 119
Gauss' method, 119
Gebhardt, W., 184
Gelatinizing minerals, 562
Gelblum, S., 227
Gelcich, E., 9

Giesenhagen, 299
Gifford, J. W., 316
Giltay, E., 133
Gladstone, J. H., 285
Glan, Paul, 167, 326, 389
Glan prism, 167
Glass polarizing prisms, 174
Glazebrook, R. T., 31, 62, 74
Gnomonic projection, 5
Goldschmidt, V., 5, 9, 250, 515, 519, 532, 543, 548
Goldschmidt separating apparatus, 548
Goldschmidt specific gravity indicators, 543
Goniometer, Double refracting, 293
Goniometer, Micro-, 301
Goniometer occular, 294
Good cleavage, 236
Govi, G., 119
Grabham, G. W., 215, 274
Grabham's explanation of the Becke line, 274
Grabham's improvements for microscope, 215
Græff, Franz F., 563
Grains, Thin sections of, 602
Grassmann, J. G., 3
Grating micrometer ocular, 289
Graton, L. C., 286
Grayson, H. J., 579, 581, 583, 592, 593, 594
Grayson's lap, 592
Great circle, 5
Greek alphabet, 619
Griffith, F. H., 152, 227
Grinding machines, 588
Grinding thin sections, 585
Grosse, W., 172, 176
Grosse prism, 172
Groth, P. von, 2, 173, 331, 413, 449, 578, 579, 589
Grundlach, Ernst, 182
Gylling, Hj., 323
Gypsum plate, See Unit retardation plate
Gypsum wedge, 365
Gypsum wedge for determining optical character of uniaxial minerals, 460

H (Half the optic axial angle) 102, 112
Haidinger, W., 2, 310, 325
Halbschattenapparate, 388
Half-shade plates, 388
Halle, Bernhard, 174
Halle, Gustav, 325
Halle prism, 174
Halos, Pleochroic, 323
Hamilton, Sir William, 99, 100, 101
Hammer belts, 606
Hammers, 605
Hanaman, C. E., 595, 603
Hand lenses, 136
Hand separation, 557
Hand specimens, 607
Harada, T., 568
Harada tube, 549
Harker, Alfred, 402
Harmonic curve, 34
Harmonic curve, Equation of, 35
Harmonic curve, Equation of velocity in, 36
Harmonic curves, Combinations of, 41
Harmonic motions, Amplitude equation, 44
Harmonic motions, Combinations of, 37
Harmonic motion, Simple, 33
Harris, W. H., 601
Hartnack, 165
Hartnack-Prazmowski prism, 165
Hastings, C. S., 62
Hastings aplanatic triplet, 138
Hauenschild, A., 553
Hauenschild separating apparatus, 553
Hausmann, 1, 2
Haushofer, K, 562, 563
Hauswaldt, Hans, 245
Hauynite, Chemical reactions on, 563
Heavy fluids, Errors resulting from their use, 556
Heavy fluids for determining specific gravities, 519 to 530
Heavy fluids for specific gravity separations, 542
Heavy fluids, Specific gravities of, 518
Heavy fluids, Tabulation of properties of, 528
Heavy melts, 545
Hecht, B., 467
Heeger, W., 567
Hemi-orthotype system, 2
Hemi-prismatic system, 2
Henniges, L., 597
Henniges' tweezers for holding cover-glasses, 597
Herschel, J. F. W., 110
Hexagonal system, 1
Highley, S., 300
Hillebrand, W. F., 556
Hilton, H., 5, 435

Himmelbauer, A., 260
Hinden, Fritz., 566
Hinden method for separating calcite from dolomite, 567
Hirschwald, J., 142, 202, 291
Hirschwald ocular, 291
Hirschwald stage, 143
Hirst, G. D., 192
Hitchcock, R., 604
Hlawatsch, C., 360, 369, 376
Hockin, C., 131
Homogeneous immersion objective, 183
Hooke, 30
Horizontal dispersion, 446
Horizontal small circles, 6
Hotchkiss, W. O., 272
Hotchkiss explanation of the Becke line, 272
Hövermann, G., 324
Hubbard, L. L., 530
Hubbard's determination of specific gravity, 530
Hull, 29
Hutchinson, A., 18, 25
Hutchinson's protractor, 18
Hutchinson's three point compass, 25
Huygens, C., 30
Huygens eyepiece, 193
Huyghens, See Huygens
Hydromagnesite, Microchemical reactions upon, 567
Hydronephelite, Microchemical reactions upon, 564
Hydrostatic balance, 515
Hydrostatic float, 534
Hydrous minerals, Sections of, 602

Iddings, J. P., 91, 292
Idiochromatic colors, 309
Illuminating apparatus, 154
Illuminating power, 181
Immersion fluids, 259
Immersion fluids, Table of, 260
Immersion objectives, 183
Immersion oil, 184
Immersion oil bottle, 184
Immersion oil, To thicken, 488
Inactive substances, 108
Incandescent gases for producing monochromatic light, 316, 317
Incident light, 51, 233
Inclined dispersion, 445
Inclined extinction, 62, 390, 392
Inclined illumination, 275
Index of refraction, 54
Index of refraction and density, Relation between, 285
Index of refraction determined by the Becke method, 271 to 283
Index of refraction determined by immersion method, 249 to 270
Index of refraction of Canada balsam, 283
Index of refraction of fluids, Determination of, 265
Indicators, Refractive index, 268
Indicators, Specific gravity, 542
Indicatrix, Equation of biaxial, 94
Indicatrix, Optical, 80, 111
Indices (Miller), 3
Indices of refraction, Principal, 94
Indistinct cleavage, 236
Ink for writing on glass, 625
Inostranzeff, A. von, 311
Intensity equation of light, 69
Intensity equation of light in interference figures, 418
Intensity equation of polarized light passing through one mineral plate, 343
Intensity equation of polarized light passing through two superposed mineral plates, 346
Intensity of light, 49, 110
Intensity of light emerging in uniaxial interference figures, 418
Intensity of light passing through calcite, 67
Intensity of light, Variation in, 59
Interference, 328
Interference between parallel nicols, 331, 333, 380
Interference colors, 348
Interference of polarized light, 337
Interference figure, 415
Interference figure, biaxial, in section cut perpendicular to the acute bisectrix, 420
Interference figure, biaxial, in section cut perpendicular to the obtuse bisectrix, 423
Interference figure, biaxial, in section cut perpendicular to an optic axis, 424
Interference figure, biaxial, in sections inclined to the bisectrix, 423
Interference figure, biaxial, in sections parallel to the plane of the optic axes, 425

Interference figures, isotropic crystals (Anomalous), 415
Interference figures, Observation of, 449
Interference figures, Orientation of image with respect to object, 456
Interference figure, uniaxial, Cause of, 416
Interference figure, uniaxial, in sections parallel to the optic axis, 419
Interference figure, uniaxial, in oblique sections, 418
Interference sphaerometer, 240
Interior conical refraction, 100
Internal conical refraction, 101
Iris diaphragm, 153
Isochronism, 33
Isogyres, 417, 421
Isogyres, Deduction from skiodromes, 434
Isogyres, Equation of, 435, 440
Isometric system, 1
Isotaques, 429
Isotropic crystals, Interference figures of, 415
Isotropic media, 48, 110
Isotropy axis, 64, 89

Jaggar, T. A., 308
James, F. L., 603
Jamin, M. J., 374
Jellett, 389
Johannsen, Albert, 18, 143, 148, 279, 304, 323, 331, 367, 455, 499
Johannsen auxiliary lens, 454
Johannsen drawing-board for stereographic projection, 18
Johannsen quartz-mica wedge, 367
Johnsen, A., 544, 549
Johnsen and Mügge's indicators, 544
Johnston-Lavis, H. J., 600
Jolly, P., 516
Jolly balance, 516
Joly, J., 291, 323, 324, 383, 530
Joly's method for determining specific gravity by immersion in paraffine, 530
Joly's method for measuring slight double refraction, 383
Joule, J. P., 517
Jullien, J., 316

Kalkowsky, E., 490, 518, 524
Karpinskij, A., 518, 524
Keilhack, K., 541
Kirchhoff, G., 413
Kirschmann, A., 316
Klein, Carl, 148, 252, 284, 302, 305, 306, 387, 450, 451, 456, 493, 508
Klein, Daniel, 521
Klein immersion fluid, 252
Klein method for observing interference figures, 450
Klein quartz plate, 387, 394
Klein'sche Lupe, 453
Klein solution, 521
Klein solution, Method of preparation, 521
Klocke, F., 508
Klönne und Müller slide marker, 612
Knight, C. W., 286
Knopf, A., 564
Knorre, V., 289
Knotenpunkte, 120
Kobell, F. von, 1, 2, 394
Kobell stauroscope, 394
Kolk, S. v. d., see Schroeder v. d.Kolk
Koch, Alfred, 289
Königsberger, J., 286, 326, 388
Köngisberger ocular, 388
Kraft, C., 331, 332
Krantz, F., Instruments made by, 26, 27, 543, 544
Kraus, E. H., 448, 517
Kreider, D. A., 555
Kreutz, St., 486, 568
Krönig, Dr., 604
Ktenas, K. A., 323
Kuznitzky, M., 196

γ, 49
L (Levy), 380
Labeling specimens, 608, 609, 610
Labels for thin sections, 610
Labels, Permanent, 609
Lacroix, A., 370, 377, 403
Lamps, Microscopic, 223
Landolt, H., 314, 316, 387, 388, 389
Lang, V. von, 325
Laps for section cutting, 591
Larsen, E. S., 263, 285
Lasaulx, A. von, 449
Lasaulx method for observing interference figures, 449
Laspeyres, H., 322, 413, 450, 451, 521, 553
Laspeyres method for observing interference figures, 451
Laspeyres separating apparatus, 553

Lateral magnification, Equation for, 121, 122
Lateral spherical aberration, 130
Laurent, L., 389
Least count, 145
Lebedew, 29
Leeson, H. B., 293, 300
Leeson prism, 293
Lehmann, J., 575, 589
Leick, W., 517
Leiss, C., 62, 143, 146, 173, 178, 203, 205, 207, 218, 240, 275, 289, 301, 302, 303, 305, 306, 319, 325, 326, 374, 376, 377, 379, 453, 454, 469, 489, 537, 580, 590, 611
Leiss prism, 172
Leitz, E., 149
Leitz, Instruments made by, 137, 141, 144, 149, 150, 152, 155, 185, 196, 200, 201, 224, 296, 297, 414
Lemberg, J., 564, 565, 566, 568
Lengths, Measurement of, 288
Lenk, H., 454
Lenk-Lasaulx method for observing interference figures, 453
Lenses, 114
Lenses, Care of, 228
Lenses, Formation of images by, Equation for, 122
Lens stands, 136, 137
Leo, Max, 286
Lépinay, J. Mace de, 374
Lépinay half-shade plate, 396
Leroux, E. P., 442
Levy, 380
Lévy, Michel-, See Michel-Lévy
Liebisch, T., 101
Light, Amount of, 227
Light for microscopic work, 223
Light, Nature of, 29
Limb of microscope, 142
Lincio, G., 149, 155
Linck, J., 518, 544, 557, 566
Lincoln, F. C., 286
Linebarger, C. E., 517
Line of collimation, 210
Line of equal birefringence, 355
Line of single normal velocity, 100
Line of single ray velocity, 99
Lippich, F., 389
Liquids, Specific gravity of, 518
Listing, J. B., 119
Lloyd, Rev. H., 101
Locating points in stereographic projection, 6
Lommel, E. von, 108, 173, 389, 442
Lommel prism, 173
Longitudinal spherical aberration, 130
Long tube microscopes, 140
Lord, C. L., 601
Lorentz, H. A., 31, 285
Lorenz, L., 285, 442
Lorenzo, G. de. 262, 277
Luedecke separating apparatus, 554
Lummer, O., 389

Madan, H. G., 171
Madan prism, 171
Magensite, Chemical reactions on, 565
Magnification, Compound microscope, 197
Magnification, Lateral, 121
Magnifying power, 133, 140, 182
Magnifying power, Proper to use, 227
Makers of microscopes, 199
Malassez, L., 182
Mallard, E., 5, 467
Mallard's constant, 468
Mallard's formula, 468
Mallard's formula, Accuracy of, 468
Mallard's method for measuring the optic axial angle, 467
Malus' law, 59
Mann, Paul, 523, 541, 602
Mann's separating instrument, 541
Manufacturers of microscopes, 199
Manufacturers of thin sections, 588
Marking thin sections, 611
Marpmann, G., 255, 263, 594
Marpmann immersion fluid, 254
Maschke, O., 249
Maschke's method for determining refractive indices, 249
Maxwell, J. Clerk, 31, 74
Measurement of areas, 290
Measurement of enlargement, 287
Measurement of extinction, 392
Measurement of field of view, 287
Measurement of lengths, 288
Measurement of plane angles, 293
Measurement of thicknesses, 293
Measurements under the microscope, 287
Measures, Table of, 623
Mechanical analysis Butte "granite," 292
Mechanical separation of rock constituents, 537

Mechanical tube length, 138
Mechanical stages, 142
Meigen, W., 568
Meigen's method for separating calcite from aragonite, 568
Melatope, 420
Melatopes, Locating biaxial, 426
Melatopes, Locating uniaxial, 425
Melilite, Chemical reactions on, 563
Meridians, 6
Merrill, Geo. P., 613
Merwin, H. E., 263, 264, 534
Merwin and Larsen's immersion fluids, 263
Merwin's method for determining specific gravity by refractive indices of fluids, 534
Methylene iodide, 525
Methylene iodide, Table showing relation between temperature and specific gravity, 526
Metz, C.. 149
Mica plate, See Quarter undulation plate.
Mica wedge, 366
Michel-Lévy, A., 16, 268, 323, 331, 355, 370, 377, 403, 406, 411, 439, 472
Michel-Lévy birefringence chart, 370
Michel-Lévy comparator, 377
Michel-Lévy method for measuring 2E, 472
Michel-Lévy refractive index indicators, 268
Microchemical filtrations, 561
Microchemical reactions, 559
Microchemical reactions, Apparatus for, 559
Microchemical reactions, Preparing the slide, 560
Micrometer, Caliper, 240
Micrometer ocular, 288
Micrometer ocular, Grating, 289
Micrometer ocular, Scale, 289
Micrometer ocular, Screw, 289
Micron, 288
Microscope, 199
Microscope, Compound, 138
Microscope lamps, 223
Microscope manufacturers, 199
Microscope, Mechanical parts of, 142
Microscope, Optical parts of, 154
Microscope, Petrographic, 141
Microscope, Selecting, 222
Microscope, Simple, 136
Miers, Henry, 5, 416
Miller, W. H., 1, 2, 3, 4, 5
Miller indices, 3
Mirror, 157
Mohl, Hugo von, 280
Mohs, 1, 2
Moigno, 362
Möller, H. J., 310
Möller, J. D., 191
Molten substances for determining specific gravity, 535
Monoclinic system, 2
Monoclinohedral system, 2
Monodimetric system, 1
Monosymmetric system, 2
Monotrimetric system, 1
Monochromatic light, 313
Monochromator, 318
Montigny, M. C., 134
Mounting thin sections, 593
Mügge, O., 323, 324, 544, 549
Murdoch, Jos., 286
Muthmann, W., 529
Muthmann heavy fluid, 529

Nachet, A., 213, 302
Nachet camera lucida, 296
Nachet, Instruments made by, 154, 213, 297, 377
Nagel, W. A., 315
Nägeli, Carl, 300
Nakamura, S., 389
Naumann, C. F., 1, 2, 3
Naumann parameters, 3
Negative biaxial crystals, 105
Negative character, Determination by rotation apparatus, 503
Negative elongation, 33, 362
Negative minerals, Determination of optical character of, 457
Negative uniaxial crystals, 70, 111
Nelson, E. M., 132, 133, 138, 149, 157, 180, 181, 182, 188
Nephelite, Chemical reactions on, 564
Neumann, F. E., 5, 351
Neutral curves, Equation of, 440
Newton, Sir Isaac, 29, 328, 330
Newton's rings, 328
Newton's scale of colors, 328, 330
Nichols, 29
Nicol, W., 158
Nicol, Cap, 176
Nicol net, 434
Nicol prism, 158
Nicol prism, Adjusting, 231

Nicol prism, Care of, 228
Nicol prism, Determination of vibration direction in, 178
Nikitin, W., 25, 385, 504
Nikitin hemisphere, 26
Nikitin's method for measuring slight double refraction, 383
Nikitin's quartz compensator, 385
Nodal points of lenses, 119, 120
Noll, F., 19
Normal dispersion, 442
Northrup, Zae, 610
Noselite, Chemical reactions on, 563
Nose piece, 147
Nowacki, A., 541
Numerical aperture, 131
Numerical aperture table, 132
Nutting, P. G., 311

O, 64, 89
ω, 64, 89, 111
Object clips, 142
Objective, 138, 180
Objective clutch, 227
Objective holder, 146
Objectives, Classification of, 185
Objectives, Comparative table of, 189
Objectives, Cost of, 193
Objectives, Testing, 191
Objectives with correction collar, 186
Object lens, 138
Oblique system, 2
Obtuse bisectrix, 105
Ocular, 138, 193
Ocular, Bertrand, 394
Ocular, Bertrand, Testing, 230
Ocular, Czapski, 453
Ocular, Demonstration, 196
Ocular dichroscope, 326
Ocular goniometer, 294
Ocular, Schwarzmann's, 470
Oculars, Comparative table of, 195
Oculars for special purposes, 195
Oebbeke, K, 549, 550
Oebbeke tube for specific gravity determinations, 550
Office work, 609
Olivine, Chemical reactions on, 565
One-fourth order mica plate, See Quarter undulation plate.
Opaque minerals, Color determined by means of a rotation apparatus, 503
Opaque minerals, Determination of color of, 311
Opaque minerals, Examination of, 285
Opening angle of nicol, 162
Optical anomalies, 508
Optical character of elongation, 361
Optical character of a mineral, 457
Optical character of a mineral, Determination by rotation apparatus, 503
Optical center of lens, 114
Optical curves, 494
Optical ellipsoid, 80, 111
Optical ellipsoid, Axes of, 61
Optical ellipsoid, Relation of, to crystallographic axes, 390
Optical indicatrix, 80, 111
Optical parts of a microscope, 154
Optical tube length, 138
Optic axes, Dispersion of, 443
Optic axes, Plane of, 105
Optic axis, 64
Optic axis, Biaxial, 93, 99, 100
Optic axis, Biaxial, Locating the point of emergence, 426
Optic axis, Determination by rotation apparatus in sections nearly parallel to the plane of the optic axes, 499
Optic axis, Determination of point of emergence, 476
Optic axis, Locating point of emergence in uniaxial crystals, 425
Optic axis, Locating one by optical curves, 494
Optic axis, Locating one by rotation stage, 489
Optic axis, Locating second by rotation stage, 495
Optic axis, Locating second when first has been determined by optical curves, 498
Optic axis, Primary, 93, 100
Optic axis, Secondary, 99
Optic axis, Simplified method for locating, 500
Optic axis, Uniaxial, 70
Optic angle, See Optic axial angle.
Optic axial angle, 102
Optic axial angle, Apparent, 102
Optic axial angle, Equation for true, 103
Optic axial angle, Measurement by means of a rotation stage, 487
Optic axial angle, Measurement of, 466

Optic axial angle, Relation between true and apparent, 104
Optic axial angle, True, 102
Optic binormals, 93, 100
Optic biradials, 99
Optic section, Principal, 70, 92
Ordinary light, 233
Ordinary ray, 64, 89
Orientation, 361
Orthographic projection, 5
Orthorhombic system, 2
Orthoscopic lenses, 130
Orthotype system, 2
Osann, A., 564
Oschatz, Dr., 573
Oscillation in ether, 31
Over-corrected lenses, 130

Packing specimens for shipment, 608
Panebianco, G., 568
Parallel extinction, 62, 390
Parallels, 6
Parameters, Weiss, 3
Parker, C. B., 604
Parkes' microscope lamp, 223
Parting, 235
Passage of light through two nicols and a mineral plate, 341
Pauly, Anton, 266
Pauly's method for determining the refractive indices of fluids, 266
Pearce, F., 326, 356, 374, 406, 407
Pearcey, F. G., 601
Pébal, L., 540
Peiser, J., 226
Penetration of objectives, 180
Penfield, S. L., 11, 16, 17, 22, 516, 546, 555
Penfield's protractor, 16
Penfield's separating apparatus for heavy melts, 555
Pennock, E., 226
Perfect cleavage, 235
Period, 33
Periodic motion, 32
Petri, R. J., 138
Petrographic microscope, 141
Pfaff, F., 599
Pfitzner, W., 603
Phase, 33
Phasal difference, Equation of, 337
Pillar of microscope, 142
Pirsson, L. V., 292
Plane of optic axes, 105
Plane of polarization, 58
Plane polarized light, 58, 107
Planimeter ocular, 291
Plano-concave lenses, 114
Plano-convex lenses, 114
Playfair, Lyon, 517
Pleochroic halos, 323
Pleochroism, 320
Pleochroism, Determination of, 325
Polarimètre á pénombre, 388
Polariscope, 413
Polarizer and analyzer, 176
Polarization, 57
Polarization angle, 58
Polarization by double refraction, 106
Polarization by reflection, 57
Polarization by refraction, 59
Polarization, Circular, 107
Polarization, Elliptical, 107
Polarization of light by lenses, 415
Polarization plane, 58, 107
Polarizing prisms, 159
Polarizing prisms, Properties of, 175
Pole, 6
Polishing rocks, 602
Poor cleavage, 236
Porous substances, Specific gravity determination of, 518
Position for work, 225
Positions of extinction, 390
Positive biaxial crystals, 105
Positive character determined by rotation apparatus, 503
Positive elongation, 33, 362
Positive minerals, Determination of character, 457
Positive uniaxial crystals, 70, 111
Posterior focal plane, 138
Post of microscope, 142
Potassium mercuric iodide solution, Use of, 519
Powders, Thin sections of, 601
Prazmowski prism, 165
Preston, T., 374, 442
Pribram, 316
Primary optic axes, 93, 100
Principal focal point, 115
Principal focal point, First, 120
Principal focal point, Second, 120
Principal focus, combined lenses, Equation for, 119

Principal focus, Equation for, 118
Principal indices of refraction, 94
Principal optic section, biaxial, 91
Principal optic section, uniaxial, 70
Principal points of lenses, 119, 120
Principal vibration axes, 61
Pringsheim, E., 316
Prismatic system, 2
Properties of polarizing prisms, 175
Protractors for stereographic projection, 14, 16, 18
Protractor, Hutchinson, 18
Protractor, Penfield, 16
Pseudo-absorption, 324
Pseudo-dichroism, 324
Pseudo-pleochroism, 324
Pumice, Thin sections of, 600
Pupil of eyepiece, 138
Pycnometer, 517
Pyramidal system, 1
Pyroxenes, Extinction angles in, 402

Quadratic system, 1
Quarter order mica plate, see Quarter undulation plate
Quarter undulation plate, 362
Quarter undulation plate for determining optical character of biaxial minerals, 462
Quarter undulation plate for determining optical character of uniaxial minerals, 457
Quartz, Separation from feldspar, microchemically, 568
Quartz wedge, 365
Quartz wedge for determining optical character of biaxial minerals, 462
Quartz wedge for determining optical character of uniaxial minerals, 460, 461, 462
Queckett, 196
Quincke, G., 331, 374

Radde's color scale, 310
Radioactivity, 323
Ramsay, W., 322
Ramsden disk, 138
Ramsden eyepiece, 194
Rapp, 521
Rath, G. vom, 564
Rauff, H., 579, 592
Ray, Extraordinary, 64, 89
Ray filters, 314
Ray front, 50
Rayleigh, Lord, 31
Ray, Ordinary, 64, 89
Ray surface, Biaxial, 94
Ray surface, Biaxial, Equation of, 97
Ray surface, Uniaxial, Equation of, 75
Ray surface, Uniaxial, Graphical development of, 76
Reactions, Microchemical, 559
Real focus, 115
Recipes, Useful, 624
Red of first order, See Unit retardation plate
Reflection of waves, 50
Refraction, 52
Refraction, Double, 61
Refraction, Double in calcite, 62
Refraction, Single, 61
Refraction through lenses, 116
Refractive index, 54
Refractive index and density, Relation between, 285
Refractive index, Determining, 237
Refractive index, Determining by the Becke method, 271 to 283
Refractive index, Determination by rotation apparatus, 504
Refractive index, Method for determining by immersion, 249 to 270
Refractive index of Canada balsam, 283
Refractive index of fluids, Determination of, 265
Refractive index, Principal, 94
Refractive index, Relation to velocity of light, 56
Regnault, 387
Regular system, 1
Reichert, Instruments made by, 151, 196, 210, 211, 275, 294, 611
Relief, 237
Replacing cross-hairs, 197
Repulsive minerals, 457
Resolving power, 181
Retardation, Equation for, 337
Retardation wedges, 365
Retgers, J. W., 252, 518, 526, 527, 531, 546, 547, 602
Retgers' determinations of specific gravity, 526, 531
Retgers' heavy fluids, 526
Retgers' immersion fluids, 252
Retrograde vernier, 145
Rhombic system, 2

Rhomohedral system, 1
Richthofen, F. von, 605
Ridgeway, R., 310
Rims around thin sections, 602
Rinne, F., 460, 463
Riva, C., 262, 277
Rogers, Austin, 516
Rogers' specific gravity balance, 516
Rohrbach, Carl, 251, 524
Rohrbach's solution, 524
Rohrbach's solution, Index of, 251
Rohrbach's solution, Method of preparation, 524
Rollet, A., 331
Rose, 1
Rosenbusch, H., 99, 237, 284, 323, 331, 374, 394, 401, 410, 460, 468, 521, 539, 557, 585, 589
Rosiwal, August, 291
Rosiwal method for measuring areas, 292
Rosenhain microscope, 221
Rotary polarization, 108
Rotary polarization in quartz, Table of, 109
Rotating drawing-board, 19
Rotating drawing stage, 478
Rotating plane of projection in stereographic projection, 24
Rotation apparatus, 300 to 308
Rotation apparatus, Adjusting, 488
Rotation apparatus for measuring $2E$, 487
Rowland, H. A., 313
Royston-Pigott, 182
Rumpf, J., 575

Sabot, R., 487
Salomon, W., 277, 279, 534
Salomon's method for computing $\omega - \epsilon$, 383
Salomon's method for determining refractive indices, 279
Salomon's method for determining specific gravity, 534
Sand, Thin sections of, 602
Sang, E., 158, 164
Sang prism, 164
Sarasin, Ed., 108
Sauer, G. A., 563, 564
Sauter, F., 9
Savart plate, 386
Savart's bands, 386
Sawing a rock slice, 583
Scale micrometer ocular, 288
Scales for stereographic projection, 14
Schaffgotsch, F. G., 519
Schaller, W. T., 284
Schieck's microscope lamp, 223
Schiefferdecker, P., 226, 611
Schiemenz, P., 296
Schistoskop, 311
Schmidt, K. E. F., 374
Schneider, J., 309
Schneiderhöhn, H., 258, 506
Schönrock, P., 396, 398
Schraf, A., 395
Schröder, Hugo, 172
Schroeder van der Kolk, 252, 255, 256, 257, 260, 267, 306, 324, 451, 529
Schroeder van der Kolk's heavy fluids, 529
Schroeder van der Kolk's immersion fluids, 255, 256
Schroeder van der Kolk's method for observing interference figures, 452
Schroeder van der Kolk's use of inclined illumination, 252, 257
Schulz, H., 175
Schuster, Arthur, 442
Schuster, Max, 392
Schwarzmann, Max, 469, 470
Schwarzmann's axial angle scale, 469
Schwarzmann's ocular, 470
Schwendener, S., 300
Screw micrometer ocular, 289, 290
Sechsgliedrige system, 1
Secondary optic axes, 99
Section boxes, 612
Section cutting machines, 574
Section grinding machines, 588
Section markers, 611
Seibert, Instruments made by, 202, 288, 537
Seidentopf, H., 384
Seidentopf quartz wedge compensator, 384
Seiffert, Dr., 597
Seiler, C., 595
Selecting a microscope, 222
Selective dispersion, 442
Sellers, C., 588
Senarmont, H. de, 363, 366
Sensitive red, 349
Sensitive tint, 349
Sensitive violet, 349
Separating apparatus, 547
Separating apparatus for heavy melts, 554
Separating by chemical means, 558
Separating by hand, 557

Separating funnel, 550
Separating thin flakes and needles, 557
Shadbolt, 131
Shade for eyes, 226
Shagreen surface, 237
Short tube microscope, 140
Sigsbee, C. D., 14
Simple microscope, 136
Simultaneous rotating nicols, 177
Sine table, 626
Single refraction, 61
Skiodromes, 430
Skiodromes, To construct for random sections, 433
Slavik, F., 285
Sleeman, P., 169
Sliding diaphragm, 152
Small circles, 6
Smeeth, W. F., 517, 552
Smeeth's method for determining specific gravity, 517
Smeeth's separating apparatus, 552
Smith, Hamilton, 604
Smith, H. L., 262
Smith, Herbert G. F., 5
Smith's method for determining the refractive indices of fluids, 265
Snell's law, 54
Société Genevoise, microscope made by, 218
Sodalite, Chemical reactions on, 563
Sokol, R., 568
Soleil, Henri, 387
Soliel bi-quartz plate, 387
Sollas, W. J., 291, 533, 534
Sollas hydrostatic float, 534
Sollas modification of the Sprengel tube, 533
Soluble minerals, Thin sections of, 602
Soluble substances, Determination of specific gravity of, 518
Sonstadt, E., 519
Sonstadt solution, 519
Sonstadt solution, Method of preparing, 519
Sommerfeldt, E., 153, 177, 397, 429, 454
Sommerfeldt condenser, 454
Sommerfeldt twinned gypsum plate, 388, 397
Sorby, H. C., 244, 247, 250, 366, 572, 573, 596, 598, 599
Sorby's method for determining refractive indices, 244
Sorby's method for showing relief, 250
Soret, J. L., 108
Souza-Brandao, V. de, 177, 205, 268, 404, 471
Souza-Brandao axial angle diagram, 471
Souza-Brandao refractive index indicators, 268
Spassky, M., 158
Specific gravity determination, 515
Specific gravity determinations by means of heavy fluids, 532
Specific gravity determinations by means of molten substances, 535
Specific gravity indicators, 542
Specific gravity separations by means of heavy fluids, 542
Specific gravity separations by means of water, 541
Specific gravity table, 544
Specimens, 607
Spezia, G., 309
Sphaerometer, 240
Spherical angles appear in their true values in stereographic projection, 9
Sprengle tube, 532
Sprockhoff, M., 285
Stages, Microscope, 142
Staining minerals, 562
Stand, Care of, 228
Stanhope lens, 136
Stark, Michael, 285, 485
Stark's modification of Becke's method for determining $2E$., 485
Stauroscope, 394
Steeg und Reuter, Instruments made by, 59, 317, 590, 612
Steenstrup, K. J. V., 589, 599
Steinheil lens, 136
Steinmann, G., 576, 581, 583, 593
Steinmann's section cutting machine, 576
Steinriede, 541
Stelzner, A., 564, 565
Stephenson, J. W., 181, 191, 251
Stereographic projection, 5
Stereographic projection, Accuracy of, 22
Stereographic projection net, 16, 17
Stevens, W. LeC., 287
Stigmatic lenses, 130
Stöber, F., 396
Stöber quartz double plate, 396
Stoë, Instruments made by, 543
Stokes, G. G., 246
Stolze, 174
Story-Maskelyne, 5

Streng, A., 530, 561, 564, 565
Streng's determination of specific gravity, 530
Strutt, R. J., 324
Subnormal color, 360
Substage, 150
Supernormal colors, 360
Surface d'élasticité, 92
Swift, Instruments made by, 214, 216
Symmetrical extinction, 391
Symmetry planes, Locating, 497

Table, 225
Tait, P. G., 158, 104, 442
Talbot, H. F., 167, 442
Talbot prism, 167
Tangent table, 628
Teall, J. J. H., 541
Teinte de passage, 365
Teinte sensible, 365
Temperature, Effect upon dispersion, 448
Ten Siethoff, E. G. A., 307, 422
Tertsch, H., 485, 486
Tertsch's modification of Becke's method for determining $2E$, 485
Tesseral system, 1
Tessular system, 1
Testing cross-hairs, 229
Test plate, Abbe, 187
Test plate, Diatom, 191
Tetarto prismatic system, 3
Tetragonal system, 1
Thick edge lenses, 114
Thickness, Measuring, 293
Thickness of a lens, 114
Thickness of a lens, Equation for, 126
Thin edge lenses, 114
Thin flakes, Separating, 557
Thin section boxes, 612
Thin sections, Preparing, 572
Thompson, S. P., 167, 172
Thompson prisms, 167
Thompson, J. J., 31
Thoulet, J., 237, 250, 519, 535, 547, 602
Thoulet's determination of the specific gravity of minerals heavier than the immersion fluid, 535
Thoulet's method for determining refractive indices, 250
Thoulet's separating apparatus, 547
Thoulet's solution, 519
Thoulet's solution, Method of preparation, 519
Thoulet's solution, Relation between specific gravity and refractive index, 250
Three point compass, 25
Thürach, Hans, 541
Tilting drawing-board, 298
Tinne, F., 415
Tomlinson, W. H., maker of sections, 588
Törnebohm, A. E., 564
Total reflection, 56
Toula, Franz, 516
Tourmaline, Absorption by, 106
Tourmaline tongs, 107
Transmitted light, 233
Traube, H., 323, 396
Traube bi-mica plate, 396
Triclinic system, 3
Triclinohedral system, 3
Trigonal system, 2
Trigonometric formulæ, 619
Trimetric system, 2
Triplets, 136
True optic axial angle, 102
Tschermak, G., 310, 325, 413, 516
Tube length, 138
Tube length of various microscopes, 140
Tube of microscope, 145
Tutton, A. E. H., 318, 326, 374, 444, 448, 593

Under corrected lenses, 130
Undulatory theory of light, 30
Unger, Professor, 573
Uniaxial crystals, 62, 70
Unit retardation plate, 349, 365, 393
Unit retardation plate for determining optical character of uniaxial crystals, 459, 461, 462
Unit retardation plate for determining optical character of biaxial crystals, 462
Unsymmetrical dispersion, 448

V (Half the optic axial angle), 102, 112
Valentin, G., 300, 362
Van Heurck, H., 191, 312
Van Werveke, L., 520, 550
Van Werveke separating funnel, 550
Velocity around a circle, Equation of, 32
Velocity of light, 49
Velocity of ray in biaxial crystals, Equation of, 98
Velocity of ray in uniaxial crystals, Equation of, 71

Velocity of wave in uniaxial crystals, Equation of, 72
Velocity, Relation to refractive index, 56
Verniers, 144
Vertex of a lens, 114
Vertical great circle, 6
Vertical small circle, 6
Vesicular rocks, Thin sections of, 600
Vibration axes, 61
Vibration axes, Biaxial, 91
Vibration directions in minerals, Determination of, 361
Vibration directions in nicol prisms, Determination of, 178
Vibration directions, Uniaxial, 73
Vibration ease, Curve of, 79
Viergliedrige system, 1
Viola, C., 274, 276, 426, 474
Viola-Becke method for determining refractive indices, 276
Viola-Becke-de Chaulnes method for determining refractive indices, 276
Viola method for determining $2E$, 474
Violet of first order, See Unit retardation plate
Virtual image, 115
Vogelsang, H, 564, 574
Voigt, W., 322
Voigt und Hochgesang, Makers of sections, 588
Vorce, C. M., 180
Vosseler, J., 604

Wahnschaffes, F., 541
Wales, W., 229
Wallerant, F., 383, 497
Wallerant's method for measuring slight double refraction, 383
Ward, R. H., 226
Ward's eye-shade, 226
Washington, H. S., 292
Wave front of light, 49, 110
Wave length of light, 35, 49
Wave lengths, Table of, 313
Wave motion, 31
Wave motion in isotropic media, 48
Wave surface, Biaxial, 98
Wave surface, Equation of biaxial, 99
Wave surface, Equation of uniaxial, 75
Wave surface of light, 49
Wave surface, Uniaxial, 76
Wave theory of light, 30
Wedges, 365
Weights, Table of, 623
Weinschenk, E., 230, 231, 360, 456, 538
Weiss, C. S., 1, 2, 3
Weiss parameters, 3
Wenham, 215
Wertheim, G., 331
Westphal balance, 533
West rotation apparatus, 301
Whewell, W., 3
Wichmann, A., 561, 599
Wiedemann, G., 395
Wiedemann double double-quartz plate, 395
Williams, G. H., 590
Winkelmann, A., 442
Winchell, 331
Witham, H., 572
Wollaston lens, 136
Wood, R. W., 442
Working distance, 182
Wrappers, 608
Wright, Sir A. E., 154, 181, 198
Wright, F. E., 91, 153, 178, 207, 222, 224, 258, 260, 263, 290, 303, 307, 311, 356, 366, 367, 385, 394, 397, 398, 454, 468, 469, 472, 473, 483, 484, 485, 491, 497
Wright artificially twinned quartz plate, 397
Wright bi-quartz wedge plate, 398
Wright combination wedge, 366, 383
Wright double combination wedge, 385
Wright immersion fluids, 263
Wright-Lasaulx method for observing interference figures, 454
Wright microscope lamp, 224
Wright's modification of Becke's method for determining $2E$, 484, 485
Wright's modification of Michel-Levy's method for measuring $2E$, 472
Wulff, George, 14, 18, 22, 358
Wulff net, 17
Wülfing, E. A., 26, 27, 99, 154, 237, 284, 319, 324, 374, 388, 394, 401, 468, 553, 557, 585, 593
Wülfing projection model, 27
Wülfing separating apparatus, 553
Wülfing wall chart for stereographic projection, 27

Young, L. J., 448
Young, Thomas, 30

Zeiss, Carl, 132
Zeiss, Instruments made by, 133, 147, 152, 184, 186, 187, 194, 208, 209, 289, 298
Zeloites, Chemical reactions on, 563
Zirkel' F., 252, 260, 284, 557, 572, 574, 585, 594, 599
Zone axis, 4
Zones, 4, 399
Zschokke, W., 129
Zwei-und-einaxige system, 1
Zwei-und-eingliedrige system, 2

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